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CONTROL STRATEGIES FOR INVENTORY MANAGEMENT

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This paper describes a control systems approach to the management of inventory. Normal inventory operation is an example of proportional control. Several control algorithms including Pseudo-Derivative Feedback (PDF), Proportional, Integral and Derivative (PID) and Feedforward control are used in this paper to produce a more sophisticated form of inventory operation that can easily reduce stock levels by up to 80% compared to 20-30% with MRP and hence reduce cost. Settling times are reduced by a factor of 50%. Modelling was achieved using the Simulink simulation package using equations developed by Ferris and Towill for a single level industrial system model rather than a conventional System Dynamics computer package. The best controller is shown to be a PID controller with Feedforward. This controller did not cause any significant oscillatory inventory level changes. These techniques compare well with other investigators using control strategies. This has special significance for JIT and MRP methods.

Key Words: Inventory Control, PID, PDF, Feedforward control algorithms

1. INTRODUCTION

The main objective of a System Dynamics (Forrester 1961) examination is to understand how the organisation of a business affects its performance. System Dynamic studies concatenate process events and treat them as a continuous system, which can be solved with any CSSL package. Principally the SD method is concerned with *causal* relationships. Which physical event leads to a decision or action? It is especially interested in establishing whether a *feedback loop* is present (i.e. if the system is open or closed) and if this is positive or negative feedback. In any system *instability* is caused by excessive *delays* in information transmission. This is true whether the system deals with hardware or economic matters.

A System Dynamics model consists of a set of difference or differential equations that can be solved either with the original computer package DYNAMO, developed at MIT by Forrester's' team or by more recent software such as STELLA, VENSIM etc. The structure of the model is normally represented as a *linear* model but the physical processes identified are *always non-linear*.

The purpose of this paper is to address the problem of inventory control. This was one of the first systems tackled by the fraternity of SD modellers. The investigation reported in this paper is a précis of the results obtained thus far indicating the potential for using control engineering principles in this field. A brief review of the previous work is given and some of the lessons learned are evaluated.

2. INVENTORY MODELS

2.1 The Control Engineers Viewpoint

Inventory analysis methods usually used are those described by statistical or operational research methods (Lewis 1981). The first use of hard system models date from Herbert A Simon (1952), Nobel Laureate in Economics. He started the examination of inventory as a control system using conventional Laplace Transform theory. This method did not find favour with the cybernetically oriented economic researchers at the time. The method was limited by the knowledge that no detailed measures of the rate of change of events was available. Forrester, on the other hand, produced the System Dynamics method to explain the cyclic behaviour of inventory systems. The System Dynamics method does not yield the same results as the operational research method.

Sharp and Henry (1979) proposed using the Zeigler-Nichols rules for evaluating control system gains to tune the performance of Proportional Integral and Derivative controller design for the inventory system. The rules are for an optimum solution based on a minimum error squared criterion. Generally these give a very oscillatory system response and are not the best or optimum solution for real systems. Towill and Yoon (1982) used two optimal methods to evaluate inventory systems including costs. The results they obtained were not as good as those presented in this paper.

Towill (1982 &1992) further developed this control model in his papers. Axsäter (1985) gave a wide-ranging review of the progress of control methods in this field to that date. His conclusion was that much promise was unfulfilled mainly due to the difficulty of influencing the process at different levels of aggregation. In answer to one of the criticisms of the System Dynamics method, Edgehill, Olsmats and Towill (1988) undertook an investigation of the sensitivity of the system to parameter variations. This study showed how important the stocking policy of the retailer was in the behaviour of the whole system.

The effect of distributors in the inventory system was investigated in detail by, Olsmats, Edgehill and Towill (1988), who showed that the relationship of parts of the organisation amplified the order rate in an unacceptable manner, causing system instability.

Ferris and Towill (1993) made a systematic investigation of the effects of various feedback and feed-forward options for inventory control. They produced a generic family of manufacturing and ordering systems that were typical of the current European industrial practice at that time. These were compared on a strength/weakness basis. The difference in performance was surprisingly small.

2.2 The System Models

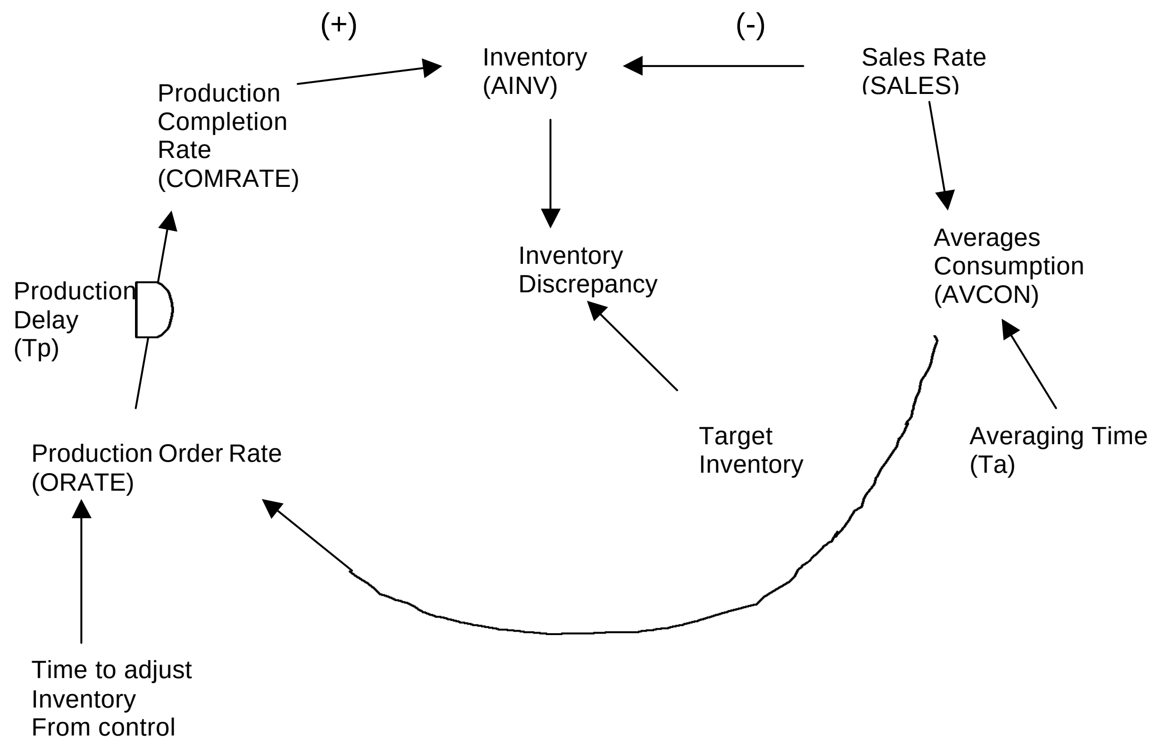
As in the early descriptions of System Dynamics delays are assumed to be exponential in form, and are described by a simple time constant T_p. The key to the way the inventory system behaves is the rate of ordering. This is a management decision that the techniques in Lewis (loc.cit.) provide a range of solutions, one of these is to balance the rate of sales. This is clearly not enough. When the sales rate increases, the inventory will become depleted because the delivery rate takes time to catch up with the different sales rate. If there is a drop in sales then the inventory level will then rise. A common decision is to add to the sales rate a portion of the difference between the desired and actual level of stock. This type of approach involves feedback and illustrates proportional control of the inventory level.

The models described by Towill validated against industrial data assume a set of deterministic inputs and a certain level of aggregation of the production process. The main structural system that has been investigated here is that of the Inventory and Order Based Production Control System (IOBPCS). The block diagram of this system is shown in figure 1. In this model sales act as a disturbance to a system that is driven by the desired inventory DINV.

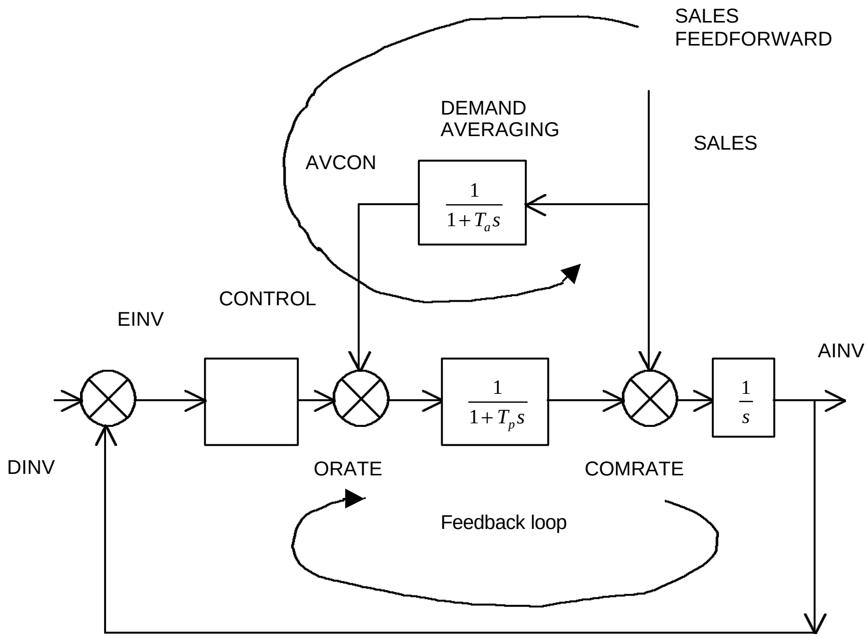
They used conventional control system models so that they could current control practice could be used to investigate the way the system behaved.
Non-linearity was found to be not as important as the major features used here.

The inventory error (EINV) is found as the difference between a fixed desired level of inventory (DINV) and the actual inventory (AINV). A smoothing function is used to obtain the average sales consumption (AVCON) as a function of the SALES rate. This value is used to obtain the order rate (ORATE) given to the production facility, wherever it is. There is generally a production delay inherent in the manufacturing process. Sales behave as a disturbance with feed-forward action to the inventory demand control system. Table 1 shows the transfer function description for the IOBPC system. It can be seen to be of third order.

The inventory level has a higher order numerator than that for the production process, ensuring a final value equal to the inventory demand after a step change in sales.



(a) System Structure



(b) Block diagram representation

Figure 1 Inventory and Order Based Production Control Block Diagram (from Ref. 11)

Table 1 Transfer function of IOPBCS

Family	$\left[\frac{\text{Production Output}}{\text{Sales}}\right]$	$\left[\frac{\text{Actual Stock Level}}{\text{Sales}}\right]$
IOPBCS	$\frac{(T_i + T_a)s + 1}{(1 + T_a s)(1 + T_i s + T_i T_p s^2)}$	$-T_i\left[\frac{(T_a + T_P)s + T_a T_p s^2}{(1 + T_a s)(1 + T_i s + T_i T_p s^2)}\right]$

Proportional control action can be obtained when:

$$U = \left(\frac{1}{T_i}\right)\text{EINV}$$

The gain of the control is $\frac{1}{T_i}$ where T_i is a variable constant.

Figure 3 shows how oscillatory simple proportional control can be. A peak undershoot of 20% is indicated whereas the peak excess in production is about 40%! However the results described here are a substantial improvement on those results. The whole process settles down in 35 weeks.

The purpose of the experiments described here was to evaluate various control strategies (human decisions) acting in the context of initially the simplest of the models observed by Towell to compare their effectiveness before trying them on much more complex multistage supply chains. These further investigations are continuing.

3. DIFFERENT CONTROL STRATEGIES

The proportional control, which is normally used (White 1996), has the disadvantage that either the required inventory level is exceeded or considerable time is spent with the stock level falling, possibly to a level where customer demand cannot be satisfied. However other control algorithms exist, which can produce a better performance. In White (2000) a Proportional Integral and Derivative (PID) controller and Optimal control were investigated. In this work both a Pseudo Derivative Feedback (PDF) controller due to Phelan (1977) and PID control with Feed-forward are compared with the earlier results. The PDF controller shown in figure 2 has a control output:

$$U = \left(K_i \int (\text{EINV})dt + K_2(\text{AINV})\right)$$

This controller has the advantage that it can cope well with systems with a time delay, sometimes better than PID controllers do.

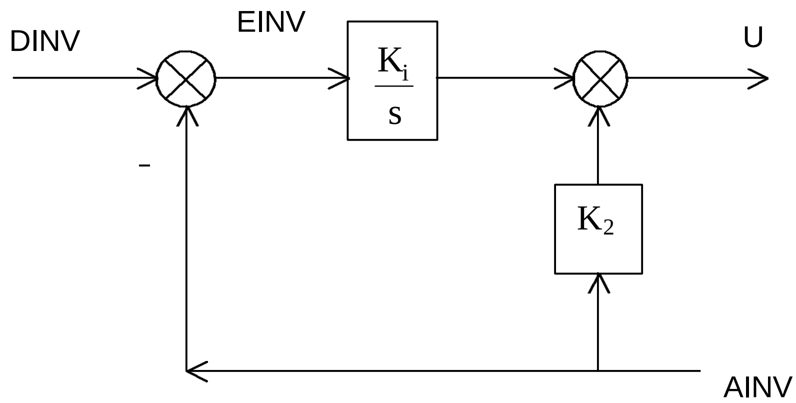


Figure 2 Block diagram of Pseudo Derivative Feedback control

The whole modelling process has been simulated in SIMULINK a MATLAB derivative.

The required inventory level is to be 400 units and a change in sales from 100 to 120 units is made after 5 days. The results are a very much smaller overshoot and undershoot in inventory compared with the proportional controller. COMRATE, the production output is seen (figures 3&4) to rise quickly to the required level of 120 units with 15% overshoot rather than 10% as before. Production rates oscillate in this solution worse than that for proportional control. The PDF controlled inventory is not as good as the PID controlled situation.

A substantial consequence of this level of control is the possibility of reducing the desired stock level to a much smaller value, or even eliminating it entirely! A stock reduction of over 80% is possible as can be seen from figures 3 and 4 since the worst case undershoot is 360 units. The effect of a variation in PID controller gains can be seen in figure 5. The time taken for any disturbance to be brought under control is reduced to about half the time previously experienced. This then has a particular relevance to JIT and MRP systems. This quality of performance is also true for a state-space optimal controller, as shown in White (1996). Both PDF and optimal controllers give better result than a proportional control system but not as good as a PID! Due to the use of a closed loop system the variation of system time constants does not make a great difference to the overall performance. However the performance of the inventory system can be dramatically altered by the use of PID with Feed-forward as in figure 6. Here the output of the system is hardly affected by the sales. The deviation of the system is around 0.25%!

The SIMULINK model is shown in figure 7. The model can be quickly altered and is very easy to use.

3.1 Are these practical methods?

Yes, they can be implemented in a spreadsheet that can deal with the orders and production directly. The only difference the supplier would see is a different number of items to be delivered. It must be remembered that what is indicated is the information flow only.

Although this is a relatively simple model it can easily be extended to cover any limits to the process or non-linearity. In particular, if the stock level goes to zero then the customer cannot be supplied. A more satisfactory model would be to include the supplier in the model; more complex models are now being simulated. Other types of control algorithm can be tried simply on the simulation.

How would this be implemented in a real business?
The first task would be to identify the time constants in the various processes. Second task would be to check the rate of input of data, too slow a data stream would mean that a discrete model rather than a continuous model should be used.

Next the model should be used to validate the business data stream and a new control algorithm could be incorporated into a spreadsheet.

This spreadsheet would then be used to provide orders to suppliers and to the production facility. This would mean that the whole process of managing the inventory could be automated, freeing the manager for more human level tasks.

4. DISCUSSION

Many companies have implemented MRP II (Materials Requirements Planning), a sophisticated computer managed system. The benefit to the manager here is that the MRP approach gives good booking accountability. All parts are labelled and booked into the database either manually or with a bar code reader. As soon as a part is produced or sold a replacement can be generated. Delays still exist in the system because no regeneration is made until the process is complete. Prediction is not usually built into the system, although some of the more recent variants do incorporate a predictor. Unlike the PID controller, which reacts to the rate of change of sales or production, which can therefore detect when the process is going out of control and generate specific remedial action. Although this is possible with MRP it is not as rate sensitive since it is replacing parts on a one-to-one basis. The built in system time constants are still dominating the process. The PID and PDF controllers change those effective time constants.

An MRP system can deliver stock reductions of around 20-30% (Waters 1995) compared to the 80% achieved with this method. MRP however, allows better planning due to the greater information that is available. It is considerably more expensive to implement than this method. MRP is not so good when sales rates vary since the master schedule has to be established in advance. It is also a very complex system and not many small companies can afford to implement this. The system described here will allow a much more rapid system response, without a large capital investment. It can be applied without major changes in plant but the real benefits will not be felt unless the manufacturing time constants are also reduced.

5. CONCLUSIONS

- A simple System Dynamics/Control Systems model of an Inventory and Order Based Production and Control System (IOBPCS) exhibits proportional control of a third order system with disturbance feed-forward. This control normally produces oscillatory stock levels, although the system can be made stable it can also be unstable.

- These large oscillations can be almost eliminated by using PDF or PID control. This can be translated to a much superior performance over that for convention operations. This algorithm can be introduced into JIT and MRP applications. Even in existing inventory systems it will allow a more rapid response to sales variations. It will also allow a better response to changes in stock demand levels. Depending on the system parameters a stock reduction of 80% is possible compared with a conventional (proportional) control.

- It can give comparable stock reduction performances to those given by JIT. From the simulation results, stock levels using PID or PDF controlled system can be in balance in half of the normal time taken by a proportional control system (IOBPCS). This increased performance can be obtained, not by adding inventory but merely by ordering the required material or components earlier, as estimated by the rate of change sensitive controller.

- In the case of PID with Feed-forward control the alteration of desired inventory with sales is almost eliminated.

- Standard Simulation packages can easily be used for System Dynamics Simulation.

6. SYMBOLS

AINV	Current Inventory level
AVCON	Average sales rate
EINV	Error in inventory level
K_I	Integral gain of controller
K_2	Feedback controller gain
ORATE	Outstanding level of orders placed with the supplier
U	Rate of ordering from supplier
COMRATE	Rate of production
S	Rate of sales
DINV	Level of planned inventory
T_a	Smoothing time constant (8 weeks)
T_i	Order constant time. (4 weeks)
T_P	Production delay time (4 weeks)

7. REFERENCES

1. Forrester, J.W. 1961. *Industrial Dynamics*, Cam. Mass., MIT press.
2. Lewis, C D. 1981. *Scientific Inventory Control*, Butterworth.
3. Simon, H.A. 1952. On the application of servomechanism theory in a study of Production Control, *Econometra* ,20, no.2, 247-26.
4. Sharp, J.A. & Henry, R.M. 1979. Designing Policies the Ziegler Nichols way, *Dynamica*, 5, no. 2**,** 79-94.
5. Towill, D.R. Yoon, S. 1981. Some Features Common to Inventory Costs and Process Controller Design, *Engineering Costs and Production Economics*, 6, 225-236.
6. Towill, D.R. 1981. Dynamic analysis of an inventory and order based production control system, *Int. J. Prod. Res*., 20, no. 6, 671-687.
7. Towill, D.R. 1992. Practical Applications of Control Theory in Manufacturing Management, *J. Systems Engineering*, 2, no. 1, 2-14.
8. Axslater, S. 1985. Control Theory Concepts in Production, *Int. J. Syst. Sci*., 16, no. 2, 161-169.
9. Edgehill, J, Olsmats, C, & Towill, D.R. 1988. Industrial case study on the dynamics and sensitivity of a close-coupled production-distribution system, *Int.J. Prod. Res*., 26, no. 10, 1681-1693.
10. Olsmats, C.M.G, Edgehill, J & Towill D.R. 1988. Industrial dynamics model building of a close-coupled production-distribution system, *Engineering costs and Production Economics*, 13, 295-310.
11. Ferris, J. & Towill, D.R. 1993. Benchmarking a Generic Family of Dynamic Manufacturing Ordering and Control Models, *J. Syst.Eng.*, 3, 170-182.
12. White, A.S. 1996. System Dynamics, inventory management and control theory, Proc. *12th Int. Conf. CAD/CAM Robotics and Factories of the Future*, 691-696.
13. Phelan, R.M. 1977. *Automatic Control Systems*, London, Cornell University Books,.
14. White, A.S. 1999. Management of Inventory using Control Systems, *Int. J Technology Management*, 17, 7/8, 847-860.
15. Waters, C.D.J. 1995. *Inventory Control and Management*, Wiley, Chichester,

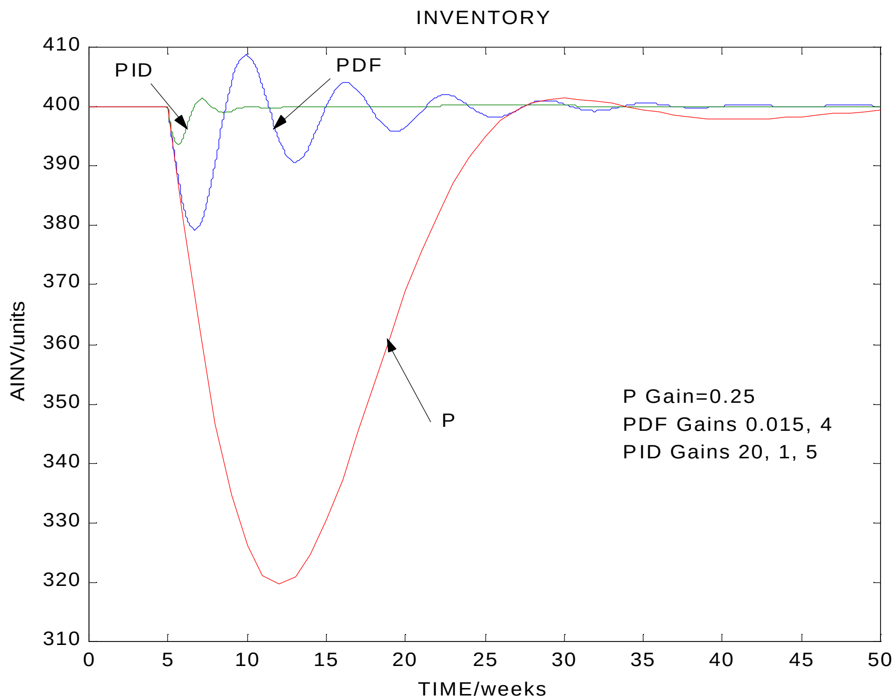


Figure 3 Control of inventory using Proportional, PID and PDF control

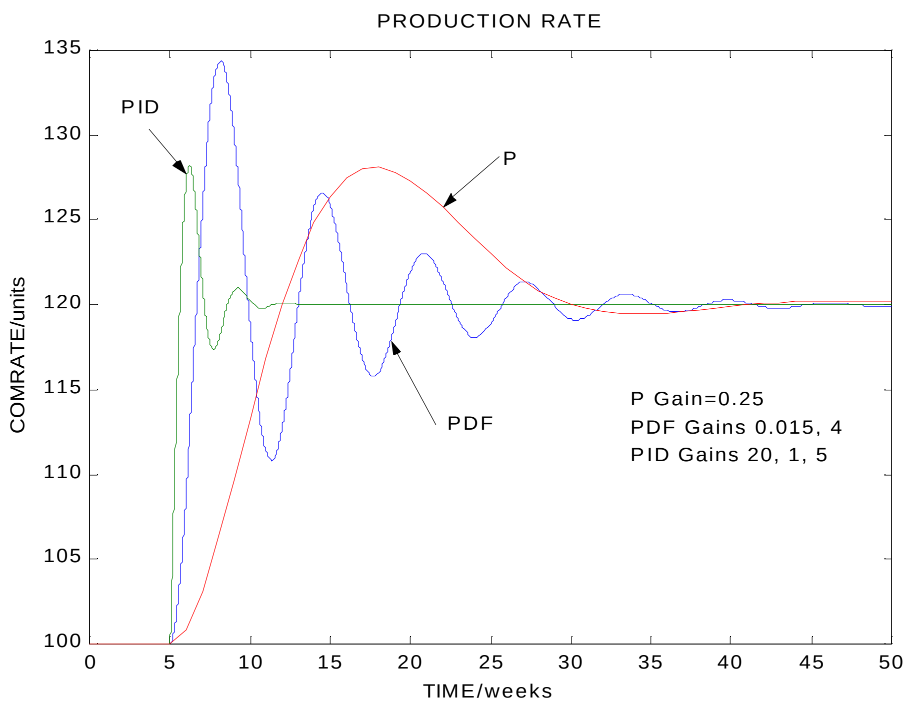


Figure 4 Production Rate under P, PID and PDF Control

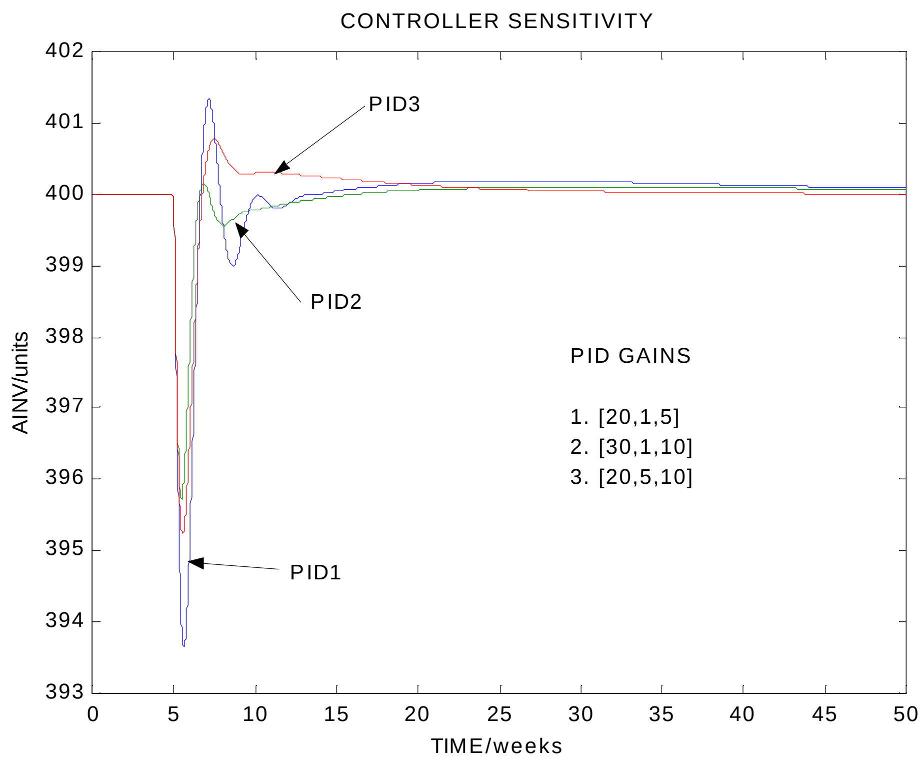


Figure 5 Sensitivity of Inventory response to PID Gains

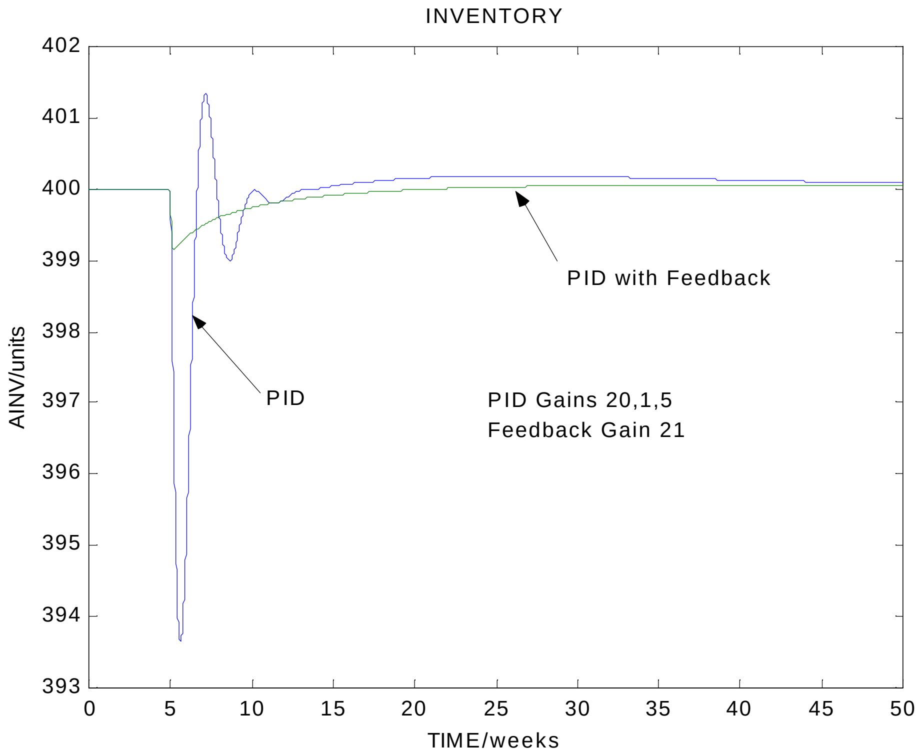


Figure 6 Inventory response with PID +Feed-forward

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Preview:
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This EPS picture will print to a
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Figure 7 SIMULINK model panel

Back to the Top