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Systems Analysis of Kondratieff Model

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Abstract: Through the analysis of the Kondratieff model, this paper discussed the causes of economic fluctuation. The analysis of the model simulating test discovered that two important factors caused the economic long wave. It can be said with certainty that an accidental event consequentially affected the economic long wave. And the social real mode of production leading to the formation of the long wave was proved.

Keywords: **Kondratieff model Economic long wave Mode of production**

1.Introduction

We have written articles to discuss a simplified long-wave model[1][2] of assets producing which simulates real ways of social production. Mathematical description of this model can be expressed by a set of nonlinear ordinary differential equations, which is:

$$\begin{aligned} y_1'(t) &= \left(\frac{1}{a_1}\cdot f_2(\beta) - \frac{1}{a_3}\cdot f_1(\beta)\right)\cdot y_2(t) + \alpha \\ y_2'(t) &= \left(\frac{1}{a_3}\cdot f_1(\beta) - \frac{1}{a_1}\right)\cdot y_2(t) - \alpha \end{aligned} \qquad 1.1.$$

All the variables in the model are the functions of time ‘t’, so according to the definition of mathematics, the equation set 1.1 is ordinary differential equations. While $f_1(b)$ and $f_2(b)$ are two nonlinear functions, therefore ordinary elementary functions cannot describe the general answer of these equations, which means it is impossible to seek the analytic solution under classical meaning. On the other hand, numerical answers of the equation are available with the assistance of computers, which will be granted in the following test. It is worth paying attention to that we got equilibrium answers from the model [1][2] in the test. From the aspects of the procedure of social production, this reflects a condition of the economic system under balanced situation. And this kind of economic condition depicts the social simple reproduction process in real economic procedures. So this paper will give systematic analysis to the reason for the system of social economy produces fluctuation.

2.Systematic Answers under Equilibrium Situations

Equilibrium is the necessary premise to sustain the society to get the most welfare from the production analysis of social macroeconomy. In the system 1.1, when production is under balanced situation, which means the potential output equals the output of willingness, then we get to

$$= \mathbf{y_1(t)/a_2+} \quad / \quad \mathbf{y_2(t)/a_3} \quad 1 \qquad 2.1$$

If we assume the demand for social consuming products be constant, social production will be in a procedure of simple reproduction and it is still easy to get an equilibrium solution 2.2 from the equation set 1.1.

$$y_1 = \frac{a_1 a_2 a}{a_1 - a_3} - a_2 a\,;\ y_2 = \frac{a_1 a_3 a}{a_1 - a_3}\,;\ x_1 = \frac{a_3 a}{a_1 - a_3}\,;\ x_2 = \frac{a_1 a}{a_1 - a_3} - a\,;\ x_3 = \frac{a_3 a}{a_1 - a_3} \qquad 2.2$$

We made an assumption that the market is under the situation of balance, that is $b \equiv 1$. While the demand for consuming products is not the function of time '*t*'. Under this circumstance, the answer for system 1.1 is as follows. Because $f_1(b) = f_2(b)$=1, we can get the following first order non-homogeneous linear differential equation

$$y_2' - l \cdot y_2 + a = 0 \qquad 2.3$$

from the equation $y_1' + y_2' = 0$, where $l = \frac{a_1 - a_3}{a_1 a_3}$ and a is the function of time '*t*'.

The general integral of the equation 2.3 is

$$y_2 = e^{lt} \int a(t) e^{-lt} dt + c_3' e^{lt} \qquad 2.4$$

And the general integral of y_1 is available by embedding 2.4 into the equation set 1.1. Considering the limitation of the market equilibrium equation 2.1, we can know $a(t) = c_3 e^{g \cdot t} - \frac{c_2 l}{a_2 g}$. So the general integral of 1.1 is

$$\begin{aligned} y_1 &= \frac{c_3}{g - l} e^{g \cdot t} - c_1 e^{l \cdot t} + c_2 \left(\frac{1 + a_2 g}{a_2 g}\right) \\ y_2 &= c_1 e^{l \cdot t} - \frac{c_3}{g - l} e^{g \cdot t} - \frac{c_2}{a_2 g} \\ a &= c_3 e^{g \cdot t} - \frac{c_2 l}{a_2 g} \end{aligned} \qquad 2.5,$$

Where $g = -\frac{a_1 + a_2}{a_1 a_2}$, and c_1, c_2, and c_3 are constant numbers. In fact, it can be concluded with certainty that c_1 equals zero because $b \equiv 1$. Then we can get the general integral of equation set 1.1 under market equilibrium situations.

$$y_1 = \frac{c_3}{g - l} e^{g \cdot t} + c_2 (\frac{1 + a_2 g}{a_2 g})$$
$$y_2 = -\frac{c_3}{g - l} e^{g \cdot t} - \frac{c_2}{a_2 g} \qquad 2.6$$
$$a = c_3 e^{g \cdot t} - \frac{c_2 l}{a_2 g}$$

Because $g < 0$, when *t* tends to be infinite, the solution showed by 2.6 is obviously converging, from which we can say, with the limitation of market equilibrium situation, social production will converge to the dynamic procedures of simple reproduction.

3.Systems' Dependence on the Condition of Initial Values

The solution of system 1.1 can be expressed as 2.6 with the restraint of equilibrium condition. From the analysis of the inner structure of systems, it was found that the equilibrium solution of systems had intimate relation with their initial values, this manufacturing system can realize the society's long-run balanced production, which means the simple reproduction of the entire society. At the moment the potential productivity of society are completely consistent with the demand of society and the social manufacturing reaches an absolute balance, E.I. $b \equiv 1$, while the so-called proper initial value is to keep the system 1.1 in equilibrium status at first ($b \equiv 1$).

In fact it can be concluded from the market equilibrium equation 2.1 that when the social producing meets the restrain conditions of equilibrium manufacture, the accumulative volume not yet completed in assets orders ($y_1(t)$) and the accumulative volume of the assets already produced ($y_2(t)$), the two conditional variables' initial values have the following relations with the system's parameters.

$$y_1(0) = \frac{a_2}{a_1} \cdot PCI \qquad y_2(0) = PCI \qquad PCI = k \bullet \frac{a_1 a_3 a}{a_1 - a_3} \qquad 3.1$$

The definitions of the parameters and the variables in 3.1 are equivalent to those of system 1.1. It is easy to conclude that this initial value condition has nothing to do with the demand for consuming products. If 3.1 are embedded into 2.1,we will get $b = \frac{a + \frac{1}{a_1} PCI}{\frac{1}{a_3} PCI}$, which can be simplified to

$$b = \frac{a_1 a_3 + a_3 \bullet PCI / a}{a_1 \bullet PCI / a} = \frac{a_1^2 a_3 - (1-k) a_1 a_3^2}{a_1^2 a_3 k} \qquad 3.2$$

Therefore when $k = 1$, we have $b \equiv 1$. In a system, the initial value condition for a society to carry on equilibrium producing is that *k* should be equal to 1. Therefore, once the system's initial value fails to meet the requirement, fluctuations will appear. Seeking the numerical solution of the system 1.1 can prove this fact. Assuming that the demand for consuming doesn't change, when *k* is 0.999 or 1.001, we can get the numerical solutions by the use of computers. And the major data outputs of our model are shown by the following Figure 3.1.

The three curves in Figure 3.1 respectively represent: the value of the model(curve 1), the Unfilled Orders for Capital "y_1" [2](curve 2) and the Production Capital "y_2" (curve 3). The periodical fluctuations of economy can be observed from the outputs. If the deviation from equilibrium is strengthened, modulating *k* to 0.99 or 1.01, a more obvious economic long-wave (Figure 3.2) can be seen from the initial value tests 3 and 4.

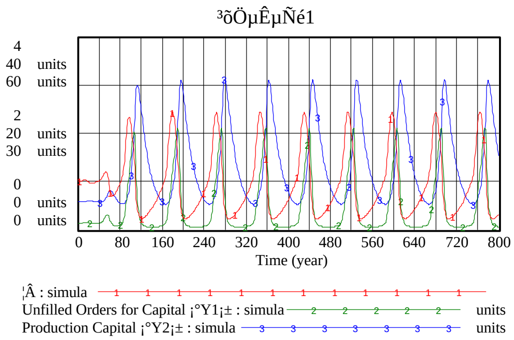

Figure 3.1 (a) *k*=0.999

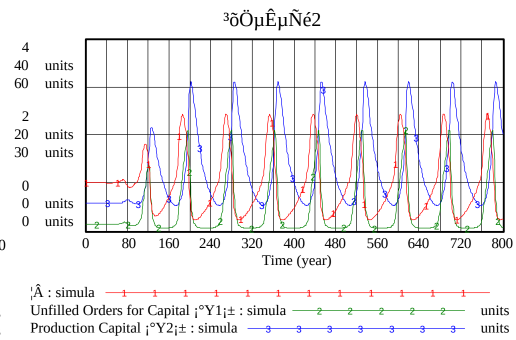

Figure 3.1 (b) *k*=1.001

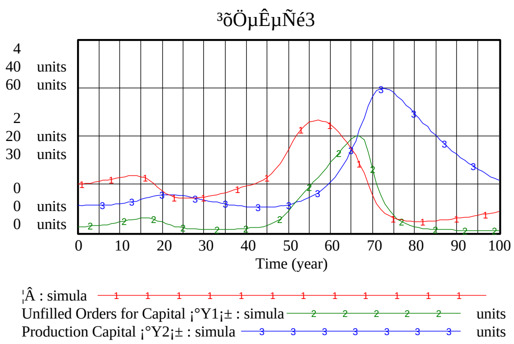

Figure 3.2 (a) *k*=0.99

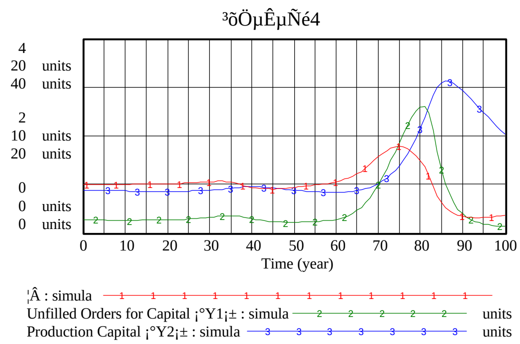

Figure 3.2 (b) *k*=1.01

4.Analysis on Systems' Inner Fluctuations

We can see from the simulated results of the system that fluctuations occur in all systems in the first 50 years under all the four circumstances. Although unchanged demand for consuming has been hypothesized, society's balanced producing is still liable to fluctuations because of the imbalance of the system at a certain moment. The four initial value tests are reasonable from the quantitative analysis of the tests. The reasons are as follows. On one hand, if the yearly increasing rate of economy is 0.1%, the economy level now is only 149 times that of 5000 years ago even though a period of 5000-year development has passed. Change the rate to 0.2% and it will be 21808 times; when it comes to 0.3%, 3196430 will be the result. Number accounting is easy. We assumed the yearly increasing rate was 1%, the general output of China 5000 years ago was only worth one cent and the Chinese population today was 1.5 billion, then the yearly output per capita still reached to 26.9 billion RMB. On the other hand, from 1952 to 2000, China's GDP per capita has increased 12 times in 48 years. As for the world scope, even the economic growth of developed countries has just boomed since the latest centuries. However, during the several thousand years before that, economy hardly increased [2]. From 1700 to 1785, the labor productivity of the Netherlands nearly didn't increased; during the time between 1785 and 1820, the Great Britain's labor productivity increased by 0.5% per year; and from 1820 to 1890 and 1890 to 1993, the labor productivity of the United States increased by 1.5% and 2.5% respectively. These three countries were the countries with the highest labor productivity level of the above mentioned time periods. From this, people posited that economic growth started from the late 18th century and the early 19th century, the period of Industrial Revolution, accelerated in the middle 19th century and reached unprecedented increasing rates in the 20th century [3]. So according to the long-run analysis, it was impossible to keep simple social reproduction when the society stepped into the industrialized period of commodity manufacturing. The balance broke and fluctuations appeared naturally. Since industrialization, the basic reason for frequent economic fluctuations is that the simple reproduction restrained by market equilibrium was crumbled down by industrialized manufacturing. Such breaking forces come from two aspects. One is the economic growth of the society itself. High-speed economic growth resulted in the collapse of original producing balance by the increase and fluctuation of social demand. The other are some unexpected events in the process of social development. As for the society and history, wars, revolutions, natural disasters, technologies and innovations are all likely to break down the temporary balance in social production. According to the initial value tests of the model, it is apparent only a little deviation of the initial value, for a certain moment, from the balanced value will lead to long-run fluctuations.

5.Conclusion

It has been proved that economic fluctuations have intimate relation with economic growth and the revolution of society with the quick economic development of recent society, the discovery of economic long wave and the empirical analysis about it[4][5][6]. Kondratieff Model shows this relation further. Therefore, as for the economic system existing now, it is reasonable to say the long-run economic fluctuations result from the mode of production. Any occasionally happened events within the system have an inevitable effect on the long-run economic fluctuations.

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