# ADAPTATION AND OPTIMAL CONTROL OF FIRM AND ITS STATE AND PARAMETERS ESTIMATION AT CHANGE OF A MARKET SITUATION

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**Abstract.** The dynamic model of one-commodity firm is described. For this model the problem of firm adaptation to change of a market situation is formulated and solved. Firm adaptation is realized by change of its parameters that are found as solution of the optimization problem subject to appropriate funds on it. The problem of optimal control by firm in conditions of a varied market situation is formulated and solved. The criterion of firm functioning efficiency representing quadratic sum of the particular criteria is constructed. For the formulated quadratic criterion optimal control as a feedback is received. As firm functioning occurs in uncertainty conditions estimation of a firm state and parameters is described according to information system of firm. Depending on information assumptions of uncertain factors it is offered to apply algorithms to processing the information the Kalman filtrations, the guaranteed and minimax-stochastic approach.

**Key words.** Model of firm, adaptation, optimal control, estimation of state and parameters

**Introduction.** The modern firm represents complex cybernetic system with many relations. Efficient functioning of such system depends on many factors. In particular, before a firm management there is a problem of operative reaction to change of a market situation. The decision of this problem is promoted by competent control of the firm activity. Therefore it is necessary to use mathematical methods for the help in correct decisions making on efficient firm control. The economy is not technical object, and, nevertheless, existing in engineering approach to construction of efficient control may be fruitful in economy.

In the present paper the mathematical model (fig. 1) of one-commodity firm including flows of orders, materials, labor, and also financial flows is constructed. For this model the problem of firm adaptation to change of a market situation is formulated and solved. Firm adaptation is realized by change of its parameters that are found as solution of the optimization problem subject to appropriate funds on it. To determine optimum control of firm, the integrated quadratic criterion of firm functioning efficiency, reflecting its total losses in a transition period caused by change of a market situation is formulated. Optimum control of firm as the law with a feedback, delivering to integrated quadratic criterion of functioning efficiency the minimal value at change of a market situation is received. In the present article change of a market situation is meant as change of demand for production, the prices for materials, time of payment for production, time of deliveries of materials and dismissal of workers at own will. As firm functioning occurs in uncertainty conditions estimation of a firm state and parameters is described according to information system of firm (FIS). Depending on information assumptions of uncertain factors it is offered to apply algorithms to processing the information the Kalman filtrations, the guaranteed and minimax-stochastic approach.

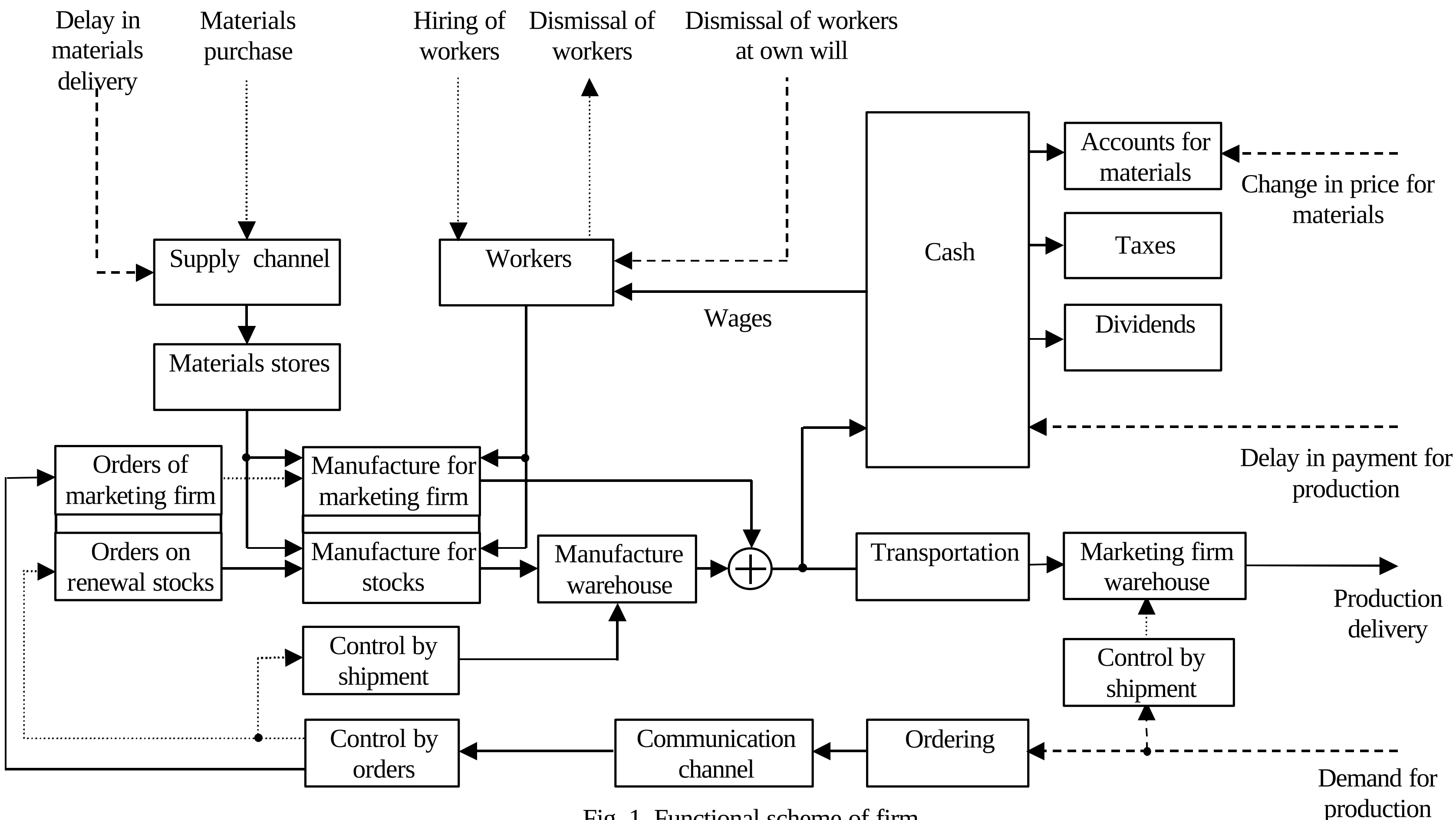



Fig. 1. Functional scheme of firm

The constructed mathematical model of firm is base: it may be changed at integration in CALS-technology, and also for realization of other researches (for example, on its basis the multicommodity model of the firm may be constructed).

**1. Dynamic economic-mathematical model of firm.** Manufacture and marketing firm are present in firm model (fig. 1). On fig. 1 material and information flows, the disturbances describing changes of a market situation and control actions that should provide optimal control of firm are designated. In marketing firm from buyers the flow of customers' orders acts. In turn, the marketing firm gives out a flow of orders for manufacture. In manufacture the part of the details, ordered marketing firm, is delivered at the expense of manufacture stocks, other part is produced specially under marketing firm orders. Therefore production volume includes release of details for satisfaction of the acted orders and for renewal manufacture stocks.

From manufacture in marketing firm the flow of finished commodity acts. These goods with some delay act on a marketing firm warehouse from which deliveries to buyers are carried out.

It is supposed that in modeled firm manufacture demands the big expenses of work and, hence, rates of manufacture are substantially specified by number of workers. In marketing firm number of workers is considered constant. The equations describing supply by materials are included in model also.

The constructed firm model is described by system of the difference equations

$$x_{k+1} = A(p)x_k + F(x_k, w_k, p) + B(p)u_k + \Gamma(p)w_k , \tag{1}$$

$$u_k = \Phi(x_k, w_k, p), \qquad k = 0, \ldots, N-1, \tag{2}$$

where $x_k$, $u_k$, $w_k$ are vectors of a state, control and disturbance; p is a vector of firm parameters; A(p), B(p), Ã(p) are constant matrixes; F($x_k$, $w_k$, p), Ô($x_k$, $w_k$, p) are nonlinear vector functions. The vector $w_k$ describes change of a market situation. As coordinates of a vector $w_k$ the following market factors act: $w_k^0$ is demand for production; $w_k^1$ is the price for materials; $w_k^2$ is time of payment for production; $w_k^3$ is time of deliveries of materials for manufacture; $w_k^4$ is disturbance in the channel of dismissal of workers (dismissal of workers at own will thus was modeled). Below separate coordinates of all vectors we shall denote by the upper index, T is the model digitization period, and function $OUT_{k+1}$ = del3($IN_k$, D) is a designation of system of the difference equations (Forrester, 1978):

$$L_{k+1}^1 = L_k^1 + T\ \left(IN_k - L_k^1/(D/3)\right), \qquad L_{k+1}^2 = L_k^2 + T\ \left(L_k^1 - L_k^2\right)/(D/3),$$

$$L_{k+1}^3 = L_k^3 + T\ \left(L_k^2/(D/3) - OUT_k\right), \qquad OUT_{k+1} = L_k^3/(D/3), \qquad k = 0, ..., N-1,$$

where $IN_k$ is rate of an entering flow; $OUT_k$ is rate of leaving flow; $L_k^1$, $L_k^2$, $L_k^3$ are auxiliary variables; D is value of delay.

<u>1.1. Ordering on manufacture.</u> The following equation determines number of the orders in ordering on manufacture $x_k^0$. To this number again acted orders are added and those orders are subtracted, the decision on which satisfaction is already accepted

$$x_{k+1}^0 = x_k^0 + T\left(x_k^{40} - \frac{x_k^0}{p^1}\right), \tag{3}$$

where $x_k^{40}$ is rate of acted orders (units per one week); $p^1$ is time of ordering on manufacture (week). Deducted in the equation (3) represents rate of ordering on manufacture. This rate is determined by the first-order lag from the general number of orders in ordering $x_k^0$.

Deliveries at the expense of manufacture stocks $x_k^1$ by marketing firm orders that are made already out, but yet are not executed, are determined by the following equation

$$x_{k+1}^1 = x_k^1 + T\left(u_k^0 - \frac{x_k^1}{p^0}\right), \tag{4}$$

where $u_k^0$ is rate of the orders which are carried out at the expense of manufacture stocks (units per one week); $p^0$ is time of goods shipment from manufacture stocks (week).

Manufacture stocks $x_k^2$ is determined by the following equation

$$x_{k+1}^2 = x_k^2 + T\left(x_k^8 - \frac{x_k^1}{p^0}\right), \tag{5}$$

where $x_k^8$ is rate of output for renewal of manufacture stocks (unit per one week). Deducted in the equations (4) and (5) represents rate of goods shipment from manufacture stocks. This rate is determined by the first-order lag from the general number of orders for shipment from manufacture stocks $x_k^1$.

Average rate of $x_k^8$ is determined by the following equation

$$x_{k+1}^3 = x_k^3 + \frac{T}{p^{15}}\left(\frac{x_k^1}{p^0} - x_k^3\right), \tag{6}$$

where $x_k^3$ is average rate of goods shipment from manufacture stocks (unit per one week); $p^{15}$ is averaging time of goods shipment from manufacture stocks (week).

Value of an average level of marketing firm orders $x_k^4$ may be received similarly (6)

$$x_{k+1}^4 = x_k^4 + \frac{T}{p^{16}}\left(x_k^{40} - x_k^4\right), \tag{7}$$

where $p^{16}$ is averaging time of marketing firm orders for manufacture (weeks).

<u>1.2. Manufacture.</u> Activity of manufacture is present by two flows (fig. 1): a flow of production for renewal a stock of manufacture and a flow of production going directly on satisfaction of marketing firm orders. In real manufacture these two flows are mutually bound, however in model they will be considered separately to reveal the necessary variables describing each of these flows. The following equation is determined with a portfolio of the orders not begun by manufacture on renewal of a stock of manufacture $x_k^5$

$$x_{k+1}^5 = x_k^5 + T\left(u_k^0 - \left(p^8 x_k^{16} - u_k^1\right)\right), \tag{8}$$

where $u_k^1$ is production direct under marketing firm orders (unit per one week); $x_k^{16}$ is number of workers on manufacture (persons); $p^8$ is labor productivity (units for one man-week). Deducted in the equation (8) represents rate of reduction of the orders not begun by manufacture on renewal of a manufacture stocks. This rate is determined as a difference of two rates: manufactures of all production $p^8 x_k^{16}$ and productions direct under marketing firm orders $u_k^1$.

The similar equation may be written down for orders $x_k^6$ carried out with the purpose of direct satisfaction marketing firm orders

$$x_{k+1}^6 = x_k^6 + T\left(\left(\frac{x_k^0}{p^1} - u_k^0\right) - u_k^1\right) . \quad (9)$$

Deducted in the equation (9) represents rate of increase of the marketing firm orders not begun by manufacture. This rate is determined as a difference between rate of ordering on manufacture $\frac{x_k^0}{p^1}$ and rate of the orders that are carried out at the expense of manufacture stocks $u_k^0$.

Process of manufacture is divided into two stages. At the first stage the firm management allocates workers on manufacture (through $u_k^1$). The second stage begins after period of production $p^2$. The equations describing the second production phase look like

$$x_{k+1}^7 = x_k^7 + T\left(p^8 x_k^{16} - u_k^1 - x_k^8\right) , \qquad x_{k+1}^8 = \mathrm{del3}\left(p^8 x_k^{16} - u_k^1,\ p^2\right) , \quad (10)$$

where $x_k^7$ are orders for renewal of manufacture stocks in a work in process (unit).

Execution of marketing firm orders is similarly determined

$$x_{k+1}^9 = x_k^9 + T\left(u_k^1 - x_k^{10}\right) , \qquad x_{k+1}^{10} = \mathrm{del3}\left(u_k^1, p^2\right) , \quad (11)$$

where $x_k^9$ are marketing firm orders in a work in process (unit); $x_k^{10}$ is rate of goods shipment under marketing firm orders (unit per one week).

<u>1.3. Materials stocks.</u> Materials stocks $x_k^{11}$ are determined by rate of their supply $x_k^{13}$ and consumption on manufacture $p^8 x_k^{16}$

$$x_{k+1}^{11} = x_k^{11} + T\left(x_k^{13} - p^8 x_k^{16}\right) . \quad (12)$$

Delivery of materials is described by the following equations of a kind

$$x_{k+1}^{12} = x_k^{12} + T\left(u_k^5 - x_k^{13}\right) , \qquad x_{k+1}^{13} = \mathrm{del3}\left(u_k^5, w_k^2\right) , \quad (13)$$

where $u_k^5$ is rate of materials purchase (equivalent units per one week); $w_k^2$ is time of delivery of materials for manufacture (weeks).

<u>1.4. Labor.</u> Providing with labor as well as regulations of its number render significant influence on manufacture. The following equation determines the workers that are starting to work $x_k^{15}$ and taking place training $x_k^{14}$

$$x_{k+1}^{14} = x_k^{14} + T\left(u_k^2 - x_k^{15}\right) , \qquad x_{k+1}^{15} = \mathrm{del3}\left(u_k^2, p^4\right) , \quad (14)$$

where $u_k^2$ is rate of hiring workers (person per one week); $p^4$ is time of training of workers on manufacture (week). In the equation (14) time constant $p^4$ is not only time of training of workers, but also that time when they to the full are not yet capable to take part in production.

Number of active workers $x_k^{16}$ is determined by the following equation

$$x_{k+1}^{16} = x_k^{16} + T\left(x_k^{15} - u_k^3 - w_k^4\right) , \quad (15)$$

where $u_k^3$ is rate of dismissal workers (person per one week); $w_k^4$ is rate of dismissal workers at own will (the person per one week).

The flow of a labor with the finished term of hiring is determined by the following equations similar (14)

$$x^{17}_{k+1} = x^{17}_k + T\left(u^3_k + w^4_k - x^{18}_k\right), \qquad x^{18}_{k+1} = del3\left(u^3_k + w^4_k, p^5\right), \tag{16}$$

where $x^{17}_k$ are dismissing workers (person); $x^{18}_k$ are the workers who have finished industrial work (the person per one week); $p^5$ is time of registration of dismissal workers (weeks). The equations (16) determine number of workers that finish the work on manufacture $x^{17}_k$ and rate of a leaving of workers $x^{18}_k$. Delay in dismissal of workers $p^5$ in this case is equal to the period during which the wages still should be charged, but workers do not participate any more in production. It may be connected to the following circumstances: the stayed period of hiring should be paid or it is the time of the lowered productivity caused by reduction of intensity of work.

The general changes in number of workers $x^{19}_k$ may be determined by the following equation

$$x^{19}_{k+1} = x^{19}_k + T\left(u^2_k + u^3_k + w^4_k\right) \, . \tag{17}$$

This variable may be considered as a parameter of the achieved degree of stability.

1.5. Period of deliveries. All entering marketing firm orders are late in divisions where they are checked and allocated on groups of orders satisfied either at the expense of manufacture stocks or at the expense of direct manufacture. After that orders satisfied at the expense of manufacture stocks undergo constant delay $p^0$ while orders satisfied manufacture undergo the variable delay dependent on volume of marketing firm orders not begun by manufacture $x^6_k$. In addition to average delay all orders undergo delay $p^1$ determined delay during which they are allocated between manufacture and manufacture stocks

$$x^{20}_{k+1} = p^1 + \frac{p^1 u^0_k}{x^0_k} p^0 + \left(1 - \frac{p^1 u^0_k}{x^0_k}\right)\left(p^2 + \frac{x^6_k}{u^1_k}\right), \tag{18}$$

where $x^{20}_k$ is variable delay of performance of orders for manufacture (week).

Value $\dfrac{p^1 u^0_k}{x^0_k}$ in the equation (18) represents a part from the general number of marketing firm orders that are satisfied at the expense of manufacture stocks and value $\dfrac{x^6_k}{u^1_k}$ is variable delay in a portfolio of outstanding orders. The general variable delay of production under marketing firm orders is determined as the sum of time constant $p^2$ and variable delay in a portfolio of outstanding orders.

The equation (18) determines period of deliveries existing at present $x^{20}_k$. The marketing firm not necessarily has it because it is transferred with the certain delay. The following equation averages a flow of the information on delay of deliveries

$$x^{21}_{k+1} = x^{21}_k + \frac{T}{p^{17}}\left(x^{20}_k - x^{21}_k\right), \tag{19}$$

where $x^{21}_k$ is variable period of deliveries under the received messages (week); $p^{17}$ is time of regulation of a flow of the information on deliveries (week). Time constant $p^{17}$

determines rate with which period of deliveries according to information received $x_k^{21}$ comes nearer to really existing period of deliveries $x_k^{20}$.

1.6. Flows of money resources. Monetary flows in model are used for an assessment of works of firm. The following equation determines a level of accounts for payment $x_k^{22}$

$$x_{k+1}^{22} = x_k^{22} + T\left( w_k^1 x_k^{13} - \frac{x_k^{22}}{p^6} \right), \tag{20}$$

where $w_k^1$ is the price of unit of materials (monetary units); $p^6$ is accounts payment time (week). In the equation (20) rate of payments for materials is determined as the certain part of number of accounts for payment $x_k^{22}$.

The money resources received for the goods are present in the other kind. Accounts for finished goods undergo the third-order lag before they will be transformed to a flow of money resources acting on manufacture

$$x_{k+1}^{23} = \mathrm{del3}\left( p^{10}\left( \frac{x_k^1}{p^0} + x_k^{10} \right), w_k^3 \right), \qquad x_{k+1}^{24} = x_k^{24} + T\left( p^{10}\left( \frac{x_k^1}{p^0} + x_k^{10} \right) - x_k^{23} \right), \tag{21}$$

where $x_k^{23}$ is rate of receipt money resources for finished goods (monetary units per one week); $x_k^{24}$ are accounts to reception on manufacture (monetary units); $w_k^3$ is time of payment by the buyer of accounts (week); $p^{10}$ is the price of a finished good (monetary units).

Total sum of the dividends $x_k^{25}$ paid to shareholders to the certain moment will make

$$x_{k+1}^{25} = x_k^{25} + T x_k^{29}, \tag{22}$$

where $x_k^{29}$ is rate of payment of dividends to shareholders (monetary units per one week).

The equation for a cash $x_k^{26}$ looks like

$$x_{k+1}^{26} = x_k^{26} + T\left( x_k^{23} - \frac{x_k^{22}}{p^6} - p^{11}\left( x_k^{14} + x_k^{16} + x_k^{17} \right) - p^{12} - p^{27} x_k^{27} - x_k^{29} \right), \tag{23}$$

where $x_k^{27}$ is rate of reception of the profit before payment of the tax (monetary units per one week); $p^{11}$ is average week wages (monetary units); $p^{12}$ is the fixed costs (monetary units per one week); $p^{27}$ is the tax to profit. In the equation (23) to money resources $x_k^{26}$ the flow of entering resources are added $x_k^{23}$ also five proceeding monetary flows are subtracted: accounts for payment in manufacture $\frac{x_k^{22}}{p^6}$, expenses on wages, constant cash charges $p^{12}$, tax deductions $p^{27} x_k^{27}$ and payment of dividends to shareholders $x_k^{29}$.

In model rate of reception of the profit before payment of the tax $x_k^{27}$ plays a role of a important parameter. It may influence decisions accepted by a firm management. Rate of reception of the profit before payment of the tax is determined with the help of the equation

$$x_{k+1}^{27}=\left(p^{10}-w_k^1-\frac{p^{11}}{p^8}\right)\left(\frac{x_k^1}{p^0}+x_k^{10}\right)-p^{12}-p^{11}\left(x_k^{14}+x_k^{16}+x_k^{17}-\frac{x_k^8+x_k^{10}}{p^8}\right). \qquad (24)$$

The first composed in the equation (24) determines total rate of manufacture of finished goods. This rate is multiplied on the price of a product minus its cost price. This value might determine the full income if the labor was used with peak efficiency. The following expression in brackets represents charges on wages minus the wages which have been taken into account in the cost price of a product.

Net profit $x_k^{28}$ will be determined in view of the rate of the tax to profit $p^{27}$

$$x_{k+1}^{28}=x_k^{28}+T\left(1-p^{27}\right)x_k^{27}\ . \qquad (25)$$

Rate of payment of dividends to shareholders $x_k^{29}$ is determined proceeding from rate of reception of the profit $x_k^{27}$. The appropriate equation looks like

$$x_{k+1}^{29}=x_k^{29}+\frac{T}{p^{18}}\left(\left(1-p^{27}\right)x_k^{27}-x_k^{29}\right), \qquad (26)$$

where $p^{18}$ is time of regulation of dividends (week).

<u>1.7. Interaction between manufacture and marketing firm.</u> Interaction between manufacture and marketing firm is carried out by means of three flows: in manufacture the flow of marketing firm orders $x_k^{40}$ acts already undergone post delay $p^{24}$; the marketing firm, in turn, receives from manufacture the information on period of deliveries $x_k^{21}$ and a flow of finished goods $\left(\frac{x_k^1}{p^0}+x_k^{10}\right)$. Number of outstanding marketing firm orders $x_k^{30}$ is determined by the following equation

$$x_{k+1}^{30}=x_k^{30}+T\left(x_k^{40}-x_k^{10}-\frac{x_k^1}{p^0}\right). \qquad (27)$$

<u>1.8. Model of marketing firm.</u> The description of marketing firm we shall start with the equations for number of customers' outstanding orders $x_k^{31}$ and stocks of production $x_k^{32}$ in a marketing warehouse. Value of outstanding orders $x_k^{31}$ is determined by the following equation

$$x_{k+1}^{31}=x_k^{31}+T\left(w_k^0-u_k^4\right), \qquad (28)$$

where $u_k^4$ is rate of goods delivery to buyers (units per one week); $w_k^0$ is demand for production (units per one week).

For maintenance of performance of customers' orders in a marketing firm there should be a stock. The following equation describes a level of marketing stocks $x_k^{32}$

$$x_{k+1}^{32}=x_k^{32}+T\left(x_k^{37}-u_k^4\right), \qquad (29)$$

where $x_k^{37}$ are the deliveries received by marketing firm (units per one week).

In connection with that demand for production $w_k^0$ will change from one day to another it is necessary for averaging to obtain data on which it would be possible to found plans concerning volume of stocks $x_k^{32}$ and orders $x_k^{31}$. For this purpose the equation of averaging of the first-order is used

$$x_{k+1}^{33} = x_k^{33} + \frac{T}{p^{21}}\left(w_k^0 - x_k^{33}\right), \tag{30}$$

where $x_k^{33}$ are the average orders to marketing firm (units per one week); $p^{21}$ is averaging time of orders to marketing firm (week).

In connection with that stocks decrease during trade it is necessary to fill up them. The equation for rate of purchases by marketing firm $x_k^{38}$ looks like

$$\begin{aligned} x_{k+1}^{38} = w_k^0 + \frac{1}{p^{22}}\Big(\left(p^{26}x_k^{33} - x_k^{32}\right) + \left(x_k^{31} - \left(p^{19} + p^{20}\right)x_k^{33}\right) + \\ + \left(\left(p^{23} + p^{24} + p^{25} + x_k^{21}\right)x_k^{33} - x_k^{34} - x_k^{35} - x_k^{36} - x_k^{30}\right)\Big), \end{aligned} \tag{31}$$

where $x_k^{34}$ are orders in marketing firm at ordering (unit); $x_k^{35}$ are the orders given by marketing firm for the purchases which are contained in a communication channel (unit); $x_k^{36}$ are the goods in a way to marketing firm (unit); $p^{19}$ is minimal time of performance of the order marketing firm (week); $p^{20}$ is average time of performance of orders by the marketing firm connected to absence in a warehouse of some goods (week); $p^{22}$ is time of regulation of stocks (week); $p^{23}$ is time of ordering in marketing firm (week); $p^{24}$ is post delay (weeks); $p^{25}$ is time of transportation of the goods from manufacture in marketing firm (weeks); $p^{26}$ is the factor describing a desirable stock in marketing firm (week).

Time constant $p^{22}$ reflects rate with what the marketing firm on the average reacts to occurrence of deficiency of production in stocks and firm channels. It is impossible to assume that the marketing firm will react immediately in full force to any theoretically possible difference between desirable and actual stocks.

If the level of a desirable stock in marketing firm will be higher or below actual $x_k^{32}$ that rate of purchases $x_k^{38}$ will be accordingly corrected (a difference in the first brackets of the equation (31)). The concept desirable a stock that can be considered as a planned stock rate is very important. Interrelation between change of stocks $x_k^{32}$ and an average orders $x_k^{33}$ is one of several most important sources of industrial activity fluctuations. Irrespective of firm stability the standard practice consists in creation or reduction of stocks according to increase or reduction of average rate of orders. It enables to measure stocks the certain number of days, during which it is possible to carry out sale at the expense of stocks. Thus interrelation between average customers' orders $x_k^{33}$ and the general stock $x_k^{32}$ accepts direct proportional relationship. The parameter $p^{26}$ represents number of weeks during which average rate of sale may be provided at the expense of a desirable stock in marketing firm.

Number of outstanding customers' orders $x_k^{31}$ is entered into the equation (31) in connection with aspiration to receive confidence that the equations will stay fair and under extreme conditions of firm activity. If goods delivery from manufacture is absent and volume of outstanding customers' orders $x_k^{31}$ is so great that induces them to abstain from the further purchases demand for production $w_k^0$ becomes equal to zero. Under such circumstances the actual stock rate $x_k^{32}$ will be deducted up to zero. The second member is a normal level of the orders not executed by retail $\left(p^{19} + p^{20}\right)x_k^{33}$.

Normal delay in marketing firm consists of two: one represents the average minimal time necessary for ordering $p^{19}$ another $p^{20}$ is caused by absence of stocks of some goods.

The difference in the third brackets of the equation (31) is similar to the members describing stocks. The average total number of orders and the goods that should be in movement on different firm channels grows out multiplication of average rate of orders $x_k^{33}$ on the general period necessary for fulfillment by the order full circulation on firm channels. Three making these periods connected to ordering $p^{23}$, with delivery of the order by mail $p^{24}$ and with transportation of the goods $p^{25}$ are constants. Period at performance of orders by manufacture $x_k^{21}$ is variable. The real orders transmitted on the channel are determined in this case as the sum of four levels of orders and the goods that have been usual in four firm channels.

Below we shall consider three separate periods in marketing firm: in accommodation of orders on manufacture, in transfer of orders by mail from marketing firm in manufacture and delivery of the goods in the opposite direction. Two equations determining the third-order lag at acceptance of decisions on purchase and accommodation of marketing firm orders in manufacture look like

$$x_{k+1}^{34} = x_k^{34} + T\left(x_k^{38} - x_k^{39}\right), \qquad x_{k+1}^{39} = \mathrm{del3}\left(x_k^{38}, p^{23}\right), \tag{32}$$

where $x_k^{39}$ are the orders given by marketing firm for purchase of the goods in manufacture (units per one week).

The exit of the delay connected to ordering serves as an entrance for post delay. Post operations also are determined by the third-order lag

$$x_{k+1}^{35} = x_k^{35} + T\left(x_k^{39} - x_k^{40}\right), \qquad x_{k+1}^{40} = \mathrm{del3}\left(x_k^{39}, p^{24}\right). \tag{33}$$

To finish the description of marketing firm it is necessary to show transportation of the goods from manufacture. For this purpose we shall write down two equations describing delay at this transportation

$$x_{k+1}^{36} = x_k^{36} + T\left(\frac{x_k^1}{p^0} + x_k^{10} - x_k^{37}\right), \qquad x_{k+1}^{37} = \mathrm{del3}\left(\frac{x_k^1}{p^0} + x_k^{10}, p^{25}\right), \tag{34}$$

where $x_k^{37}$ is rate of delivery of production from manufacture in marketing firm (units per one week).

**2. Influence of demand variation on firm behavior.** The main external factor influencing firm is demand for its production $w_k^0$ which changes were considered as entrance influence. For studying properties of the constructed model jump of demand for 20% was used in comparison with an initial level $w_0^0$ (fig. 2). The behavior variable model looks like fading fluctuations which transition period makes 5 years.

**3. Adaptation of firm to change of demand for its production.** Improvement of technical and economic characteristics of firm may be made by a finding of optimal values of its parameters p that gives extensive opportunities of application of optimization methods for adaptation to demand for production variation. For this purpose we shall consider the value of missed benefit $L_k$ (Baev 2001) showing as far as precisely the firm fulfils change of demand for its production $w_k^0$. It characterizes firm lost profit caused by a backlog demand and also additional expenses for storage of production.

Expression for value of the missed benefit looks like

$$L_{k+1} = L_k + T\left|w_k^0 - u_k^4\right|, \qquad k = 0, \ldots, N-1, \tag{35}$$

where $L_k$ is value of the missed benefit; $w_k^0$ is demand for production; $u_k^4$ is rate of delivery of production to buyers.

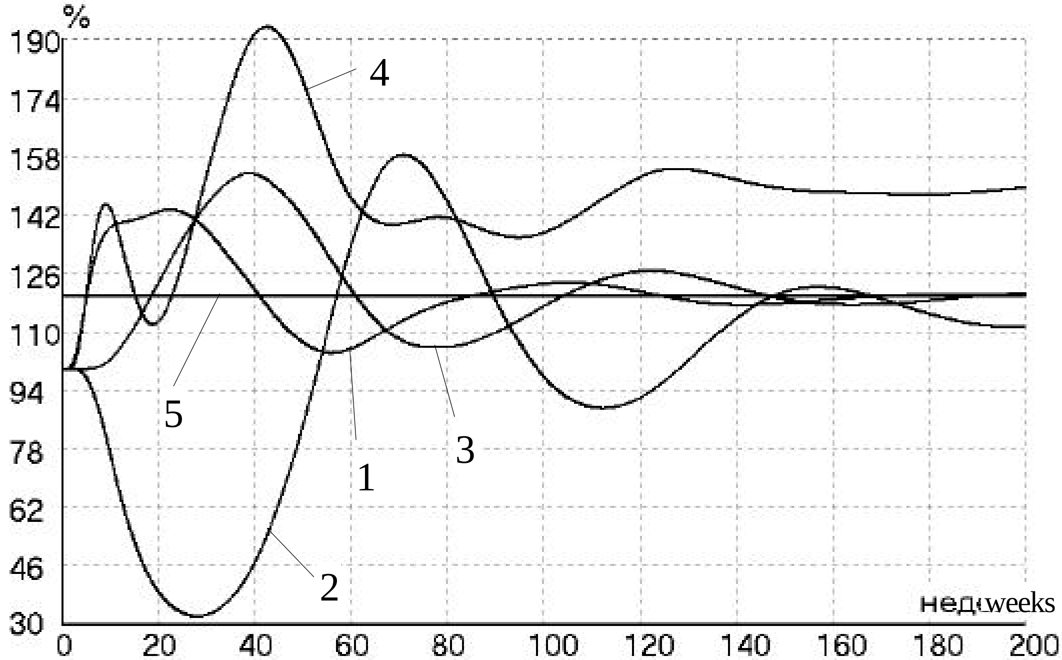



Fig. 2. Reaction of firm on 20% jump of demand for its production

1 – orders in ordering on manufacture; 2 – manufacture stocks; 3 – number of workers; 4 – rate of reception of the profit; 5 – demand for production

Rate of delivery of production to buyers $u_k^4$ depends on firm parameters p and also from demand $w_k^0$. At change of demand generally these sizes do not coincide that is connected to the delays caused by dynamic processes both in firm as a whole and in its separate divisions. In result there is an accumulation of the missed benefit. For its reduction it is necessary to achieve smaller transient on period in firm at change of demand for production. Thus the value of the missed benefit depends on demand and firm parameters and precisely knowing values of some firm parameters and having an opportunity to change them it is possible to reduce value of the missed benefit. Thus any change of firm parameters is connected to expenses and there is a number of restrictions on change of firm parameters.

Let's formulate optimization problem. On trajectories of firm model (1), (2) it is necessary

$$f(p) \quad \min \tag{36}$$

at restrictions on parameters

$$p_1^i \le p^i \le p_2^i, \qquad i = 0, \ldots, s-1, \qquad \sum_{i=1}^{s} c^i (p_0^i - p^i) \le R,$$

where f is criterion function; $p^i$ are firm parameters; R are means for change of parameters; $c^i$ are expenses for change of parameters; $p_1^i, p_0^i, p_2^i$ are the minimal, initial and maximal values of firm parameters ($p_1^i \le p_0^i \le p_2^i$); s is number of changeable firm parameters. As criterion function f it is possible to consider value of the missed benefit

(35). Therefore at its substitution on a place f in (36) it is received optimization problem. As a result of its solution optimal values of firm parameters in sense of criterion (35) also will be found. We shall emphasize that search of the optimal solution is made in view of limits of means R selected on firm adaptation.

Optimization was made by a method of Franc-Wolf that was used only for finding local extremes. For search global extremes the initial point in this method got out from allowable set at random for each of which the algorithm repeated again. After several pass the point appropriate minimal from found values of function (35) got out. The solution of a problem (36) gives value of firm parameters providing its optimum performance at change of a market situation in sense of given criterion.

Let's compare behavior of firm at not optimal and optimal parameters (fig. 2 and 3). For this purpose we shall consider reaction of firm with optimal parameters on 20% increase of demand for production in comparison with initial value $w_0^0 = 1000$ units per one week. Optimal firm parameters $p^i$ are received as a result of the solution of a optimization problem (36) for criterion of the missed benefit (35) at means for change of parameters R=20 of units. From fig. 3 follows that after optimization the maximal deviations from the established values and duration of a transition period have considerably decreased.

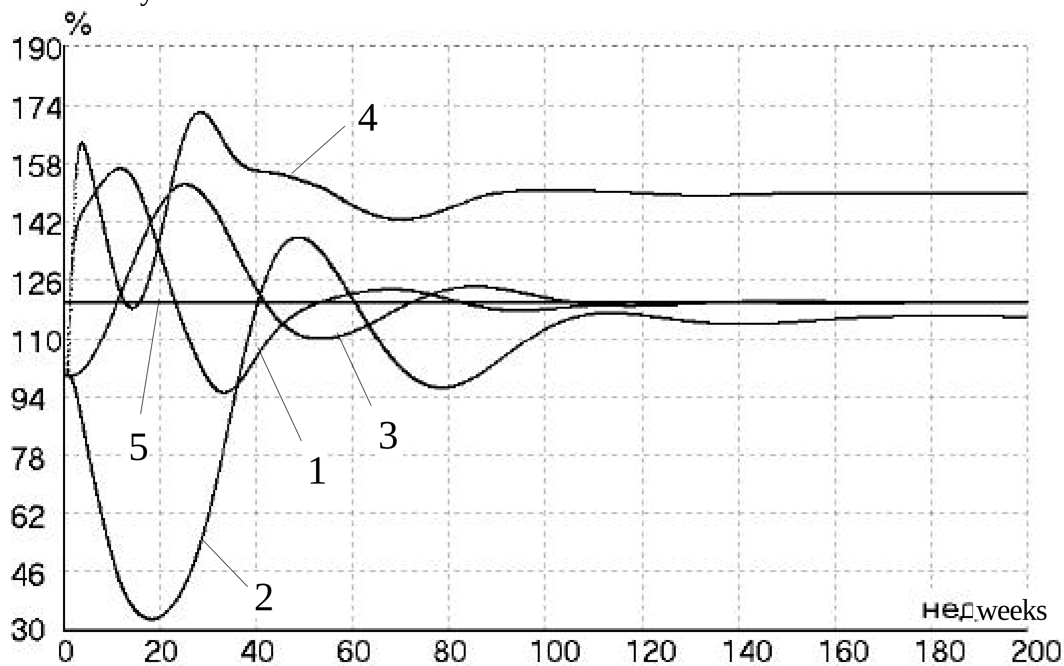



Fig. 3. Reaction of firm with optimal parameters on 20% jump of demand for its production

1 – orders in ordering on manufacture; 2 – manufacture stocks; 3 – number of workers; 4 – rate of reception of the profit; 5 – demand for production

On fig. 4 and in tab. 1 results of modeling are given at 20% change of demand in two cases: 1) restriction on means R=20 of units (the price of change of parameters: $c^2$=10 units; $c^i$ =1 unit, $i \neq 2$); 2) restrictions on means are not present (is really spent 45.2 units). Period of modeling is 200 weeks. Initial values of firm parameters $p_0^i$ get out equal to their top borders $p_2^i$ (they get out as not optimal). Value of missed benefit

$L_N$ in the first case has made 173.45 units, in the second – 170.47 units (for comparison: the value of missed benefit $L_N$ at firm with not optimal parameters makes 863.5 units.).

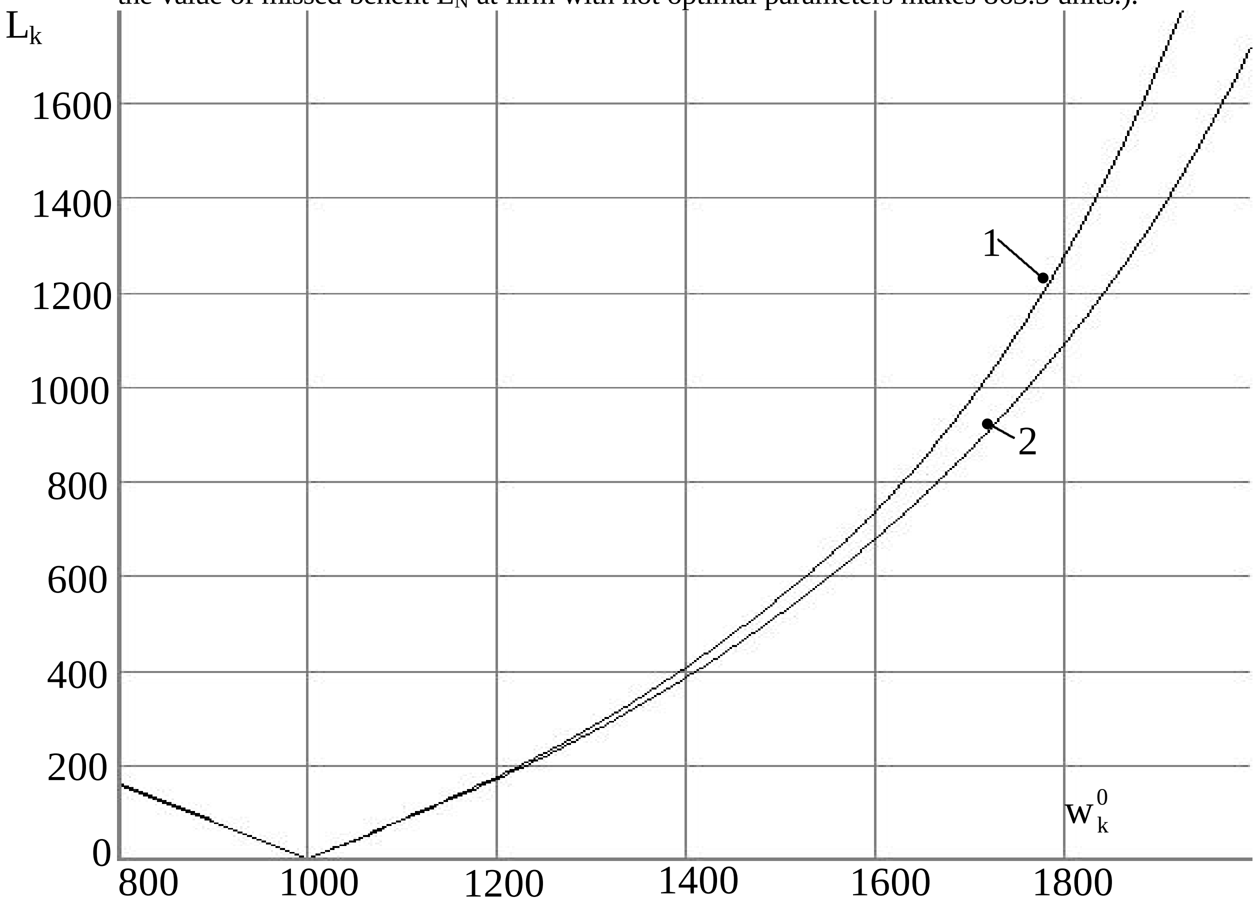



Fig. 4. Relationship between missed benefit and actual value of demand for production at optimal parameters and 20% jump of demand

1 – appropriate funds R = 20 units; 2 – appropriate funds is not restricted

Table 1

Optimal firm parameters at change of demand for 20 %

| Name of parameter | Range | Optimal value (restriction on means is 20 units) | Optimal value (restriction on means is not present) |
|---|---|---|---|
| $p^0$ Delay of goods shipment on manufacture | 0.5 .. 1 | 0.5 | 0.5 |
| $p^1$ Delay in ordering on manufacture | 0.5 .. 1 | 0.5 | 0.5 |
| $p^2$ Period of production | 4 .. 6 | 6 | 4 |
| $p^3$ Delay in a normal portfolio of orders on manufacture | 2 .. 4 | 4 | 4 |
| $p^4$ Time of training of workers | 1 .. 3 | 1 | 1 |
| $p^{16}$ Averaging time of orders on manufacture | 7 .. 15 | 12.2 | 7 |
| $p^{19}$ Minimal time of performance of customers' orders marketing firm | 0.6 .. 1 | 0.6 | 0.6 |

| | | | |
|---|---|---|---|
| $p^{20}$ Average delay of performance of orders by the marketing firm connected to absence in a warehouse of some goods | 0.1 .. 0.4 | 0.1 | 0.1 |
| $p^{23}$ Time of ordering in marketing firm | 0.4 .. 3 | 0.4 | 0.4 |
| $p^{24}$ Delay in communication channel of marketing firm | 0.2 .. 0.5 | 0.2 | 0.2 |
| $p^{25}$ Delay of transportation of the goods in marketing firm | 0.4 .. 1 | 0.4 | 0.4 |

Firm parameters $p^i$ share on 3 groups (tab. 1): the parameters accepting the top values $p_2^i$; the parameters accepting the bottom values $p_1^i$; the parameters accepting intermediate values (their occurrence it is connected to insufficiency of means R for change of firm parameters). Aspiration to the bottom border $p_1^i$ is observed at all parameters except for parameter $p^3$ (delay in a normal portfolio of orders on manufacture). Really, at the increase of demand the firm has no sufficient number of workers for performance of orders on manufacture at small delay $p^3$ that attracts longer transient in firm in comparison with the big value $p^3$. On fig. 4 it is easy to see that attraction of additional means for change of firm parameters allows to reduce value of missed benefit $L_N$ however it is necessary to pay attention that such investments should not exceed economic benefit of firm functioning optimization.

**4. Optimal control of firm at change of a market situation.** To formulate and solve a control problem the equations (2) were removed from model. Except for it the nonlinear equations were linearized. In result the mathematical model of firm (1), (2) corresponds as system of the linear difference equations

$$x_{k+1} = Ax_k + Bu_k + \tilde{A}w_k \ , \qquad k = 0, \ldots, N-1, \tag{37}$$

where $x_k$, $u_k$, $w_k$ are vectors of a state, control and disturbances accordingly; A, B, Ã are known constant matrixes. The vector $w_k$ describes the current market situation. For firm model (37) its initial condition $x_0$ and a sequence of vectors $w_k$, k=0, ..., N–1 are given also.

<u>4.1. Quadratic criterion of firm functioning efficiency.</u> During firm functioning several purposes are simultaneously put. Some of these purposes under the tendency of their realization may be inconsistent. In the present paper for model of firm (37) the quadratic criterion of functioning efficiency that reflects total losses of firm in a transition period caused by change of a market situation is formulated. The constructed criterion has the following kind

$$x'_N S x_N + \sum_{k=1}^{N-1} \left( x'_k Q x_k + u'_k R u_k \right) , \tag{38}$$

where Q, S are symmetric nonnegative defined matrixes; R is the symmetric positive defined matrix; N is number of digitization intervals.

The criterion firm functioning efficiency (38) is divided on three parts: deviation of the profit from some desirable value (the half–received profit); production costs; losses of marketing firm. To each component in criterion of efficiency (38) there corresponds the weighting factor $\lambda^i, i = 0,1,2$. The concrete set of weighting factors reflects structure of preferences of the persons accepting administrative decisions. Division of criterion of efficiency into separate parts reflects discrepancy of the purposes of heads of separate structural divisions of firm and also determines the basic strategy of a firm survival in the market. Production costs reflect the purposes of its head. For him it is more important to reconstruct manufacture under the current market situation having avoided thus of the big fluctuations in number of the workers on manufacture, a portfolio of outstanding orders etc. Losses of marketing firm reflect the purposes of its head. For him it is more important to reconstruct marketing firm under the current market situation having avoided thus of the big fluctuations of outstanding orders.

To result all parts in criteria (38) in one scale the cost estimation of everyone is used. Thus elements of matrixes R, Q and S serve as factors of translation in monetary units of those units in which the appropriate variable are measured. For achievement of the purposes of firm the criterion (38) should be minimized. Below we shall consider separate composed criterion (38).

The natural order to the profit is maximization of its final value. It can be achieved if to include in criterion (38) following composed

$$\lambda^0 \left(x_N^{28} - M\right)^2 ,$$

where $x_N^{28}$ is final profit (monetary units); M is big number (monetary units); $\lambda^0$ is weighting factor of the half–received profit. It is possible to take for example the target profit increased on number greater of unit

The situation in manufacture should correspond to a situation in marketing firm. The manufacture head has a parameter that is the entrance characteristic for manufacture. It is orders that are showed by marketing firm to manufacture ($x_k^{40}$). The following expression determines a deviation of orders in ordering on manufacture $x_k^0$ from a normal level

$$\lambda^1\alpha\left(\frac{x_k^0}{p^1} - x_k^{40}\right)^2 ,$$

where $p^1$ is time of ordering on manufacture (week); $\lambda^1$ is weighting factor of production costs; $\alpha = \left(p^{10}T\right)^2$; $p^{10}$ is the price of unit products (monetary units). The normal level of orders during ordering on manufacture is determined only by marketing firm orders $x_k^{40}$ and parameter $p^1$.

Orders for shipment from manufacture stocks $x_k^1$ also are adjusted by means of expression

$$\lambda^1\alpha\left(\frac{x_k^1}{p^0 p^{13}} - x_k^{40}\right)^2 ,$$

where $p^0$ is time of goods shipment from manufacture stocks (week); $p^{13}$ is a normal part of the general number of orders which is satisfied at the expense of manufacture stocks.

The following expressions adjust volume of portfolios of outstanding marketing firm orders $x_k^5$ and $x_k^6$

$$\lambda^1\alpha\left(\frac{x_k^5}{p^3 p^{13}} - x_k^{40}\right)^2 , \qquad \lambda^1\alpha\left(\frac{x_k^6}{p^3\left(1 - p^{13}\right)} - x_k^{40}\right)^2 ,$$

where $p^3$ is delay in a normal portfolio of orders (week).

Similar expressions may be written down for regulation of levels of orders in a work in progress $x_k^7$ and $x_k^9$ and the appropriate rates of manufacture $x_k^8$ and $x_k^{10}$

$$\lambda^1\alpha\left(\frac{x_k^7}{p^2p^{13}}-x_k^{40}\right)^2, \qquad \lambda^1\alpha\left(\frac{x_k^8}{p^{13}}-x_k^{40}\right)^2, \qquad \lambda^1\alpha\left(\frac{x_k^9}{p^2\left(1-p^{13}\right)}-x_k^{40}\right)^2, \qquad \lambda^1\alpha\left(\frac{x_k^{10}}{1-p^{13}}-x_k^{40}\right)^2,$$

where $p^2$ is period of production (weeks).

Stocks of materials in manufacture $x_k^{11}$ are adjusted by means of expression

$$\lambda^1\alpha\left(\frac{x_k^{11}}{p^9}-x_k^{40}\right)^2,$$

where $p^9$ is factor of a desirable stock of materials in manufacture (week).

Regulation of number of workers in manufacture $x_k^{16}$ is carried out by means of the expression determining a deviation of number of workers on manufacture from a normal level

$$\lambda^1\alpha\left(p^8x_k^{16}-x_k^{40}\right)^2,$$

where $p^8$ is labor productivity (units for one man-week).

The situation in marketing firm should correspond to a market situation. The head of marketing firm has a parameter that is the entrance characteristic for marketing firm. It is demand for production ( $w_k^0$ ). Then a deviation of number of outstanding customers' orders $x_k^{31}$ from a normal level it is determined by expression

$$\lambda^2\alpha\left(\frac{x_k^{31}}{p^{19}+p^{20}}-w_k^0\right)^2,$$

where $p^{19}$ is minimal time of performance of the order marketing firm (week); $p^{20}$ is average delay of performance of orders by the marketing firm, connected to absence in a warehouse of some goods (week); $\lambda^2$ is weighting factor of losses of marketing firm.

Similar expressions may be written down for regulation of levels of orders in various channels of marketing firm

$$\lambda^2\alpha\left(\frac{x_k^{34}}{p^{23}}-w_k^0\right)^2, \qquad \lambda^2\alpha\left(\frac{x_k^{35}}{p^{24}}-w_k^0\right)^2, \qquad \lambda^2\alpha\left(\frac{x_k^{36}}{p^{25}}-w_k^0\right)^2,$$

where $x_k^{34}$ are orders in marketing firm in ordering (unit); $x_k^{35}$ are the orders given by marketing firm for the purchases which are taking place in an communication channel (unit); $x_k^{36}$ are the goods in a way to marketing firm (unit); $p^{23}$ is time of ordering in marketing firm (week); $p^{24}$ is post delay (weeks); $p^{25}$ is time of transportation of the goods in marketing firm (weeks).

Similar expression may be written down for reception of orders of manufacture $x_k^{40}$

$$\lambda^2\alpha\left(x_k^{40}-w_k^0\right)^2.$$

In criterion (38) matrix R should be nonsingular. Hence all coordinates of control vector $u_k$ should enter in criterion (and without weighting factor). Deviations of rate of orders $u_k^0$ carried out at the expense of manufacture stocks and production $u_k^1$ by marketing firm orders from a normal level look like

$$\alpha\left(\frac{u_k^0}{p^{13}}-x_k^{40}\right)^2, \qquad \alpha\left(\frac{u_k^1}{1-p^{13}}-x_k^{40}\right)^2.$$

The following expressions represent deviations of rates of hiring $u_k^2$ and dismissals of workers $u_k^3$ in manufacture from a zero level

$$\alpha\left(u_k^2\right)^2, \qquad \alpha\left(u_k^3\right)^2.$$

Following composed the criterion includes a deviation of delivery of production to buyers $u_k^4$ from demand (the missed benefit)

$$\alpha\left(u_k^4-w_k^0\right)^2.$$

By last composed the criterion includes a deviation of rate of purchases of materials $u_k^5$ from the orders showed by marketing firm to manufacture

$$\alpha\left(u_k^5-x_k^{40}\right)^2.$$

4.2. Statement and the solution of a problem of optimal control. The problem of optimal control of firm at change of a market situation can be formulated as follows. There is a linear system of the difference equations (37) describing behavior of firm. The initial firm state $x_0$ and a sequence of vectors $w_k$, k=0, ..., N–1 are given. The quadratic criterion of firm functioning efficiency (38) reflecting total losses of firm in a transition period is given also. It is required to find firm control $u_k$, k=0, ..., N–1 which delivers to criterion (38) minimal value.

The formulated problem of optimal control is solved by dynamic programming approach (Kwakernaak, 1972). Thus optimal control turns out as

$$u_k = -K_k^1 x_k - K_k^2 w_k, \quad k=0,\ldots,N-1, \tag{39}$$

where matrixes $K_k^1$ and $K_k^2$ are determined from

$$K_k^1 = Z_k T_{k+1}^1 A, \qquad K_k^2 = Z_k\left(T_{k+1}^1\tilde{A}+T_{k+1}^2\right), \qquad Z_k = \left(R+B' T_{k+1}^1 B\right)^{-1} B',$$

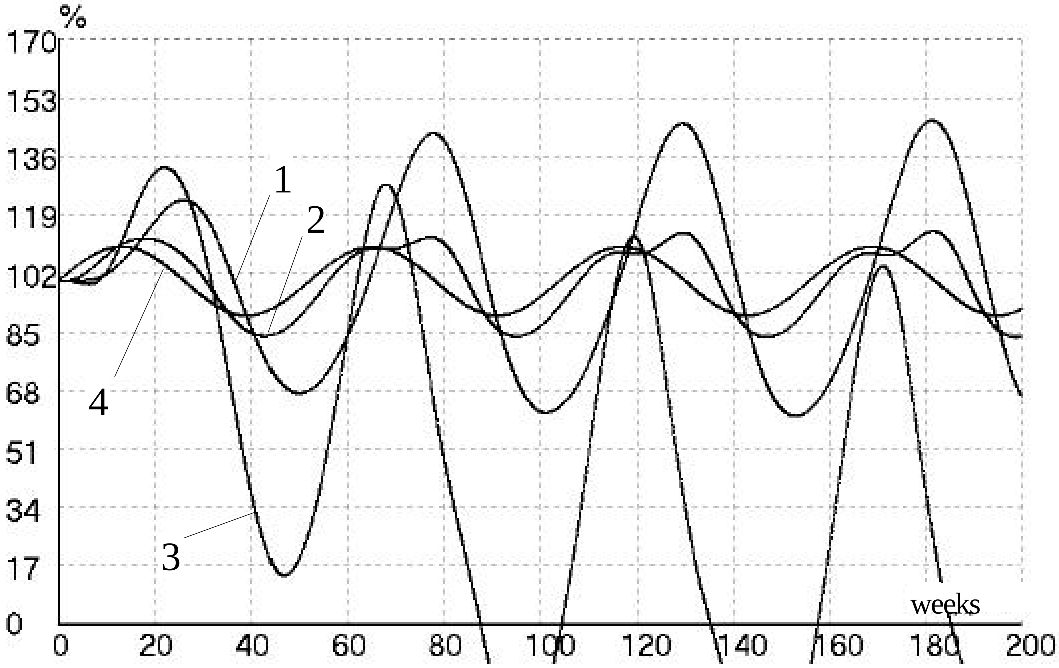
and matrixes

$$T_k^1 = Q + A' T_{k+1}^1\left(A-BK_k^1\right), \qquad T_k^2 = \left(A-BK_k^1\right)'\left(T_{k+1}^1\tilde{A}+T_{k+1}^2\right),$$

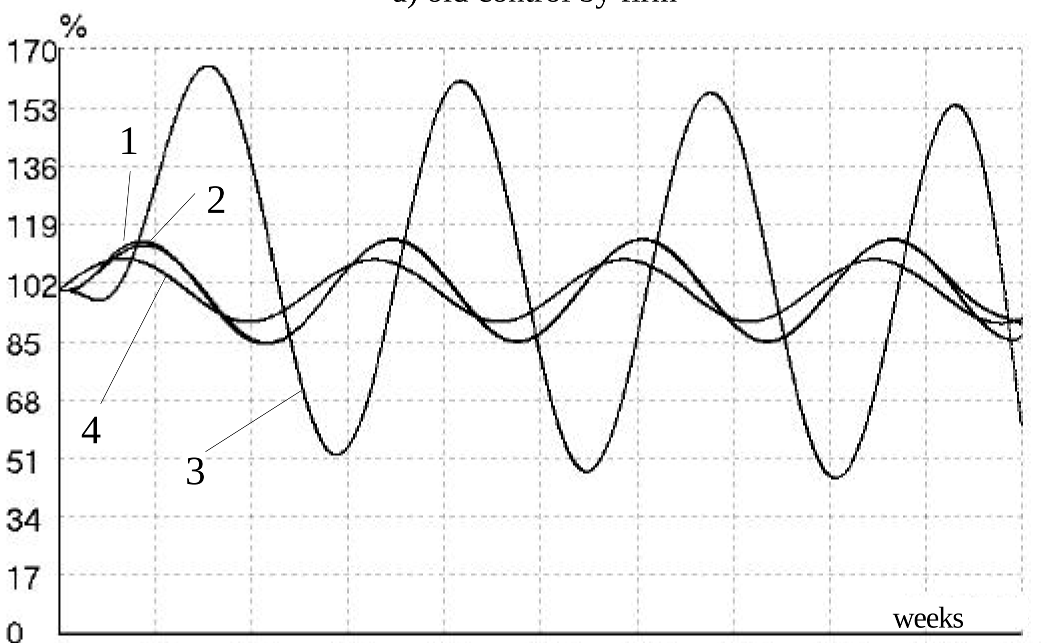
are solutions of the equations such as Riccatti with boundary conditions $T_N^1 = S$, $T_N^2 = 0$.

At unknown disturbances $w_k$ and absence of the information on exact current values of a firm state vector $x_k$ (i.e. at control in uncertainty conditions) in (39) it is necessary to use estimations of vectors $w_k$ and $x_k$. Below some ways of reception of such estimations will be stated.

At construction of optimal control (39) for firm (37) as parameters the optimal parameters received after its adaptation were taken. For studying behavior of firm at optimal control as weightings of components in quadratic criterion the following factors were taken: weighting of the profit $\lambda^0$ = 0.01; weighting of production costs $\lambda^1$ = 1; weighting of losses of marketing firm $\lambda^2$ = 1. The model digitization period was taken T = 0.05 weeks. Modeling was carried out for 200 weeks. On fig. 5 the behavior separate variable manufactures is shown at sine wave fluctuation of demand with amplitude of 10% and the period in 52 weeks (at initial value of demand $w_0^0$=1000 units per one week).




a) old control by firm




b) optimal control by firm

Fig. 5. Firm reaction at sine wave change of demand for production on 10% with the period of 52 weeks

1 – accounts for payment; 2 – accounts to reception; 3 – cash; 4 – demand for production

To have an opportunity to estimate quality of new control rules except for quadratic criterion (38) other criteria were counted up also: missed benefit $L_N$ (35), final profit of firm $x_N^{28}$ (25) and final dividends of shareholders $x_N^{25}$ (22). In tab. 2 values of criteria for optimal control (39) and for old control rules (2) are given. Values of all criteria at optimal control of firm (39) became better.

Table 2

The comparative table of criteria
at sine wave change of demand for production on 10% with the period of 52 weeks

| Criterion | Old control rules | New control rules | Change of (%) |
|---|---|---|---|
| The missed benefit (goods) | 577.89 | 518.07 | 10.35 |
| Profit (mon. units) | 1412677.63 | 1518080.42 | 7.46 |
| Dividends (mon. units) | 1483807.12 | 1546335.33 | 4.21 |
| Quadratic (mon. units$^2$) | 117476957272.75 | 9846474362.40 | 91.62 |

Any deterioration in any of three these criterions conducted and to deterioration of value of quadratic criterion. It speaks that the constructed quadratic criterion really may be used for an estimation of quality firm functioning.

**5. Estimation of firm parameters and its state in conditions of uncertainty.** The solution of optimal control problem and to decision making is preceded with a stage of definition of its current state. In firm state and dynamics of processes proceeding in it are characterized by parameters and state vector. Changes of a firm state are not predetermined. Therefore at construction of mathematical model uncertain sizes with which it is necessary to deal in practice are represented through the determined (known) and casual components. Estimation of firm is carried out in conditions of incomplete information.

In such situation it is possible representation of economic-mathematical model of firm in terms of state space and, at transform to linear model, application of Kalman filter (Brammer and Siffling, 1989) providing an unbiased state estimation with the minimal dispersion of error. Besides other approaches to the solution of a problem firm state estimation that are not supposing presence in model of casual components are possible.

The firm state $x_k$ is observed by information system. Measurements $y_k$ carried out with error $\xi_k$ act further in the estimation block where on the basis of the aprioristic information on the economic-mathematical model known with error $x_k$ a priori uncertain parameters are specified and carried out estimation of the current condition $\hat{x}_k$ firms with use Kalman filter for example. On a basis a posteriori information on a firm state is made a decision expressed in control influence $u_k$ on firm.

The equation of firm (1) can be presented as

$$x_{k+1} = A(p)x_k + f(x_k, p) + B(p)u_k + \Gamma(p)\xi_k, \quad k = 0, \ \dots, N-1, \tag{40}$$

where $x_k$, p, $u_k$, $\xi_k$ are vectors of a state, parameters, control and disturbances accordingly; $A(p), B(p), \Gamma(p)$ are matrixes; $f(x_k, p)$ is nonlinear vector function.

Vector of disturbances $\xi_k$ characterizes inexact knowledge of the firm equations. Concerning a vector $\xi_k$ various assumptions on the used approach to estimation may be made. For example at use of Kalman filter the assumption of its casual character is made. The vector of parameters p is supposed constant and unknown.

It is considered that to FIS are accessible to measurement only a part of variables and probably firm parameters. For example such state variable as orders which have been not executed by marketing firm ($x_k^{31}$). It is necessary to note that for firms at which the vector of parameters p is not a constant and if its instability renders essential influence on firm functioning knowledge of all coordinates of a vector p may justify expenses for FIS creation.

Measurements are made with errors. For example materials in channels of supply $x_k^{21}$ is measured with error $\eta_k^{21}$ and on FIS output we have

$$y_k^{21} = x_k^{21} + \eta_k^{21}, \; k = 0, \; \dots, \; N.$$

So let FIS work is described by the following equations

$$y_k = G\left[x_k, \; p\right]' + \eta_k, \; k = 0, \; \dots, \; N, \tag{41}$$

where G is a constant matrix; $y_k$, $\eta_k$ are vectors of measurements and measurements errors. As well as for a vector of disturbance $\xi_k$ interpretation of a vector of errors $\eta_k$ depends on the approach used for estimation of firm state and its parameters.

Matrix G in (41) determines what information on firm from all variable conditions $x_k$ and, probably, parameters p is accessible to observation. Its volume will differ strongly in cases if firm management is interested in reception of the information on a firm state, for example, and, on the other hand, any external observer wishing to receive more full than popular information on firm state. It is obvious that the observer estimating a state within of firm has more full information on its current state. In model it is expressed in increase of dimension of a vector $y_k$ in (41) in comparison with observation from the outside and, hence, in improvement of quality estimation.

Below the following approaches to the solution of a estimation of state and unknown firm parameters problem will be considered: based on use of algorithm of Kalman filter (Brammer and Siffling, 1989), the guaranteed approach (Kurzhanskii 1996) and also the minimax-stochastic approach (Shiryaev 1994.).

<u>5.1. Estimation by Kalman filter.</u> We shall consider all over again a case when firm parameters are known and there is a problem of reception of firm state estimation by results of the measurements made by FIS (41). We shall consider that nonlinearity influence in (40) can be neglected. Then the equations of firm and FIS can be presented as

$$x_{k+1} = Ax_k + Bu_k + \Gamma\xi_k, \; k = 0, \; \dots, \; N-1, \tag{42}$$

$$y_{k+1} = Gx_k + \eta_k, \; k = 0, \; \dots, \; N-1. \tag{43}$$

The initial condition $x_0$ is given with some error. $\xi_k$, $\eta_k$ are random variables with normal distribution, zero average and given covariance matrixes $Q_k$ and $R_k$ accordingly. The equation of Kalman filter looks like

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + K_{k+1}\left(y_{k+1} - G\hat{x}_k\right), \; k = 0, \; \dots, \; N-1, \tag{44}$$

where $K_k$ is optimum filter coefficient; $\hat{x}_k$ is an estimation of a firm state $x_k$. Coefficient $K_k$ is determined as

$$K_k = [A_{k-1}P_{k-1}A'_{k-1} + \tilde{A}Q_{k-1}\tilde{A}]G'_k[G_k(A_{k-1}P_{k-1}A'_{k-1} + \tilde{A}Q_{k-1}\tilde{A}')G'_k + R_k]^{-1}, \tag{45}$$

For a linear case (42) equations of firm $A_k = A$, $k = 0, ..., N$.

Estimation of a firm state occurs as follows. The initial estimation of a state vector $x_0$ and covariance matrix $P_0$ is set. Then the optimum filter coefficient $K_1$ (45) is calculated and on the basis of measurement $y_1$ the estimation $\hat{x}_1$ of state vector $x_1$ is received. On the given step of algorithm it is possible to judge estimation errors on the appropriate diagonal elements of matrix $P_1$. Further the algorithm repeats while errors estimation do not become small enough.

Lack of algorithm of Kalman filter for estimation of firm state is that the filter should be constructed on the basis of the aprioristic data about statistics of considered process that in this case corresponds to presence of the aprioristic information on statistical characteristics of vectors $\xi_k$ and $\eta_k$. The guaranteed approach (Kurzhanskii 1996) is relieved of the given lack.

In case of the nonlinear equations of firm expanded Kalman filter (Brammer and Siffling, 1989) is used. In this case the equations of firm can be presented as (40). Expanded Kalman filter uses the equations that are linearized concerning last estimation. In this case the equation for optimum filter coefficient $K_k$ also looks like (45). Matrix $A_k$ in this case is calculated as follows:

$$A_k = \left.\frac{\partial f}{\partial x}\right|_{x_k=\hat{x}_k}, \qquad k = 0, \dots, N, \tag{46}$$

where each element of this matrix represents value of a partial derivative an element i of the appropriate vector function on an element j of a state vector calculated at $x_k = \hat{x}_k$. After linearization of the equations are resulted in a kind (42). The equation of the filter (44) in this case looks like

$$\hat{x}_{k+1} = f\left(\hat{x}_k, p, u_k\right) + Bu_k + K_{k+1}\left(y_{k+1} - G\hat{x}_k\right), \qquad k = 0, \dots, N.$$

That besides state variable were calculated estimations of unknown parameters the state vector is supplemented with elements of a vector of parameters. In result we have a vector

$$X_k = [x_k,\ p]'.$$

which is named the expanded vector. Accordingly model (40) will be added to the following equations

$$p^i_{k+1} = p^i_k + \xi^{i+n}_k, \qquad i = 0, \dots, s,$$

where $\xi^{i+n}_k$ is drift of value parameter i; n is dimension of a vector $x_k$; s is number of firm parameters. The standard algorithm of expanded Kalman filter further is applied to a vector $X_k$.

Practice of application of Kalman filter for nonlinear systems shows that linearization (in Kalman filter) transforming it to a linear kind not always provides good approximation of initial model (fig. 6, curve 2). In other words values of state vector of firm model (40) and linear model (42) in due course differ more and more. It depends not only on a kind of the equations (40) but also is determined by dimension of system. Therefore it is meaningful to carry out decomposition of the given system on rather independent subsystems of smaller dimension and to solve estimation problem already in each of them. In separate subsystems flows (information, orders, material etc.) may be allocated both divisions of firm. Then the model of everyone subsystem i will be written down as

$$x^i_{k+1} = f^i\left(x^i_k, p^i, z^i_k\right) + \xi^i_k, \qquad y^i_{k+1} = g^i\left(x^i_{k+1}\right) + \eta^i_{k+1}, \quad k = 0,\ \dots,\ N-1,$$

where $z^i_k$ is a known vector of variables from other divisions.

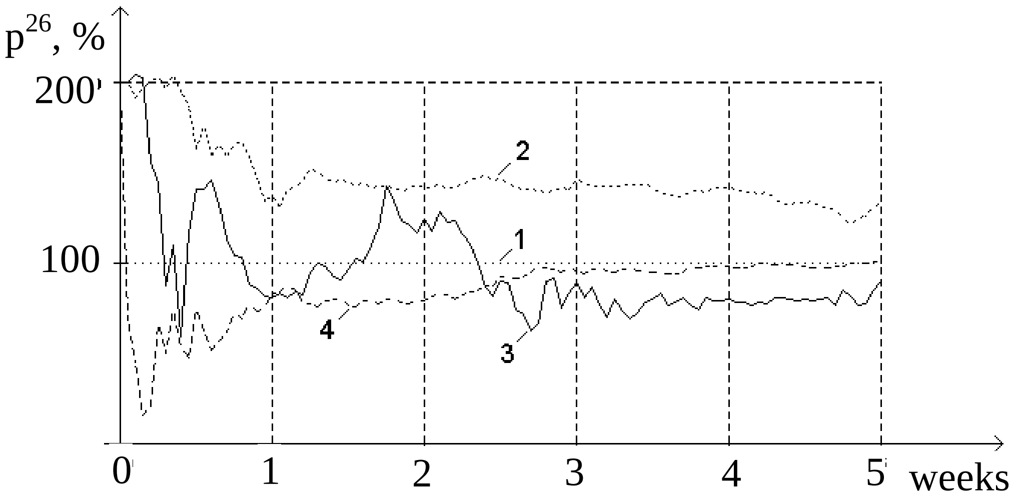



Fig. 6. Coefficient describing a desirable stock in marketing firm

As a result of a linearization error decrease that results in reception of more exact estimations of firm parameters (fig. 6, curve 3). If it is necessary to estimate parameters only parts of subsystems it is possible to not waste period and resources on estimation of firm parameters. But it is obvious that the best estimation results in the certain parameter are achieved if its value is accessible to measurement of FIS (fig. 6, a curve 4).

5.2. Application of the guaranteed approach for estimation of firm state. In the guaranteed approach (Kurzhanskii 1996) it is considered that a priori for firm model and FIS as (42), (43) the estimation of firm state $x_0$ as the given set $\hat{X}_0$ to which possesses value of this vector is known only $x_0 \in \hat{X}_0$. During measurements it is specified and the received guaranteed estimations of $\hat{X}_k$ are used for control. It is similarly considered that values of disturbances $\xi_k$ acting on firm and a vector of measurements errors $\eta_k$ belong to some known sets: $\xi_k \in W_k$, $\eta_k \in V_k$.

On the basis of the previous guaranteed estimation of vector of a firm state $\hat{X}_k$ and the accepted administrative decision $u_k$ the prediction locus $X_{k+1/k}$ of vector of a firm state $X_{k+1/k} = A\hat{X}_k + Bu_k + \tilde{A}W_k$, k = 0, …, N–1. Obviously all operations here are made above sets (linear transformations of sets and the sum of sets). The sum of sets is understood in Minkovskii sense: A+B={ñ: ñ=a+b, a∈A, b∈B}. By results of measurement $y_{k+1}$ the set of vectors $Y_{k+1}$ compatible to results of measurements is calculated

$$Y_{k+1} = \{x_{k+1} : y_{k+1} = Gx_{k+1} + V_{k+1}\}, \qquad k = 0, 1, …, N–1.$$

These two sets are used for calculation of the current guaranteed estimation of a firm state $\hat{X}_{k+1}$

$$\hat{X}_{k+1} = X_{k+1/k} \cap Y_{k+1}, \qquad k = 0, 1, …, N–1,$$

where $\cap$ is operation of sets crossing. If necessary for a point estimate of a vector $\hat{X}_{k+1}$ may be taken the Chebyshev center of this set (it is a point the maximal distance from which up to any point of set is minimal).

One of the most widespread ways of the definition of sets is their representation as convex polyhedrons. The polyhedron may be set either a vertex set or system of linear inequalities. In this case operations above sets do not leave a class of convex polyhedrons. Other widespread way of representation of sets is by means of ellipsoids. It is possible to write down an inequality specifying ellipsoids as

$$\left(Q^{-1}(x-a),(x-a)\right)\le 1,$$

where a is a n-dimensional vector (the ellipsoid center); Q is the symmetric positive defined matrix of dimension $n\times n$; brackets ( , ) designate scalar product of vectors.

Each of these ways has the merits and demerits. So approximation of sets by polyhedrons system of the big dimension results in the big number of vertex that demands increase of volume of operative memory but at the same period allows to specify an estimation beyond all bounds. This algorithm has essential advantages before algorithm of construction ellipsoid estimations as it enables at restrictions on computing capacities to use coarsened estimations of sets. All operations above polyhedrons result in polyhedrons that is not true for ellipsoids. Realization of the basic operations above approximating ellipsoid by ellipsoids is significant coarsened received results. However when ellipsoids approach provides acceptable results it is more preferable. On fig. 7 estimation errors for two state variables by results of 100 measurements are given. On the given interval appropriate to two days of real time the estimation errors has decreased as a minimum four times that is good result.

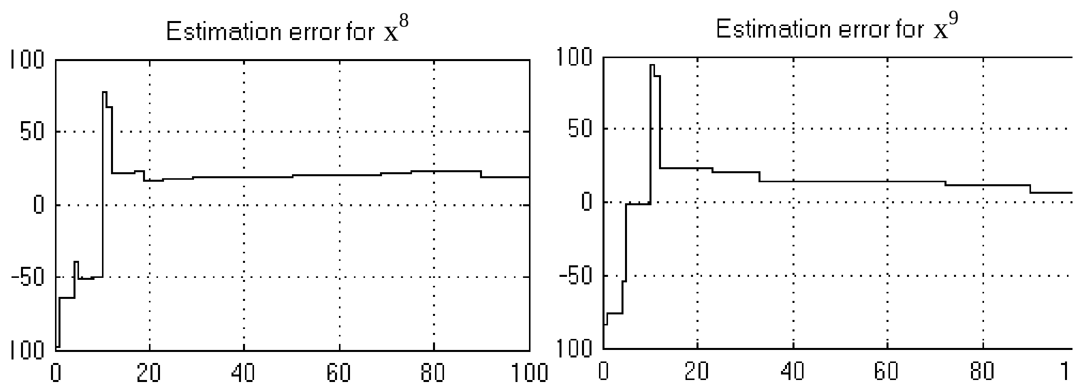



Fig. 7. Estimation errors

<u>5.3. Application of the minimax-stochastic approach for estimation of firm state.</u> In this case the aprioristic information on an initial firm state, disturbances and handicaps has the mixed character (Shiryaev 1994.), i.e. simultaneously there are both components with known distribution laws and components concerning which areas of their change are known only.

Let in linear approximation of the equation of firm and FIS look like

$$x_{k+1}=A_k x_k+B_k u_k+\tilde{A}_k w_k+\hat{\imath}_k,\; y_{k+1}=G_{k+1}x_{k+1}+H_{k+1}v_{k+1}+ç_{k+1},\qquad k=0,1,\ldots \quad (47)$$

Here $\hat{\imath}_k$, $\eta_k$ are independent normal casual vectors with zero average and known covariance matrixes $Q_k$, $R_k$, accordingly. Initial firm state $\overline{x}_0$, disturbances $w_k$ and handicaps $v_k$ may accept any value from given convex compact is known only

$$\overline{x}_0\in\overline{X}_0,\;\; w_k\in W_k,\;\; v_{k+1}\in V_{k+1},\;\; k=0,1,\ldots.$$

All matrixes in (47) are known an index k at them below falls for reduction of record. Initial value covariance matrix $P_0$ also is given.

At minimax estimation on measurements the information set is constructed

$$\overline{X}_k=\{\hat{x}_k=M[x_k\mid y_k(\cdot),\zeta_k(\cdot)]\in D_k\},$$

where $\zeta_k(\cdot)=\{\hat{x}_0, w_{k-1}(\cdot), v_k(\cdot)\}$ are the three from an initial condition sequences of disturbances $w_{k-1}(\cdot)=\{w_0,\dots w_{k-1}\}$ and handicaps $v_k(\cdot)=\{v_1,\dots v_k\}$;
$D_k=\{\zeta_k(\cdot): \bar{x}_0 \in \overline{X}_0,\ w_i \in W_i,\ v_{i+1} \in V_{i+1},\ i=0,1,\dots k-1\}$; $M[x_k \mid y_k(\cdot), \zeta_k(\cdot)]$ is the operator of a conditional expectation; $\hat{x}_k = M[x_k \mid y_k(\cdot), \zeta_k(\cdot)]$ is the optimum root-mean-square estimation calculated at fixed value $\zeta_k(\cdot)$.

For information set the estimation from above looks like

$$\overline{X}_{|k} = \tilde{X}_k + L_k,\ k=0,1,\dots,\ \tilde{X}_0 = \overline{X}_0,\ L_0 = 0 \tag{48}$$

where

$$\tilde{X}_k = \overline{A}_{k-1}\tilde{X}_{k-1} + \overline{\tilde{A}}_{k-1}W_{k-1} + \Lambda_k H_k(-V_k) \tag{49}$$

is the aprioristic information set and the sum of sets as well as in the guaranteed approach is understood in Minkovskii sense; a vector

$$L_k = \overline{A}_{k-1}L_{k-1} + \overline{B}_{k-1}u_{k-1} + \Lambda_k y_k \tag{50}$$

is determined by results of measurements $y_k$. Matrixes in (48), (49) look like
where I is an unitary matrix; $\Lambda_k = P_k G' R_k^{-1}$; a matrix $P_k$ is from Riccatti equation

$$P_k = M_k G'(GM_kG' + R_k)^{-1}GM_k,\ M_k = AP_{k-1}A' + Q_{k-1}\ .$$

At receipt of the next measurement $y_k$ vector $L_k$ is from (50) and information set $\overline{X}_{|k}$ turns out from (48). We shall note that if sets $D_k$ is convex polyhedrons sets $\tilde{X}_k$ also will be convex polyhedrons and a problem of construction of set $\tilde{X}_k$ may be shown to construction of a convex hull of the appropriate vertex set. Approximation of sets is possible also $\tilde{X}_k$ by ellipsoid. The sizes of set $\tilde{X}_k$ will determine errors estimation of vector $x_k$. To similarly guaranteed approach for a point estimation may be taken the Chebyshev center of this set.

* * * *

In the present paper the dynamic economic-mathematical model of firm including flows of orders, materials, a labor, and also the finance and accounts department is constructed. The analysis of change of demand for its production on behavior of firm is carried out.

The problem of firm adaptation to change of a market situation as a problem of parametrical optimization is formulated and solved. Influence on values of optimal firm parameters of the means restriction allocated on adaptation and changes of demand for production are investigated.

For the solution of optimal control problem by the firm at change of a market situation formulates integrated quadratic criterion of firm functioning efficiency reflecting its total losses at change of a market situation. The integrated criterion is divided on three parts to each of which there corresponds the weighting factor. The concrete set of weighting factors reflects structure of preferences of the persons accepting administrative decisions. By means of integrated criterion of efficiency the problem of optimal control by firm is formulated and solved. Thus control is received as feedback that makes its more flexible and sensitive to the processes proceeding inside firm and provides operative firm management at change of a market situation. Comparison of changes of other firm functioning efficiency criterions (the missed benefit, the profit and dividends of shareholders) with changes of integrated quadratic criterion shows that the constructed criterion really may be used as an estimation of a firm state.

The problem estimation of firm state and its parameters by means of firm information system in conditions of disturbances acting on firm and information system is solved. A number of approaches to the solution of estimation problem are described: estimation by Kalman filter (stochastic disturbances), application guaranteed (disturbances for which the range of change is known only) and minimax-stochastic approaches (disturbance of both kinds). Last approach is the most effective for the solution of estimation problem and considerably allows improving quality of accepted administrative decisions in conditions of uncertainty. Application of a method decomposition of mathematical model of firm according to its structural divisions has allowed lowering orders to computing resources at realization of algorithms in real period for nonlinear model of the big dimension.

The developed mathematical model, algorithms of adaptation, optimal control and estimation firm parameters and its state and the software may be used in CALS-technologies in the integrated firm control systems

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