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A Historical Fit of a Model of the US Long Wave

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Abstract

This paper refines a theoretical model of capitalist reproduction. A compact state-space form of this model defines a hypothetical Law of capital accumulation. The state variables are the unit wage, employment ratio, gross unit rent, man-made capital-output ratio, natural capital-output ratio, indicated natural capital-output ratio, and unit depreciation of the natural capital. An application of an extended Kalman filtering to the US macroeconomic data 1958-1991 identifies unobservable components of this Law. It is shown that long wave is a dominant non-equilibrium quasi-periodic behavioural pattern of the US capital accumulation. Evaluating the historical fit through appropriate summary statistics and long-range forecasting strengthen confidence in the model.

Keywords: Capital accumulation, long wave, extended Kalman filtering, forecasting

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Introduction

Analysts have been pondering whether the current global slowdown is just a correction of a brief period of overindulgence or a beginning of a long-term economic recession. It is reported that a former chief executive of Invensys, the UK-based engineering group, said (*The Financial Times*, July 25, 2001: 15): "I had been projecting a mild downturn in the US, which would have a limited impact on the rest of the world. In fact it has turned into the most serious recession…in the US past 30 years."

This paper asserts that the present US downturn is the beginning of the recession of the long wave. Taking as a point of departure (Ryzhenkov 2000), next sections present an upgraded system dynamics model that reflects these phenomena and enables futuristic projections.

The model assumptions

A capitalist economy is restricted by natural resources. Produced capital is an embodiments of knowledge and, similarly, natural capital is a stock of information. Fixed assets, labour and natural assets are essentially complementary to each other and are also substitutes to some degree depending on relative price changes. The other important premises are such:

(1) two social classes (capitalists and workers); the state enforces property rights, yet the cost of such an enforcement is not treated explicitly;

(2) three factors of production -- labour force, man-made capital, natural capital -- are homogenous and non-specific;

(3) only one aggregated good is produced for consumption, investment and circulation, its price is identically one;

(4) production (supply) equals effective demand;

(5) all wages consumed, the resource rent and a part of profits saved and invested;

(6) steady growth in the labour force that is necessarily not fully employed;

(7) a growth rate of the real wage rises in the neighbourhood of full employment;

(8) a change in capital intensity and technical progress are not separable due to a flow of invention and innovation over time;

(9) a qualification of the labour force corresponds to technological requirements.

This model abstracts from over-production of commodities inherent in over-production of capital during certain phases of industrial cycles. The assumption (5) corresponds to the immediate aim of capitalist production.

The assumption (6) means that the labour force grows exponentially over time. This assumption could be substituted by an assumption of an asymptotic growth or by another hypothesis.

The model equations

The model is formulated in continuous time. Time derivatives are denoted by a dot, while growth rates will be indicated by a hat. The model consists of the following equations:

$$P = K/s \tag{1}$$

$$a = P/L \tag{2}$$

$$u = w/a \tag{3}$$

$$\hat{a} = m_1 + m_2(\widehat{K/L}) + m_3 y(\hat{v}) + m_5 \widehat{F/L}, \tag{4}$$

where $y(\hat{v}) = \mathrm{SIGN}(\hat{v})\mathrm{ABS}(\hat{v})\wedge j$, $m_1 \geq 0$, $1 \geq m_2 \geq 0$, $m_3 \geq 0$, $m_5 \geq 0$, $0 < j$

$$(\widehat{K/L}) = n_1 + n_2 u + n_3(v - v_c) + n_5(Z/P), \tag{5}$$

$n_2 \geq 0$, $n_3 \geq 0$, $n_5 \geq 0$, $1 > v_c > 0$,

$$v = L/N \tag{6}$$

$$N = N_0 e^{nt}, \quad n = const \geq 0, \; N_0 > 0 \tag{7}$$

$$\hat{w} = -g + rv + b(\widehat{K/L}) + q\widehat{F/L}, \quad g \geq 0, \; r > 0 \tag{8}$$

$$P = C + \dot{K} + Y = wL + (1-k)M + \dot{K} + Y \tag{9}$$

$$\dot{F} = Y - Z \tag{10}$$

$$Z = eP, \quad 0 < e < 1 \tag{11}$$

$$\dot{y} = (o_1(c - f) + o_2\hat{f})y, \quad y = Y/P \geq 0 \tag{12}$$

$$\hat{X} = i \tag{13}$$

$$f = F/P \tag{14}$$

$$c = X/P \tag{15}$$

$$\hat{e} = \hat{P}(e_1/e - 1), \quad e \quad e_1 > 0 \tag{16}$$

$\dot{K} = kM = k[(1 - w/a)P - Y] = k[(1 - u)P - Y],$ (17)
where $0 < k \leq 1$.

Eq. (1) postulates a technical relation between the capital stock (K) and net output (P). The variable s is called produced or man-made capital-output ratio. Eq. (2) relates labour productivity (a), net output (P) and labour input or employment (L). Eq. (3) describes the shares of labour in net output (u).

Eq. (4) is an extended technical progress function. It includes: the rate of change of produced capital intensity, K/L, the direct scale effect, $m_3 y(\hat{v})$, and the rate of change of natural capital intensity, F/L. ABS(x) is absolute value of x that is non-negative, x^j is x raised to the j-th power, SIGN(x) is a sign of x. The parameter j has been randomised in a univariate sensitivity analysis (Ryzhenkov 2001).

Eq. (6) outlines the rate of employment (v) as a result of the buying and selling of labour-power. Labour force grows exponentially in (7). In the Eq. (8), the rate of change of the real wage (w) depends on the employment rate (v), as in the usual Phillips relation, and on the rates of change of capital intensity (K/L) and (F/L), additionally. The capital intensity (K/L) is a proxy for qualification.

In the Eq. (9), a sum of net export, final private and public consumption is $C = P[u + (1-k)(1- u - y)]$. The net formation of produced fixed capital is $\dot{K} = kM$, where K is man-made fixed assets. The gross accumulation of natural assets Y equals the gross resource rent in monetary (or information value) terms. Eq. (9) and Eq. (17) show that profit ($M = (1-u - y)P$) and incremental man-made capital ($\dot{K}$) are not equal in monetary (or information value) terms if the investment share $k < 1$. Considering the latter as a variable and reflecting the workers saving is left for a future research.

In the Eq. (10), $\dot{F}$ is a net accumulation (loss) of the natural capital (F). Z is the net environmental damage in the Eq. (11), i.e., depletion and degradation of non-produced natural assets (land, soil, landscape, eco-systems) due to economic uses above the regeneration rate.

The rate of change of capital intensity (K/L) in the Eq. (5) is a function of the unit wage (u), difference between real employment ratio and some base ('natural') magnitude ($v - v_c$), depletion/degradation of natural capital in relation to net output (Z/P). The rate of growth of capital intensity depends on the environmental damage per unit of output (an application of the principle 'a pollution prevention pays'), in particular. A high wage share and high employment ratio foster mechanisation (automation).

The indicated natural capital, X, may remain constant or change in the Eq. (13). The Eq. (12) defines an investment policy that is aimed to develop the natural capital in accordance with the indicated natural capital (y is the investment ratio for the natural capital). A combination of proportional and derivative control over the investment in natural capital is used hereby.

The stock of environmental assets is not treated explicitly in this model. The natural capital-output ratios -- real, f, and indicated, c, in Eq. (14) and Eq. (15) -- belong to the state variables of the model.

We assume that the unit depletion (degradation) of the natural capital asymptotically declines due to substitution and structural change as in (16) where for $\hat{P} > 0$, $\dot{e} < 0$. The higher the rate of economic growth, the faster the reduction of eco-intensity (or the promotion of eco-efficiency in the narrow meaning). The Eq. (16) is, likely, a better approximation than $e = \text{const} > 0$. An approximation of a higher order can be easily implemented in the future work.

Three profit rates are defined for this economy. The first is the *average* rate of return to man-made capital (1- u - y)/s. The second is a *general* one, it measures a ratio of the economic surplus to the total value of produced and natural capital (1 - u - e)/(s + f). The third is a *biased* profit rate (1- u)/s that is more easily calculated based on the statistics with incomplete data on the natural resources.

The rate of net rent is the ratio of net unit rent to natural capital – output ratio, (y - e)/f. The general rate of profit is a weighted average of the rate of return to man-made capital and the rate of net rent: (1 - u - e)/(s + f) = [s/(s + f)](1- u - y)/s +[f/(s + f)](y - e)/ f.

The model and law in a deterministic state space form

To get a compact model we need the following transformations.

$$\hat{s} = (K\hat{/}P) = (K\hat{/}L) - (P\hat{/}L)$$

$$= n_1 + n_2u + n_3(v - v_c) + n_5(Z/P) - (m_1 + m_2\, K\hat{/}L + m_3y\,(\hat{v}) + m_5F\hat{/}L)$$

$$= n_1 + n_2u + n_3(v - v_c) + n_5e - (m_1 + m_2(n_1 + n_2u + n_3(v - v_c) + n_5e) + m_3y\,(\hat{v}) + m_5(\hat{F} - \hat{L}))$$

$$= -m_1 + (1 - m_2)(n_1 + n_2u + n_3(v - v_c) + n_5e) - m_3y\,(\hat{v}) - m_5\,(\hat{F} - \hat{K} + n_1 + n_2u + n_3(v - v_c) + n_5e)$$

$$= -m_1 + (1 - m_2)(n_1 + n_2u + n_3(v - v_c) + n_5e) - m_3y\,(\hat{v}) - m_5(\hat{f} - \hat{s} + n_1 + n_2u + n_3(v - v_c) + n_5e)$$

$$= -m_1 + (1 - m_2 - m_5)(n_1 + n_2u + n_3(v - v_c) + n_5e) - m_3y\,(\hat{v}) - m_5(\hat{f} - \hat{s});$$

$$\hat{v} = (L\hat{/}N) = \hat{K} - (n_1 + n_2u + n_3(v - v_c) + n_5e) - n$$

$$= k\,\frac{1 - u - y}{s} - (n_1 + n_2u + n_3(v - v_c) + n_5e) - n;$$

$$\hat{u} = \hat{w} - \hat{a}$$

$$= -g + rv + b(n_1 + n_2u + n_3(v - v_c) + n_5e) + qF\hat{/}L - (m_1 + m_2(n_1 + n_2u + n_3(v - v_c) + n_5e) + m_3y\,(\hat{v}) + m_5(\hat{F} - \hat{L}))$$

$$= -g + rv - m_1 + (b - m_2 - m_5)(n_1 + n_2u + n_3(v - v_c) + n_5e) - m_3y\,(\hat{v}) - m_5(\hat{f} - \hat{s}) + q(\hat{f} - \hat{s} + n_1 + n_2u + n_3(v - v_c) + n_5e)$$

$$= -g + rv - m_1 + (b + q - m_2 - m_5)(n_1 + n_2u + n_3(v - v_c) + n_5e) - m_3y\,(\hat{v}) + (q - m_5)(\hat{f} - \hat{s});$$

$$\hat{f} = \hat{F} - \hat{P} = \frac{y - e}{f} - \hat{a} - \hat{L}$$

$$= \frac{(y - e)}{f} - (m_1 + m_2(n_1 + n_2u + n_3(v - v_c) + n_5e) + m_3y\,(\hat{v}) + m_5\,(\hat{F} - \hat{L})) - \hat{L}$$

$$= (1 - m_5)\,\frac{(y - e)}{f} - m_1 - m_2(n_1 + n_2u + n_3(v - v_c) + n_5e) - m_3y\,(\hat{v}) - (1 - m_5)\,(\hat{v} + n);$$

$$\hat{c} = \hat{X} - \hat{P} = i - \hat{K} + \hat{s} = i - k\frac{1 - u - y}{s} + \hat{s};$$

$$\hat{y} = (o_1(c - f) + o_2\hat{f});$$

$$\hat{e} = \hat{P}(e_1/e - 1) = (P\hat{/}L + \hat{v} + n)(e_1/e - 1) = (k\,\frac{1 - u - y}{s} + m_1 + (m_2 + m_5 - 1)(n_1 + n_2u + n_3(v - v_c) + n_5e) + m_3y\,(\hat{v}) + m_5(\hat{f} - \hat{s}))(e_1/e - 1).$$

In a compact form, the model consists of the seven non-linear ordinary differential equations (17) -- (23) that define the *hypothetical Law of capital accumulation*:

$$\dot{s} = -\frac{1}{(1-m_5)}(m_1 + (m_2 + m_5 - 1)(n_1 + n_2 u + n_3(v - v_c) + n_5 e) + m_3 y(\hat{v}) + m_5 \hat{f})s, \qquad (17)$$

$$\dot{v} = (k\frac{1-u-y}{s} - (n_1 + n_2 u + n_3(v - v_c) + n_5 e) - n)v, \qquad (18)$$

$$\dot{u} = (-g + rv - m_1 + (b + q - m_2 - m_5)(n_1 + n_2 u + n_3(v - v_c) + n_5 e) - m_3 y(\hat{v}) + (q - m_5)(\hat{f} - \hat{s}))u, \qquad (19)$$

$$\dot{f} = ((1 - m_5)\frac{(y-e)}{f} - m_1 - m_2(n_1 + n_2 u + n_3(v - v_c) + n_5 e) - m_3 y(\hat{v}) - (1 - m_5)(\hat{v} + n))f, \qquad (20)$$

$$\dot{c} = (i - k\frac{1-u-y}{s} + \hat{s})c, \qquad (21)$$

$$\dot{y} = (o_1(c - f) + o_2 \hat{f})y, \qquad (22)$$

$$\dot{e} = (k\frac{1-u-y}{s} + m_1 + (m_2 + m_5 - 1)(n_1 + n_2 u + n_3(v - v_c) + n_5 e) + m_3 y(\hat{v}) + m_5(\hat{f} - \hat{s}))(e_1 - e), \qquad (23)$$

where $s > 0$, $1 \geq v > 0$, $1 \geq u > 0$, $f > 0$, $c > 0$, $1 > y$, $1 > e$. The requirement for denominators to be positive is skipped. If $\dot{K} > 0, \dot{F} > 0$ for every instant of time, the system (17) -- (23) defines a *strongly sustainable development.*

A nontrivial stationary state is defined as

$$\mathrm{E_a} = (s_a, v_a, u_a, f_a, c_a, y_a, e_a), \qquad (24)$$

where

$$s_a = s_0,$$
$$v_a = (g + (1 - b - q)(d - n))/r,$$
$$u_a = (d - n - n_1 - n_3(v_a - v_c) - e_a n_5)/n_2,$$
$$f_a = (1 - u_a - e_a)/d - s_a/k,$$
$$c_a = f_a,$$
$$y_a = e_a + d f_a,$$
$$e_a = e_1,$$
$$i = d;$$

the positive s_0 is exogenous, y_a is the gross unit rent, e_a is the unit depreciation of the natural capital. This stationary state is not locally stable for reasonable parameters' values.

The man-made capital stock, natural capital and net output increase thereby at the rate $\hat{K}_a = \hat{F}_a = \hat{X}_a = \hat{P}_a = d = \frac{m_1}{1 - m_2 - m_5} + n > 0$ that is equal to the net rent rate and less than (for $k < 1$) or equal to the average profit rate (for $k = 1$). The stationary average profit rate equals $(1 - u_a - y_a)/s_a = d/k$. The both quantities ($d$ and d/k) are used as benchmarks in a long-range forecasting below.

An Extended Kalman Filtering (EKF)

The Kalman filter is a particular powerful tool for estimating unobservable part of a model (parameters and meta-parameters like variances) in one operation. Although the

Kalman filter itself does not estimate the unknown parameters of the model, it provides a one-step-ahead prediction error with its covariance matrix. The prediction error decomposition of the likelihood function utilises this information. Maximisation routines can be used to determine the unknown parameters (Cuthbertson et al. 1992: 210-225). The presentation of EKF below is an abridged version from (Peterson 1980) for a stationary white sequence with a time invariant covariance.

State equations:
$\mathbf{x}(n) = \mathbf{f}\,[\mathbf{x}(n\text{-}1)] + \mathbf{w}(n)$.
Measurement equations:
$\mathbf{z}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{v}(n)$.
Index of data samples:
$n = 1, 2, \ldots N$.
Initial conditions:
$\mathbf{x}(0) = \mathrm{N}[x_0, \Psi]^*$.
Equations errors (driving noise):
$\mathbf{w}(n) = \mathrm{N}[0, \mathbf{Q}]^*$.
Measurement errors:
$\mathbf{v}(n) = \mathrm{N}[0, \mathbf{R}]^*$.
Discrepancies $\mathbf{v}(n)$ and $\mathbf{w}(n)$ are modelled as zero mean uncorrelated random vectors with time invariant covariance matrices
$\mathrm{E}\{\, \mathbf{x}(n_1)\mathbf{v}'(n_2)\,\} = 0,\ n_1 \neq n_2$ and $n_1 = n_2$
$\mathrm{E}\{\, \mathbf{x}(n_1)\mathbf{w}'(n_2)\,\} = 0,\ n_1 \neq n_2$ and $n_1 = n_2$
$\mathrm{E}\{\, \mathbf{v}(n_1)\mathbf{v}'(n_2)\,\} = \mathbf{R},\ n_1 = n_2$
$\mathrm{E}\{\, \mathbf{v}(n_1)\mathbf{v}'(n_2)\,\} = 0,\ n_1 \neq n_2$
$\mathrm{E}\{\, \mathbf{w}(n_1)\mathbf{w}'(n_2)\,\} = \mathbf{Q},\ n_1 = n_2$
$\mathrm{E}\{\, \mathbf{w}(n_1)\mathbf{w}'(n_2)\,\} = 0,\ n_1 \neq n_2$.
Linearization about old estimate (estimated state):

$$\tilde{\mathbf{F}}(n) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x} = \hat{\mathbf{x}}(n-1|n-1)},$$

Filter Equations

Predicted state:
$\hat{\mathbf{x}}(n|n-1) = \mathbf{f}[\hat{\mathbf{x}}(n-1|n-1)]$.
Predicted measurement:
$\hat{\mathbf{z}}(n|n-1) = \mathbf{H}\hat{\mathbf{x}}(n|n-1)$.
Residuals (innovations):
$\delta_z(n|n-1) = \mathbf{z}(n) - \hat{\mathbf{z}}(n|n-1)$.
Predicted state covariance:
$\Sigma_x(n|n-1) = \tilde{\mathbf{F}}(n)\Sigma_x(n-1|n-1)\tilde{\mathbf{F}}'(n) + \mathbf{Q}$.

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* N[**m**, **c**] denotes a normal, white process with mean **m** and covariance matrix **c**.

Predicted measurement covariance:
$\Sigma_Z(n|n-1) = \mathbf{H}\Sigma_X(n|n-1)\mathbf{H}' + \mathbf{R}$.
Normalised predicted measurement residuals:
$\tilde{\delta}_Z(n|n-1) = \delta_Z(n|n-1)\sqrt{\Sigma_Z(n|n-1)^{-1}}$.
Updated state covariance:
$\Sigma_X(n|n) = [\Sigma_X{}^{-1}(n|n-1) + \mathbf{H}'\mathbf{R}^{-1}\mathbf{H}]^{-1}$.
Filter Gain:
$\mathbf{K}(n) = \Sigma_x(n|n-1)\mathbf{H}'\Sigma_z{}^{-1}(n|n-1)$.
The gain depends on the estimate $\hat{\mathbf{x}}(n-1, n-1)$ and thus cannot be pre-computed. The full equation has to be solved in real time.
Updated state estimate:
$\hat{\mathbf{x}}(n|n) = \hat{\mathbf{x}}(n|n-1) + \mathbf{K}(n)\delta_z(n|n-1)$.
Define
α: vector containing all the unknown parameters in **f**, **R**, **Q**, Ψ.
$p(\mathbf{z}_N : \alpha)$: probability density of $\mathbf{z}_N = (\mathbf{z}(1),\ldots, \mathbf{z}(N))$ for given α.
$\xi(N : \alpha) = \ln p(\mathbf{z}_N : \alpha)$: log likelihood function (Schweppe 1973: 434).
$\hat{\alpha}(N)$ = value of α which maximises $\xi(N : \alpha)$ for particular $\mathbf{z}_N$. Thus $\hat{\alpha}(N)$ is the maximum likelihood estimate given the observations $\mathbf{z}(1),\ldots, \mathbf{z}(N)$.
Log likelihood is defined recursively:
$\xi(n) = \xi(n-1) - \frac{1}{2}\tilde{\delta}'_z(n|n-1)\tilde{\delta}_z(n|n-1) - \frac{1}{2}\ln|\Sigma_z(n|n-1)|$.
Initial conditions:
$\hat{\mathbf{x}}(0|0) = \mathbf{x}_0$,
$\Sigma_X(0|0) = \Psi$,
$\xi(0) = 0$.
The matrices **R** , **Q,** Ψ are positively (semi)-definite.

The basic *one-step* prediction problem is to calculate $\hat{\mathbf{x}}(n|n-1)$, the estimate of $\mathbf{x}(n)$, using $\mathbf{z}(1),\ldots, \mathbf{z}(n-1)$. The Vensim output contains a file 1Step.err that equals $-\delta_z(n|n-1) = \hat{\mathbf{z}}(n|n-1) - \mathbf{z}(n)$. For the my stochastic model in the state space form, N = 34, dim $\mathbf{z}(n)$ = 7, dim $\mathbf{x}(n)$ = 8, the indicated natural capital-output ratio, c, is not observable.

Confidence Tests from the Optimal Filter

The whiteness of $\delta_z(n|n-1)$ is sometimes viewed as a basic property of an optimum estimator. It means the *correction* $\delta_z(n|n-1)$ made using $\mathbf{z}(n)$ cannot be predicted.

A statistical test for whiteness of the prediction residuals using an estimator based on the EKF belongs to the class of model behaviour tests. The EKF-based estimator does involve a model simulating in computing a likelihood function. The VENSIM professional soft-ware has served for performing such an extended Kalman filtering (EKF) in this paper below.

It is argued (Peterson 1980) that the whiteness property of residuals (innovations) provides an independent test of model validity since is not employed directly in maximising the log likelihood. The whiteness may be tested by computing correlation measures of the residuals. Each residual (in the case of multiple-dimensional measurements) should have a correlation coefficient of one with respect to itself, zero correlation with respect to lagged value of itself, zero cross correlation with all other residual processes.

Examination of the two basic assumptions of the extended Kalman filtering (EKF)

Two crucial initial assumptions of the EKF are the conjectures that

- there are no structural errors in the model;
- the "disturbance terms" in dynamic and measurement equations are random variables.

The EKF is usually supported and lightened by auxiliary assumptions on a simple distribution law imposed on input and measurement "disturbance terms" as random variables, for example, in a form of the Gaussian multivariate normal distribution. The EKF is maintained by another subordinate assumption that the discrepancy terms are not only random variables, but even random variables of the special kind called white noise.

It has been already shown in the literature (Blatt 1983: 339) that "in all actual models, specification errors are the price one must pay for having any sort of workable model at all." *Truncation* error, *lag structure* error, *aggregation* error, *omitted variable* error, *unknown variable* error belongs to the mostly important specification errors that are practically unavoidable (Ibid., 342).

The residuals from the mis-specified relationships are not due entirely, or largely, to pure random influences. On the contrary, these residuals contain a highly systematic, non-random component while data limitations make it impossible to eliminate these errors.

Typically, a process of maximising the log likelihood function by a hill-climbing algorithm with random multiple starts cannot be finished. This process should be normally terminated by a researcher at a simulation run relying not only on logic but intuition as well. Therefore to find a genuine optimal solution is hardly possible in practice. Linearization of models' equations, integration technique, rounding-off contributes to the systematic errors additionally. So besides specification errors, *identification* errors in the parameters values are unavoidable.

In macroeconomic applications, the input $\mathbf{w}(n)$ and measurement disturbances $\mathbf{v}(n)$ are not white processes but rather *structural* processes; i.e., they are time-correlated discrepancies. For example, the shorter quasi-cycles are the most important factor of autocorrelation of these discrepancies in the model of long wave that abstracts from other fluctuations in its deterministic part. Of course, we cannot exclude quasi-periodic patterns longer than the long wave relying on the available data.

The model discrepancies, termed in the literature as disturbances, have uncertain values, their mean values can change over time. It has been pointed out (Barlas 1989: 60) that 'noise terms of system dynamic models are not necessarily independent and /or normally distributed, behaviour patterns generated by typical system dynamic models are highly auto-correlated, and they are in general non-stationary in the mean and sometimes non-stationary in variance. Such characteristics of system dynamic models violate three fundamental assumptions of standard statistical tests: normality, independence and stationarity. Therefore, standard hypothesis tests (such as *t*-test, F-test, χ^2 – test) are not directly applicable to system dynamic behaviour validation… "

The presence of non-white (time-correlated) disturbances, errors of specification and identification makes passing the test for whiteness essentially impossible for a system dynamic model of a macroeconomic system based on real data.

Pre-whitening of the noise and dealing with non-random input and measurement errors through augmentation of a model is recommended for physical applications. Unfortunately these procedures are either not warranted in macroeconomics by available data or are to be continued *ad infinitum* in view of non-linear interactions of multiple quasi-periodic processes rooted in circulation and turnover of the social and individual capital.

The EKF, based on abstraction and idealisation, is inconsistent to some degree with macroeconomic data. Specification and identification errors, non-randomness and/or non-whiteness of discrepancies *exclude* whiteness of residuals in practical macroeconomic applications. Still parameters of the stochastic dynamic model, expected values of the system states, residuals and their covariance matrices can be roughly estimated by the EKF. Revealing a time-pattern in residuals could give additional insights into actual regularities thereby. If the test for whiteness of residuals cannot be passed fundamentally, a researcher is to apply other validity and consistency tests that are more appropriate.

A Macroeconomic Application of the Extended Kalman Filtering

The compiled macroeconomic data in the Table 1 cover the period 1958-1999. The information on natural capital relates to a significant part of natural capital, namely the stocks of proved mineral reserves for 1958-1991.

It is known that the likelihood computation is exact only in the case of linear systems with Gaussian noise for driving and measurement errors (Peterson 1980). This paper reports only about quasi-optimal estimates obtained so far for the presented non-linear model.

The data from the Table 1 feed the model for a shorter period 1958-91. It is assumed that there are high measurement errors and great uncertainty about the initial system state. The famous book (Morgenstern 1963) has prompted this assumption**.** Of course, neither Morgenstern's inquiry nor our previous discussion suggests that errors in the economic statistics are purely random. In fact, the both types of errors contain quasi-cyclical components related to fluctuations that are not treated explicitly.

The dozens of marathon runs culminated in the final optimisation output (File 1). In this macroeconomic application of EKF, we consider the economic subjects' measurement errors (variances associated with the data stream they face) as the variables' weights in the pay-off definition file *pay-off.vpd*. The filter control file specifies a covariance matrix for a driving noise and initial covariance of the state vector (*kalman.prm* in the Appendix).

EKF realised in the VENSIM software has enabled to estimate the unobservable components of the compact model (17) – (23) and of an additional equation for the level of labour productivity, a = INTEG (a , a_0), after this model has been transformed into the canonical stochastic form. The number of measured levels is seven since the indicated natural capital (c) is not observable directly.

The given estimates of the model parameters and meta-parameters are not as exact as required by optimal filtering. Still it may be interpreted an advantage rather than a drawback if we recall that the economic application of the optimal filtering assumes that learning agents use the given information optimally. It is clearly a very strong idealisation for the world of bounded rationality. Without this idealisation the identification is even more complicated. It is a matter of combined theoretical and empirical research to find out how efficiently the information is really used by the economic agents. If it can be proved than a quasi-optimal usage of available information is more likely than optimal, then the common pragmatic usage of quasi-optimal estimates (in this paper, in particular) is justified theoretically.

It is not excluded that economic subjects could learn the system states from their own practice with a higher precision than the precision of the statistics that mirror these states. So magnitudes of (co-)variances in a lower range are relevant for other computing experiments outside the current presentation. Smaller weights in the definition of the EKF pay-off would be especially reasonable if the quality of the data could be improved.

The compact model with the identified quasi-optimal parameters is run for a long term forecast. This forecast projects the internal tendencies of capital accumulations into the XXI century. Our forecasting uses no real data after 1991.

Scweppe wrote (Scweppe 1973: 254-5): “Pattern recognition can be viewed as a special case of hypothesis testing. In pattern recognition, an observation **z** is to be used to decide what pattern caused it. Each possible pattern can be viewed as one hypothesis. The main problem in pattern recognition is the development of models for the **z** corresponding to each pattern (hypothesis).” Our main hypothesis is that the real economy exhibits the long wave as a dominant quasi-cycle. A subordinate hypothesis is that the residuals are not white due to other quasi-cycles and non-random factors.

A Weak Behaviour Reproduction Test

According to (Sterman 1984: 52), “a historical fit of a model is a weak test… it is nonetheless an extremely important one.” My model focuses on long-run cyclical growth and explicitly excludes the business cycle in its original deterministic form. Driving and measurement noise is the only exogenous input in the final non-linear stochastic model. The behaviour of this stochastic model and its ability to replicate historical data, to capture the long wave and its turning points are mostly the result of interaction of the endogenous variables.

I have simulated the model for 1958-1991 with the Time Step of one year. This simulation enables a comparison of the model output to the observed behaviour of the economy over the all 34 years. A simulation with the Time Step of 0.588893 (*File 1.out*) calculates estimated states only at 29 time points and is omitted here therefore.

As expected, the residuals are not white and auto-correlated. Figure 1 shows the auto-correlation plot for the variable u. The x-axis on this plot represents the number of lags, where a lag is one SAVEPER (here one year). The X-axis starts at 0 where, by definition, the auto-correlation is 1. On the Figure 1, the auto-correlation exceeds 0.65 for the lag of one year. There is also a high correlation between residuals for $a(t)$ and data of the employment ratio with the lag of one year $v(t\text{-}1)$ as seen on the Figure 5.

The residuals are not due entirely, or largely, to pure random influences. On the contrary, these residuals contain highly systematic, non-random components. The predicted residuals are plotted against the independent variable on Figures 2-4. The subscript A stands for the actual data, S – for simulated. The dynamics of the residuals, i.e., one-step-ahead prediction errors, are determined at least partially by the shorter quasi-periodic growth cycle(s), as expected. Thus although our model does not ‘pass’ the test for whiteness of residuals, this ‘failure’ supports empirically long wave as the behavioural *pattern*.

Tables 2 and 3, Figures 6 – 19 report on other aspects of the error analysis for the seven variables. Our calculations use the Theil inequality statistics (Theil 1966, Sterman

1984). The root-mean-square percent error (RMSPE) provides a normalised measure of the magnitude of the error:

$$\sqrt{\frac{1}{n}\sum_{t=1}^{n}[(S_t - A_t)/A_t]^2} \ ,$$

where S_t and A_t are simulated and actual quantities, respectively.

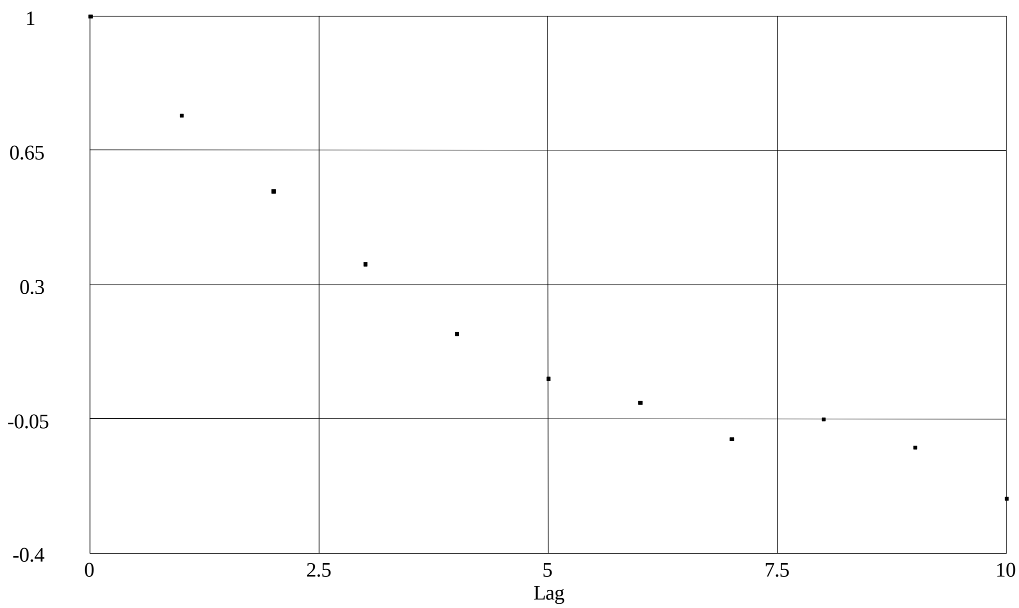

Figure 1. Auto-correlation for the residuals for *u*

The mean-square-error (MSE) and inequality statistics provide a measure of the total error and how it breaks down into bias, unequal variation, and unequal covariation components:

$$\frac{1}{n}\sum_{t=1}^{n}(S_t - A_t)^2 = (\bar{S} - \bar{A})^2 + (S_S - S_A)^2 + 2(1-r)S_S S_A$$

where $\bar{S}$ and $\bar{A}$ are means of *S* and *A*, i.e.

$\bar{S} = \frac{1}{n}\sum S_t$ and

$\bar{A} = \frac{1}{n}\sum A_t$, respectively;

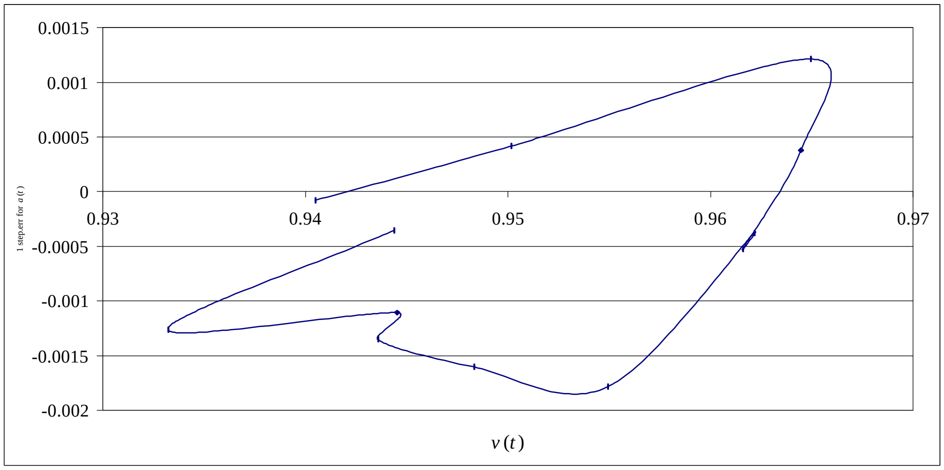

Figure 2. $v_A(t-1)$ versus the VENSIM 1 step.err for $a(t)$, 1961-1972. Clockwise

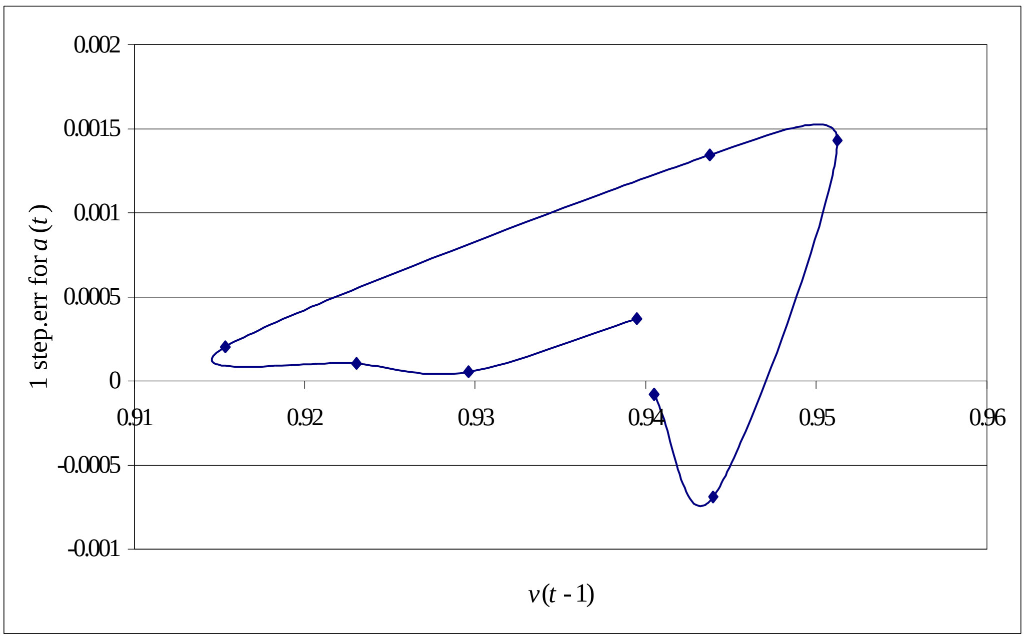

Figure 3. $v_A(t-1)$ versus the VENSIM 1 step.err for $a(t)$, 1972-1978. Clockwise

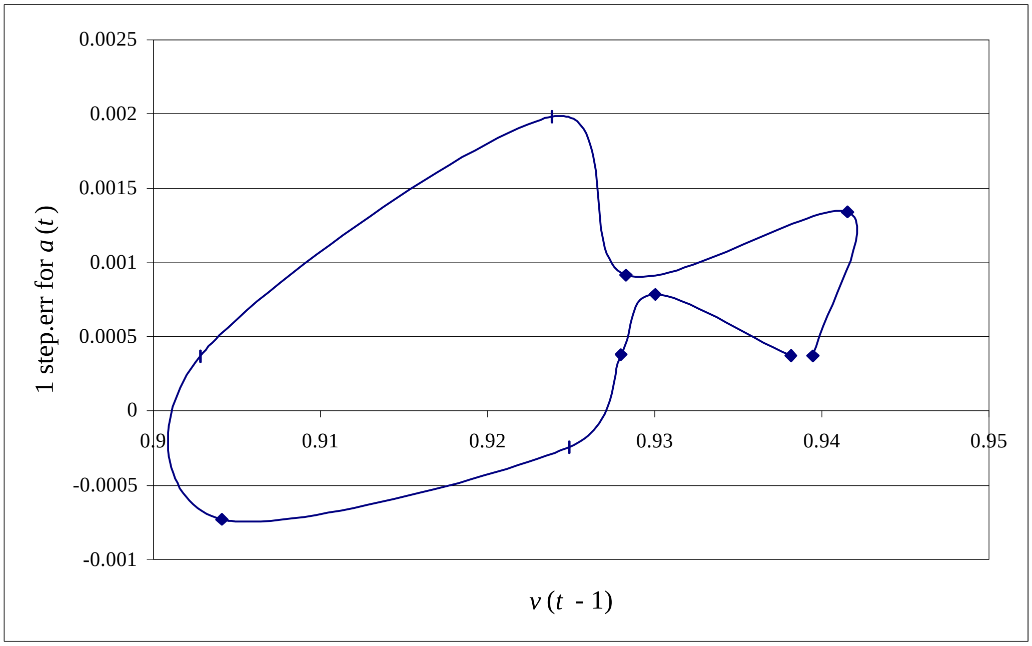

Figure 4. $v_A(t-1)$ versus the VENSIM 1 step.err for $a(t)$, 1979-1987. Clockwise

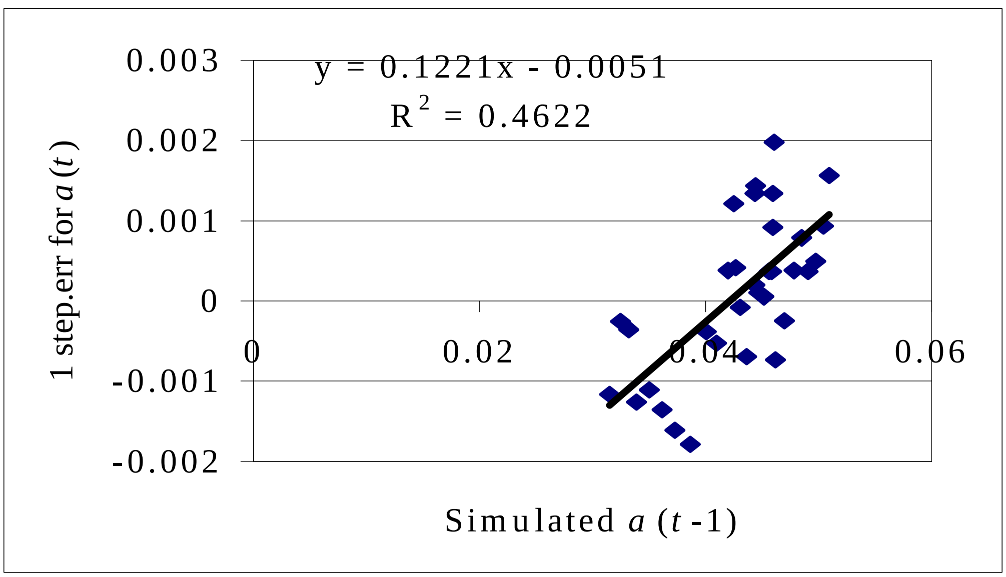

Figure 5. $a_S(t-1)$ versus 1 step.err for $a(t)$, 1959-1991

S_S and S_A equal the standard deviations of S and A, i.e.,

$S_S = \sqrt{\frac{1}{n}\sum(S_t - \bar{S})^2}$ and

$S_A = \sqrt{\frac{1}{n}\sum (A_t - \bar{A})^2}$, respectively.

The term $(\bar{S} - \bar{A})^2$ measures the bias between simulated and actual series. The term $(S_S - S_A)^2$ is the component of the MSE due to a difference in the variances of simulated and actual series, and measures the degree of unequal variation between the two series. Finally, the term $2(1-r)S_S S_A$ is the component of the error due to incomplete covariation between the two series.

By dividing each of the components of the error by the total MSE, the "inequality proportions" are derived:

$$U^M = \frac{(\bar{S} - \bar{A})^2}{\frac{1}{n}\sum (S_t - A_t)^2},$$

$$U^S = \frac{(S_S - S_A)^2}{\frac{1}{n}\sum (S_t - A_t)^2},$$

$$U^C = \frac{2(1-r)S_S S_A}{\frac{1}{n}\sum (S_t - A_t)^2}.$$

By the definition of the correlation coefficient

$r = \frac{COV(S, A)}{S_S S_A}$, where the covariance

$$COV(S, A) = \frac{1}{n}\sum [(S_t - \bar{S})(A_t - \bar{A})]$$

$= \frac{1}{n}\sum (S_t A_t) - \bar{S}\ \bar{A}$ (Sterman 1984: 63).

Of course, $U^M + U^S + U^C = 1$, so U^M ,U^S , and U^C reflect the fraction of the MSE due to bias, unequal variance, and unequal covariance, respectively.

To compare the means of the simulated and observed behaviour patters, we compute the *percent error in the means* E1 as

E1= $\frac{|\bar{S} - \bar{A}|}{|\bar{A}|}$.

This quantity tells how large is a discrepancy between the means.

Similarly, the *percent error in the variations* is calculated as

E2= $\frac{|S_S - S_A|}{|S_A|}$.

In addition to the sample estimates E1 and E2, the amplitudes of the real and simulated quasi-cycles will be measured and compared (Table 3). A summary measure of overall behaviour discrepancy is finally the *discrepancy coefficient*, a version of Henry Theil's 'inequality coefficient':

$$U = \frac{\sqrt{\sum (S_i - \bar{S} - A_i + \bar{A})^2}}{\sqrt{\sum (A_i - \bar{A})^2} + \sqrt{\sum (S_i - \bar{S})^2}}$$

$$= \frac{S_E}{S_A + S_S}$$

which is insensitive to additive constants being between zero ('perfect predictions') and one ('worst predictions').

Table 2. Error Analysis of the Model for 1958-91

Variable	RMSPE (%)	MSE (units^2)	U^M	U^S	U^C	S_A	S_S
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
a	1.37	3.31E-07	0.010	0.304	0.686	0.005	0.005
e	7.13	7.91E-07	0.000	0.533	0.467	0.003	0.002
f	3.95	6.16E-05	0.001	0.059	0.940	0.076	0.078
s	3.65	0.006	0.026	0.156	0.818	0.149	0.118
u	1.89	0.000180	0.013	0.175	0.812	0.010	0.016
v	1.19	0.000123	0.045	0.006	0.949	0.015	0.016
y	28.76	2.83E-05	0.000	0.400	0.600	0.008	0.004

Totals of the columns (4)-(6) may not add exactly to 1 due to rounding.

Table 3. Indicators of Discrepancy for 1958-91

Vari-able	Min		Max		Range		E1	E2	*U*
	real	simu-lated	real	simu-lated	real	simu-lated			
a	0.031	0.031	0.050	0.051	0.019	0.020	0.001	0.062	0.054
e	0.008	0.009	0.018	0.018	0.01	0.01	2E-04	0.211	0.161
f	0.119	0.109	0.375	0.375	0.256	0.266	0.001	0.025	0.051
s	1.845	1.910	2.417	2.297	0.573	0.387	0.006	0.208	0.289
u	0.685	0.683	0.731	0.728	0.047	0.045	0.002	0.564	0.523
v	0.903	0.919	0.965	0.969	0.062	0.050	0.003	0.056	0.352
y	0.006	0.008	0.046	0.023	0.041	0.014	0.001	0.435	0.439

The RMSPE of *s* is 3.65%. Three variables, *u*, *v*, *a* have the RMSPE under 2%. Exceptionally, the RMSPE of *e* is 7.13%, the RMSPE of *y* is 28.76%, whereas the RMSPE for *f* is only 3.95%.

While the small total errors in most variables show the model satisfactorily tracks the major variables, the two larger errors of *e* and *y* might raise questions about the internal consistency of the model or the structure controlling those variables. A preliminary conclusion is that the model underrates changes of *y*, *e* but represents well the dynamic of the natural capital-output ratio *f*.

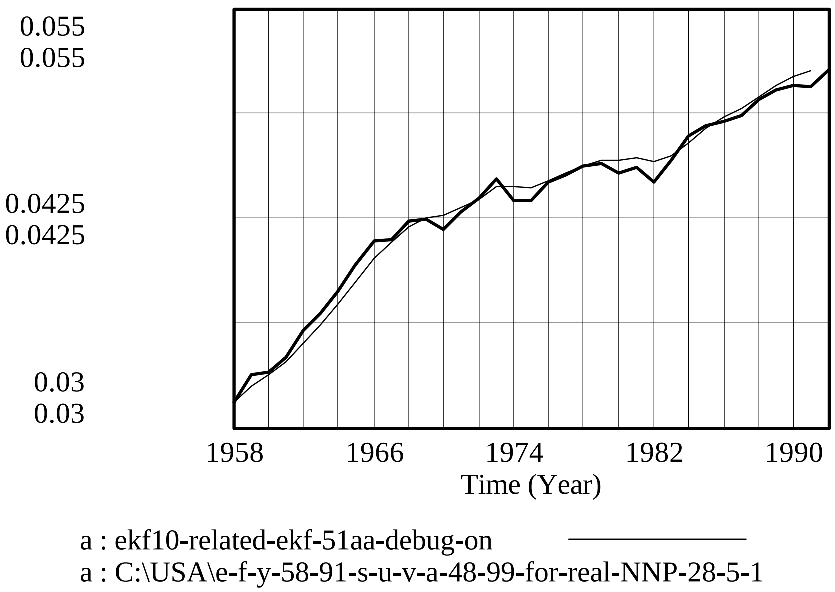

Figure 6. a_A (thick) versus a_S (thin), 1958-1991

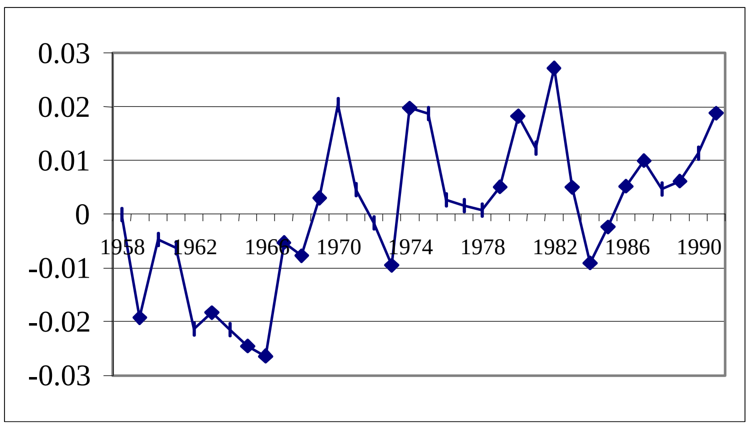

Figure 7. $\dfrac{a_S - a_A}{a_A}$, 1958-1991

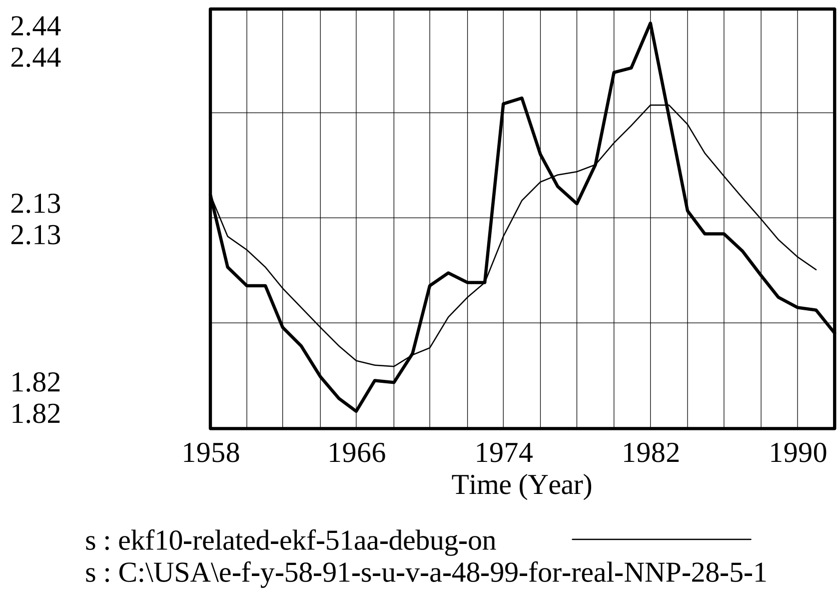

Figure 8. s_A(thick) versus s_S (thin), 1958-1991

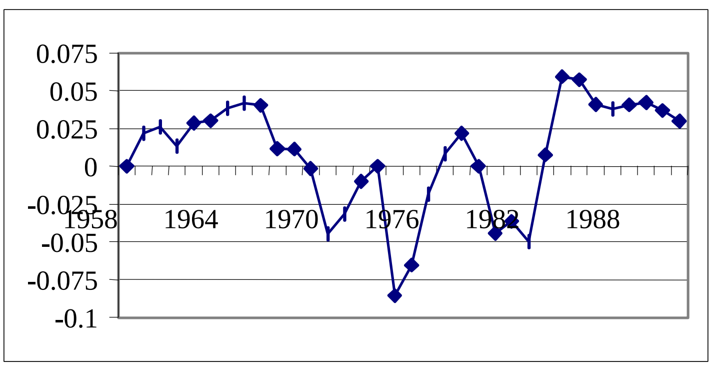

Figure 9. $\dfrac{s_S - s_A}{s_A}$, 1958-1991

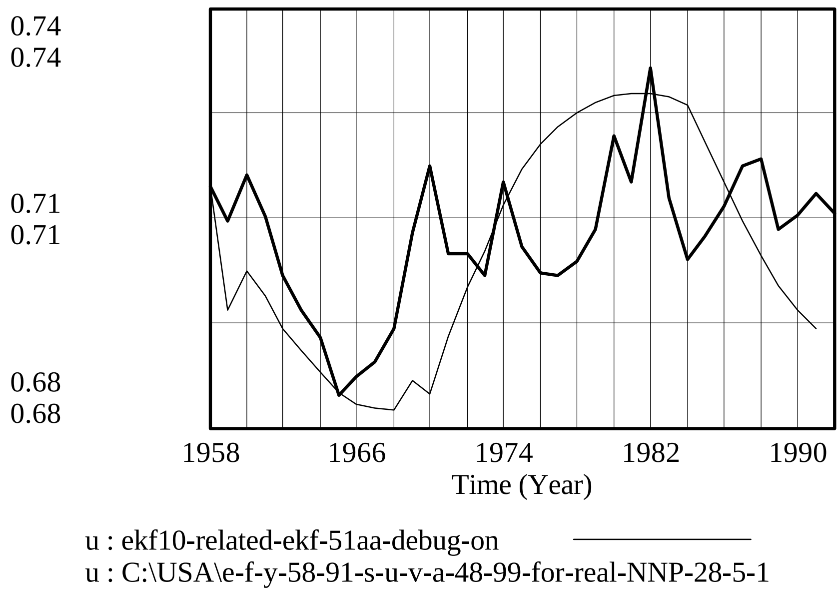

Figure 10. u_A (thick) versus u_S (thin), 1958-1991

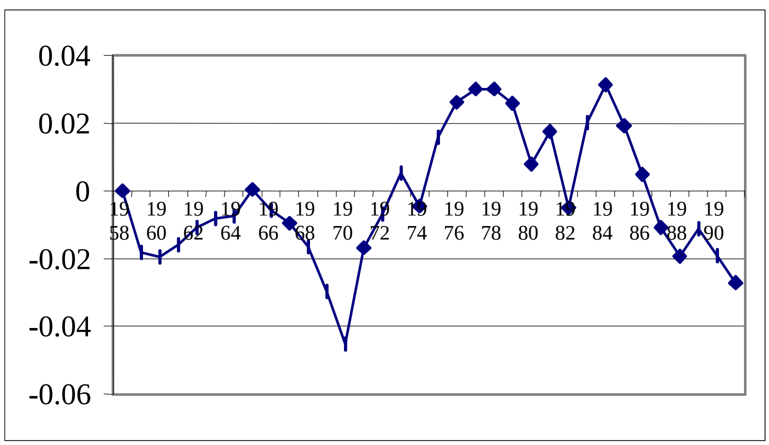

Figure 11. $\frac{u_S - u_A}{u_A}$, 1958-1991

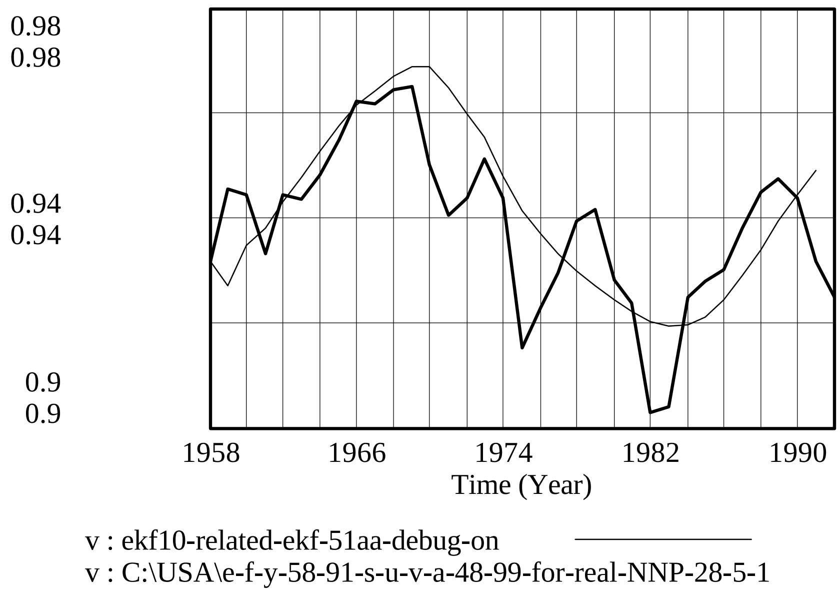

Figure 12. v_A (thick) versus v_S (thin), 1958-1991

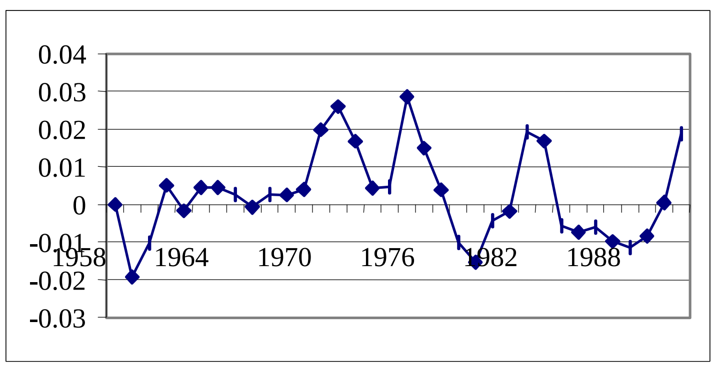

Figure 13. $\frac{v_S - v_A}{v_A}$, 1958-1991

The error decomposition gives additional insights. Consider *e*. Plotting the two series and their residuals (Figures 14 and 15) shows that they diverge point-by-point. It causes unequal covariation. However the tracking of declining *e* is not very bad. The bias is negligible, unequal variation (nearly 53%) accounts for approximately one half of the total while unequal covariation (nearly 47%) accounts roughly for the other half. More importantly, actual *e* fluctuates with the shorter quasi-cycle(s) that are not captured by the model.

The largest RMS percent error, nearly 29%, shows up in the *y*. Still the bias is negligible. The model reflects the declining tendency of *y* rather well. Error decomposition shows the majority of MSE to be due to the unequal covariance (60%) and unequal variance (40%). Additions (*Y*) to the stocks of subsoil minerals (*F*) experience more violent changes than the stock itself and depletion (*Z*). Proven reserves quantities sometimes change dramatically because previously uncertain reserves are found to be economic (e.g., Alaskan oil in 1970). The experts conclude (Nordhaus and Kokkelenberg

1999: 86): “The basic problem is valuing non-reserve resources. BEA intends ultimately to include unproved resources as a part of non-produced environmental assets.”

As pointed out by the experts, the reported BEA’s initial estimates of the natural capital should be considered preliminary and tentative at this time (Nordhaus and Kokkelenberg 1999: 168). In the absence of observable market prices for reserves, BEA estimated mineral reserve and flow values using five valuation methods. BEA’s results show clearly the potential margin for error among different techniques, for they give widely different estimates. In some cases, the net change in the value of reserves even has a different sign under different valuation techniques (Table 4). Therefore the data on *y*, *e* and *f* are suffering from particularly high measurement errors.

Table 4. A Fragment of the IEESA Asset Account, 1987, for Subsoil Assets, the Highs and Lows of the Range Based on Alternative Valuation Methods

Opening stocks, *F*(*t*-1)	Change				Closing stocks, *F*(*t*)
	Total, net	Depletion, degradation, *Z*(*t*)	Capital formation, *Y*(*t*)	Revaluation and other changes	
(1)	(2) = (3)+(4)+(5)	(3)	(4)	(5) = (6)-(1)+(3)-(4)	(6)
270.0↔1067	57.8↔-116.6	-16.7↔-61.6	16.6↔64.6	58↔-119.6	299.4↔950.3

Notice opposite signs in column 2, 5.
Source: *Survey of Current Business*, April 1994: 41.

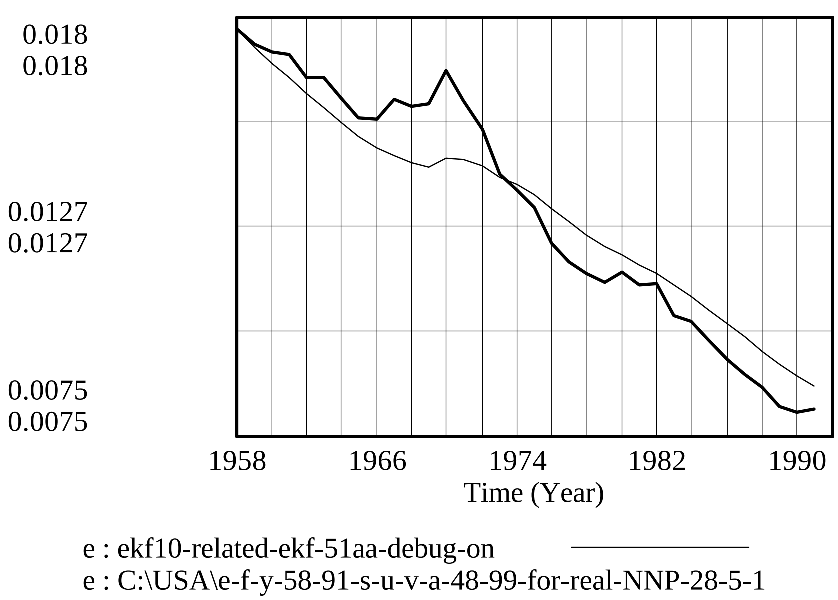

Figure 14. e_A(thick) versus e_S (thin), 1958-1991

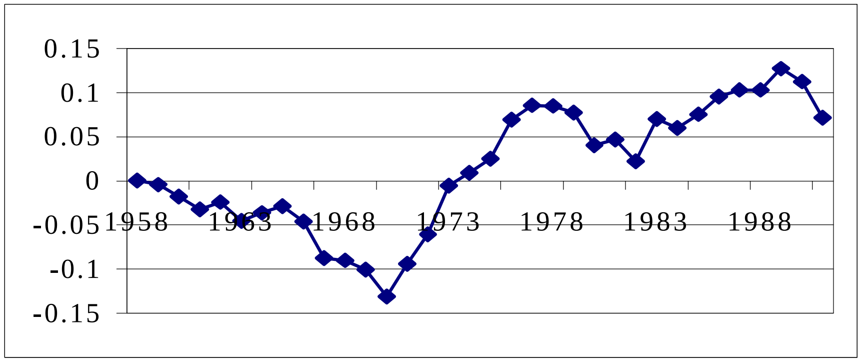

Figure 15. $\dfrac{e_S - e_A}{e_A}$, 1958-1991

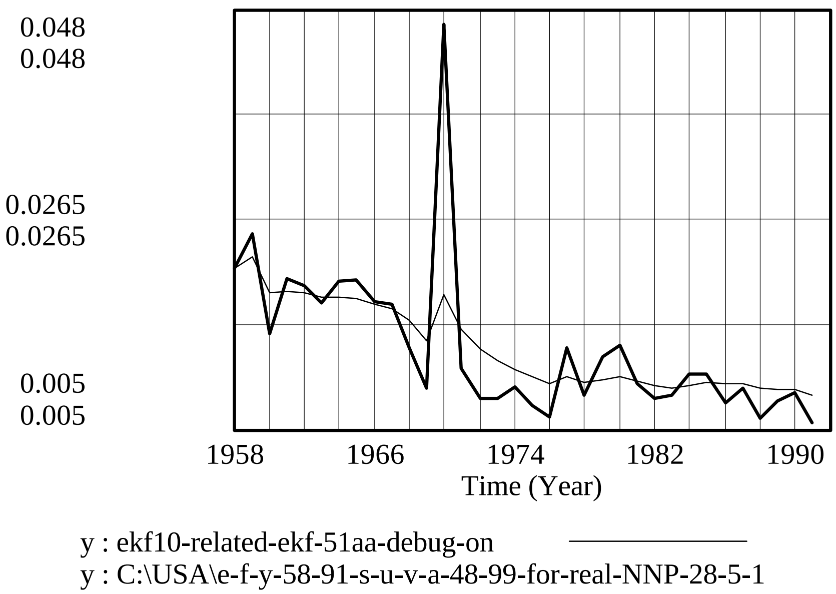

Figure 16. y_A (thick) versus y_S (thin), 1958-1991

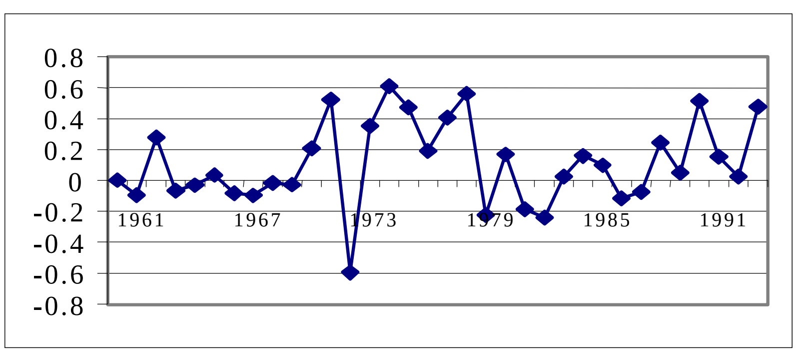

Figure 17. $\dfrac{y_S - y_A}{y_A}$, 1958-1991

Besides the large measurement errors, unequal variance is likely due to a visible change in ecological policy after 1970-1973. The policy was drastically directed to raise eco-efficiency. Thus our postulate of the dynamic law for *e* is more appropriate for the period after 1970, while *e* = const would be more realistic for 1958-1970. A simplifying assumption on investment policy in the natural capital could be partially responsible for the large difference in variation of these observed and simulated data too.

It has been shown via Monte Carlo experiments (Barlas 1989: 72) that *U* can be as large as 36% even if a model has a perfect structure with moderate noise. In the Barlas experiments with 'synthetic real systems', E1 does not go beyond 5%, whereas E2 becomes as large as 30% and *U* changes between 35 and 70%. Experiments show that, even for models with no systematic errors, *U* can be as large as 70% (Ibid.: 77-78).

The analogous indicators in Table 3 are closer the lower values of these bounds: E1 is less than one percent in all cases, E2 exceeds 30% only for *y* (44%) and *u* (56%), while *U* is higher than 36% also only for *y* (44%) and *u* (52%). Still the unequal variance term (U^S) for *u* is only about 17.5% with the bias term (U^M) less than 1.5% (Table 2). Thus the discrepancy between u_S and u_A is mostly due to fluctuations with higher frequencies and divergence on a point-by-point basis.

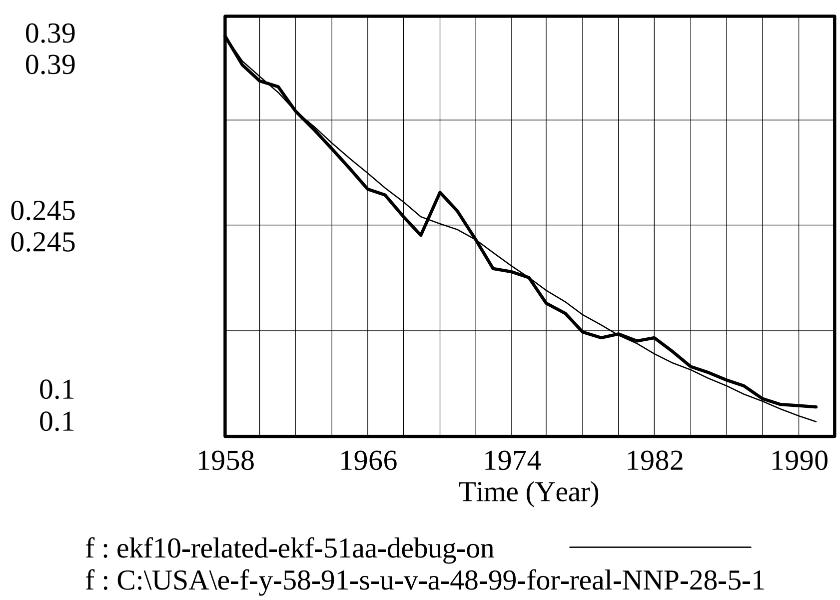

Figure 18. f_A (thick) versus f_S (thin), 1958-1991

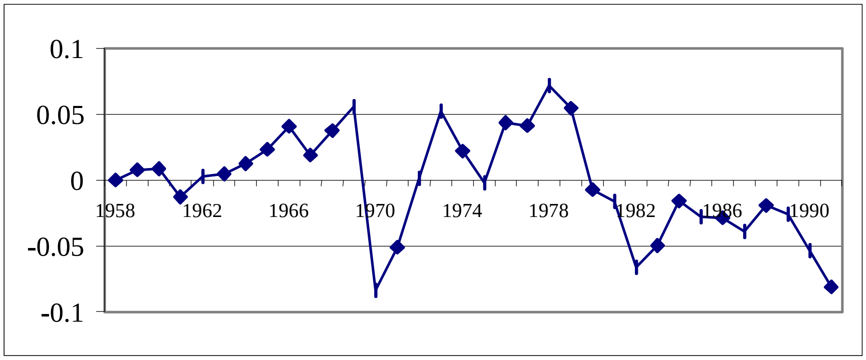

Figure 19. $\frac{f_S - f_A}{f_A}$, 1958-1991

The extraordinary measurement errors for *y*, *e* and *f* preclude a substantial improving of the model fit, particularly in respect to these variables. Still the errors of the retrospective forecast are substantially lower than the relative standard deviations of the measurement errors and higher than the relative standard deviations of the equation errors (Table 5). The reader is not to overlook that the same data have been used to estimate the parameters and to test the historical fit.

Table 5. The Accuracy of the Retrospective Forecast for 1958-1991

Variable	Innovation mean δ_z	The Forecast relative error $\sqrt{MSE}/\bar{A}$ (%)	$\sqrt{\mathbf{Q}_{ii}}/\bar{A}$ (%)	$\sqrt{\mathbf{R}_{ii}}/\bar{A}$ (%)
a	-0.0001	1.3	1.3	2.6
c	…	…	0	…
e	2.46e-006	6.7	2.7	17.1
f	-0.000605495	3.6	0.014	14.4
s	-0.0139	3.8	1.5	7.3
u	0.0015	1.9	0.01	7.1
v	-0.0025	1.2	0.08	7.0
y	3.89e-005	39.6	14.8	43.6

Note. The unit of measurement of the innovation mean coincides with that of the respective variable. The suboptimal magnitudes of dispersion $\mathbf{Q}_{ii}$ and $\mathbf{R}_{ii}$ are taken from File 1.out of the Appendix.

The exploratory scenario 1991-2107: a falling rate of capital accumulation

The characteristic of this first scenario is its opportunity rather than normative orientation. An extrapolation results from the compact model, probabilistically identified by EKF. EKF has created a VENSIM data file with estimated values of the model variables and constants. The 4th-order Runge - Kutta integration with a variable step size has been selected. The parameters from this file are used in a simulation with the state vector given by EKF for the year 1991: $a_0 \approx 0.0512$, $s_0 \approx 2.052$, $c_0 \approx 13.98$, $f_0 \approx 0.1091$, $e_0 \approx 0.0087$, $y_0 \approx 0.0083$, $v_0 \approx 0.9475$, $u_0 \approx 0.6948$. The reader sees that these magnitudes differ from the observation posted in the Table 1 for the year 1991. The growth rate of the indicated natural capital $i \approx 0.0374 < d \approx 0.0384$ while $o_1 < 0$ (these parameters are defined in the equations (21), (22) and (24)).

There is no locally unstable stationary state with a limit cycle nearby. The long wave is perceived as a quasi-periodic trend and stochastic attractor.

Capitalists accumulate profit and expand output. The economic growth is punctuated by recurrent slowdowns. New more advanced machinery is a source of increased labour productivity. As wages increase, profits relatively decrease. The spiral of accumulation is almost periodically arrested by the relative shortage of labour. A quasi-period of fluctuations is about 29-33 years. Accelerated introduction of the living labour-saving machinery, shedding of labour enables to overcome these temporally hindrances. The current downswing in the long wave manifests itself in the growing produced capital-output ratio and unit wage, declining profitability and employment ratio.

There is a secular profit squeeze and deceleration of economic growth in spite of the steady reduction of the eco-intensity and labour productivity growth. Worsening profits slow the growth in productivity that inhibits profits, in turn. The both profit rates $(1 - u - e)/(s + f)$ and $(1- u - y)/s$ tend to be lower and lower than the benchmark $d/k \approx 0.144$ (Figure 20).

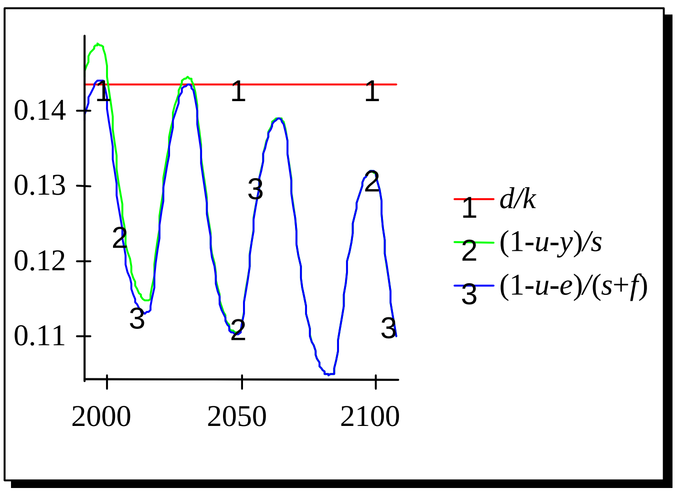

Figure 20. The average profit rate, $(1 - u - y)/s$, and general profit rate, $(1 - u - e)/(s + f)$, relative to the benchmark (d/k) in the exploratory scenario

The net rent rate is negative over the long term. A depletion of the natural capital slows the economic growth. The rate of economic growth deviates from the benchmark (d) downward. The rate of net rent falls from zero to about -0.045, that is lower then the positive benchmark ($d \approx 0.0384$). The quasi-cyclical deceleration of the labour productivity and real wages is relentless.

An univariate sensitivity analysis for 1991-2034 has shown that the long term business upturn will not probably happen until 2012 or even 2018. It will proceed thereafter up to the beginning of the next long term downturn in 2035-2040 (Ryzhenkov 2001).

The above scenario does not satisfy the all necessary conditions for sustainable development: the society accumulates the produced capital; however its activity brings about the excessive depletion of the natural capital. To avoid this, a more efficient policy is necessary.

A real development will differ from the offered description because of remaining specification and identification errors. Still the model parameters can be adjusted by EKF and the forecast can be updated each period, based on new information.

The normative scenario 1991-2107: extending the natural capital

The second scenario corresponds to a rather strong criterion of sustainable development. In particularly, the gross rent is increased step-wise in the year 2003:

$$\dot{y} = (o_1(c - f) + o_2\hat{f})y \ + \text{STEP}(0.0018, 2003). \qquad (22a)$$

At the end of the year 2002 or beginning of 2003, $y_{2003} \approx 0.00656$; at the end of the year 2003 or beginning of 2004, $y_{2004} \approx 0.00757$. This modification does not exclude other possible alterations for achieving sustainable development. Still it addresses the critical shortcoming of the exploratory scenario, namely the depletion of the natural capital.

In the normative scenario, the economic growth is quasi-cyclical with a period of about 31-33 years. The maximum employment is firstly achieved in the year 1999, it declines thereafter until the year 2011, then growth again until the year 2028.

The increase in the unit gross rent is achieved by a reduction of the unit wage by about the same quantity. Yet the labour productivity and real wage of an employee increase faster than in the previous (exploratory) scenario. The natural capital-output ratio declines, while the total capital-output ratio goes up (see for details Ryzhenkov 2001).

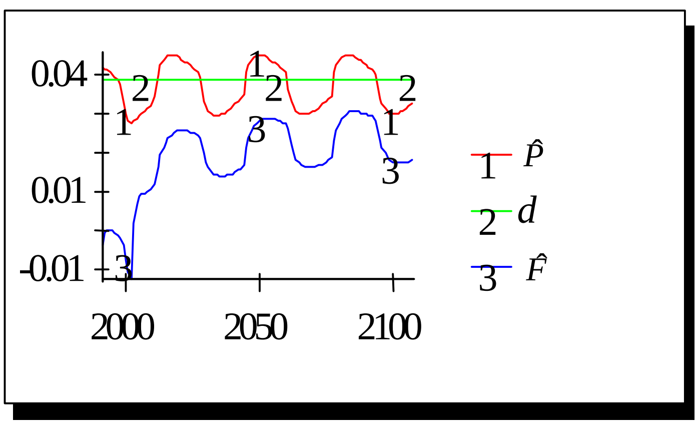

Figure 21. The rates of growth of net output ($\hat{P}$), growth rate of natural capital ($\hat{F}$) versus the benchmark (d) in the normative scenario

It is reasonable to stress that the substitution of the Eq. (22) by the Eq. (22a) is the single alteration of the above compact model. Only 0.18 per cent of the NNP is invested additionally in the natural capital each year since 2003. Still this partial redistribution of the NNP produces desirable positive effects over the whole period on the average:

- the rate of the economic growth rate is increased (Figure 21);

- the natural capital is extended;
- the average and general rates of profit are raised without any apparent tendency to fall;
- there are gains in the employment rate.

Conclusion

The suggested conscious alteration of the societal relationship with the natural capital is probably pivotal for converting the unsustainable evolution of the exploratory scenario into the strongly sustainable development of the normative scenario.

A drop in the unit wage is the condition for the higher employment ratio and profit rates, growing natural capital in the normative scenario. A complete win-win solution with an increased unit wage would require a substantial reduction in the produced capital-output ratio (*s*). Historically, lifting this ratio is a leverage against an excessive growth of the real wage. New patterns of the production relations and new technologies are prerequisite for greater equity.

The present downturn in the big quasi-cycle of conjuncture is not only a regularly recurrent phase of the long wave that strengthens the tendency of the profit rates to fall. Additional pains of this downturn are characteristic of childbirth of the natural capitalism. The system dynamics approach could be helpful for shortening and lessening disorder and distress of this evolution. There is a potential for a greater social advantage of the positive feedback between sustainable development and unbiased profitability, revealed in this paper.

The long wave is comprehended as the stochastic attractor and non-equilibrium quasi-periodic trend. The above analysis strengthens confidence in the presented model.

Appendix (The real data and sources)

Table 1. The author's estimates for U.S. real macroeconomic data, 1958-1999

	f	*u*	*s*	*v*	*a*	*y*	*e*
1958	0.37537	0.71458	2.16474	0.93162	0.03149	0.02143	0.01768
1959	0.35589	0.70956	2.05753	0.94542	0.03312	0.025	0.01728
1960	0.34513	0.71619	2.02956	0.94438	0.03333	0.01489	0.01712
1961	0.34072	0.71012	2.03005	0.93321	0.03413	0.02047	0.01703
1962	0.32338	0.70172	1.96848	0.94452	0.03579	0.01964	0.01646
1963	0.31121	0.69675	1.93987	0.94357	0.03679	0.01788	0.01647
1964	0.29774	0.69298	1.89615	0.94833	0.03813	0.0202	0.01595
1965	0.28434	0.68468	1.86284	0.95494	0.03962	0.02035	0.01545
1966	0.26946	0.68737	1.84462	0.96215	0.04117	0.01802	0.0154
1967	0.26558	0.68933	1.88948	0.96158	0.04122	0.01777	0.01591
1968	0.25116	0.69404	1.888	0.96446	0.04231	0.01337	0.01574
1969	0.2379	0.70784	1.92997	0.96493	0.04238	0.00928	0.01581
1970	0.26829	0.71733	2.03042	0.95015	0.04184	0.04639	0.01663
1971	0.25499	0.70482	2.04807	0.94048	0.04288	0.01131	0.01589

1972	0.23468	0.70488	2.03457	0.94396	0.04371	0.00819	0.01517
1973	0.21493	0.70169	2.03485	0.95124	0.04485	0.00813	0.01407
1974	0.21282	0.71508	2.29921	0.94375	0.04351	0.0094	0.01367
1975	0.20938	0.7058	2.30697	0.91533	0.04352	0.0074	0.0132
1976	0.19149	0.70207	2.22419	0.92306	0.04461	0.00623	0.01232
1977	0.18417	0.70181	2.17677	0.92962	0.04509	0.01332	0.01185
1978	0.17141	0.70391	2.15084	0.93948	0.0456	0.00845	0.01154
1979	0.16713	0.70826	2.20955	0.94155	0.04575	0.01244	0.01136
1980	0.17061	0.7217	2.3441	0.92829	0.04513	0.0136	0.01158
1981	0.16563	0.71519	2.35172	0.92384	0.04554	0.00966	0.01126
1982	0.16738	0.73149	2.41718	0.90281	0.04462	0.00815	0.01132
1983	0.1583	0.71291	2.27873	0.90411	0.04595	0.00842	0.01053
1984	0.14766	0.70398	2.14105	0.92487	0.04743	0.01072	0.01037
1985	0.14381	0.70729	2.10606	0.928	0.04797	0.01065	0.00991
1986	0.13807	0.71158	2.10566	0.93004	0.04827	0.00776	0.0094
1987	0.13395	0.71733	2.08067	0.93814	0.04856	0.00921	0.00903
1988	0.12582	0.71845	2.04407	0.94496	0.04952	0.00607	0.00871
1989	0.12148	0.70831	2.01227	0.94733	0.05011	0.00786	0.00823
1990	0.12008	0.71042	1.99851	0.9439	0.05037	0.00879	0.00808
1991	0.11898	0.71355	1.99378	0.93162	0.05028	0.00572	0.00815
1992	…	0.71058	1.95891	0.92497	0.05132	…	…
1993	…	0.70712	1.94959	0.9309	0.05196	…	…
1994	…	0.70476	1.94537	0.93914	0.0526	…	…
1995	…	0.70051	1.94455	0.94402	0.05325	…	…
1996	…	0.69304	1.92755	0.94602	0.05426	…	…
1997	…	0.68867	1.89862	0.95064	0.05524	…	…
1998	…	0.69822	1.88958	0.95494	0.05656	…	…
1999	…	…	…	0.95782	0.05769	…	…

Units of measurement: *u*, *v*, *e* and *y* [dimensionless], *c*, *f* and *s* [years], *a* [billions of chained 1996 dollars per 1000 civil persons employed per year]. In calculating *f* and *a*, constant 1996 dollars are used for the nominators and denominators; calculations of *u* and *s* are done with the nominators and denominators valued in current prices. The employment ratio *v* is for the civil labour force.

This paper assumes that the annual value of the labour force of a proprietor is equal to the annual earnings of a hired employee. Calculating the relative labour income (*u*) requires two steps:

1) estimating the ratio of the total labour force to employees M = (CLF + AFP)/(CLF + AFP - SE), where CLF is the civil labour force, AFP is Armed Force Population, SE is a number of self-employed in all industries; this account does not covers the part of the

defence related personnel outside the AFP because of incompleteness of the official statistics available for the author;

2) estimating the labour share *u* = (W+S+NWC)*M/NNP, where W, S, NWC are accruals for wage and salary income and disbursements for other labour income, NNP is net national product.

The natural capital is represented only by the value of the closing stocks of proved mineral reserves (*Survey of Current Business*, April 1994: 58-60). Current rent method I and current rent method II are net present value methods based on the Hotelling valuation principle. The stock valuations, given by the BEA current rent method I, in billions of 1987 dollars have been converted into billions of 1996 dollars through multiplication by the NNP price index for 1987-1996. This index is calculated as (NNP 1987, billions of 1996 dollars)/(NNP 1987, billions of 1987 dollars) = 5460/4029 = 1.355.

The book (Nordhaus and Kokkelenberg 1999: 59-105) and discussion paper (Ryzhenkov 2001) offer explanations of an upward bias in the mineral-resource values calculated with the Hotelling valuation principle, especially if the BEA current rent method II is applied. To avoid this bias as much as possible, this paper does not use the stock valuations based on the latter. Still due to the usage of the Hotelling valuation technique by the BEA, the current rent method I gives biased estimates too.

The new BEA estimates of non-residential fixed assets for 1998 and revised estimates for 1958-1997 are used in calculating produced capital – output ratio (*Survey of Current Business*, April 2000). The other data are compiled from the site http://www.economagic.com and issues of “Statistical Abstract of the USA” for 1999 and 2000 published by the U.S. Department of Commerce.

The VENSIM optimisation files

The following specifications of the noise and optimisation pay-off are in agreement with the VENSIM format:

pay-off.vpd

a/meas a variance
e/meas e variance
f/meas f variance
s/meas s variance
u/meas u variance
v/meas v variance
y/meas y variance

kalman.prm

a/ a dr variance/a ISC
c/c dr variance/c ISC
e/e dr variance/ e ISC
f/f dr variance/f ISC
s/s dr variance/S ISC
u/u dr variance/u ISC
v/v dr variance/v ISC
y/y dr variance/y ISC

All variances from the file kalman.prm and variances of the measurement noise (payoff.vpd) for the same seven state variables have been included in the list of parameters to be estimated.

File 1.out

```
:COMSYS After 2123 simulations
:COMSYS Best payoff is 930.511
:COMSYS User terminated multiple search session
:OPTIMISER=Powell
:SENSITIVITY=Payoff Value
:MULTIPLE_START=Random
:RANDOM_NUMER=Linear
:OUTPUT_LEVEL=2
:TRACE=2
:MAX_ITERATIONS=10000
:PASS_LIMIT=2
:FRACTIONAL_TOLERANCE=0.0003
:TOLERANCE_MULTIPLIER=21
:ABSOLUTE_TOLERANCE=1
:SCALE_ABSOLUTE=1
:VECTOR_POINTS=1.24418e-306
0 <= b = 0.621019  <= 1
0 <= c0 = 13.2528
0 <= e1 = 0.0054249
0 <= g = 0.0531989  <= 1.5
0 <= m1 = 0.0149761  <= 0.02
0 <= m3 = 0.0105916  <= 0.1
0 <= m5 = 0.0888489  <= 0.3
0 <= n = 0.0199143  <= 0.022
0 <= n2 = 0.353022  <= 0.5
0 <= n3 = 0.5  <= 0.5
0 <= n5 = 0.0106352  <= 1
0 <= r = 0.0609304
0.001 <= f ISC = 0.001  <= 0.004
0.001 <= MEAS f VARIANCE = 0.001  <= 0.004
0.0025 <= MEAS u VARIANCE = 0.0025  <= 0.01
0.0025 <= u ISC = 0.0025  <= 0.01
0.0043 <= MEAS v VARIANCE = 0.0043  <= 0.0174
0.00435 <= v ISC = 0.00435  <= 0.0174
0.0234 <= MEAS s VARIANCE = 0.0234  <= 0.0936
0.0234 <= s ISC = 0.0234  <= 0.0936
0.05 <= j = 0.211049  <= 1
0.1 <= m2 = 0.1  <= 0.75
0.125 <= TIME STEP = 0.588893  <= 3
0.2 <= k = 0.267234  <= 0.5
0.75 <= vc = 0.92536  <= 0.99
1.25e-006 <= a ISC = 1.25e-006  <= 5e-006
1.25e-006 <= MEAS a VARIANCE = 1.25e-006  <= 5e-006
1e-009 <=  a DR  VARIANCE = 2.93202e-007  <= 0.001
```

c DR VARIANCE = 0
c ISC = 1
1e-009 <= e DR VARIANCE = 1.22235e-007 <= 1e-006
1e-009 <= f DR VARIANCE = 1e-009 <= 0.001
1e-009 <= s DR VARIANCE = 0.000929877 <= 0.01
1e-009 <= u DR VARIANCE = 5.51477e-009 <= 0.001
1e-009 <= v DR VARIANCE = 5.16068e-007 <= 0.001
1e-009 <= y DR VARIANCE = 3.9415e-006 <= 2e-005
5e-006 <= e ISC = 5e-006 <= 2e-005
5e-006 <= MEAS e VARIANCE = 5e-006 <= 0.0002
5e-006 <= MEAS y VARIANCE = 3.41319e-005 <= 0.0002
5e-006 <= y ISC = 5e-006 <= 2e-005
i = 0.0373606
n1 = -0.24223 <= 0.02
o1 = -0.0299728
o2 = -9.93389
q = -0.0084833

Notice: The File 1 contains the best payoff so far, the reason the optimiser stopped, and the values of the search parameters needed to achieve that payoff. It has been assumed additionally that c ISC equals one, c dr variance equals zero.

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