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AGGREGATE VERSUS COHORT MODELS; A POLICY SENSITIVITY ANALYSIS

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Abstract

The purpose of this paper is twofold. First, we compare two representations of a fish stock: a complex cohort (age-class) model and a simple aggregate (surplus growth) model. A key question is whether the aggregate model is an appropriate simplification of the more complex model. The comparison is made with respect to the optimal fishing strategies that follow from each representation. With proper economic mechanisms in place, the difference between the two harvesting policies is surprisingly small. Second, we see the comparison as an example of an advanced sensitivity analysis, where sensitivity in terms of policy is considered rather than in terms of behavior. Assuming the ultimate aim of modeling is better policies, our choice seems a proper one. However, we also recognize that in many cases, less costly tests of behavioral sensitivity may prove more practical.

1. Introduction

The purpose of this paper is twofold. First, we compare two representations of a fish stock, a complex cohort (age-class) model and a simple aggregate (surplus growth) model. A key question is whether the potential model simplification is appropriate. The comparison is made with respect to the optimal fishing strategies that follow from each representation. That is, we ask if the two representations lead to similar or different policy recommendations. In case they differ, the choice of model representation is an important one. Second, we see the comparison as an example of an advanced sensitivity analysis, where we consider sensitivity in terms of policy rather than in terms of behavior. Since the ultimate aim of modeling is better policies, it seems that our choice is the better one.

The next two sections present the two model representations of the fishery. The case is cod (*Gadus Morhua*) in the Barents Sea, for which case model parameters are found. Interestingly, the parameter estimates in the aggregate model are found to be sensitive to the historical fishing strategy. For those who do not want to read these two sections, Figures 1 and 2 give quick introductions to the biological parts of the two models. In the fourth section we discuss the optimization method. In short, this section explains a method to find optimal or near-to-optimal policies linking observations of fish biomass to harvesting decisions. In the fifth section we discuss our approach to sensitivity analysis. In particular we consider under what conditions traditional behavior sensitivity is more appropriate and when policy sensitivity is to be prefered. Then we present the results. When important economic feedbacks are removed from the models, we find that the models lead to quite different policies (constant target escapement versus pulse fishing). However, when the economic feedbacks are in place, we can no longer be confident that the policy recommendations differ.

2. The cohort model

The Barents Sea cod fishery is managed by yearly quotas. Therefore we focus on a strategy for quota setting. The quota or harvest will be found as a function of total biomass. Cohort models used to find optimal fishing strategies, e.g. Mendelssohn (1978), Naqib and Stollery (1982), Spulber (1983), and Spulber (1985), typically limit themselves to rather simple representations since according to Mendelssohn: “The large increase in analytic complexity caused by the addition of even the simplest interaction term is cause for both consternation and challenge.” Since our method allows for greater model complexity, we introduce “interaction” terms to capture vital feedback mechanisms, e.g. recruitment and weight dynamics. On the other hand, we will, different from the above papers, make the simplifying assumption that harvesters do not target specific age-classes. This is largely consistent with current fishery policies, which do not change restrictions on gear and which do not make major changes in allocations between fishing grounds and vessel segments depending on stock size.

We want to maximize the expected net present value

$$V = E\sum_{t=0}^{\infty} r^t \left\{ (p_0 - p_1 H_t) H_t - \left(c_0 + (c_1 - c_0)(\frac{e_t}{e_0})^a \right) e_t - c_2 e_0 \right\} \tag{1}$$

for the cod fishery in the Barents Sea. The discount factor is denoted by r, e_t is the applied fishing effort, and e_0 reflects the fishing capacity. Price of fish is a linear function of harvest H_t with parameters p_0 and p_1. Unit variable costs equal c_0 at zero effort, and they equal c_1 when effort equals e_0. Increasing marginal costs are ensured by assuming a>0. The per unit leasing cost of capacity is c_2. We explicitly avoid maximizing a social welfare function for the fishing nation. Most of the harvest is exported and domestic prices reflect export prices.

In the following we use capital letters to denote fish in biomass terms (million tons), while lower case letters are used to denote numbers (billion fish). Total effort

$$e_t = X_0 \{ (\frac{X_t}{X_0})^{1-b} - (\frac{X_t - H_t}{X_0})^{1-b} \} / (1-b) \tag{2}$$

is derived from harvest using a standard instantaneous catch per unit effort relationship, $h = e(X / X_0)^b$, where $X = X_0$ is the biomass for which effort is defined equal to harvest. The expression is found by solving the catch per unit effort relationship for *e* and by integrating over *X* from $X_t - H_t$ to X_t, see Clark (1985).[1]

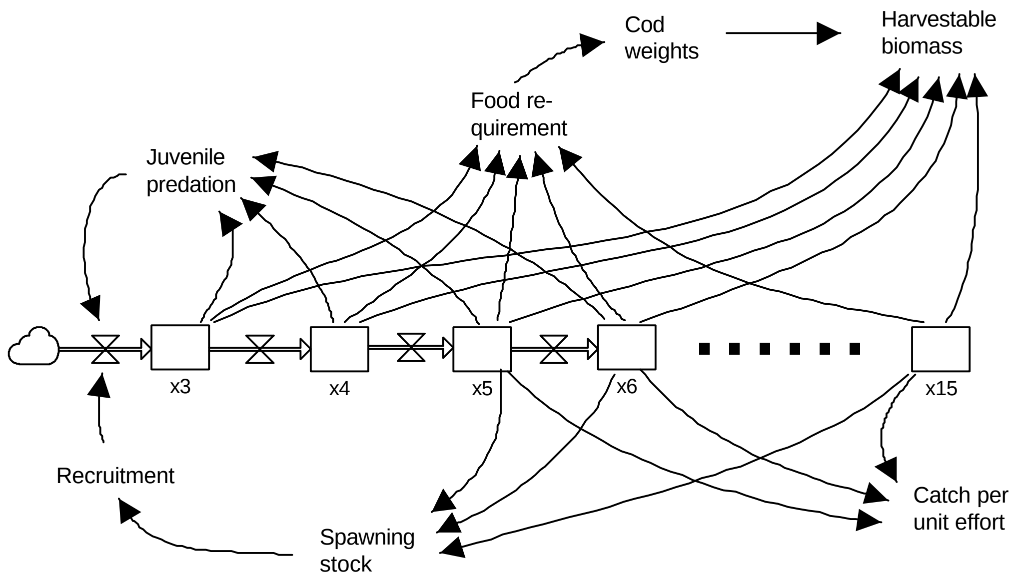

Figure 1: Illustration of the biological part of the cohort model, not showing rates of natural mortality and harvest out of each age class.

A cohort model is used to describe the biology of cod, see Figure 1. The number of fish in the different age classes are given by the following equations:

$$x_{3,t} = S_{t-3} \exp(r_0 + r_1 S_{t-3} + r_2 J_t + v_{r,t}) < r_s \tag{3}$$

$$x_{i+1,t+1} = x_{i,t} \exp(-m_i v_{i,t}) - h_{i,t} \exp(-m_i v_{i,t} / 2) \qquad i=3,4,...,13 \tag{4}$$

$$x_{15,t+1} = x_{14,t} \exp(-m_{14} v_{14,t}) - h_{14,t} \exp(-m_{14} v_{14,t} / 2) + x_{15,t} \exp(-m_{15} v_{15,t}) - h_{15,t} \exp(-m_{15} v_{15,t} / 2) \tag{5}$$

where $x_{3,t}$ represents recruitment of three year old cod. S_{t-3} is the biomass of the spawning stock at the appropriate point in time, J_t is a measure of cannibalistic cod juveniles, $v_{r,t} \sim N(0, s_r)$ represents random recruitment variability and r_s is the maximum recruitment. Yearclass harvest is denoted by $h_{i,t}$, m_i is the natural mortality

1 Ideally, there should have been a stochastic variable in this equation since the effort needed to catch a given quota is likely to vary from year to year. However, a test shows that such an extra random variable is of little importance for the optimal harvesting policy. The catch per unit effort relationship should also be expected to be related to the selectivity of the gear. This is no problem since we will keep the selectivity constant. However, in future studies it is important to reconsider the catch per unit effort relationship if selectivity is allowed to vary.

for yearclass i, and $v_{i,t} \sim N(1, s_m)$ represents random variations in natural mortality. Even though one might expect natural mortalities for age classes to be influenced by some of the same environmental forces, we disregard this possibility here and assume independence. Suitability matrices (based on stomach content analyses) indicate that there is a certain cannibalism on three year old cod. We ignore this direct relationship since the bulk of cannibalism is supposed to be captured by the recruitment function.

Due to the choice of total harvest as the decision variable, it is most practical to use age-class harvests and not fishing mortalities in these equations. To facilitate this, we have made use of Pope's approximation, i.e. harvest is assumed to take place in the middle of the year. This approximation is thought to yield good results for cod[2]. Equation 5 shows that the survivors of age class 15 are re-entered into this age class. This is not a perfect way to represent fish older than 15 years of age since fish weight does not increase with further aging. With normal fishing activity, however, there are very few fish in the upper age classes such that this approximation should be of little concern.

The spawning stock biomass is given by ogives o_i, age class body weights $w_{i,t}$, and age class numbers $x_{i,t}$.

$$S_t = \sum_{i=3}^{15} o_i w_{i,t} x_{i,t} \tag{6}$$

The total biomass of harvestable fish (3 years and older) is

$$X_t = \sum_{i=3}^{15} w_{i,t} x_{i,t} \tag{7}$$

Juveniles J_t represent a weighted average of biomass in lower age classes

$$J_t = \sum_{i=4}^{15} s_i w_{i,t} x_{i,t} \tag{8}$$

The weights s_i reflect suitability of pre-recruitment cod for these age groups. The harvest from each age class

$$h_{i,t} = \frac{H_t q_i x_{i,t}}{\sum_{i=3}^{12} q_i x_{i,t} w_{i,t}} \tag{9}$$

is derived from the total harvest H_t, the policy variable. Here q_i represents the selectivity of the fishing gear. One can easily see that the sum of harvests from individual age classes equals in H_t (multiply by $w_{i,t}$ on each side of the equation and sum over all

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2 Personal communication with Bjarte Bogstad at the Institute of Marine Research, IMR, Bergen.

i) .[3] Based on observed patterns seen in VPA data, harvesting selectivities are given by a logistic function:

$$q_i = e^{u_{i,t}} / \{1 + (q_h / i)^{q_e}\} \qquad i=3,4,...,15 \tag{10}$$

For older age classes, q_i tends towards 1.0, q_h denotes the age at which q_i equals 0.5, and the exponent q_e influences the steepness of the function. Selectivities are also influenced by natural variation, $u_{q,t} \sim N(0, s_q)$.

Nearly all model studies we have come across ignore the effect of intraspecies competition in terms of the effect of own stock biomass on own weight. One exception is Ault and Olson (1996). We assume that the weight of each age class is given by a reference weight for this age class $w_{i,0}$ times a weight index w_t:

$$w_{i,t} = w_{i,0} w_t \tag{11}$$

where the weight index

$$w_t = B_t^{j/(1-j)} e^{v_{w,t}} < w_s \tag{12}$$

depends on the cod biomass and a random variable $v_{w,t} \sim N(0, s_w)$. To avoid simultaneous equations, we have replaced the actual cod biomass X_t by an approximation based on the reference weights for each weight class, $B_t = \sum_{i=3}^{15} w_{i,0} x_{i,t}$. This is not a problem because we need a measure of the food requirement, and not the actual biomass. Weight is assumed to stay below an upper limit w_s in case cod biomass becomes very low. Using one common weight index implies that we ignore possible differences between age classes. We also ignore time delays in the effect of intraspecies competition. It seems however that the delays are short and of little importance.

Tables 1 and 2 give a summary of the parameter values used in the cohort model. The parameter values are found by a variety of methods, e.g. catch-at-age analysis, OLS and direct observation, see Moxnes (1999).

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3 When the selectivity varies over age classes, Equation 9 does not ensure that harvests will be less than population numbers in all of the age classes. This problem is most easily seen in the case that $H_t = X_t$. In this case all age classes should be harvested completely. However, harvests in age classess with higher than normal values of q_i will be greater than the corresponding population numbers x_i. The problem is caused by discretization in time. In a continuous world the q_i's could stay constant while the population numbers gradually decrease. In turn the declining populations numbers would serve to limit harvests to what is available. Fortunately this is only a problem when H_t is close to X_t. Since any reasonable harvesting policy will keep a good distance, the weakness of the formulation usually presents no problem.

Table 1: Parameter values in cohort model.

Parameter	Symbol	Value	Unit
Discount factor	r	0.95	
Price at zero harvest	p_0	10.0	NOK/kg
Price reduction with harvest	p_1	2.0	
Lower unit variable cost	c_0	3.7	NOK/kg
Unit variable cost when $e_t = e_0$	c_1	4.5	NOK/kg
Leasing cost of capital	c_2	1.8	NOK/kg
Exponent for variable costs	a	2.0	
Biomass where effort equals harvest	X_0	1.0	Mill.tons
Exponent for catch per unit effort	b	0.6	
Recruitment, constant	r_0	0.85	
Recruitment, effect of spawning stock	r_1	0.7	per mill.tons
Recruitment, effect of juveniles	r_2	-0.25	per mill.tons
Recruitment, maximum	r_s	2.0	Billion
Recruitment, standard deviation	s_r	0.64	
Mortality	m	0.2	
Mortality, standard deviation	s_m	0.35	
Selectivity, half value	q_h	4.75	
Selectivity, exponent	q_e	6.25	
Selectivity, standard deviation	s_q	0.25	
Weight, elasticity w.r.t. biomass	j	-0.2	
Weight, standard deviation	s_w	0.34	
Maximum weight index	w_s	1.3	

Table 2: Parameters that are distributed over age classes.

Age class	Reference fish weights [kg] $w_{i,0}$	Spawning stock ogives o_i	Juvenile distribution s_i	Expected initial populations [Billions] $x_{i,00}$
3	0.8	0.00	0.30	0.500
4	1.3	0.02	0.80	0.403
5	1.9	0.08	1.00	0.305
6	2.7	0.28	1.00	0.207
7	3.8	0.57	1.00	0.129
8	5.2	0.79	0.63	0.077
9	6.8	0.90	0.26	0.045
10	8.3	0.96	0.00	0.026
11	9.8	1.00	0.00	0.015
12	11.5	1.00	0.00	0.009
13	12.7	1.00	0.00	0.005
14	13.5	1.00	0.00	0.003
15	16.8	1.00	0.00	0.002

3. The aggregate model

A discrete version of the surplus growth model, Schaefer (1954), is used:

$$X_{t+1} - (X_t - H_t) = a(X_t - H_t) + b(X_t - H_t)^2 + x_t \qquad (13)$$

where X_t denotes total biomass and H_t is total harvest measured in biomass, i.e. similar notation to the one used in the cohort model. The surplus growth function is illustrated in Figure 2. The economic part is identical to the one for the cohort model, Equations 1 and 2.

Parameter estimates for Equation 13, based on 50 year long historical time-series, are shown in Table 3, Moxnes (1999). As can be seen, estimates are obtained with large t-ratios.

As a preliminary test of the similarity of the models, we use the cohort model to produce synthetic time-series data, from which we estimate another set of aggregate model parameters. Actually, we produce two sets of synthetic data because it turns out that the estimates obtained are sensitive to the choice of fishing policy in the cohort model. Parameter estimates are shown in Table 3 for a historical policy ($H_t = 0.28 * X_t$) and for the best possible proportional policy ($H_t = 0.21 * X_t$). To get precise results we used 1000 years of synthetic data, which explains the very high t-ratios. Figure 2 shows the three surplus growth models.

Table 3: OLS estimates for aggregate models (t-ratios in parentheses).

Data used	*a*	*b*	s_A
Historical data (51 years of VPA data from IMR)	0.89	-0.25	0.30
(later referred to as aggregate-historical)	(11.1)	(-7.5)	
Cohort model output (1000 years, $H_t = 0.28 * X_t$)	0.94	-0.27	0.71
	(26.1)	(-18.9)	
Cohort model output (1000 years, $H_t = 0.21 * X_t$)	1.03	-0.23	1.08
(later referred to as aggregate-simulated)	(28.4)	(-24.6)	

The model obtained from historical data (solid line) is nearly identical to the one obtained from synthetic data with the historical policy (dotted line). The close fit is somewhat arbitrary since we have observed that estimates of the surplus growth curve based on only 50 year long synthetic time-series move around quite a bit. The important point here is that the two curves are not statistically different. We do note however that the estimates of the residuals s_A are significantly different (Chi square test). One possibility is that there is too much natural variation in the cohort model.[4] Another possibility is that the cohort model produces data that are less consistent with the

4 In the opposite direction we have underestimated the variation in the cohort model because we used a fixed policy when simulating the cohort model rather than a policy with a certain element of randomness (observations of historical policies always deviate from fixed one-dimensional policies).

surplus growth model than what the real system does. Both explanations indicate a certain improvement potential for the cohort model.

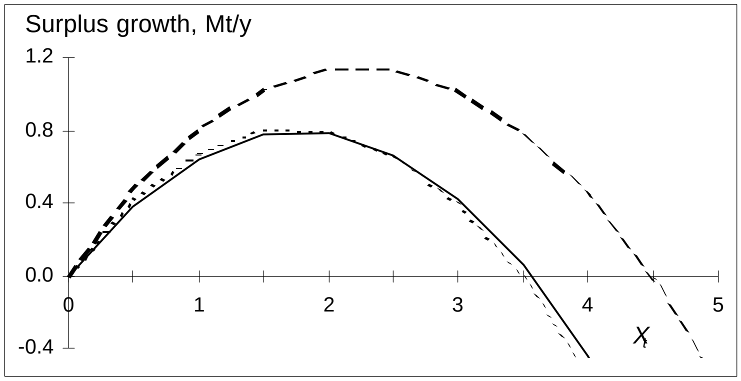

Figure 2: Surplus growth: Historical data: solid line, simulated data with historical policy: dotted line, and simulated data with the best proportional policy: dashed line.

Another interesting observation is that the estimates of the aggregate surplus growth model are sensitive to assumptions about the harvesting policy in the cohort model. The best possible proportional policy, which implies more careful harvesting and higher average fish stocks, leads to a higher estimate of the surplus growth curve (dashed line). This is an unfortunate feature of the aggregate model, at least when its parameters are based on data from historical periods with over- or underfishing compared to some optimal policy. Thus, in the remaining part of this paper we will consider both the aggregate model based on historical data, and the one based on synthetic data from the cohort model using the best possible proportional policy. Comparing optimal policies for these two models, we will get a sense of the importance of this estimation problem.

4. Stochastic optimization in policy space, SOPS

Cohort models are characterized by a large number of states. Hence, a direct application of stochastic dynamic programming, SDP, is ruled out by the 'curse of dimensionality'. Some sort of model reduction would be needed to use SDP. While such simplifications are conceivable, they would lead to less transparent models, and to models that are not known in the decision making environment. To maintain familiarity and to ensure that important effects are captured by the model, we rely on a method termed 'stochastic optimization in policy space', SOPS. In this case it is primarily the policy that is simplified in order to obtain solutions, and not the model.

Optimization in policy space has been proposed, used, and implemented in various settings, e.g. Walters (1986), Bertsekas and Tsitsiklis (1996), Ermoliev and Wets (1988), and Polyak (1987) and can to some extent be performed by simulation programs like Powersim and Vensim. Here we rely on a practical adaptation to stochastic problems presented in Moxnes (forthcoming).

In short the method transforms the problem of stochastic dynamic programming into a problem of non-linear static optimization. I.e. maximize

$$W(\mathbf{è}) = \frac{1}{M}\sum_{m=1}^{M}\ \sum_{t=0}^{T} r^{t}\left\{\left(p_0 + p_1 H_t\right)H_t - \left(c_0 + (c_1 - c_0)(\frac{e_t}{e_0})^{a}\right)e_t - c_2 e_0\right\} \tag{14}$$

were W is an estimate of the expected net present value V (Equation 1), and where **è** is a vector of policy parameters in a given type of fishing strategy

$$H_t = f\left(X_t, \mathbf{è}\right) \geq 0 \tag{15}$$

where X_t represents the biomass of the fish. W is produced by M Monte Carlo simulations of the respective models with the proposed fishing strategies implemented. The models are as described earlier except that the random variables with subscripts t, now appear with subscripts *tmn*, e.g. $v_{r,t}$ becomes $v_{r,tmn}$. Thus besides varying with time t, the random variables also vary over Monte Carlo runs m=1, 2, ..M and over n=1, 2, ..N separate searches for the policy parameters **è**. Each new parameter search starts with different initial policy parameters, $\mathbf{è} = \mathbf{è}_{\mathbf{0,n}}$, which are drawn from uniform distributions. Normally we use M=100 Monte Carlo runs, a time horizon of T=50 years, while N varies with the need for repetitions.

For the cohort model each Monte Carlo run starts out with randomly chosen initial age class populations

$$x_{i,0} = x_{i,00}e^{v_{x,0}} \tag{16}$$

where $v_{x,0} \sim N(0, s_x)$ and $s_x = 0.4$. Initial conditions vary similarly for the aggregate model.

$$X_0 = X_{00}e^{v_{X,0}} \tag{17}$$

where $v_{X,0} \sim N(0, s_X)$ and $s_X = 0.4$.

To find the parameter vector **è** that maximizes W, a hill-climbing search procedure is used (Fletcher-Powell variable metric). The search routine provides accurate parameter values judged by variations between repeated searches with different starting points for the parameter set $\mathbf{è}_{\mathbf{0}}$ (ignoring occasional solutions that are not close to the global optimum). Naturally, accurate parameters are only found in subsets of the state space that are visited and where the policy is of importance for the criterion.

Since we do not know what function characterizes the optimal solution, we rely on a flexible policy function, which does not restrict the solution very much. For the one dimensional policies to be used here a good numerical approximation can be obtained by interpolating between five grid points and extrapolating beyond the end grid points. The policy H_t is given by

$$0 \le H_t = q_k (k - (X_t - j)/d) + q_{k+1}((X_t - j)/d - (k-1)) \le X_t - X_l \tag{18}$$

where j is the location of the first grid point, d is the distance between grid points, and the policy parameter q_k denotes harvest at grid point *k* determined by

$$1 \le k = \text{int}((X_t - j)/d) + 1 \le 4 \tag{19}$$

Compared to the discrete representation in dynamic programming, we note that the fish stock X_t and the policy H_t are continuous variables. The grid points denote the kinks in the piecewise linearized policy. Moxnes (forthcoming) shows how linear interpolation can be extended into higher order policy surfaces.

The more complex the model, the greater the need to seek simplifications of the policy. By restricting ourselves to infinite horizon problems, time is left out of the policy function.When all states are measured perfectly, the ideal optimal policy is a function of all states. Since the aggregate model has only one state, its policy will be one-dimensional. The cohort model has many states and the ideal policy is very complicated. However, as shown in Moxnes (1999), a one-dimensional policy gives a nearly perfect result compared to higher order policies. The main reason for this is that the fishing selectivities are fixed. In models with targeted harvesting of all age classes, one dimensional policies would not suffice, Mendelssohn (1978) and Spulber (1983).

By repeated searches and varying the initial policy parameters, we increase the probability that a global rather than some local optimum is found. The fact that the method identifies correctly global solutions to problems with known solutions also increases our confidence in the method.

Finally we note that SOPS is an interesting method also from a more practical point of view. The method allows for the use of simulation models and assumptions familiar to decision makers. This is an advantage to the extent that decision makers distrust overly simplified models, Gulland (1991). On the other hand, large models may require that policy functions are simplified. However, this may also be perceived as desirable. According to Walters (1986): "..we will have to find ways to visualize [policy] functions when there are many [state] variables, since it would be silly to expect any real decision maker or manager to blindly plug numbers into such a function and then follow its prescription", p.243. SOPS could be used to find the best possible simplified and "visualizable" policies. This approach may also provide an attractive alternative to the intuitive blending of two or more exact results from simplified models to come up with a best possible policy for practical management. That this can be a complicated task is exemplified by the at times surprising effects of adding new nonlinearities, stochastic variables, and feedbacks to existing models. Empirical evidence of this difficulty is presented in Brekke and Moxnes (forthcoming). If decision makers are not able to untangle complexity, they are left with uncertainty about received results. Such uncertainty is believed to be the major obstacle to diffusion of technologies and policies, Rogers (1995). In this regard, it may also be an advantage of the method that it does not require knowledge of more sophisticated techniques than simulation and search.

By pointing to potential advantages of the method, we do not claim that SOPS is a panacea. For instance, other methods are needed to guide efficient problem formulation, to judge the likelihood that proper solutions are obtained, and to help explain why policies turn out the way they do. In highly complex cases SOPS will only provide improvement, which is also the rationale behind various related methods to tackle highly complex problems, e.g. neuro-dynamic programming and reinforcement learning, Bertsekas and Tsitsiklis (1996).

5. Policy sensitivity analysis

The traditional and frequently used form of sensitivity analysis has been to vary model parameters and to observe how behavior changes. This is a very useful procedure for model testing, learning, and validation. Using optimization models, one can in addition observe how the optimal policy changes due to variations in model parameters. This is what we do here, and what we refer to as policy sensitivity testing. Below we give a motivation for the use of policy sensitivity testing and we discuss limitations of the approach.

The main purpose of modeling is problem solving. In the light of double-loop learning models, Argyris and Schön (1978), problem solving can take quite different forms. At one level, the main challenge is to convince managers, politicians or their electorate, that a problem exists and that improvements are possible. Once the problem is acknowledged and proper institutions are in place, the problem is often dealt with at another level where more detailed and advanced policy analysis may be appropriate.

For these two modes of problem solving, different types of sensitivity analysis is needed. To convince that a problem exists, sensitivity analysis could be used to show that basic (problem) behavior modes are insensitive to large variations in model parameters. This is for instance the type of sensitivity analysis Jay W. Forrester refers to when discussing his world dynamics model, Senge (1973) p.5-18. The underlying assumption is that as long as the basic problem behaviors persist, proper policies stay approximately the same.

At the level of more detailed and fine-tuned policy analysis, the assumption that policies stay the same becomes more questionable. The fact that model behavior is sensitive to parameter change, may or may not imply that appropriate (optimal) policies are sensitive to the same parameters. This can be a difficult question to answer because it may be just as hard to identify proper policies as it is to intuitively predict behavior in complex dynamic models.

Two examples serve to illustrate these two situations. First one example where policy sensitivity is not likely to be very important. In Moxnes (1998) and Moxnes (2000) laboratory experiments show that decision makers misperceive the dynamics of reindeer pastures. The resulting mismanagement is likely to persist under a wide range of system parameters. This is illustrated by observations of similar modes of problem behavior in real management of differing reindeer pastures around the world, Moxnes et al. (2002). The same rough policy improvement is likely to lead to quite satisfactory management in all these cases.

The second example deals with policy analysis in a setting where management institutions have been in place for a long time. In Moxnes (forthcoming) the importance of errors in fish stock assessments is investigated. A traditional sensitivity analysis, keeping the harvesting policy constant, shows that the total payoff from the fishery is highly sensitive to the amount of assessment error. The apparent policy conclusion is that it is profitable to increase measurement or assessment accuracy at “any cost”. However, when using policy sensitivity analysis, it turns out that the optimal policy is sensitive to the error level. When the policy is allowed to vary with the error level, around ¾ of the earlier estimate of the value of accuracy disappears. Still accuracy is valuable, however, for a start, it is a cheaper option to change the harvesting policy. Furthermore, a pilot laboratory experiment indicates that optimization is truly needed for this type of analysis because the participants in the experiment were not very successful in adjusting policies to account for variations in assessment accuracy.

As said, we use optimization to estimate the sensitivity of policies to model parameter changes. Using optimization, there will be no random element of judgment when comparing policies. Thus, the comparison will be “fair”, showing optimal policies for all parameter changes. Since this type of policy sensitivity test requires that optimal policies can be found, the direct application of the method breaks down in complex models. One possibility is to resort to near-to-optimal policies. Such policies will have an element of error since they represent simplifications. If this error component is not sensitive to the model parameter one is changing, policy simplification is of little concern. The approximate policy will change similar to the truly optimal policy. However, one will rarely know to what extent the policy error depends on the actual model parameter, and therefore this solution is of little help. On the other hand, if the simplified policy is the best one can do, and it is the policy one will use in practical management, parameter dependent errors are of less concern. Then the policy sensitivity will say how the practical policy varies with changes in model parameters. Still the comparison is fair and of practical value, while it is of less value from a more theoretical point of view.

One can also think of types of policy sensitivity analysis where optimization is not needed by definition. An examples is presented in Andersen (1980). In two cases, policies resulting from two different models were compared. Andersen found that the policy conclusions were sensitive to the choice of modeling paradigm. When comparing the models, Andersen did not compare optimal policies. Rather, he took for granted the policies suggested by those who performed the original studies, implicitly assuming that the policy recommendations were representative for the different paradigms. Thus, comparing modeling paradigms it seems fair to include potential (if representative) shortcomings of the analysts.

Our problem, comparing an aggregate and a cohort model, seems particularly well suited for policy sensitivity testing. Then we test if optimal policies are sensitive to not only a parameter change but an entire change in model concept. The two models are not easily compared by other means. Not only do the aggregate model collapse all the age classes, it also combines recruitment, growth, and mortality into one variable for surplus growth. Hence variables in one model are not easy to interpret relative to those in the other model, and it also seems difficult to conduct eigenvalue-based linear analysis, see Eberlein (1989). Our earlier comparison of parameter estimates for the aggregate model based on historical and synthetic cohort model data provides an indication of similarity between the biological parts of the models. However, this comparison does not give strong indications about the policy sensitivity. Actually, we will see that the policy sensitivity is also shaped by the exact formulation of the (identical) economic parts of the two models.

6. Comparing optimal policies for the two models

We use the optimization method (SOPS) described in section 3 to find policies for the two models, using linear interpolation. For both types of models we also search for the optimal fixed harvesting capacity e_0, represented as a separate parameter in the policy parameter vector **è** .

We compare the two models under two different conditions. First we maintain the assumptions in Equation 1 that fish price declines with increasing harvests (p_1=2.0) and that unit costs increase with effort (a=2.0). Later we assume that price and unit costs are constant. We find policies for two versions of the aggregate model, one in which the parameters are based on historical data (aggregate-historical) and one in which the best possible proportional harvesting policy is used when generating synthetic data with the cohort model (aggregate-simulated), see Section 3.

Figure 3 and Table 4 show the resulting policies. The thin solid line in the figure shows the optimal policy for aggregate-historical. This policy is very close to the one for aggregate-simulated (thin dashed line). Hence the fact that the aggregate model parameters depend on the historical fishing strategy has a limited effect on policies. Next and of key interest in this paper, the harvesting policy for the cohort model (thick line) is somewhat less aggressive than the policies for the aggregate model. However, the similarity is more striking than the difference.

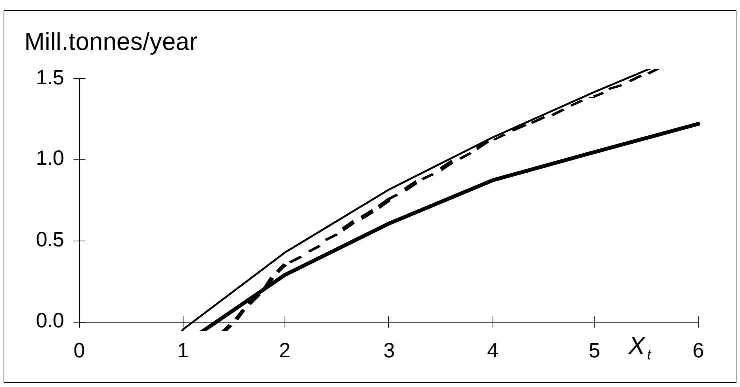

Figure 3: Policies for models with variable price and unit costs. Thick line: Cohort model. Thin solid line: aggregate-historical. Thin dashed line: aggregate-simulated.

What is the value of using the more complicated cohort model? Assuming that the cohort model is the correct one, we can find the expected net present value for this model with the policies derived from the aggregate models. Fist we note that the policy for the cohort model yields an expected net present value of NOK 73.8 billion. Using the policy for aggregate-historical, the net present value is reduced by NOK 5.7 billion or 7.7 percent. Using the policy for aggregate-simulated, the present value is reduced by NOK 3.9 billion or 5.3 percent. These numbers should not be taken too literally however. A priori we cannot rule out the possibility that the aggregate model is the better representation of reality.

Table 4: One-dimensional policies for cohort and aggregate models (averages of 20 searches).

Model/policy	φ_1	φ_2	φ_3	φ_4	φ_5	e_0	W
Variable price and unit costs							
Cohort	-0.14	0.30	0.61	0.88	1.05	0.37	73.8
Aggregate-historical	-0.04	0.43	0.81	1.14	1.41	0.40	55.2
Aggregate -simulated	-0.34	0.34	0.76	1.13	1.40	0.46	70.5
Constant price and unit costs							
Cohort*	-2.00	-1.13	-1.17	0.04	1.49	0.5	91.4
Aggregate -historical	-0.84	0.20	1.20	2.19	3.18	0.5	48.3
Aggregate -simulated	-1.75	-0.39	0.59	1.58	2.59	0.5	90.8

* Additional grid point, φ_6=3.32.

Optimal fishing capacities are higher for the aggregate models with their more aggressive harvests at high stock levels, see e_0 in Table 4. Aggregate-simulated has a somewhat higher optimal capacity than aggregate-historical consistent with its higher surplus growth curve, see Figure 2. The capacity in aggregate-simulated is 24 percent higher than in the cohort model. Also note that the total value of the fishery, W in Table 4, is quite close for the aggregate-simulated model and the cohort model. Aggregate-historical has a considerably lower value consistent with its lower surplus growth curve.

Then we turn to a similar comparison of policies when the fish price is set constant (p_1=0) and where unit variable costs do not vary with capacity utilization (α=0). When α=0 it is no longer meaningful to search for the optimal fishing capacity, hence we simply set e_0 equal to 0.5 million tons per year. The resulting policies are shown in Figure 4 and Table 4.

Both aggregate models (thin lines) now show the well known constant target escapement policy, Reed (1979). When biomass is above the target, harvest is set such that the biomass is reduced exactly to the target. The difference between the policies for aggregate-historical (thin solid) and aggregate-simulated (thin dashed) is more pronounced than when fish prices and unit costs were variable. The distance between the lines reflects the distance between the peaks for the respective surplus growth curves in Figure 2.

The policy for the cohort model[5] (thick line) portrays the "pulse-fishing" property found in studies of cohort models, Spulber (1983). No fishing takes place for biomasses below approximately 4 million tons. For higher biomasses, the harvest increases faster than the

5 The pulse fishing strategy is determined with somewhat lower precision than the other policies since the policy becomes sensitive to the exact distribution of high fish stocks. However, all the standard deviations for average policy parameters over 20 policy searches (N=20) are less than 0.12.

biomass, such that the harvest reduces the biomass to a level below 4 million tons. Then it is likely that a period with no fishing is needed before stocks again exceed 4 million tons and harvesting is again allowed. In this case the cohort model policy is very different from the aggregate model policies.

Price elasticity and increasing marginal costs have similar effects. At low harvest rates, the fishery becomes more profitable, prices increase and unit costs decrease. At high harvest rates the opposite happens. This explains why the policies in Figure 3 are less steep than the policies in Figure 4. Hence, the feedback through price and unit costs are important for policies.

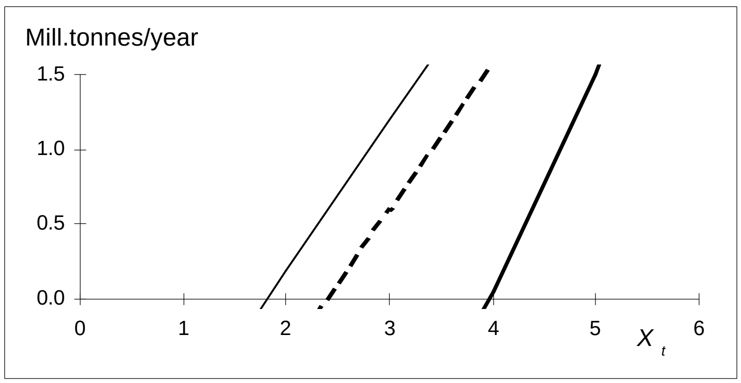

Figure 4: Policies for models with constant price and unit costs: Thick line: policy for cohort model. Thin solid line: policy for aggregate-historical. Thin dashed line: policy for aggregate-simulated.

Why is the effect of prices and unit costs different for the two models? A rather superficial, however simple answer is that the cohort policy is much more sensitive to the two feedbacks. There is almost no difference between the expected values W in Table 4 between the cohort policy and the aggregate -simulated policy. This gives a rough indication that there is little to gain by choosing a pulse-fishing strategy when there is no economic feedback. Consistent with this, the policy changes quickly as the feedback is introduced.

7. Conclusions

We have compared a cohort model of a cod fishery to a more simple aggregate surplus growth model of the same fishery. We have argued that an appropriate comparison in this case should involve policy sensitivity analysis, both because policy is the ultimate purpose of the models and because alternative methods to judge model simplification seem less appropriate. To estimate the sensitivity of harvesting policies to model choice, we have used stochastic optimization in policy space (SOPS) to find policies. Without optimization the comparisons would have included an element of judgment because it is not trivial to find optimal policies.

If price and unit costs are set constant in both models, harvesting policies are found to differ considerably. The apparent conclusion is that the choice of model is important.

However, even though the constant target escapement policy found for the aggregate model and the pulse-fishing strategy found for the cohort model have been analyzed in the economics literature, these solutions do not present themselves are very realistic due to the large variations they imply in fishing effort from year to year. Therefore, it is more interesting to focus on the comparison made of the two models when prices and unit costs are allowed to vary with effort and harvest. In this case the policies do not differ very much.

If the cohort model is considered correct, a loss of about 5 percent would result if the policy from the aggregate model is used in the cohort model rather than the optimal policy. However, it is not obvious that the cohort model is the better one of the two. This implies that we cannot say for sure that there is a significant difference between the two policy recommendations.

Considering the costs of developing large models and the reduced possibilities for learning and information diffusion using large models, the conclusion could be that the aggregate model is the best choice. The smaller and less significant the fish stock is economically, the more correct this conclusion will be. This conclusion may not carry over to multi-species models and to situations where fishing selectivities become part of the harvesting policy. There is also the possibility that the cohort model has a larger potential for improvement than the aggregate model. Further research is needed to answer these questions. However, until improved models become available for practical use, aggregate models may be preferable.

If aggregate models are used for policy making (quota setting), a couple of adjustments may be appropriate. Since aggregate model parameters are sensitive to historical harvesting policies, parameter estimates should ideally be made from time periods with near-optimal policies in place. If one suspects historical over- or under-fishing, one may consider a certain adjustment of the parameters suggested by the detailed results of this study. Furthermore, if one thinks that the cohort model is the better model after all, one might also consider minor adjustments in policies derived from aggregate models in the direction of lower quotas at high fish stocks.

In many cases formal policy sensitivity analysis is not likely to be a practical choice. Over time, better tools for policy optimization, be it SOPS or other methods, will make it more practical to perform this type of sensitivity analysis. Irrespective of technical difficulties, the qualitative insights from policy sensisitivity analysis can be useful for practical modeling. Keeping in mind that it is policy sensitivity that really matters, could help modelers throw out model details that are of little or no importance for final policy recommendations, even though these details may have some impact on model behavior.

References

Andersen, D.F. (1980). “How Differences in Analytic Paradigms Can Lead to Differences in Policy Conclusions.” Pp. 61-75 in *Elements of the System Dynamics Method*, edited by Randers, J. Cambridge: MIT Press.

Argyris, C., and Schön, D. (1978). *Organizational learning: A theory of action perspective*: Addison-Wesley.

Ault, J.S., and Olson, D.B. (1996). “A multicohort stock production model.” *Transactions of the American Fisheries Society* **125**(3):343-363.

Bertsekas, D.P., and Tsitsiklis, J.N. (1996). *Neuro-Dynamic Programming*. Belmont Massachusetts: Athena Scientific.

Brekke, K.A., and Moxnes, E. (forthcoming). “Do numerical simulation and optimization results improve management? Experimental evidence.” *Journal of Economic Behavior and Organization* .

Clark, C.W. (1985). *Bioeconomic Modelling and Fisheries Management.* New York: John Wiley & Sons.

Eberlein, R.L. (1989). “Simplification and understanding of models.” *System Dynamics Review* **5**(1):51-68.

Ermoliev, Y., and Wets, R.J.-B. (1988). “Stochastic programming, an introduction.” Pp. 1-32 in *Numerical Techniques for Stochastic Optimization*, edited by Ermoliev, Y. and Wets, R.J.-B. Berlin: Springer-Verlag.

Gulland, J.A. (1991). “Under what conditions will multispecies models lead to better fisheries management?” *ICES Mar. Sci. Symp.* **193**:348-352.

Mendelssohn, R. (1978). “Optimal harvesting Strategies for Stochastic Single-Species, Multiage Class Models.” *Mathematical Biosciences* **41**:159-174.

Moxnes, E. (1998). “Overexploitation of renewable resources: The role of misperceptions.” *Journal of Economic Behavior and Organization* **37**(1):107-127.

Moxnes, E. (1999). “Near-to-optimal harvesting strategies for a stochastic multicohort fishery.” Report No. 57/99. Bergen: SNF.

Moxnes, E. (2000). “Not only the tragedy of the commons: misperceptions of feedback and policies for sustainable development.” *System Dynamics Review* **16**(4):325-348.

Moxnes, E. (forthcoming). “Uncertain measurements of renewable resources: Approximations, harvest policies, and value of accuracy.” *Journal of Environmental Economics and Management*.

Moxnes, E., Danell, Ö., Gaare, E., and Kumpula, J. (2002). “Reindeer husbandry: a practical decision-tool for adaptation of herds to rangelands.” forthcoming. Bergen, Norway: SNF.

Naqib, F., and Stollery, K. (1982). “Optimal control of a multi-cohort fishery.” *Economic Letters* **10**:385-390.

Polyak, B.T. (1987). *Introduction to optimization*. New York: Optimization Software, Publications Division.

Reed, W.J. (1979). “Optimal escapement levels in stochastic and deterministic models.” *Journal of Environmental Economics and Management* **6**:350-363.

Rogers, E.M. (1995). *Diffusion of innovations*. New York: The Free Press.

Schaefer, M.B. (1954). “Some aspects of the dynamics of populations important to the management of commercial marine fisheries.” *Bulletin of the Inter-American Tropical Tuna Commission* **1**:25-56.
Senge, P. (1973). “Issues in social modeling.” Transcripts of faculty seminars 1972-1973. Boston: Sloan School of Management, MIT.
Spulber, D.F. (1983). “Pulse-fishing and Stochastic Equilibrium in the Multicohort Fishery.” *Journal of Economic Dynamics and Control* **6**:309-322.
Spulber, D.F. (1985). “The Multicohort Fishery Under Uncertainty.” *Marine Resource Economics* **1**(3):265-282.
Walters, C.J. (1986). *Adaptive Management of Renewable Resources*. New York: Macmillian.

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