Optimal Dynamical Balance of Raw Materials – Some Concept of Embedding Optimization in Simulation on System Dynamics Models and Vice Versa

Elżbieta Kasperska, Elwira Mateja-Losa, Damian Słota
Institute of Mathematics
Silesian University of Technology
Kaszubska 23, Gliwice 44-100, Poland
e-mail: elakaspe@polsl.gliwice.pl, elwimat@polsl.gliwice.pl, damslota@polsl.gliwice.pl

Abstract

The purpose of this paper is to present the optimal dynamic balance of raw materials in two formulations. The model DYNBALANCE(3-1) appears in two cases, named I and II. The main idea of case I of model DYNBALANCE(3-1)concist in optimal control of supplying three raw materials according to productivity of three technologies, by taking into consideration possibilities of sources of raw materials and actual demand for product on market. This optimal control lies in optimal balancing of raw materials, according to plans: cost of raw materials, cost of production and according to forrecast of demand. The objective function in optimizing process (in Coyle's sense) is consisted of three main components and one component which has "penalty" function. The optimization experiments were performed usying COSMOS package (Computer – Oriented System Modelling Optimisation Software), which is software tool automatically linking a dynamic simulation model to an optimization package. In this paper authors present the whole structure of model DYNBALANCE(3-1) case I, with the legend and list of variables and parameters. The equations of the model reader can find in program output in Appendix B.

The main idea of case II of model DYNBALANCE(3-1) concist in replecement of problem of solving of balance of raw materials, which minimize the norm $||Ax - b||$ (at the condition $x \geqslant 0$) by the problem of solving pseudosolution of extended balance of raw materials, which minimize the norm $||\overline{A}x - \overline{b}||$ (without condition $x \geqslant 0$). The benefit of such solution of problem is that we can embedding optimization in simulation on System Dynamics model by using language Dynamo. In each step during simulation the program solves the x by the Legras formula: $(\overline{A}^T\overline{A})^{-1}\overline{A}^T\overline{b}$, step by step. Such a solution (balance of raw materials) we can named "dynamic", so the embedding the optimization in simulation, on System Dynamics models, has dynamical character.

Many interesting experiments were undertaken by authors and discussed in paper. At the end of paper some conclusions were formulated, specially concerning the problem of simplificationes of models.

Keywords: System Dynamics Method, Optimization Embedding in Simulation, Simulation during Optimization, Balance of Raw Materials.

1 Introduction

The idea of extending System Dynamics method was undertaking already by authors in paper [5]. The point was that some known methods of optimization can be embedding in simulation on System Dynamics model and vice versa. The authors were occupied with dynamical balance of production. Now, in this paper, the problem is opposite. The balance relates to three raw materials and one product. The two cases of model, called DYNBALANCE(3-1), were created by Kasperska and many interesting experiments were performed using Professional Dynamo 4.0 and COSMIC and COSMOS by Mateja-Losa and Słota.

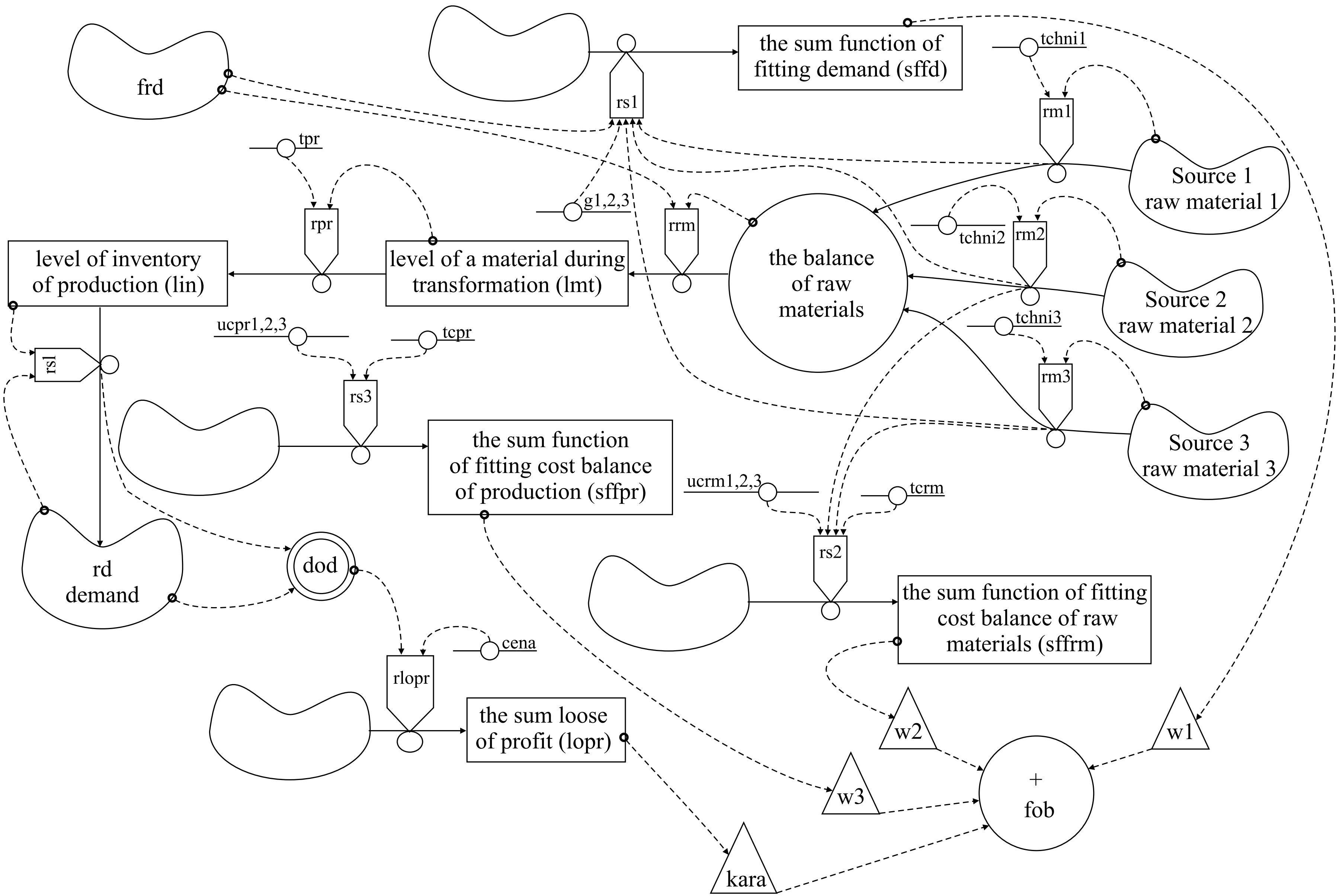


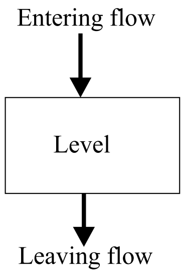
Figure 1. The optimal dynamics balance of raw materials (model DYNBALANCE(3-1))

2 Some examples of the optimal balance of raw materials

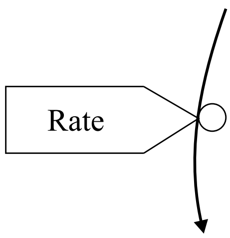
In this paper we present two cases of model DYNBALANCE(3-1). Generally speaking, in the case I the model has the structure, which allow optimization in sense of Coyle [2–4]. In the case II the model has the structure which allow embedding optimization in simulation on System Dynamics model, in the sense of authors.

Lets, at first, present on Figure 1 the structure of model DYNBALANCE(3-1) in case I. The graphic convention is based on Łukaszewicz' symbols [7, 8], supplemented by own idea of authors, and is explained in legend to the figure.

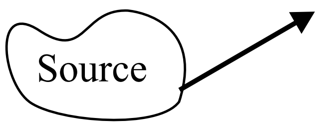
Legend to the Figure 1



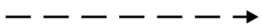
classical level in Forrester sense;



classical rate (action) in Forrester sense;

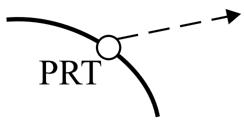


classical source (independent variable);

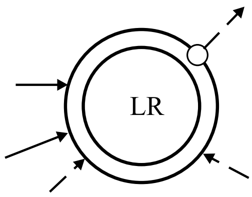


flow of information;

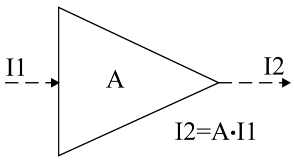
flow of materials;



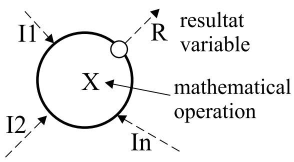
parameter;



symbol in double frame (local relationship in model, for example the optimal balance of raw material);



direct amplification (A – amplification coeficient);



auxiliary variable.

The main idea in case I of model DYNBALANCE(3-1) consist in optimal control of supplying three raw materials according to productivity of three technologies, by taking into consideration possibility of source of raw materials and actual demand for product on market. Optimal control lies in optimal balancing of raw materials, according to plans: cost of raw materials, cost of production and according to forrecast of demand. The objective function in optimizing process (in Coyle's sense [2–4]) is consisted of three main components and one component which has "penalty" function. Lets explain the variables and parameters, to clear the role of objective function.

Variables and parameters of model DYNBALANCE(3-1)

Levels:

- lmt – level of material during transformation;
- lin – level of inventory of production;
- $sffd$ – level which sum up the discrepancies between the mass plan of production (equal to forecasted DEMAND – independent variable in that model) and its realization (production are product from three raw materials, according to three technologies) , this level authors named the "sum function of fitting demand";
- $sffrm$ – level which sum up the discrepancies between the planed cost of raw materials (total cost plan: parameter $tcrm$) and its realization (which depends on unitary cost of raw materials and actual calculated values of three flows of raw materials);
- $sffpr$ – level which sum up the discrepancies between the planed cost of production (technology and peaples) (total cost plan: parameter $tcpr$) and its realization (which depends on unitary cost of production from three technologies and actual calculated values of three flows of raw materials);
- $lopr$ – level which sum up the loose of profits, when actual sale is lower then demand.

Rates:

- rrm – rate of raw material (in unit of production) which enter the level of material during transformation, is calculated by appropriate summing of optimised three flows of raw materials;
- $rm1$ – rate of raw material number 1 (which depends on optimal productivity of first technology and actual cindition of source of raw material 1);
- $rm2$ – rate of raw material number 2 (which depends on optimal productivity of second technology and actual cindition of source of raw material 2);
- $rm3$ – rate of raw material number 3 (which depends on optimal productivity of third technology and actual cindition of source of raw material 3);
- rpr – rate of production (we assume that level of production (material during transformation) is delay of first order);
- rsl – rate of sale (depends on actual rate of demand and possibility of inventory of production);
- $rs1$ – rate of discrepancies between the mass plan of production and its actual realization (input to level $sffd$);
- $rs2$ – rate of discrepancies between the cost plan of raw materials and its actual realization (input to level $sffrm$);
- $rs3$ – rate of discrepancies between the cost plan of production and its actual realization (input to level $sffpr$);
- $rlopr$ – rate of loose of profits, when actual sale is lower then demand (we assume the constant price of product).

Auxiliaries:

$source1$	–	actual possibility of supply of source of raw material 1;
$source2$	–	actual possibility of supply of source of raw material 2;
$source3$	–	actual possibility of supply of source of raw material 3;
rd	–	demand for product (we assume sinusoidal character of curve of demand);
frd	–	forecasted demand for product (we assume sinusoidal character of curve of forecast);
dod	–	difference of demand from sale (measures the difference between actual demand and its realization (sale));
fob	–	objective function (sum of $sffd$, $sffrm$, $sffpr$ and penalty which measures the loos of profits, when the actual sale is lower then demand; The components of the sum have weight factors.

Parameters:

$q1$	–	product multiplier from the first raw material;
$q2$	–	product multiplier from the fsecond raw material;
$q3$	–	product multiplier from the third raw material;
$tchn1$	–	productivity of first technology;
$tchn2$	–	productivity of second technology;
$tchn3$	–	productivity of third technology;
tpr	–	time of production;
po	–	parameter step of input (demand);
$p1$	–	parameter of amplitude of sinusoidal input (demand);
$perd$	–	parameter period of input (demand);
$w1$	–	weight factor for component $sffd$ of function *fob*;
$w2$	–	weight factor for component $sffrm$ of function *fob*;
$w3$	–	weight factor for component $sffpr$ of function *fob*;
$p2$	–	parameter of amplitude of sinusoidal characteristic of forecast of demand (frd);
$ucr1$	–	unitary cost of raw material 1;
$ucr2$	–	unitary cost of raw material 2;
$ucr3$	–	unitary cost of raw material 3;
$ucpr1$	–	unitary cost of production from raw material 1 (technology 1);
$ucpr2$	–	unitary cost of production from raw material 2 (technology 2);
$ucpr3$	–	unitary cost of production from raw material 3 (technology 3);
$tcrm$	–	total cost of raw materials (plan);
$tcpr$	–	total cost of production (technology and people) (plan);
$cena$	–	price of product on market;
$kara$	–	coeficient of gain of loose of profit (weight factor for *lopr*) (see: objective function).

The optimization experiments were performed usying COSMOS [1]. COSMOS (the Computer – Oriented System Modelling Optimisation Software) is a software tool which automatically links a dynamics simulation model to an optimisation package. The type of optimisation we used were so called "Direct Optimisation", which requires:

– an objective function,

– a group of parameters to be searched,

– the permissible range for each parameters.

The objective function ia a variable which summarise the undesirable characteristic of the system. The parameters are usually constant in the model. The author of COSMOS package, prof. G. Coyle, didn't discover which optimisation technique were used in package. We suppose that it could be, for example, method COMPLEX for nonlinear optimisation. To apply COSMOS we not need the knowledge of method but only some rules of optimisation dialoque, which has form like on Figure 2.

```
Name of Objective Function

Minimise or maximise?

Parameters and ranges

Optimisation control

     Parameter value printed

     Length of simulation

     Number of iteration

     Step multiplier

     Number of output lines

Type of optimisation

     Base vector

     Simplifier

     Planning horizon

     If none of these them

     Direct optimisation

Main command mode

     or

Rerun mode
```

Figure 2. General form of optimization dialoque (from Coyle [1])

Now, lets pay attention on case II of model DYNBALANCE(3-1). Some of the variables and parameters don't require the explanation (see: list of variables and parameters of model, case I), but there are some new elements (see Figure 3). Theirs roles are easily visible on Figure 2 and in Appendix B.

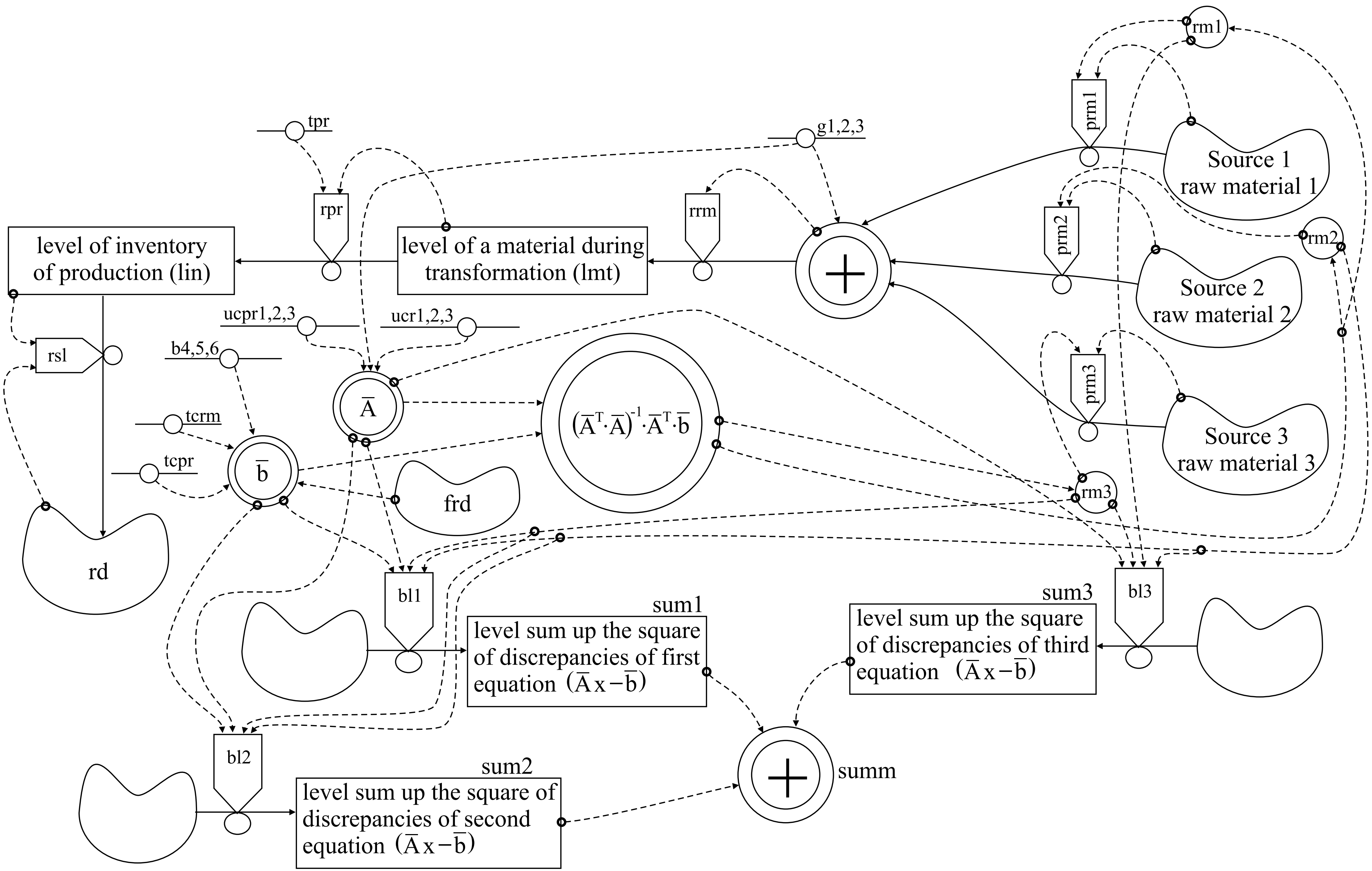


Figure 3. Structure of model DYNBALANCE(3-1) – case II (with the elements of optimization in sense of authors)

The main idea of the method used is the replacement of the one problem named (1) by the second problem named (2). The problem (1) consists in minimizing the norm $||Ax - b||$ by the condition $x \geqslant 0$. The matrix A has dimensions 3×3, the matrixes x and b have dimensions 3×1. The norm takes the form:

$$||Ax - b|| = \sqrt{\sum_{i=1}^{3} \left(\sum_{j=1}^{3} a_{ij} x_j - b_i \right)^2} .$$

In our example the matrixes A and b are:

$$A = \begin{pmatrix} g_1 & g_2 & g_3 \\ ucr_1 & ucr_2 & ucr_3 \\ ucpr_1 & ucpr_2 & ucpr_3 \end{pmatrix}, \qquad b = \begin{pmatrix} frd \\ tcrm \\ tcpr \end{pmatrix}.$$

The looked up solution x is:

$$x = \begin{pmatrix} rm_1 \\ rm_2 \\ rm_3 \end{pmatrix}.$$

The problem (2) consists in minimizing norm $||\overline{A}x - \overline{b}||$ without limitation on x. Matrix $\overline{A}$ is created by extending matrix A by matrix E, where:

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Matrix $\overline{b}$ is created by extending matrix b by three elements of large value (b_4, b_5, b_6). In our example the matrixes $\overline{A}$, $\overline{b}$ and x are:

$$\overline{A} = \begin{pmatrix} g_1 & g_2 & g_3 \\ ucr_1 & ucr_2 & ucr_3 \\ ucpr_1 & ucpr_2 & ucpr_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \overline{b} = \begin{pmatrix} frd \\ tcrm \\ tcpr \\ b_4 \\ b_5 \\ b_6 \end{pmatrix}, \qquad x = \begin{pmatrix} rm_1 \\ rm_2 \\ rm_3 \end{pmatrix}.$$

In literature of mathematics we can find (see [6]) the statement that: minimizig the norm of system $||\overline{A}x - \overline{b}||$ is compensated by calculating matrix x by the formula:

$$x = (\overline{A}^T \cdot \overline{A})^{-1} \cdot \overline{A}^T \cdot \overline{b}. \qquad (\star)$$

So, is the idea applied by authors in presented example. The benefits of such solution of problem are that we can embedding optimization in simulation on System Dynamics model by using language Dynamo. The technical details reader can find in program in Appendix A.

In each step during simulation, variable *frd* is changed, so the program simulats the solving of x, by the formula $(\star)$, step by step during horizon of simulation. Such the solution x we can named "dynamic". So the embedding the optimization in simulation on System Dynamics model, has dynamical character.

Table 1.

Parameter	Final value	Original value	Lower limit	Upper limit
$tchni1$	15.182	20.0	0	40
$tchni2$	0.000	10.0	0	40
$tchni3$	6.045	20.0	0	40
Initial value of objective function *fob*				$0.8425 \cdot 10^{10}$
Final value of objective function *fob*				$0.1421 \cdot 10^{9}$
Final value of $sffd$				$53.3649 \cdot 10^{4}$
Final value of $sffrm$				$12.5733 \cdot 10^{7}$
Final value of $sffpr$				$38.3082 \cdot 10^{5}$
Final value of $lopr$				$65.8087 \cdot 10^{5}$

3 The results of simulation and optimization

At first, let us present some of the results of the experiments that have been obtained from simulation during optimization (using COSMIC and COSMOS).

Tabel 1 compiles the results of the first experiment. The number of iteration was 30.

Some of these results are presented in an illustrative form in Figures 4, 5 and 6.

Now, let us present some of the results of the experiments that we achieved from embedding optimization in simulation (using Professional Dynamo 4.0). In Figures 7, 8 and 9 the characteristics of rates: $rm1$, $rm2$, $rm3$ and characteristics of variables $sum1$, $sum2$, $sum3$ and $summ$ are shown. It is interesting to compare the obtained value of objective function *fob* (in experiments in COSMOS) and the obtained value of variable $summ$ (in experiments in Dynamo). These variables are derived from two different philosophies. In one word, we can say that: *fob* like as the classic objective function measures some aspects of the behavior of the system in the whole simulation horizon. On the contrary, the variables $sum1$, $sum2$, $sum3$ measure, "step by step", the differences between the equations of balance and, surely, the variable $summ$ is the summarizer of this differences in the whole horizon. To compare the dynamics of both: *fob* and $summ$, see Figures 4 and 9.

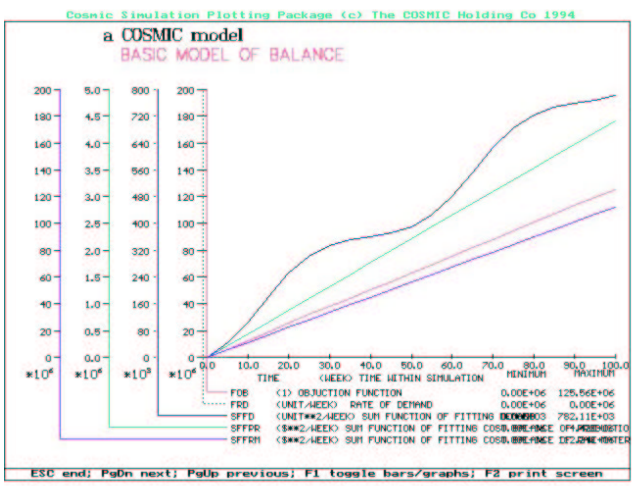


Figure 4. The dynamics of the characteristics of variables $sffd$, $sffpr$, $sffm$ and *fob* (experiments in COSMOS)

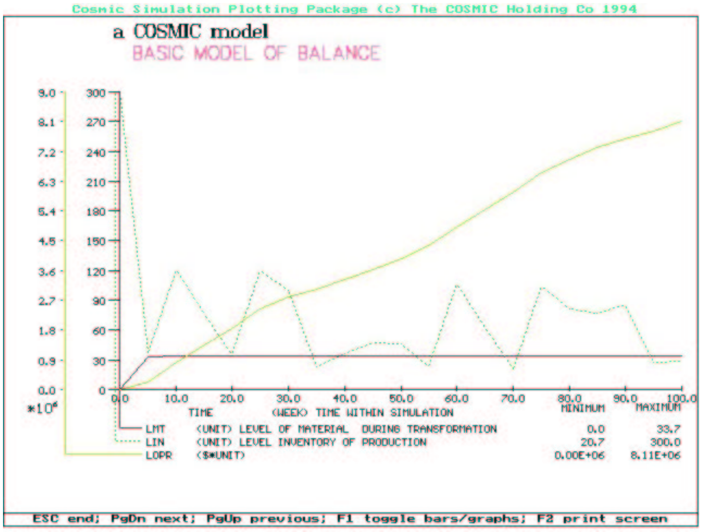


Figure 5. The dynamics of the characteristics of main levels of system (experiments in COSMOS)

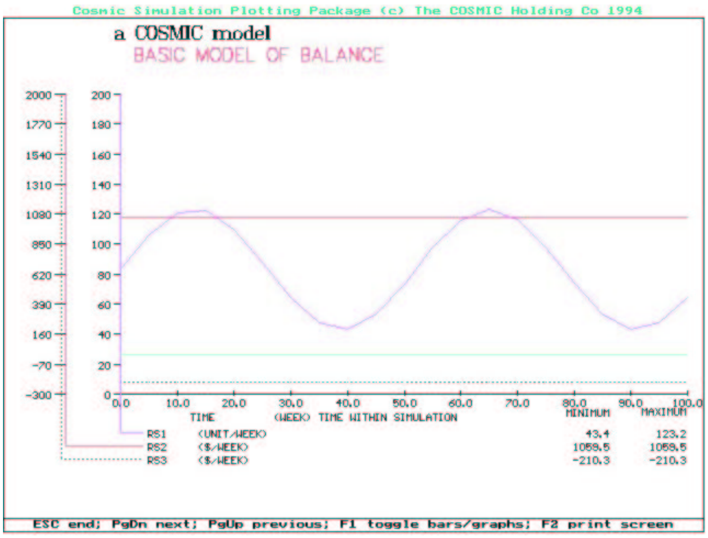


Figure 6. The dynamics of the characteristics of variables $rs1$, $rs2$, and $rs3$ (experiments in COSMOS)

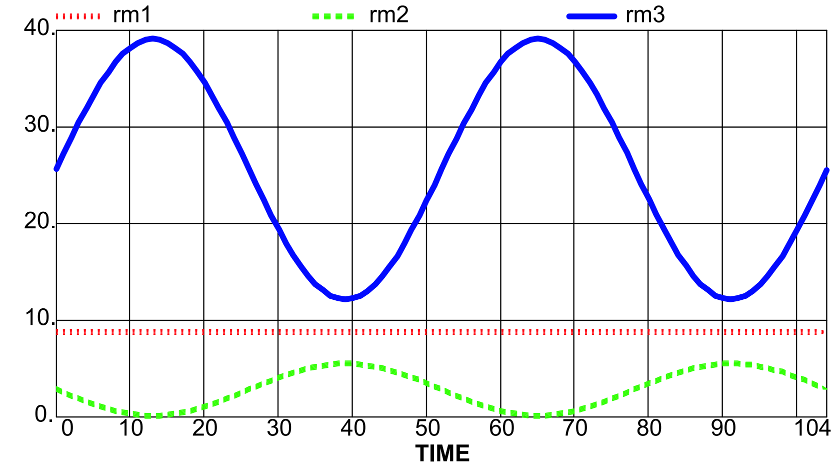


Figure 7. The dynamics of the characteristics of variables $rm1$, $rm2$ and $rm3$ (experiments in Professional Dynamo 4.0).

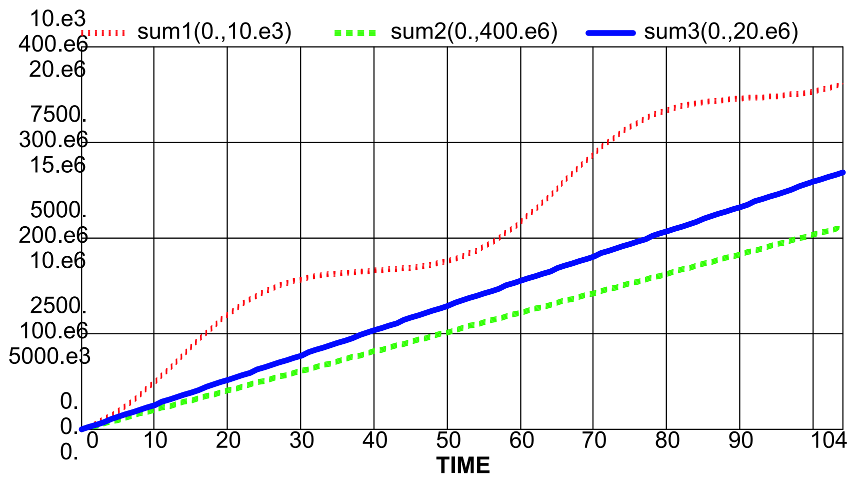


Figure 8. The dynamics of the characteristics of variables $sum1$, $sum2$ and $sum3$ (experiments in Professional Dynamo 4.0).

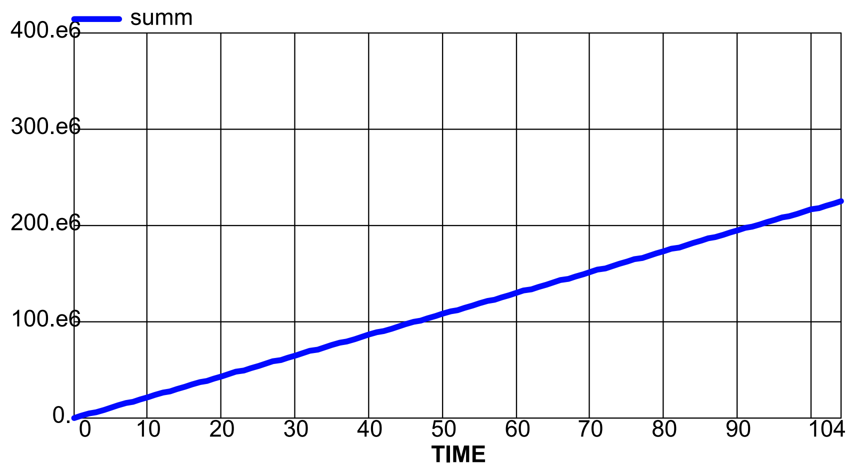


Figure 9. The dynamics of the characteristics of variable $summ$ (experiments in Professional Dynamo 4.0).

The economics interpretation of the results presented on Figure 7 we try to explain on example of the value of variables $rm1$, $rm2$ and $rm3$ in time "day 35". The variables takes the values:

$$\begin{aligned} rm1 &= 8.75, \\ rm2 &= 5.22, \\ rm3 &= 13.74. \end{aligned}$$

The first simulated equation of system $\overline{R} = \overline{A}x - \overline{b}$ gives:

$$8.75 + 2 \cdot 5.22 + 3 \cdot 13.74 - 64.62 = -4.21, \qquad (I)$$

the second equation gives:

$$100 \cdot 8.75 + 50 \cdot 5.22 + 10 \cdot 13.74 - 2700 = 1273.6, \qquad (II)$$

the third equation gives:

$$500 \cdot 8.75 + 500 \cdot 5.22 + 100 \cdot 13.74 - 8000 = 359. \qquad (III)$$

The result (I) has such the meaning that optimizing solution requires the limitation of production (in relation to plan (forecasted) values). The result (II) indicates that optimized solution gives cheaper costs of raw material (the value of "savings" is 1273.6). The result (III) has the meaning that optimizing solution (in sense of norm $||\overline{A}x - \overline{b}||$), gives the little expensive cost of production (peoples and technology, the value of 359). In summary, the optimizing solution allows to save the value:

$$1273.6 - 359.0 = 914.6.$$

The one problem is that the limitation of production can lead to decrease of inventory and in consequences, sometimes, can make impossible to satisfy the demand on market. But such is the benefit of connecting the optimization with simulation on System Dynamics models, that the simulation allows to study the influences of optimized solution on dynamics of a whole system.

4 The concept of embedding optimization in simulation on System Dynamics models and vice versa – Conclusions

The relations between simulation and optimization are illustrated in Figures 10 and 11.

In block diagrams in Figure 12 and 13 the most important elements of both formulations are contained. The differencies between formulations may be easily observed.

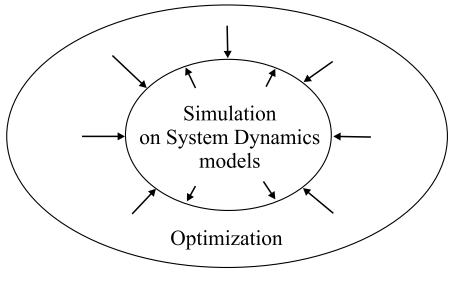


Figure 10. Embedding simulation in optimization (in Coyle's sense)

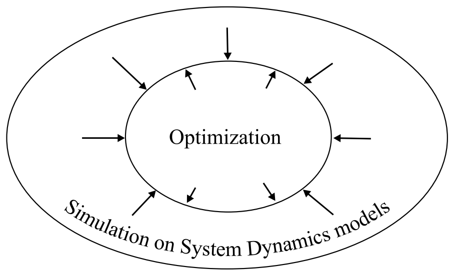


Figure 11. Embedding optimization in simulation on System Dynamics models (in the authors' understanding)

After presenting this synthetic form of the concept of embedding simulation in optimization and vice versa, we have come up to the following conclusions:

1^o Both formulations have different possibilities and require different tools for their realization.

2^o The actual, simulated in our experiments, version of model DYNBALANCE(3-1) has many simplification, for example:

a) productivities of technology 1, 2, 3 can be defined like tables (not parameters) and the optimization procedure will find the fully dynamic characteristic of optimal solution (supplies of three raw materials in case I);

b) sources of raw materials can be defined like any dynamics characteristic (not only "step" like in case I and II);

c) price of product (in case I) can be variable not parameter in whole horizon of simulation;

d) weight factors in objective function (in case I) can take optional values to model authors preferencies to the factors pf objective function (of course the preferencies can change in horizon of simulation).

3^o The minimizing of objective function *fob* in case I of model DYNBALANCE(3-1) is not the only possibility to ask the question of optimization in problem of balance of raw materials. The contrary we can formulate the problem in oposite way: maximizing the profit from sale at the minimum discrepancies from plans ("mass" plan, cost plans).

4^o The tools for simulation experiments, chosen by authors, seems to be adequate to taking the problem of optimization in both cases. Professional Dynamo 4.0 make possible to optimizing in simulation on System Dynamics models, and COSMIC and COSMOS allows to embedding simulation in optimization in Coyle's sense.

5^o Authors are at the begining of the way of incorporation the other methods with System Dynamics and expects the discussion on that subject.

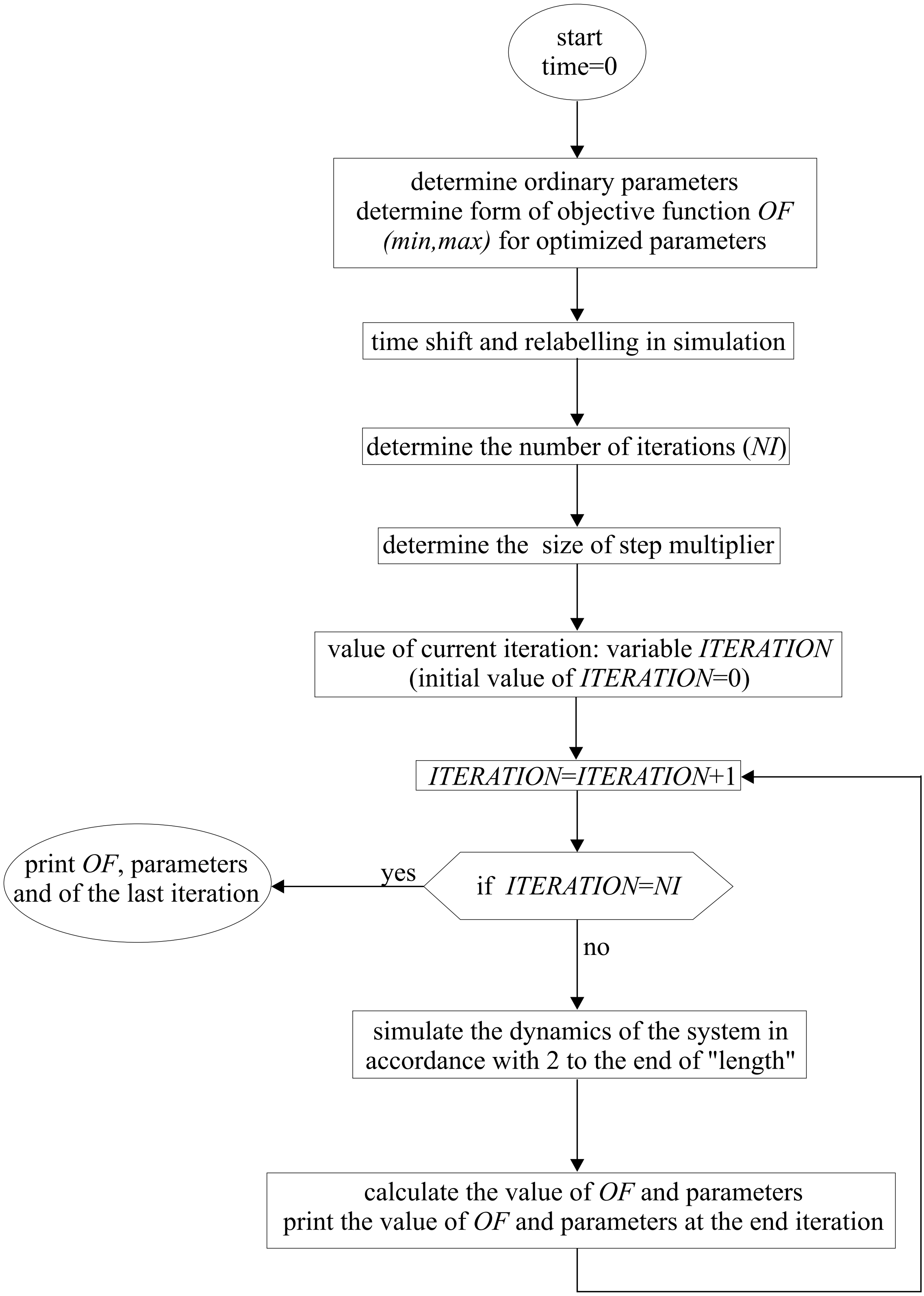


Figure 12. Block diagram of simulation during optimization

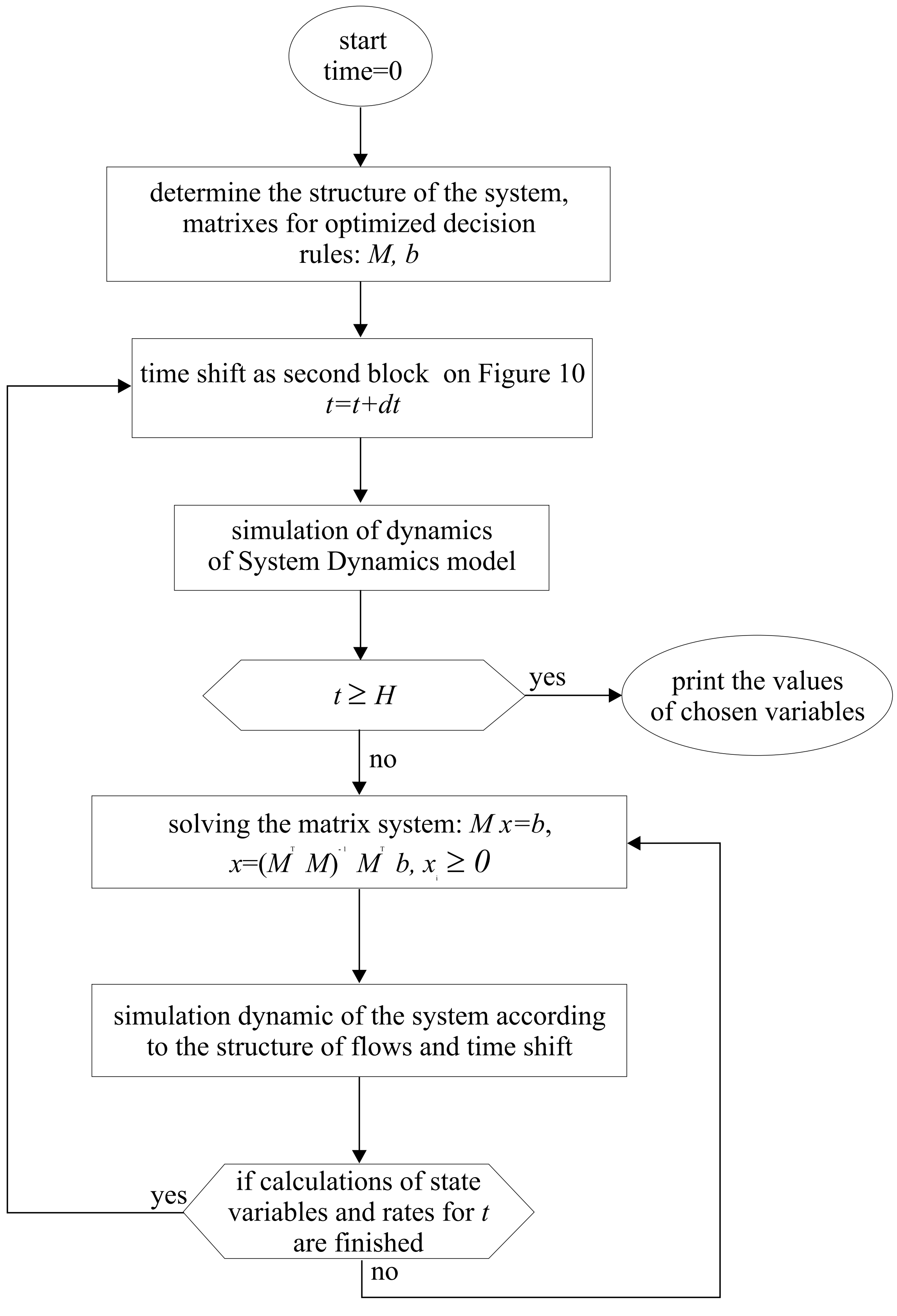


Figure 13. Block diagram of embedding optimization in simulation on System Dynamics models

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Appendix A. Program in DYNAMO

```
*  Balance of 3 raw materials
note
note  level of raw material during transformation
note
n lmt=300
l lmt.k=lmt.j+dt*(rrm.jk-rpr.jk)
note
note input rate to lmt (rrm)
note
r rrm.kl=g1*prm1.k+g2*prm2.k+g3*prm3.k
c g1=1
c g2=2
c g3=3
note
note proofed rate of 1st raw material (rm1)
note proofed rate of 2nd raw material (rm2)
note proofed rate of 3rd raw material (rm3)
note
a prm1.k=clip(rm1.k,source1.k,source1.k,rm1.k)
a prm2.k=clip(rm2.k,source2.k,source2.k,rm2.k)
a prm3.k=clip(rm3.k,source3.k,source3.k,rm3.k)
a source1.k=step(100,0)
```

```
a source2.k=step(100,0)
a source3.k=step(100,0)
note
note output rate from lmt (rpr)
note
r rpr.kl=lmt.k/tpr
note
note time of production (tpr)
note
c tpr=2
note
note level of inventory of production (lin)
note
n lin=0
l lin.k=lin.j+dt*(rpr.jk-rsl.jk)
note
note output rate from lin - rate of sale (rsl)
note
r rsl.kl=clip(0,rd.k,0,lin.k)
note
note rate of demond (rd)
note
a rd.k=p0+p1*sin((6.28*time.k)/perd)
c p0=100
c p1=30
c perd=52
note
note
note
a frd.k=p0+p2*sin((6.28*time.k)/perd)
c p2=40
note
note unit cost of production of 1st raw material (ucr1)
note unit cost of production of 2nd raw material (ucr2)
note unit cost of production of 3rd raw material (ucr3)
note
c ucr1=100
c ucr2=50
c ucr3=10
note
note unit cost of production from 1st raw material (ucpr1)
note unit cost of production from 2nd raw material (ucpr2)
note unit cost of production from 3rd raw material (ucpr3)
note
c ucpr1=500
c ucpr2=500
c ucpr3=100
note
note total cost of raw material - plan (tcrm)
note total cost of production - plan (tcpr)
```

```
note
c tcrm=2700
c tcpr=8000
note
note first three row of matrix a
note
a a11.k=g1
a a12.k=g2
a a13.k=g3
a a21.k=ucr1
a a22.k=ucr2
a a23.k=ucr3
a a31.k=ucpr1
a a32.k=ucpr2
a a33.k=ucpr3
note
note vector b
note
a b1.k=frd.k
a b2.k=tcrm
a b3.k=tcpr
a b4.k=8000
a b5.k=80000
a b6.k=16000
note
note vector bb=at.b
note
a bb1.k=a11.k*b1.k+a21.k*b2.k+a31.k*b3.k+b4.k
a bb2.k=a12.k*b1.k+a22.k*b2.k+a32.k*b3.k+b5.k
a bb3.k=a13.k*b1.k+a23.k*b2.k+a33.k*b3.k+b6.k
note
note matrix c=at.a
note
a c11.k=1+a11.k*a11.k+a21.k*a21.k+a31.k*a31.k
a c12.k=a11.k*a12.k+a21.k*a22.k+a31.k*a32.k
a c13.k=a11.k*a13.k+a21.k*a23.k+a31.k*a33.k
a c21.k=c12.k
a c22.k=1+a12.k*a12.k+a22.k*a22.k+a32.k*a32.k
a c23.k=a12.k*a13.k+a22.k*a23.k+a32.k*a33.k
a c31.k=c13.k
a c32.k=c23.k
a c33.k=1+a13.k*a13.k+a23.k*a23.k+a33.k*a33.k
note
note  determinant of matrix c=at.a
note
a detc.k=-c13.k*c22.k*c31.k+c12.k*c23.k*c31.k+c13.k*c21.k*c32.k-^
  c11.k*c23.k*c32.k-c12.k*c21.k*c33.k+c11.k*c22.k*c33.k
note
note matrix d=Det[c]*Inverse[c]
note
```

```
a d11.k=c22.k*c33.k-c23.k*c32.k
a d12.k=c13.k*c32.k-c12.k*c33.k
a d13.k=-c13.k*c22.k+c12.k*c23.k
a d21.k=-c21.k*c33.k+c31.k*c23.k
a d22.k=-c13.k*c31.k+c11.k*c33.k
a d23.k=c13.k*c21.k-c11.k*c23.k
a d31.k=-c22.k*c31.k+c21.k*c32.k
a d32.k=c12.k*c31.k-c11.k*c32.k
a d33.k=-c12.k*c21.k+c11.k*c22.k
note
note rate of 1st raw material (rm1)
note rate of 2nd raw material (rm2)
note rate of 3rd raw material (rm3)
note
note rm=(d*bb)/Det[c]
note
a rm1.k=(bb1.k*d11.k+bb2.k*d12.k+bb3.k*d13.k)/detc.k
a rm2.k=(bb1.k*d21.k+bb2.k*d22.k+bb3.k*d23.k)/detc.k
a rm3.k=(bb1.k*d31.k+bb2.k*d32.k+bb3.k*d33.k)/detc.k
note
note
note
a bl1.k=(b1.k-a11.k*rm1.k-a12.k*rm2.k-a13.k*rm3.k)**2
a bl2.k=(b2.k-a21.k*rm1.k-a22.k*rm2.k-a23.k*rm3.k)**2
a bl3.k=(b3.k-a31.k*rm1.k-a32.k*rm2.k-a33.k*rm3.k)**2
note a bl4.k=(b4.k-rm1.k)**2
note a bl5.k=(b5.k-rm2.k)**2
note a bl6.k=(b6.k-rm3.k)**2
note a functio.k=bl1.k+bl2.k+bl3.k+bl4.k+bl5.k+bl6.k
note a fun13.k=bl1.k+bl2.k+bl3.k
note
note
l sum1.k=sum1.j+dt*bl1.j
l sum2.k=sum2.j+dt*bl2.j
l sum3.k=sum3.j+dt*bl3.j
n sum1=0
n sum2=0
n sum3=0
note
a summ.k=(sum1.k+sum2.k+sum3.k)
note
note  parameters of simulation
note
spec length=104/dt=1/savper=1
save rm1,rm2,rm3,sum1,sum2,sum3,summ
```

Appendix B. Program in COSMOS

```
note ================================
note balance of three raw materials
note ================================
l lmt.k=lmt.j+dt*(rrm.jk-rpr.jk)
n lmt=0
r rrm.kl=q1*rm1.kl+q2*rm2.kl+q3*rm3.kl
c q1=1
c q2=2
c q3=3
r rm1.kl=clip(tchn1,source1.k,source1.k,tchn1)
a source1.k=step(100,0)
c tchn1=20
r rm2.kl=clip(tchn2,source2.k,source2.k,tchn2)
a source2.k=step(100,0)
c tchn2=10
r rm3.kl=clip(tchn3,source3.k,source3.k,tchn3)
a source3.k=step(100,0)
c tchn3=20
r rpr.kl=lmt.k/tpr
c tpr=2
l lin.k=lin.j+dt*(rpr.jk-rsl.jk)
n lin=300
r rsl.kl=clip(0,rd.k,rd.k*dt,lin.k)
a rd.k=p0+p1*sin(6.28*time.k/perd)
c p0=100
c p1=30
c perd=52
c w1=1
c w2=1
c w3=1
c scale1=1
c scale2=1
c scale3=1
c scale4=1
l sffd.k=sffd.j+dt*((rs1.jk)**INT(2))
n sffd=wp1
c wp1=0
l sffrm.k=sffrm.j+dt*((rs2.jk)**INT(2))
n sffrm=wp2
c wp2=0
l sffpr.k=sffpr.j+dt*((rs3.jk)**INT(2))
n sffpr=wp3
c wp3=0
r rs1.kl=frd.k-q1*rm1.kl-q2*rm2.kl-q3*rm3.kl
r rs2.kl=tcrm-ucr1*rm1.kl-ucr2*rm2.kl-ucr3*rm3.kl
r rs3.kl=tcpr-ucpr1*rm1.kl-ucpr2*rm2.kl-ucpr3*rm3.kl
a frd.k=p0+p2*SIN(6.28*time.k/perd)
c p2=40
```

```
c ucr1=100
c ucr2=50
c ucr3=10
c tcrm=2700
c ucpr1=500
c ucpr2=500
c ucpr3=100
c tcpr=8000
c cena=1000
c kara=1
l lopr.k=lopr.j+dt*(rlopr.jk)
n lopr=0
a dod.k=clip(rd.k-rsl.kl,0,rd.k-rsl.kl,0)
r rlopr.kl=dod.k*cena
note =================================================
note objective function
note =================================================
a fob.k=((w1*sffd.k)/scale1)+((w2*sffrm.k)/scale2)
x +((w3*sffpr.k)/scale3)+((kara*lopr.k)/scale4)
note =================================================
note output control sector
note =================================================
c dt=1
c length=104
c prtper=5
c pltper=5
print 1)rm1,rm2,rm3,fob
print 2)lmt,lin,lopr,rlopr
print 3)sffd,sffrm,sffpr
print 4)rrm,rsl,rs1,rs2,rs3
plot fob=1,frd=2/sffd=3(0,1000)/sffpr=4(0,90E+08)/sffrm=5
note plot rm1=1,rm2=2,rm3=3
plot lmt=1(0,10000),lin=2/lopr=3
note plot sffd=1,sffrm=2,sffpr=3
plot rs1=1(-60,60)/rs2=2,rs3=3
run basic model of balance
note ===================================================
note definition of variable
note ===================================================
d lmt=(unit) level of material  during transformation
d rrm=(unit/week) rate of material
d rpr=(unit/week) rate of  material
d tpr=(week) time of production
d rd=(unit/week) actual rote of demand
d lin=(unit) level inventory of production
d sffd=(unit**2/week) sum function of fitting demand
d scale1=(unit**2/week)
d sffrm=($**2/week) sum function of fitting cost balance of raw material
d scale2=($**2/week)
d sffpr=($**2/week) sum function of fitting cost balance of production
```

```
d scale3=($**2/week)
d rs1=(unit/week)
d rs2=($/week)
d rs3=($/week)
d frd=(unit/week)  rate of demand
d rm1=(unit/week) source of raw material
d rm2=(unit/week) source of raw material
d rm3=(unit/week) source of raw material
d source1=(unit/week)
d source2=(unit/week)
d source3=(unit/week)
d tchn1=(unit/week) productivity of technology 1
d tchn2=(unit/week) productivity of technology 2
d tchn3=(unit/week) productivity of technology 3
d tcrm=($/week) total cost of raw material
d tcpr=($/week) total cost of production technology and people
d fob=(1) objective function
d q1=(1) fraction of material 1
d q2=(1) fraction of material 2
d q3=(1) fraction of material 3
d rsl=(unit/week)
note ========================================================
d perd=(week) parametr period of output
d time=(week) time within simulation
d length=(week) simulated period
note ========================================================
d p0=(unit/week) parametr step of input
d p1=(unit/week) parametr of amplitude of sinusoidal input
note d g1=(1)
note d g2=(1)
note d g3=(1)
d ucr1=($/unit)
d ucr2=($/unit)
d ucr3=($/unit)
d ucpr1=($/unit)
d ucpr2=($/unit)
d ucpr3=($/unit)
d cena=($)
d kara=(1)
d w1=(1)
d w2=(1)
d w3=(1)
d wp1=(unit**2/week)
d wp2=($**2/week)
d wp3=($**2/week)
d dod=(unit/week)
d rlopr=($*unit/week)
d lopr=($*unit)
d scale4=($*unit)
```