

Multi-Period Portfolio Assessment

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Abstract

This study explores the prospects of implementing a multi-period mathematical portfolio planning model for security trading in the Istanbul Stock Exchange(ISE), and compares the performance of different models for this purpose.

1. Introduction

Investment decisions involve questions about how to select the securities to invest in, how extensive the investment should be and when the investment is to be made; which is the core of portfolio planning. Most analytical approaches to portfolio management fall into two categories; (i) the mean-variance approach and (ii) scenario based approaches. The mean-variance approach uses return and risk as input parameters and tries to determine ‘optimal portfolios’. In this process, an optimal portfolio is defined either to be the portfolio with the minimal risk for a given level of return, or the portfolio showing maximal average return for a given level of risk. A common but unrealistic assumption in most portfolio management models is that we have a single period problem. Scenario models on the other hand, are multi period models that consider the long-term in order to improve investment performance. These are return-based, expected utility optimization models since they maximize an expected utility computed over a set of future scenarios that can be expected during the planning horizon. Sequential risk arises as part of the portfolio risk as time goes on and the models consider portfolio revision as part of the problem. Such problems are usually constructed as stochastic programming models. Scenarios may be generated for key economic parameters for a portfolio investor, such as future interest rates, foreign exchange rates or stock prices. They must depict the investor’s expectations about the future and also be realistic. Rather than resolving the stochasticity of future returns in two dimensions only – return and risk –

scenario models take many representative points on the probability distributions with the help of scenario generation equations. Stochastic programming provides a general purpose-modelling framework: it can address real-world features such as turnover constraints, transaction costs, risk aversion, limits on groups of assets and other considerations.

In this study we solve a multi-period portfolio model utilizing a set of scenario generation equations and compare the results of the base run of this model obtained for different planning horizons. We also solve this multi-period model using different objective functions representing risk preferences of the investor, and compare expected performance under different scenario paths.

The models used in this study are stochastic models with recourse. For such models, scenario trees for possible future outcomes of stochastic variables must first be constructed. A scenario can be defined as a single deterministic realization of future uncertainties of stochastic variables. It is therefore necessary first to determine the stochastic variables on which scenarios will be based. In our analysis interest rates paid for monthly bank deposits and the real security returns are chosen as the stochastic variables, since for a portfolio planning problem these are the most usual variables that affect decisions.

2. Scenario Tree generation

We first define our scenarios using a stochastic equation for only one economic factor. Possible security returns for each scenario are then calculated employing a general relationship equation between the interest rate and each security return. The utilization of stochastic equations helps us define relationships between security returns and the economic factor and generate scenarios in a structured manner, and prevents any bias that could be caused by the personal anticipations of the scenario generator.

The analysis is carried out by considering just a few outcomes for these random variables to exhibit the features of the model rather than enumerating all possible outcomes since the actual process of determining all possible outcomes would require consideration of social, political and other economical factors.

The stochastic equation defining monthly interest rates is as follows:

$$dr_t = (\mathbf{a} + \mathbf{b}r_t)dt + \mathbf{s}r_t dZ$$

where

r_t denotes the observed real rate of monthly bank deposits interest rates

dZ is a standard Brownian motion (Normal(0,1)), and

$\mathbf{q} = (\mathbf{a}, \mathbf{b}, \mathbf{s})$ is a vector of unknown parameters.

This equation was used by Mulvey et al.(1999) for defining short-term interest rate scenarios, where returns of fixed-income assets, such as treasury bonds, were assumed to depend on short-term interests. For our study, it implies that the change in the monthly interest rate at any time depends on the level of interest rate at that specific time, some parameters and a variance term.

In order to estimate the unknown parameters of this equation we apply the generalized method of moments (GMM), using the observed monthly bank deposit rate data between September 1990- September 2000. A discrete time approximation of the stochastic equation, is applied to this sample in order to estimate the unknown parameter vector, \mathbf{q} . Thus the stochastic differential equation becomes;

$$r_{t+1} = \mathbf{a} + (1 + \mathbf{b})r_t + \mathbf{s}r_t dZ$$

Denoting the random error, $\mathbf{s}r_t dZ$ as ε_t for simplicity, the stochastic equation becomes;

$$r_{t+1} - r_t = \mathbf{a} + \mathbf{b}r_t + \mathbf{e}_{t+1}$$

This equation now implies that, the interest rate of the next period depends linearly on the level of the present interest rate plus some random error term. In order to get good estimates for the unknown parameters, we use the normality assumption for dZ and solve the system of nonlinear equations obtained from the generalized method of moments;

$$J(\mathbf{q})'W_T(\mathbf{q})GMM_T(\mathbf{q}) = 0$$

where,

$W_T(\mathbf{q})$ is a positive symmetric weighting matrix;

$\theta = (\alpha, \beta, \sigma)$, and

$J(\mathbf{q})$ is the Jacobian Matrix of GMM

We solve this system of equations, using real world data taken from the Istanbul Stock Exchange and the monthly interest rates between 1990-2000 under different assumptions and obtain several different estimates of the parameter vector, θ . We take additional steps such as:

- In one of the solutions the interest rate values during the 1994 crisis are considered to be outliers and are excluded from the data set.

- In order to obtain a model that generates a meaningful series of interest rates, we restrict the values of unknown parameters (α, β, σ) , using lower and upper bounds and solve the system several times with different bounds. For each set of bounds we obtain different optimal values for the parameters.

The model that best fits the interest rate data is then selected by testing for normality of each model's random element, dZ . This is achieved by applying the models to forecast interest rates for the previous 120 periods, and then obtaining the random error element between the forecast and the actual values. As a result the stochastic equation for real interest rates can be written as:

$$r_{t+1} = 0.803 + (1 - 0.533)r_t + 1.599r_t dZ$$

Now we want to construct a scenario tree for the next T periods. We use the stochastic equation for generating the scenarios for periods t in the planning horizon starting from $t = 0$ (October 2000). At the beginning of October 2000 although we know the exact value of the nominal interest rate, the real rate of interest for the period is not known, since the inflation rate is unknown at the beginning. Thus the available data of nominal interest rate for the initial period cannot be used. Instead, the real rates of interest for every period including the initial one are determined for every scenario using the stochastic equation above. Our scenarios for October –December 2000 are assumed to evolve on a tri-nomial lattice of interest rates as shown in the scenario tree of Figure 1.

At instance t_0 (beginning of October 2000) three scenarios are anticipated, and by instance t_1 (beginning of November 2000) this uncertainty is resolved. We denote the three scenarios by s_0^O, s_0^N , and s_0^P , where O stands for optimistic, N stands for normal and P stands for pessimistic; although scenarios do not represent any optimistic or pessimistic anticipation of the future, we use such notation for convenience. At time t_1 three more scenarios are anticipated, s_1^O, s_1^N and s_1^P and so on. A complete path is denoted by a tuple of scenarios. For example there are nine paths from t_0 to t_2 denoted by the pairs $(s_0^O, s_1^O), (s_0^O, s_1^N), (s_0^O, s_1^P), (s_0^N, s_1^O), (s_0^N, s_1^N), (s_0^N, s_1^P), (s_0^P, s_1^O), (s_0^P, s_1^N), (s_0^P, s_1^P)$. It should be noted that for each node on the scenario tree there is a different interest rate value. In order to generate these interest rates we first define the scenarios and scenario paths formally as follows;

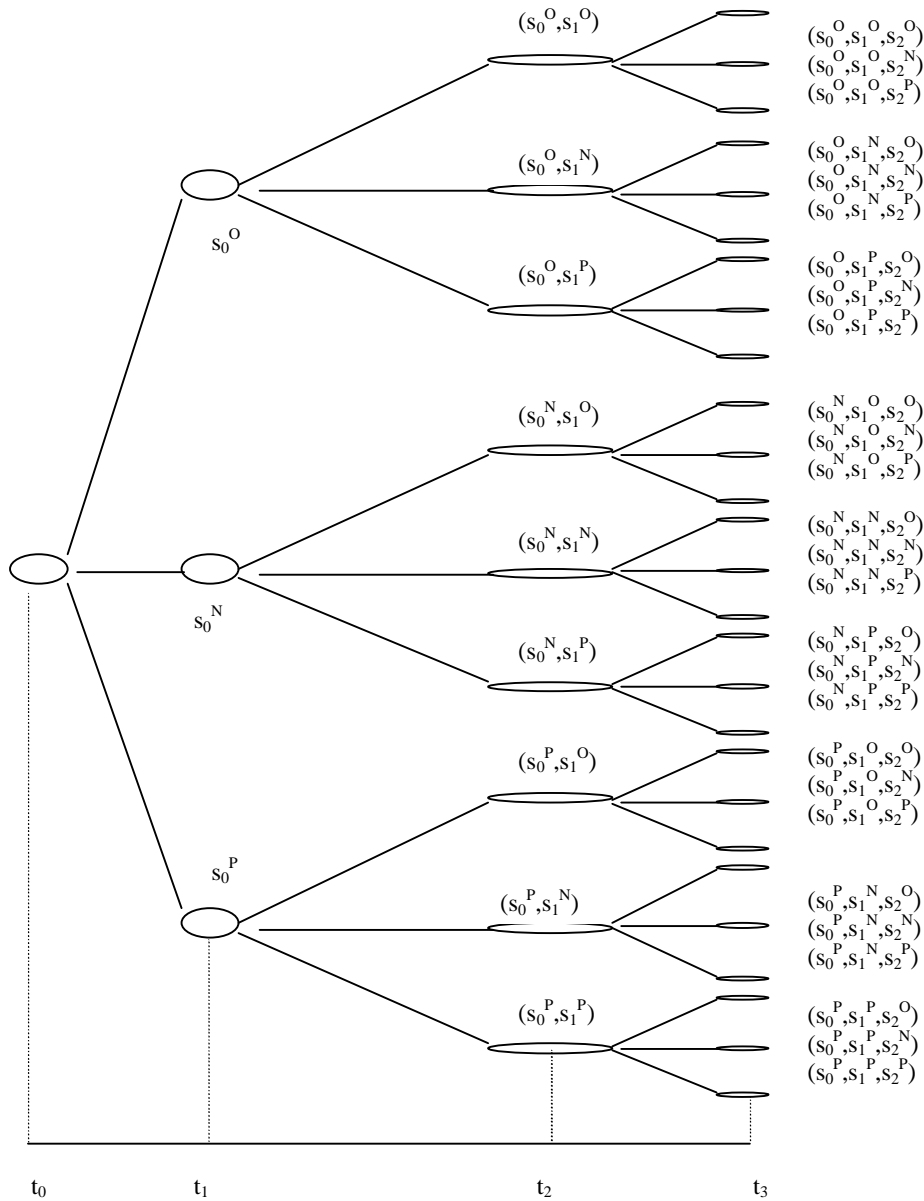


Figure 1. Scenario Tree Representation for October-December 2000

If S_t is the set of scenarios anticipated at time t , for $t=0,1,\dots,T$, we use s_t to index scenarios from the set S_t . Let I_t denote paths of scenarios that are resolved until period t , where $t=1,2,\dots,T$. Paths are denoted by $I_t=(s_0,s_1,\dots,s_t,\dots,s_{t-1})$ where $s_n \in S_n$. With each path we associate a probability π_{I_t} , and let P_t denote all paths that can be constructed by combining scenarios from the scenario sets S_0, S_1, \dots, S_{t-1} .

The interest rate in any node of the scenario tree in Figure 3.2 depends on the path of the scenarios followed from 0 to t. Thus we denote the anticipated real interest rate for monthly bank deposits for time period t as $INTSCEN_t(l_t, s_t)$ if l_t is the resolved scenario path from period 0 to t and s_t is the anticipated scenario for period t. Since we know the interest rate for September 2000, the interest rates for the first period are reduced to $INTSCEN_0(s_0^O)$, $INTSCEN_0(s_0^N)$ and $INTSCEN_0(s_0^P)$ and are computed replacing the interest rate term for the previous period in the equation, $INTSCEN(l_{-1}, s_{-1})$, with the known value of interest rate for September 2000.

Once this model is initialized the only difference between the scenarios that have the same scenario path l_t , is generated by assigning random values to dZs. Since our model employs a tree form with each node splitting into three branches every period, three different dZ values must be specified over the probability distribution of the random variable dZ which is distributed normally. These points are chosen so that the probability masses falling on these three points are equal as explained in Pflug (2000).

3. Estimation of scenario dependent parameters

The second set of stochastic variables of our model is the expected real return of each security. Having generated scenarios based on interest rates, the second step is to compute the security returns corresponding to each scenario. Rather than computing the security returns directly we assume that they are related to a basic economic factor, the interest rates. It should be noted here that there are also factors that affect security prices, such as inflation and foreign exchange rates among others. Instead of considering other factors, we take into account only the interest rates, assuming that interest rates directly influence the security returns. Interest rate or the monthly bank deposit rate is an investment alternative for most of the investors who do not want to undertake more risk by investing in stocks. Thus when the interest rate increases, more investments are shifted to bank deposits, which causes a decrease in the demand for securities and a fall in the prices of the securities and vice versa. Although this relationship is reasonable, a model that generates scenarios considering other factors would probably perform better in representing the reality.

We assume that the real return rs_{jt} of an asset j at period t is related with the real interest rate r_t according to a model proposed by Grinold (1999);

$$rs_{j,t} = r_t + \mathbf{m}_j + \mathbf{S}_j z_{j,t}$$

where,

$rs_{j,t}$ denotes the real rate of return for stock j in period t,

r_t denotes the real rate of monthly bank deposits for period t,

\mathbf{m}_j is the unconditional expected excess return for asset j,

\mathbf{s}_j is the standard deviation for security j, and,

$Z_{j,t}$ is the standardized random element of the return that represents the uncertainty in returns and is assumed to be normally distributed.

This model implies that the return on a security at a given time can be divided into three parts when compared to interest rates; interest rate dependent part of the security return, unconditional expected excess return of the security which is independent of the interest rate and a standardized random element. The parameters of the above equation are derived from historical data on security returns between periods September 1990 and September 2000. Accordingly the estimated return on each security under any scenario is computed using;

$$SCENRETS_{j,t}(l_t, s_t) = INTSCEN_t(L_t, s_t) + \mathbf{m}_j + \mathbf{s}_j z_{j,t}(L_t, s_t)$$

where, $SCENRETS_{j,t}(l_t, s_t)$ is the expected percent real return on security j at time period t if l_t is the resolved scenario path between periods 0 to t and s_t is the anticipated scenario for period t.

4. The Model

We build an SPR model which will optimize portfolio decisions for an investor whose investment alternatives are a set of securities and monthly interest rates. Thus generation of scenarios is required at the first step of the analysis. The interest rate and rate of return scenarios for the planning horizons October-December 2000 and October 2000 and March 2001 are generated. These interest rates ($INTSCEN_t(l_t, s_t)$) and security returns ($SCENRETS_{j,t}(l_t, s_t)$) are used in the SPR model.

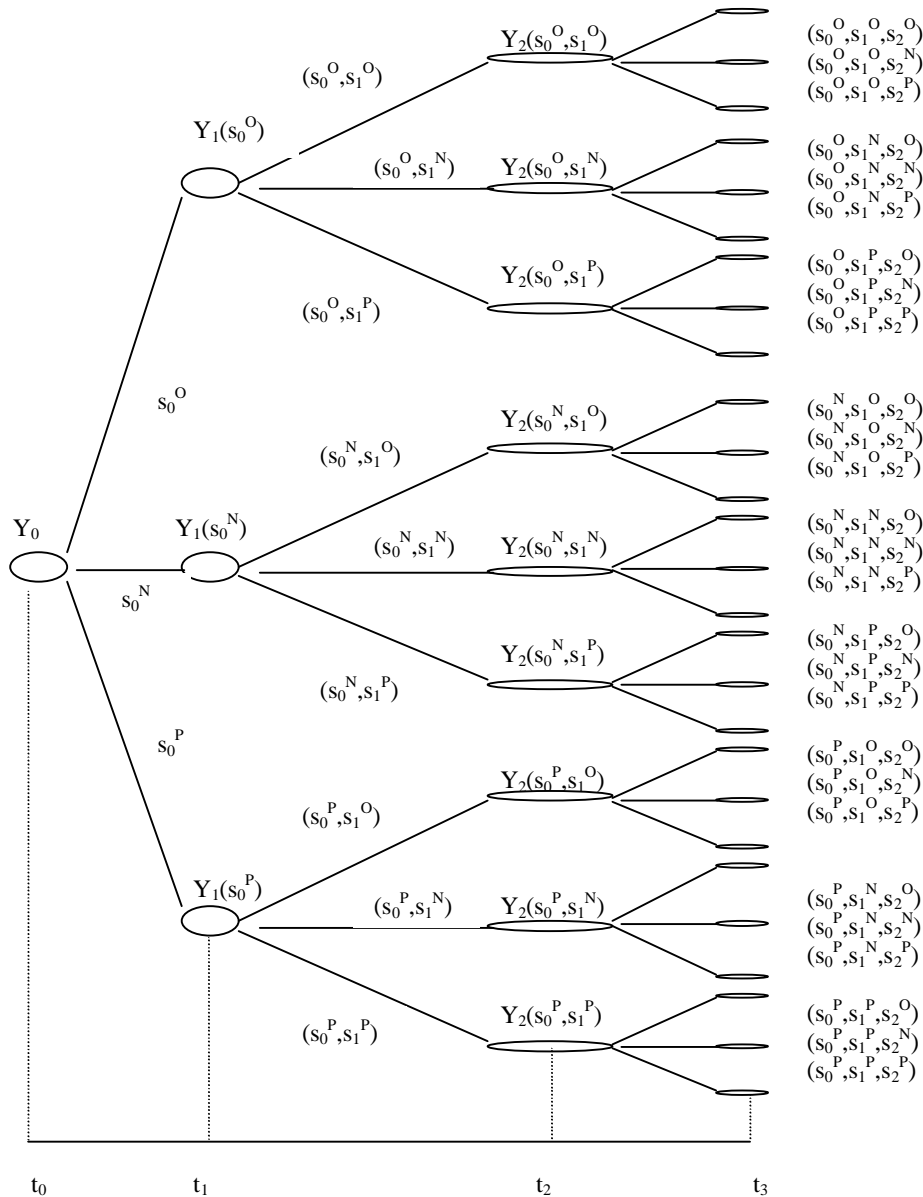


Figure 2 The Decision Tree for SPR

It is assumed that real interest rates unfold on a tri-nomial lattice, thus our scenario tree is similar to that given in Figure 2. in which every node denotes a different level of discretized interest rate in that period. Note also that each node in every period has one link with only one node of the previous period and three links with three different nodes of the next period, meaning that every realized scenario is assumed to unfold into three different anticipated scenarios in the next period.

The tree represented in Figure 2 is the same as the one given in Figure 1 except we now show the vectors of conditional decision variables of investments Y_t at each node. As can be seen from this representation decisions at any node depends on the scenario path from $t=0$ to that node. Thus for example for $t=2$ there are nine sets of decision variables whereas we assume a total of 27 scenarios at the same time interval. Thus the general frame of decision making assumes that scenarios at a period are realized after the investment decision is made for that period on the basis of information available at that time. Using the same notation given in Figure1, decision variables at any node t will depend on only the scenario path I_t between $t = 0$ and that specific node. We assume that the investment alternatives for the investor, are the 45 representative securities of ISE, and cash that is to be deposited at the monthly interest rate in TL (Turkish Lira) accounts. Our model does not allow any lending or borrowing. The securities used in the analysis are selected from those with sufficient historical data in the period September 1990-September 2000. In this model nominal monthly bank deposit rate is known and fixed at the beginning of the planning horizon($t = 0$), but, since the inflation rate is unknown for the subsequent periods, real interest rates as well as the deflated stock returns are stochastic. Possible outcomes of the stochastic variables (namely the real interest rate and security returns) are determined by examining the monthly returns data of each security and monthly nominal bank deposit interest rates between September 1990 and September 2000. All the data used for estimating the parameters for our model are deflated accordingly, thus the model uses real interest rates, and expected scenario returns. Now we formulate the SPR model.

i) Objective Function: The objective of the SPR basically is to maximize expected terminal wealth. Although this represents, by itself, an admissible objective, other specifications that allow for more specific risk perception are also possible. In this study three different specifications are considered. We first explain the model specification that maximizes expected terminal wealth minus a weighted penalty cost of downside and upside deviations from expected terminal wealth computed over each path starting from $t = 0$ and terminating at $t = T$. In order to be able to write the objective function explicitly we first must define expected terminal wealth and deviations.

ii) Computation of Expected Terminal Wealth: After the last portfolio decisions are made at time instance $T-1$ and the anticipated scenarios s_{T-1} are observed, the terminal wealth at the beginning of T is computed. $W(I_T)$ is actually the value of the portfolio at the end of the planning horizon computed for each scenario path I_T between $0-T$. This value depends on the path I_{T-1} and every last period scenario s_{T-1} , on the composition of the portfolio and the value

of the securities at T. In our scenario tree for three periods(Figure 2) we will have 27. $W(l_T)$ values since there are 27 terminating paths, l_T s. Then an expected terminal wealth computed over all l_T s can be computed using the equation:

$$EXPW = \sum p(l_T)W(l_T)$$

where, $\pi(l_T)$ is the probability for the path l_T defined between $t = 0$ to $t = T$

Deviations from the Expected Terminal Wealth

In order to incorporate risk aversion in our model we first introduce a risk measure by writing:

$$W(l_T) + NEGDEVS(l_T) - POSDEVS(l_T) = EXPW$$

where, $NEGDEVS(l_T)$, $POSDEVS(l_T)$ are deviations from the expected terminal wealth and the terminal wealth of each l_T . When the terminal wealth of one path is below the overall expected terminal wealth then this scenario path will produce a downside deviation, $NEGDEVS$. On the other hand if the terminal wealth of one path is above the expected terminal wealth this scenario path will produce an upside deviation, $POSDEVS$. According to the risk perception of the investor these deviations can be given weights and be inserted into the objective function to penalize investment plans that generate deviations from expected returns over the planning horizon. Having defined the terminal wealth and deviations we now define the objective function as:

$$\max \left[EXPW - \sum_{l_T} \left[F1 \times NEGDEVS(l_T) + F2 \times POSDEVS(l_T) \right]^2 \right]$$

where, $F1$ and $F2$ are the risk aversion constants which can be set to different levels by different investors. For an investor, who considers both downside and upside deviations from expected terminal return or wealth as a risk, $F1$ and $F2$ can be given positive values. On the other hand if an investor only considers downside deviation as risk, we can employ a positive $F1$ value while setting $F2$ to zero. Finally if the investor is risk neutral we set $F1=F2=0$. Having defined the objective function we formulate the constraints.

iii) Budget Constraints: These constraints equate the value of portfolio holdings at the end of each period minus the transaction cost due to sale of securities to the value of portfolio holdings at the beginning of the following period. At the first stage ($t=0$), the initial capital equals the amount invested in securities plus the amount invested in stochastic monthly cash account interest rate

$$\sum_j (Y_{j0}) + FREINT_0 = CAPITAL$$

Decisions made at any period t, after t = 0 depend on the path l_t meaning that they are conditioned on the realized scenarios until t, (s_0, s_1, \dots, s_{t-1}). Hence at each time instance t there is one constraint for each path l_t ;

$$\begin{aligned} & \sum_j \{Y_{j,t-1}(l_{t-1})[1 + SCENRETS_{j,t-1}(l_{t-1}, s_{t-1})]\} + \\ & FREINT_{t-1}(l_{t-1})[1 + INTSCEN_{t-1}(l_{t-1}, s_{t-1})] = \\ & FREINT_t(l_t) + Y_{j,t}(l_t) + \sum_j [TRCOST_j \times SELL_{j,t}(l_t)] \end{aligned} \quad \forall l_t$$

iv) Inventory Balance Constraints: Inventory balance for each security is assured at each decision making point, for every resolved path and every anticipated scenario. According to the inventory balance constraints the stock of each security sold or remaining in the portfolio should be equal to the stock of the same security at the end of previous period, plus any amount purchased at the beginning of the current period.

$$\begin{aligned} & Y_{j,t-1}(l_{t-1})[1 + SCENRETS_{j,t-1}(l_{t-1}, s_{t-1})] + BUY_{j,t}(l_t) \\ & = SELL_{j,t}(l_t) + Y_{j,t}(l_t) \end{aligned} \quad \forall j, t, l_t \text{ and } s_t$$

v) Cash Flow Constraints: Cash flow constraints equate the amount invested in the purchase of new securities and in monthly bank deposits plus total transaction costs paid, to cash generated from the sale of securities plus income generated from the maturity of previous month's bank deposits. There is one constraint for each l_t , s_t and t.

$$\begin{aligned} & \sum_j [BUY_{j,t}(l_t)] + FREINT_t(l_t) + \sum_j [TRCOST_j (SELL_{j,t}(l_t))] \\ & = FREINT_{t-1}(l_{t-1})[1 + INTSCEN_{t-1}(l_{t-1}, s_{t-1})] + SELL_{j,t}(l_t) \end{aligned} \quad \forall t, l_t \text{ and } s_t$$

vi) Arbitrage Constraints: The model should not allow arbitrage, that is simultaneous purchase and sale of a security at any time over the planning horizon. We satisfy this condition with the help of the following constraint;

$$BUY_{j,t}(l_t) \times SELL_{j,t}(l_t) = 0 \quad \forall j, t \text{ and } l_t$$

vii) Non-negativity Constraint: Since we assume that no short sales and no borrowings are allowed all of our decision variables should be non-negative.

All variables ≥ 0 .

5. Solutions of the model

In an SPR model choice of the planning horizon makes a difference. In this study, the model is solved for two planning horizons; first over three months and then over six months starting in October 2000. The interest rate scenarios were generated in section 2 for the horizon October-December 2000. This scenario tree is unfolded into three further periods in order to obtain the interest rate scenarios for the six month planning horizon model solution. Then security returns are computed in section 3 for each scenario based upon the interest rates.

We solve the SPR model for three different objective functions as explained above. In the first function we utilize risk aversion coefficients of $F1=F2=10$ for both horizons. This objective function corresponds to an investor who wants to obtain robust terminal wealth values that do not fluctuate too much under different scenario paths. Thus this investor can be assumed to be both downside and upside risk averse. For the second set of solutions of the model we set $F1=10$ and $F2=0$. This case corresponds to an investor who does not like returns below expected terminal wealth and penalizes only downside deviations. Finally the third objective function utilizes $F1=F2=0$ which is the objective function of a risk neutral investor, who only maximizes her expected terminal wealth regardless of the sign of the deviations under different scenarios. Thus we solve the SPR model for the three objective functions given below;

$$\max \left[EXPW - \sum_{l_T} [10 \times NEGDEVS(l_T) + 10 \times POSDEVS(l_T)]^2 \right] \quad (1)$$

$$\max \left[EXPW - \sum_{l_T} [10 \times NEGDEVS(l_T)]^2 \right] \quad (2)$$

$$\max [EXPW] \quad (3)$$

Altogether, the SPR model is solved for six specifications, using combinations of three different objective functions and two different planning horizons.

Solving the model for each of the six cases we obtain the results given in Appendices 1,2,3 and 4. Our conclusion regarding these results are;

- Choice of objective function and length of the planning horizon is crucial in portfolio optimization with SPR models. A model penalizing both downside and upside deviation will perform better when a longer planning horizon is used.
- On the other hand for the short planning horizon choices, a model with only downside deviation is more robust and does not result in any losses for the whole horizon.
- Although a risk neutral model does not provide diversified portfolios for the initial period of the planning horizon, it results in the highest returns for most of the terminal scenarios.
- Finally we suggest that SPR models should be solved for longer planning horizons than 6 months which might be more promising for the ISE market, since such a model will consider more information about future realizations of the stochastic variables.

5. Future Research

These results should not be generalized without caution since we solve the model only for the initial period and do not roll over the model throughout the planning horizon. Also our scenario generation is very simple and does not consider many random variables of the economy such as inflation rates and exchange rates. Thus the scenarios employed by our model may not be entirely realistic.

When ISE data is considered, for multi-period model, based on calculations of rolling over an initial investment of 1 T.L. over 21 months, it appears that utilization of downside risks in the objective function mostly resulted in better performances, thus implementation of downside risk models are more promising in the ISE market. However more work is needed to test different downside risk measures.

Models employing more detailed and realistic scenario generation can be developed. These models may consider movements of more economic variables and their correlation. Also other constraints such as minimum transaction lots can be included. Other methods such as scenarios other than those generated can be used for measuring the performance of multi-period portfolio models.

Appendix 1

Table 4.9 Scenarios and Actual Values of Security Returns for October 2000 (% return)

Stocks	Scenarios for October 00			Actual Returns
	SOP	SON	S00	
AEFES	-18.747	11.975	5.331	-2.253
AKBNK	-9.651	6.296	-24.052	-23.480
AKCNS	12.598	13.404	-14.637	20.438
AKGRT	-20.686	-6.282	-19.989	16.984
AKSA	-21.740	20.892	-0.627	10.625
ALARK	4.640	-15.042	2.789	17.778
ALCTL	-20.805	-7.574	2.893	-72.634
ARCLK	-10.330	-14.085	-2.244	-9.737
ASELS	-16.383	-9.268	14.787	16.496
AYGAZ	-10.900	-8.163	11.085	27.734
DISBA	-17.772	-2.691	-4.000	-46.867
DOHOL	-38.276	-1.732	-15.101	28.500
DYHOL	1.285	6.599	-7.757	14.005
ECILC	-31.882	7.260	-8.838	24.807
ECZYT	-28.426	-6.475	-7.975	21.259
EFES	-0.55	-11.670	-14.433	43.959
ENKA	2.936	-11.785	-4.000	24.729
EREGL	-10.827	-12.632	42.359	10.058
FROTO	1.887	-8.451	-5.425	-44.052
GARAN	-35.628	-3.095	-16.659	-60.521
GIMA	-12.636	9.188	-17.802	10.108
GLMDE	-33.883	0.627	25.195	-23.498
HURGZ	-27.754	-3.200	-7.351	-25.625
IHLAS	-14.227	-2.681	-15.739	10.536
ISCTR	-17.516	14.998	12.052	-23.550
ISGYO	-1.999	-1.473	-2.525	-9.231
KCHOL	-24.125	-5.282	-10.845	9.196
KIPA	17.529	1.512	0.022	20.269
KRDMD	4.125	-0.152	4.054	-7.698
MIGRS	-21.070	-13.975	22.148	15.071
NETAS	-7.882	9.522	-22.984	31.606
NTHOL	-16.628	-21.431	4.215	-6.167
OTKAR	-17.985	17.769	-24.393	-23.264
PETKM	-19.613	10.859	25.600	34.109
PRKTE	-0.55	-13.685	-18.602	-40.137
PTOFS	-22.930	-2.878	29.333	10.108
SAHOL	-9.467	14.505	-3.748	-6.699
SISE	-6.261	3.868	-7.992	19.481
THYAO	-7.178	-35.144	13.457	47.187
TNSAS	17.384	-17.705	1.592	31.630
TOASO	-32.663	1.207	14.883	42.978
TRKCM	-14.596	-12.719	-6.913	-19.243
TUPRS	-21.095	-2.978	31.459	3.025
VESTEL	-7.260	8.795	-2.485	11.206
YKBNK	-13.693	12.186	7.926	-21.156
INTEREST	1.535	1.009	0.482	0.358

Appendix 2: Portfolio Compositions of the Models for October 2000

	OBJ. FUNCTION=(1)		OBJ. FUNCTION=(2)		OBJ. FUNCTION=(3)	
	% for 3-m P.H.	% for 6-m P.H.	% for 3-m P.H.	% for 6-m P.H.	% for 3-m P.H.	% for 6-m P.H.
STOCK						
AEFES	0	0,071	0	0	0	0
AKBNK	0	0	0	0	0	0
AKCNS	0	0	0	0	0	0
AKGRT	0,73312	0	0	2,373	0	0
AKSA	0	0,393	0	0	0	0
ALARK	0	0,017	0	0	0	0
ALCTL	0	0	0	0	0	0
ARCLK	0	0	0	0	0	0
ASELS	0	0	0	0	0	0
AYGAZ	0	0	0	0	0	0
DISBA	0	0,198	0	0	0	0
DOHOL	27,67563	92,842	0	0,219	0	0
DYHOL	0	1,529	0	0	0	0
ECILC	0	0	0	0	0	0
ECZYT	5,59609	0	0	0	0	0
EFES	0	1,946	0	0	0	0
ENKA	0	0	0	0,242	0	0
EREGL	0	0	7,922	0	0	0
FROTO	0	0	0	0	0	0
GARAN	6,54585	0	0	0	0	0
GIMA	0	0	0	0	0	100
GLMDE	0	0,432	0	0,939	0	0
HURGZ	0	0,294	0	0	0	0
IHLAS	0	0	0	0	0	0
ISCTR	0	0	0	0	0	0
ISGYO	0	0	0	92,971	0	0
KCHOL	1,27822	0	0	0	0	0
KIPA	0	0	19,725	0	0	0
KRDMD	0	0	0	3,016	0	0
MIGRS	0	0,126	0	0	0	0
NETAS	0	0	0	0	0	0
NTHOL	9,59073	0	0	0	0	0
OTKAR	0	0,438	0	0,108	0	0
PETKM	0	0	0	0	0	0
PRKTE	35,00974	1,208	0	0	0	0
PTOFS	0	0	0	0	0	0
SAHOL	0	0,292	0	0	0	0
SISE	0	0	0	0	0	0
THYAO	8,97306	0,214	0	0	0	0
TNSAS	0	0	0	0	0	0
TOASO	0	0	0	0	0	0
TRKCM	4,59757	0	0	0	0	0
TUPRS	0	0	0	0,132	0	0
VESTEL	0	0	0	0	0	0
YKBNK	0	0	0	0	0	0
INTEREST	0	0	72,353	0	100	0

Appendix 3: Expected and Realized Portfolio Values at the End of October 2000

Scenario	OBJ. FUNCTION=(1)		OBJ. FUNCTION=(2)		OBJ. FUNCTION=(3)	
	3-m P.H. Portfolio value	6-m P.H. portfolio value	3-m P.H. portfolio value	6-m P.H. portfolio value	3-m P.H. portfolio value	6-m P.H. portfolio value
SOP	81,92	62,61	103,33	97,33	101,01	87,36
S0N	88,25	98,10	100,40	98,47	101,54	109,19
S0O	88,78	85,03	103,71	97,51	100,48	82,20
Overall	86,31	81,92	102,48	97,77	101,01	92,91
Realized	94,06	128,64	105,05	91,47	100,36	110,11

Appendix 4: Terminal Wealth($W(l_3)$) of the Models for Each Scenario Path

Scenario Path l_3	OBJ. FUNCTION (1) 3-M. P.H.	OBJ. FUNCTION (2) 3-M. P.H.	OBJ. FUNCTION (3) 3-M. P.H.
$S_0^P .S_1^P .S_2^P$	50.450	109.440	88.391
$S_0^P .S_1^P .S_2^N$	63.206	109.182	103.990
$S_0^P .S_1^P .S_2^O$	52.899	109.510	131.494
$S_0^P .S_1^N .S_2^P$	52.507	109.418	156.686
$S_0^P .S_1^N .S_2^N$	58.577	108.243	118.778
$S_0^P .S_1^N .S_2^O$	57.377	105.251	135.528
$S_0^P .S_1^O .S_2^P$	51.133	136.733	167.170
$S_0^P .S_1^O .S_2^N$	51.499	118.914	116.061
$S_0^P .S_1^O .S_2^O$	36.771	139.718	153.565
$S_0^N .S_1^P .S_2^P$	48.959	109.446	117.201
$S_0^N .S_1^P .S_2^N$	44.523	109.044	116.772
$S_0^N .S_1^P .S_2^O$	65.705	108.654	116.344
$S_0^N .S_1^N .S_2^P$	66.090	109.490	126.662
$S_0^N .S_1^N .S_2^N$	86.319	109.418	113.749
$S_0^N .S_1^N .S_2^O$	88.502	114.713	144.560
$S_0^N .S_1^O .S_2^P$	86.896	109.197	114.989
$S_0^N .S_1^O .S_2^N$	82.738	109.235	165.670
$S_0^N .S_1^O .S_2^O$	63.773	109.507	169.563
$S_0^O .S_1^P .S_2^P$	51.733	109.134	82.479
$S_0^O .S_1^P .S_2^N$	44.353	108.566	82.628
$S_0^O .S_1^P .S_2^O$	59.896	109.450	73.269
$S_0^O .S_1^N .S_2^P$	61.538	109.450	91.242
$S_0^O .S_1^N .S_2^N$	55.502	108.633	111.533
$S_0^O .S_1^N .S_2^O$	53.122	118.625	97.142
$S_0^O .S_1^O .S_2^P$	95.335	117.432	194.447
$S_0^O .S_1^O .S_2^N$	75.180	123.611	201.919
$S_0^O .S_1^O .S_2^O$	70.795	105.587	131.850

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