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The origins of business cycles

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Abstract

In this paper, we apply eigenvalue analysis to a classical inventory-workforce interaction model in order to identify the sources of oscillations (i.e. business cycle) in the model. The inventory-workforce interaction constitutes the core of the short-term business cycle. The inventory-workforce model used in this paper is explained in detail in John Sterman's book "Business Dynamics" (Sterman 2000, chapter 19). As John Sterman puts it: "However, explaining the behavior by saying that production oscillates because the system contains negative loops with delays is not sufficient. Good modelers must strive for a deep understanding of the causes for the behavior observed in their models…Understanding model behavior goes beyond the invocation of simple archetypes such as 'the oscillation is caused by negative loops with delays'…While true, these statements don't provide the deep insight into model structure and behavior required to develop your intuition about dynamics or your ability to identify high leverage policies".

Keywords: Model analysis, business cycle**,** oscillation, inventory management, human resource management.

1. Managing the behavior of complex, dynamic systems

In this paper, we investigate the relationship between the behavior and the underlying structure of complex, dynamic models. The purpose of this investigation is to develop a method whereby managers can find ways to effectively influence the behavior of complex, dynamic systems. The method is based on a system dynamics approach, and thus relies heavily on the utilization of modeling and analysis. The method is based on a recognition of the fact that systems behavior is composed of a number of modes of behavior, each one characterized by its relative significance with respect to the total behavior. Moreover, it is recognized that, in nonlinear systems, the relative significance of each such mode of behavior changes over time. Finally, it is recognized that the

structural origin of each such mode of behavior can be identified so that the impact of the various structural components of a system can be identified.
When a person is confronted with the task of managing a complex dynamic system, we propose that a system dynamics model of that system be built and that an analysis of that model be conducted by way of the method presented in this paper. In general, managers exercise their roles by influencing the gains associated with the causal relationships that altogether constitute the structure of a system. For that purpose, the manager should rely on the identification of the modes of behavior -- eigenvalues, λ_k, -- and their associated elasticities, ε_{kj}, that, at any point in time, characterize the impact of fractional changes in gains, g_j, on the modes of behavior.

In figure 1(a), we describe the relationship between the gains characterizing the structure, and the convergence/divergence characterizing the behavior of a monotonic mode of behavior. Note that associated with monotonic modes of behavior are real eigenvalues and real elasticities.

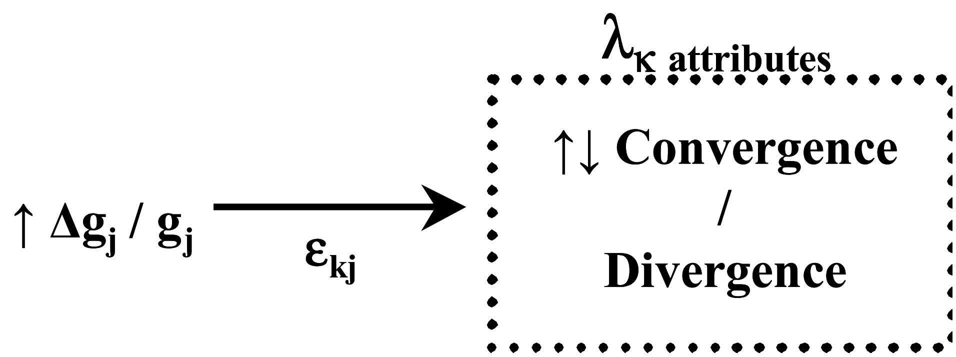

Fig. 1(a): The relationship between the gains and the convergence/divergence attribute of a monotonic mode of behavior

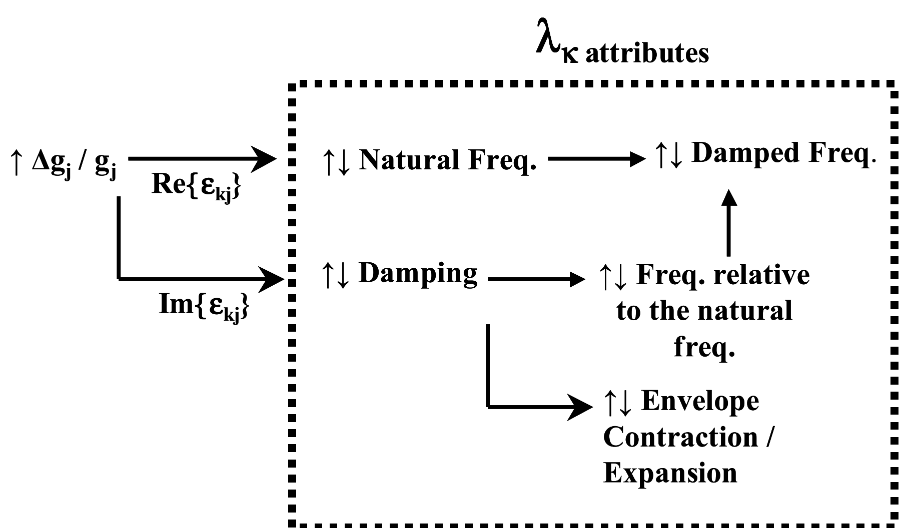

Fig. 1(b): The relationship between the gains and the frequencies and the envelope contraction/expansion attributes of an oscillatory mode of behavior

In figure 1(b), we describe the relationship between the gains characterizing the structure, and the frequencies and the envelope contraction/expansion characterizing the behavior of an oscillatory mode of behavior. Note that associated with oscillatory modes of behavior are complex eigenvalues and complex elasticities. As indicated by figure 1(b), the imaginary component of the elasticity, $\mathrm{Im}\{\varepsilon_{kj}\}$, impacts the damping (or amplification); and the real component of the elasticity, $\mathrm{Re}\{\varepsilon_{kj}\}$, impacts the natural frequency.

Figure 1 constitutes the framework for this paper linking structure to behavior through eigenvalues and their elasticities.

In the first part of the paper (sections 2 and 3), we will outline the method of analysis of such models. This method centers around an eigenvalue characterization of a mode of model behavior and a characterization of an eigenvalue based on the link gains characterizing the underlying structure of the model. In the second part of the paper (sections 4 and 5), we illustrate the use of the method applied to a well-recognized model of business cycles.

2 A characterization of model behavior

2.1 Model behavior

In system dynamics, the pattern of behavior of a model is typically characterized by the trajectory of the net rates (slope vector) of the state variables of the model. Using eigen-value analysis, we can consider the dynamics of each slope trajectory of a model made up of a number of behavior modes, each associated with a particular eigenvalue. The relative significance of each of these modes of behavior is determined by the state of the model -- determining the current slope vector -- and the directions of the right eigenvectors associated with the eigenvalues (as explained below).

In our analysis, we will focus on the following:

In linear models, exhibiting steady state behavior, we are able – using an analytical method - to identify which modes of behavior that significantly influence the model behavior; i.e. to identify the dominant modes of behavior. In linear models exhibiting transient behavior, and in nonlinear models in general, we are also able – this time using an empirical method - to determine which modes of behavior that significantly influence the model behavior (Saleh & Davidsen, 2000); i.e. to identify the dominant modes of behavior. Each mode of behavior is characterized by a particular eigenvalue. Moreover, in general (i.e. in all cases), we can - using an analytical method –determine which link gains that most significantly influence such an eigenvalue (Forrester, 1983). Thus we are able to identify the structural components that most significantly influence the most significant modes of behavior. By governing these structural components, we can influence the modes of behavior that govern the model behavior and thus manage the model.

This paper concerns how to identify the behavior modes that are of significance and to how to govern the model. Aside from the mode of behavior itself, it remains to find out why a particular mode of behavior takes on such significance. As indicated above, this is related to the state of the system and to the direction of the right eigenvectors. We will briefly comment on the implications of the state of the model (leading to a distinction between linear models in steady state and such models in a transient phase) and leave the discussion of the right eigenvectors for further elaboration in a subsequent paper.

In linear systems, the eigenvalues are constant and so are the modes of behavior that altogether make up the total model behavior. In a non-linear model the eigenvalues vary and so do the modes of behavior. Consequently, we need to distinguish between linear and non-linear models.

The transient behavior of a linear model depends on the current state of the model, - more specifically on the direction of the slope vector (determined by the current state) relative to the directions of the right eigenvectors (constant and determined by the model structure). Consequently, we need, also, to distinguish between linear models in steady state, and such models in a transient phase. In general, the steady state and transient behavior of a nonlinear model depends on the current state of the model. In section 2.2, therefore, we discuss the steady state behavior of linear models, while as in section 2.3, we discuss the transient behavior of linear models and the behavior of nonlinear models in general.

2.2 The steady state behavior of linear models

In steady state, the behavior of a linear model is characterized by a monotonic mode of behavior (associated with a single, real eigenvalue) or an oscillatory mode (associated with a pair of complex conjugate eigenvalues). I.e. in steady state, all other modes of behavior have faded out compared to the dominant one. Associated with such a dominant mode of behavior, there is a dominant eigenvalue, and, associated with that, right and left eigenvectors

Moreover, in general, the steady state behavior of a linear model is independent of its initial state, except under a particular condition: If we set the model in an initial state so that the slope vector (the direction of the model behavior) is orthogonal to the *left* eigenvector associated with the eigenvalue that, in general, is considered dominant, then *this* (generally considered dominant) eigenvalue will have no significance throughout the entire life of the model, and there will be no trace of the associated mode of behavior in the model behavior itself.

To illustrate this exceptional case, we use a simple second order model where the structure consists of a single positive feedback loop, portrayed in figure 2. The equations of the model are listed in table 1.

Note that in this particular model, the left and right eigenvectors, associated with each eigenvalue, have the same direction.

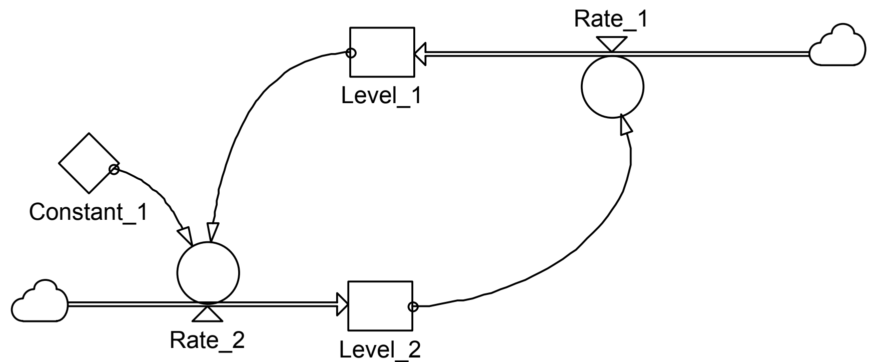

Fig. 2: Stock and flow diagram of the simple second order model

flow Level_1 = Rate_1
flow Level_2 = Rate_2

Rate_1 = 0.1* Level_2
Rate_2 = (0.1* Level_1)+Constant_1

Constant_1 = -1.32

Table 1: simple second order model, equations

The mode of behavior generally expected to dominate this model is divergent (exponential growth or decline). Yet, by initiating the model in (11.966,1.234) the resulting slope vector, (0.1234, -0.1234), is orthogonal to the right eigenvector, $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, associated with the "generally considered" dominant eigenvalue (which represents a divergent mode of behavior). Consequently, one does not observe any trace of this divergent mode (that is generally considered the dominant mode) in the model behavior (see figure 3).

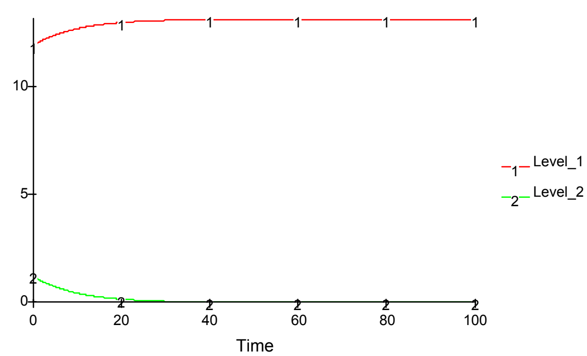

Fig. 3: model behavior, initial state (11.966,1.234)

Note that the right eigenvectors are solely determined by the structure of the model. The direction of each right eigenvector relative to the slope vector (which is determined by the current state of the model) collectively determine the contributions of the various modes of behavior to the slope vector, i.e. to the dynamics of the model as a whole. In this case, the direction of the right eigenvector associated with the "generally considered" dominant eigenvalue is orthogonal to the slope vector (see figure 4). Consequently, the projection of the slope vector onto that right eigenvector amounts to 0, and so is the contribution to the total behavior in the direction of that eigenvector. In the meanwhile, the other right eigenvector, associated with the "generally considered" non-dominating eigenvalue is equal to ($1/\sqrt{2}$,- $1/\sqrt{2}$), i.e. it is totally aligned with the slope vector (see figure 4). Consequently, in this case, the model (throughout its entire life) will be dominated by the "generally considered" non-dominating eigenvalue (which represents a convergent behavior in this model).

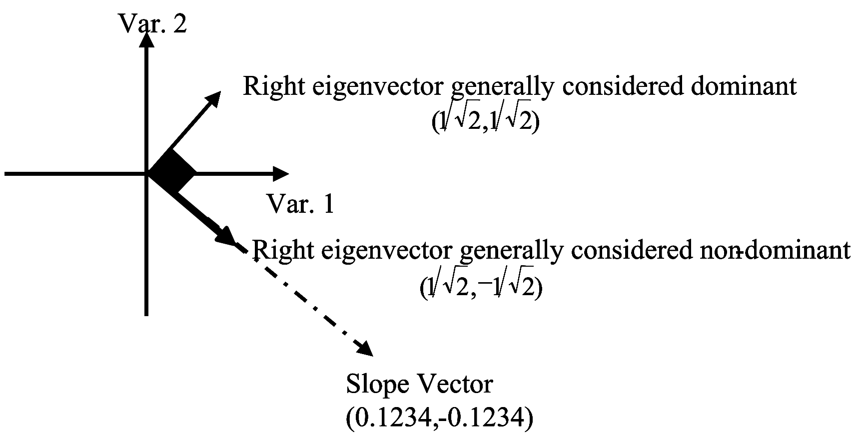

Fig. 4: Orientation of the slope vector relative to the right eigenvectors

If we change the initial state of the model infinitesimally, say to (11.966,1.234**1**), then the projection of the slope vector on the right eigenvector ($1/\sqrt{2}$, $1/\sqrt{2}$) will initially be infinitesimally small (yet not 0), and so will the contribution to the model behavior along that right eigenvector. However, since the mode of behavior associated with that right eigenvalue is divergent, then this infinitesimally small contribution will grow over time, and, finally, in the long-term it will dominate the model behavior (as is generally the case) (see figure 5). I.e. in steady state, the slope vector will be totally aligned to that right eigenvector. Thus, throughout the entire life of the model the slope vector will rotate 90^o (counter-clock) from its initial position (see figure 6). Note that, in steady state (i.e. after the slope vector has completed its 90^o rotation), the direction of the slope vector will be fixed; yet its length will increase indefinitely.

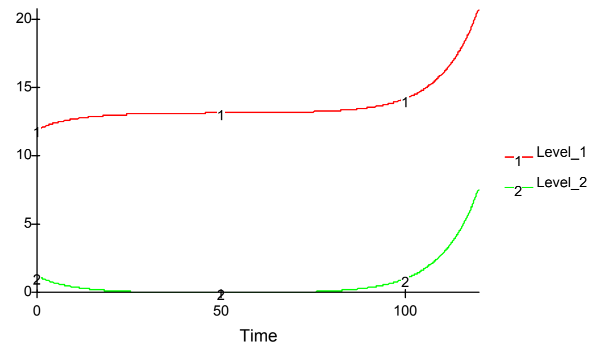

Fig. 5: Model behavior, initial state (11.966,1.2341)

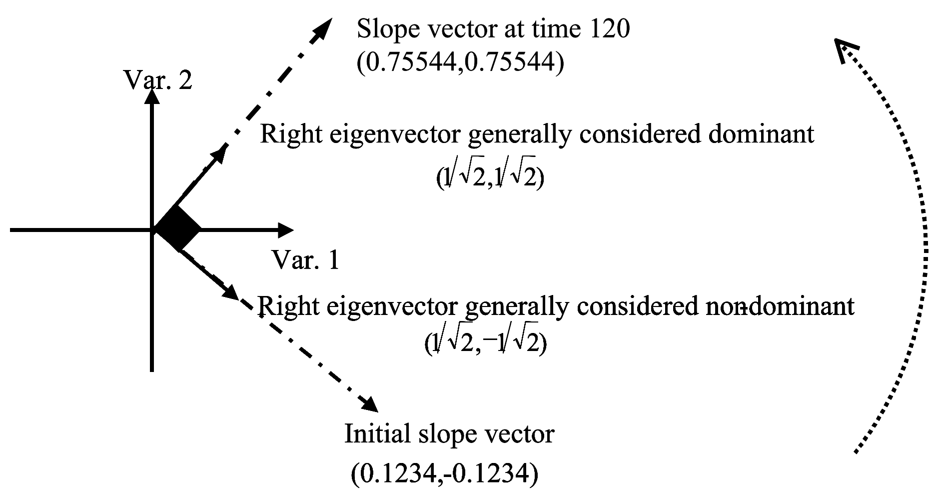

Fig. 6: The rotation of the slope vector

2.3 Transient behavior of linear models and the behavior of non-linear models

2.3.1 Modes of behavior and their significances

The modes of behavior of a model are characterized by the eigenvalues of the model. In a linear model the eigenvalues are constant and the model exhibits a permanent set of behavior modes. In non-linear models the eigenvalues vary over time and so do the modes of behavior that altogether make up the model behavior. This is summarized in the first columns in tables 2 and 3.

The relative significance of a mode of behavior (that contribute to the model behavior) is determined by the alignment of the right eigenvector, associated with that mode, with the model behavior (i.e. with the direction of the current slope vector), relative to the corresponding alignments of all the other right eigenvectors (each associated with a specific mode of behavior). The significance of a particular mode of behavior is, in other words, determined by the angle between the associated eigenvector and the current slope vector, relative to all the other such angles. In a linear model in a transient phase, the relative significances of the various modes of behavior vary due to a change in direction of the slope vector alone, - a change caused by a change in the state of the model. In that case, the eigenvectors, and thus their directions, are constant. In a nonlinear model, the relative significances of the modes of behavior may vary, in part, due to a change in the direction of the slope vector (as a consequence of a change in the state of the model), and, in part, due to changes in the directions of the eigenvectors (as a consequence of changes in the gains characterizing the structure of the model). This is summarized in the second and third columns in tables 2 and 3.

	Eigenvalues	**Direction of the slope vector**	**Right eigenvectors**
Originating from	Structure	Behavior (The state of the model)	Structure
Characterizing	Modes of behavior	Relative significances of the modes of behavior	Relative significances of the modes of behavior

Table 2: The origin and roles of the eigenvalues, the slope vector and the right eigenvectors.

	Eigenvalues	**Direction of the slope vector**	**Right eigenvectors**
Linear model in steady state	Constant	Constant	Constant
Linear model in transient phase	Constant	Variable	Constant
Nonlinear model in general	Variable	Variable	Variable

Table 3: The characteristics of the eigenvalues, the slope vector and the right eigenvectors in the various kinds of models.

2.3.2 Three dimensions of model behavior

2.3.2.1 Introduction

In general, the behavior of a model can be characterized along three different dimensions (behavior aspects) (figure 7):

(1) One may consider the model behavior over a set of individual time intervals.

(2) One may consider individually the behavior of each of the state variables of the model.
(3) One may consider individually each of the modes of behavior that altogether constitute the behavior of each state variable.

In this paper, we will make use of these aspects so as to obtain a clear exposition of the method we apply. In general, this method allows us to deal with complex, dynamic models, whether the model exhibits a transient or a steady state behavior. To underscore this issue, we will briefly discuss each of these aspects individually:

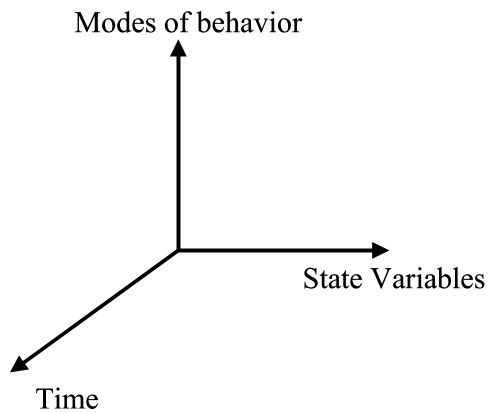

Fig. 7: Three dimensions of model behavior

2.3.2.2 The time dimension

As indicted in section 2.2, the behavior of a linear model in *steady state* will exhibit the same mode of behavior over the entire time interval that behavior unfolds. Consequently, an analysis of its behavior applies to that entire time interval. However, a linear model exhibits a *transient* behavior where shifts in mode dominance can take place. When a linear model is in its *transient phase*, it may exhibit a variety of behavior mode combinations over time. Thus we can only instantaneously determine the dominant mode(s) of behavior. Consequently, we will need to consider the model and its behavior over infinitely small time intervals. In practice, we emulate such a process by iterating the procedure described above over a finite number of sufficiently small time intervals. Here is the reason why we need to reconsider the behavior of a linear model in a transient state within each of the time intervals. The state of the model changes the direction of the slope vector relative to the right eigenvectors. Thus the relative significance of the various modes of behavior, each characterized by an eigenvalue, changes, depending upon the degree of alignment of that eigenvector with the current slope vector. This is true also for nonlinear models in general, whether in a *steady state* or a *transient phase*.

2.3.2.3 The state variable dimension

Moreover, suppose we consider the model behavior within the time interval under investigation. Then, generally, in a linear model in *steady state,* the behavior of all state variables can be characterized by a single mode of behavior, and, consequently, the behavior of *all* these variables may be investigated in a single analysis. When a linear model is in a *transient phase*, however, the behavior of each state variable *may* be dominated (and thus characterized) by an individual combination of modes of behavior, and thus exhibit a *unique* behavior over time.

As we mentioned in section 2.3.1, the relative significance of a mode of behavior is determined by the alignment of the right eigenvector, associated with that mode, with the model behavior (i.e. with the direction of the current slope vector), relative to the corresponding alignments of all the other right eigenvectors (each associated with a specific mode of behavior). The theme of section 2.3.1, was the relative significance of a mode of behavior to the overall (total) model behavior; i.e. the relative significance of a mode of behavior to the behavior of the majority of state variables in the model. It can be the case, however, that a dominant mode of behavior significantly affects the behavior of the majority of state variables in the model, yet at the same time it has absolutely no impact on a particular state variable. This peculiar case occurs, if the right eigenvector associated with this dominant mode of behavior, is orthogonal to the standard axis -- in the standard space -- associated with that particular state variable. In general, the orientation of a right eigenvector relative to the standard axes (in the standard space) determines the relative contributions of the mode of behavior (associated with that right eigenvector) to the behavior of the state variables in the model. I.e. in other words, the relative contributions of a mode of behavior to the states variables are determined by the projections of the right eigenvector -- associated with this mode -- on the standard axes (each associated with a state variable), respectively. (Recall that the elements of a right eigenvector represent the projections of the right eigenvector on the standard axes.) Note that the sum of squares of those projections always equals unity. I.e. the sum of squares of relative contributions is always unity; thus one can conclude that the effects of the dominant –i.e. the most significant— behavior mode must materialize along some (if not the majority) of state variables in the model. In general, for a mode of behavior to greatly influence the behavior a certain state variable there are two conditions; first the mode itself must be of high significance (which is determined by the relative alignment of the right eigenvector associated with this mode with the slope vector); second the relative contribution of the mode to the behavior of the state variable (which is determined by the projection of the right eigenvector on the standard axis associated with the state variable) must also be high. I.e. in short, it is the multiplicative effect of significance and contribution that, finally, determines the influence of a mode of behavior on the behavior of a certain state variable.

Thus in conclusion, we *may* need to consider the behavior of each of the state variables individually over time. This is true also for nonlinear models in general, whether in a *steady state* or a *transient phase*.

2.3.2.4 The behavior mode dimension

Finally, suppose we consider the behavior of a state variable within the time interval under investigation. In a linear model in *steady state,* the behavior of such a state variable can be characterized by a monotonic mode of behavior, associated with a constant real eigenvalue, or an oscillatory mode of behavior, associated with a pair of complex conjugate eigenvalues. Consequently, the investigation of that behavior may focus on that particular mode of behavior. When a linear model is in a *transient phase*, however, the state of the model changes and, with it, the relative significances of the behavior modes that altogether make up the model behavior. Hence, within a specific time interval – depending on the current state of the model - the behavior of a state variable *may* be dominated by a particular subset of behavior modes, each associated with an eigenvalue. Consequently, we *may* need to consider, individually, each of the various modes of behavior that altogether make up the composite behavior. This is true also for nonlinear models in general, whether in a *steady state* or a *transient phase*.

3 Eigenvalue analysis

3.1 An eigenvalue characterization of a mode of behavior

Since the dominant eigenvalue characterizes a dominant mode of behavior, our aim is to relate a characterization of the structure of the model to that eigenvalue. Note that there might, in other models, be several eigenvalues each associated with a mode of behavior that, in a steady state (or transient phase), significantly impact the overall model behavior. The analysis proposed in this section applies to one such eigenvalue and the associated mode of behavior.

The structure of the model is made up of the relationships of the model. These can be considered individually, as links, or as sequences of links, forming open or closed loops. The links as well as the loops are characterized by the associated gains. Thus we aim towards relating the gains, characterizing the model structure, to an eigenvalue that characterizes a dominant mode of behavior. For this purpose, we will later, in section 3.2, introduce eigenvalue elasticities. But first we will offer a brief characterization of various modes of model behavior by way of the associated eigenvalues.

In general, we distinguish between the following modes of behavior, each characterized by the associated eigenvalue:

- Monotonic convergent mode of behavior (exponential decay), associated with a real, negative eigenvalue.
- Monotonic divergent mode of behavior (exponential growth or decline), associated with a real, positive eigenvalue.
- Sustained oscillation, a mode of behavior associated with a complex conjugate pair of eigenvalues with zero real parts.
- Convergent oscillation, a mode of behavior associated with a complex conjugate pair of eigenvalues with negative real parts.

- Divergent oscillation, a mode of behavior is associated with a complex conjugate pair of eigenvalues with positive real parts.

The interpretation of a real eigenvalue is straightforward: A positive eigenvalue characterizes the growth fraction of an exponentially divergent behavior; the larger the eigenvalue, the faster the divergence. The doubling time of this divergent behavior is equal to ($\ln 2 / \lambda$). A negative eigenvalue characterizes the decay fraction of an exponentially convergent behavior; the larger the eigenvalue, the faster the convergence. The half-life time of this convergent behavior is equal to ($-\ln 2 / \lambda$). In short, a larger absolute value of a *real eigenvalue* implies a *faster* exponential growth, i.e. a smaller doubling time, if the eigenvalue is positive, and a *faster* exponential decay, i.e. a smaller half-life time, if the eigenvalue is negative.

Complex eigenvalues characterize oscillatory behavior, and for such eigenvalues, we offer an empirical and an analytical interpretation:

The *empirical* interpretation relies on the *real* and the *imaginary components* of an eigenvalue (Forrester, 1983; Ogata, 1997). The real component corresponds to the *fractional expansion* (growth) or *contraction* (decay) of the envelopes within which the oscillatory behavior is seen to unfold (see figure 8). One can empirically identify the doubling time or half-life time associated with these envelopes. The imaginary component of an eigenvalue corresponds to the *frequency empirically observed* in a convergent or divergent oscillatory behavior. This frequency is called the damped frequency of the model. This is because this damped frequency results from a frequency reduction relative to the natural frequency of the model. Thus, in general, we can estimate the components (real and imaginary) of the eigenvalues based on empirical evidence from the observed behavior.

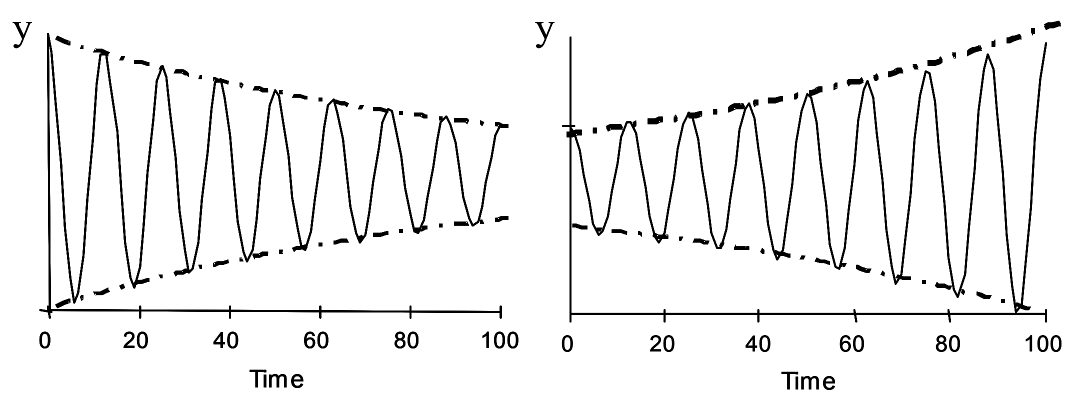

Fig. 8: Oscillatory behavior associated with complex eigenvalues

The *analytical* interpretation of a complex eigenvalue relies on the *absolute value* and the *angle*, - i. e. on a *polar representation* of such an eigenvalue (Forrester, 1983; Ogata, 1997):

The *absolute value* (length) of a complex eigenvalue constitutes the natural frequency, ω_n, of the associated behavior.

> "*… the distance of the pole [a root of the characteristic equations, i.e. an eigenvalue] from the origin is determined by the undamped natural frequency* ω_n " (Ogata, 1997, p. 413).

The natural frequency is the frequency at which the model would oscillate if the attenuation (damping) or amplification (negative damping), i.e. the real component of the eigenvalue, had been reduced to 0. A larger absolute value (length) of a *complex eigenvalue* implies a higher natural frequency in the oscillations exhibited by the model. Consequently, ceteris paribus (constant angle), the model oscillates with a higher frequency, whether these oscillations are amplified, sustained or attenuated.

The *angular location* of a complex eigenvalue is specified by the angle θ, which is measured counter-clockwise from the imaginary axis (see the figure below).

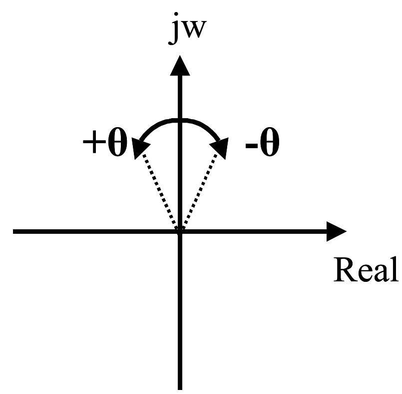

Fig. 9: The angular location of a complex eigenvalue.

Note that the angular location of the eigenvalue has *no* effect on the natural frequency, ω_n, of the model.

If the angle, θ, is 0, then the model behavior does not exhibit any attenuation or amplification. The resulting behavior is a sustained oscillation at the natural frequency of the model, - solely determined by the absolute value (length) of the eigenvalue, i.e. in this case the actual, observed frequency will be the natural frequency.

As the angle changes, from 0, we observe two behavioral effects, associated with the real and the imaginary components of the eigenvalue, respectively:

With respect to the *real components*, we have two alternatives (figure 10):

A. If the angle increases (from 0, changes counter-clockwise), the real component of the eigenvalue (the projection of the eigenvalue on the real axis) is decreased from 0 (takes larger negative values), and thus the *contraction* (decay) of the envelope within which the oscillatory behavior is seen to unfold is increased, resulting in a stronger attenuation (i.e. less envelope half-life time).

B. If the angle decreases (from 0, changes clockwise), the real component of the eigenvalue (the projection of the eigenvalue on the real axis) is increased from 0 (takes larger positive values), and thus the fractional expansion (growth) of the envelope within which the oscillatory behavior is seen to unfold is increased, resulting in a stronger amplification (i.e. less envelope doubling time).

In both cases (A and B), the *imaginary component* of the eigenvalue (the projection of the eigenvalue on the imaginary axis) is being reduced, and thus the damped frequency with which the model actually oscillates is reduced relative to its natural, unaffected frequency.

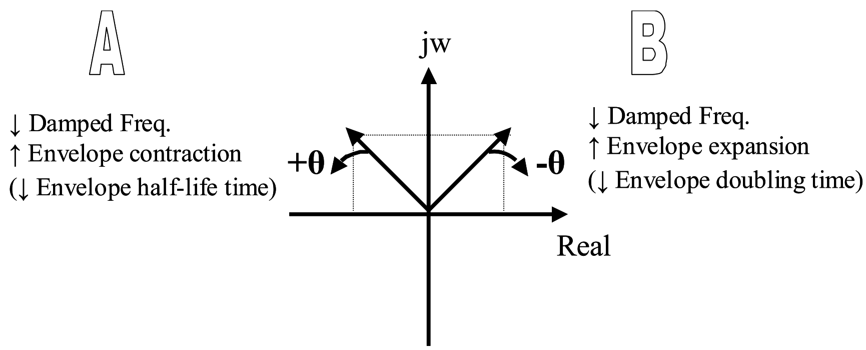

Fig. 10: The effect of a change in the angular location of a complex eigenvalue on the damped frequency and envelope expansion/contraction

This investigation has, so far, left us with a qualitative characterization of a mode of behavior based on;

(a) the polar representation of an eigenvalue, i.e. the absolute value (length) and the angle of the eigenvalue;

(b) the effect of the angle on the real and the imaginary components of the such an eigenvalue; and

(c) the interpretation of the absolute value (length) as well as the real and the imaginary components of the eigenvalue with respect to a mode of behavior of the model.

As we will describe in section 5, we rely on the concept of the *damping ratio* in the analysis and management of complex, dynamics models. The damping ratio, ζ, is associated with the *angular location* of a complex eigenvalue as follows;

$$\zeta = \sin \theta$$

(where, again, θ is measured counter-clockwise from the imaginary axis):

> "*The damping ratio determines the angular location of the poles [roots of the characteristic equations, i.e. eigenvalues] …* " (Ogata, 1997, p. 413)

The relationship between the damping ratio and the angular location of a complex eigenvalue is be portrayed in figure 11:

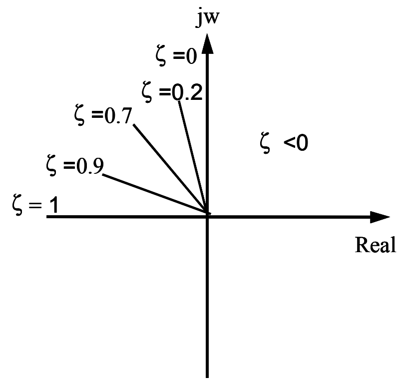

Fig. 11: The relationship between the damping ratio and the angular location of a complex eigenvalue

In managing complex, dynamic models, we typically affect the damping ratio monotonically, i.e. in a positive or a negative direction. Let us now summarize the effect of a monotonic increase in the damping ratio, as portrayed in figure 12:

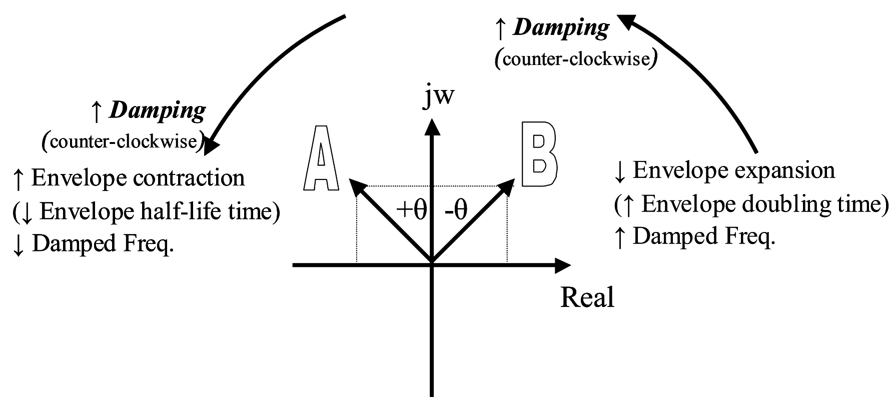

Fig. 12: The effect of a change in damping on envelope expansion/contraction and the damped frequency

In the quadrant A, the angular location is positive and an increased damping causes the angular location to increase (i.e. to diverge from 0). This has two consequences: It increases the *negative*, real component of the eigenvalue and causes the oscillations to contract more forcefully, i.e. less half-life time. Moreover, a larger positive angle will simultaneously reduce the imaginary component of the eigenvalue and the damped frequency of the oscillation will be reduced relative to the natural frequency of the model (determined by the length of the eigenvalue).

In the quadrant B, the angular location is negative and an increased damping causes the angular location to increase (i.e. to approach 0). This has two consequences: It decreases the *positive*, real component of the eigenvalue and causes the oscillations to expand less forcefully, i.e. larger doubling time. Moreover, a negative angle approaching 0 will simultaneously increase the imaginary component of the eigenvalue and thus the damped frequency of the oscillation will be increased towards the natural frequency of the model (determined by the length of the eigenvalue).

In table 4, we summarize the findings with respect to the relationship between damping and the envelope of an oscillatory mode of behavior:

If the eigenvalue has a negative real part (indicating a contracting envelope within which the oscillation takes place), then;
- increasing damping results in a marginal increase in the envelope contraction, and thus in faster decaying amplitude (i.e. a reduction in the envelope half-life time); and
- decreasing damping results in a marginal decrease in the envelope contraction, and thus in slower decaying amplitude (i.e. an increase in the envelope half-life time).

If the eigenvalue has a positive real part (indicating an expanding envelope within which the oscillation takes place), then;
- increasing damping results in a marginal decrease in the envelope expansion, and thus a slower growing amplitude (i.e. an increase in the envelope doubling time); and
- decreasing damping results in a marginal increase in the envelope expansion, and thus a faster growing amplitude (i.e. a reduction in the envelope doubling time).

	Negative real part of the eigenvalue	**Positive real part of the eigenvalue**
Increased damping	↑in the envelope contraction (↓in the envelope half-life time), i.e. faster decaying amplitude	↓in the envelope expansion. (↑in the envelope doubling time), i.e. slower growing amplitude
Decreased damping	↓in the envelope contraction (↑in the envelope half-life time), i.e. slower decaying amplitude	↑in the envelope expansion. (↓in the envelope doubling time), i.e. faster growing amplitude

Table 4: The amplification (↑)/attenuation (↓) of the envelope associated with a complex eigenvalue resulting from of a change in damping

In table 5, we summarize the findings with respect to the relationship between damping and the *empirically observed* damped frequency; ceteris paribus, i.e. assuming fixed natural frequency:

If the eigenvalue has a negative real part (indicating a contracting envelope within which the oscillation takes place), then;

- increasing damping results in a marginal decrease in the damped frequency relative to the natural frequency.
- decreasing damping results in a marginal increase in the damped frequency relative to the natural frequency.

If the eigenvalue has a positive real part (indicating an expanding envelope within which the oscillation takes place), then;

- increasing damping results in a marginal increase in the damped frequency relative to the natural frequency.
- decreasing damping results in a marginal decrease in the damped frequency relative to the natural frequency.

	Negative real part of the eigenvalue	Positive real part of the eigenvalue
Increased damping	↓	↑
Decreased damping	↑	↓

Table 5: The change in the empirical (damped) frequency relative to the natural frequency, resulting from of a change in damping

3.2 Link elasticities associated with an eigenvalue

We are now ready to relate the gains characterizing the model structure to an eigenvalue that characterizes a mode of behavior, by way of the eigenvalue elasticities, - in short elasticities. The elasticity of a link is the fractional change in the eigenvalue, λ_k, resulting from an infinitesimal fractional change in the gain, g_j, of the link.

$$\varepsilon_{kj} = (\delta\lambda_k / \lambda_k) \quad / (\delta g_j \; / g_j)$$

Where $\delta\, g_j \rightarrow 0$ (i.e. $\delta\, g_j$ limits to zero).

Note that an elasticity associated with a real eigenvalue is a real number, and the one associated with a complex eigenvalue is a complex number.
Nathan Forrester (Forrester, 1982; Forrester, 1983) suggested the magnitudes (absolute values) of elasticities as measures of the relative significance (dominance) of links to a certain mode of behavior.

In this paper, we extend our investigation beyond the magnitudes of the elasticities, in the sense that we consider *individually* the real and the imaginary components of the elasticities associated with a dominant complex eigenvalue and their impact on the *length* (absolute value) and the *angle* of that eigenvalue. We will first consider real eigenvalues and the associated real elasticities:

Since a positive, real eigenvalue characterizes the fractional growth of an exponential mode of behavior, a link elasticity associated with that eigenvalue characterizes the marginal impact of a fractional change in the gain of that link on such a growth. I.e. the higher the absolute value taken by the elasticity, the more significantly will a fractional change in the gain impact that growth. If the elasticity is positive, then a fractional gain increase will cause an increase in growth, i.e. a decrease in the doubling time. If the elasticity is negative, it will cause a decrease in growth, i.e. an increase in the doubling time (see table 6).

Since a negative, real eigenvalue characterizes the fractional decay in an exponential mode of behavior, a link elasticity associated with that eigenvalue characterizes the marginal impact of a fractional change in the gain of that link on such a decay. I.e. the higher the absolute value taken by the elasticity, the more significantly will a fractional change in the gain impact that decay. If the elasticity is positive, then a fractional gain increase will cause an increased decay, i.e. a decrease in the half-life time. If the elasticity is negative, then the decay will decrease, i.e. an increase in the half-life time (see table 6).

Thus, for real eigenvalues and elasticities, we can characterize the impact of a fractional change in a link gain on a particular mode of behavior, and thus identify the relationship between the structure and behavior of a model. These findings are summarized in the following table:

	Positive Eigenvalue	**Negative Eigenvalue**
Positive Elasticity	Increase in growth fraction (Decrease in doubling time)	Increase in decay fraction (Decrease in half-life time)
Negative Elasticity	Decrease in growth fraction (Increase in doubling time)	Decrease in decay fraction (Increase in half-life time)

Table 6: The growth and decay fraction resulting from a fractional gain increase conditioned upon the elasticity and the real eigenvalue

Second, we consider complex eigenvalues and the associated complex elasticities. We can now utilize the following properties of the real and the imaginary components of the link elasticities (Forrester, 1983):

(a) The real component of a link elasticity characterizes the impact of a fractional change in the gain on the absolute value (length) of the associated eigenvalue. I.e.:

$$\mathrm{Re}\{\varepsilon_{kj}\} = (\delta|\lambda_k| / |\lambda_k|) \quad / (\delta g_j \ / g_j)$$

(b) The imaginary component of a link elasticity characterizes the impact of a fractional change in the gain on the angle of the associated eigenvalue. I.e.:

$$\mathrm{Im}\{\varepsilon_{kj}\} = \delta\theta_\kappa / (\delta g_j \ / g_j)$$

In section 3.1, we described the relationship between the absolute value and the angle of an eigenvalue, on the one hand, and the associated mode of model behavior on the other. Thus, also for complex eigenvalues and elasticities, we can now characterize the impact of a fractional change in a link gain on a particular mode of behavior, and thus identify the relationship between the structure and behavior of a model.

From (a) above, we can draw the following conclusion: Since the absolute value (length) of a complex eigenvalue characterizes the natural frequency of an oscillatory mode of behavior, the real component of a link elasticity (associated with that eigenvalue) characterizes the marginal impact of a fractional change in the gain on that natural frequency. I.e. the higher the magnitude of the real component of the elasticity, the more significantly will a fractional change in the gain impact that natural frequency. If the real component (of elasticity) is positive, then a fractional gain increase will cause an increased natural frequency, - if the real component is negative, it will cause a decrease (see figure 13).

From (b) above, we can draw the following conclusion: Since the angle of a complex eigenvalue characterizes the damping ratio of an oscillatory mode of behavior, the imaginary component of a link elasticity (associated with that eigenvalue), characterizes the marginal impact of a fractional change in the gain on that damping ratio. I.e. the higher the magnitude of the imaginary component of the elasticity, the more significantly will a fractional change in the gain impact that damping ratio. If the imaginary component (of elasticity) is positive, then a fractional gain increase will cause an increased damping, - if the imaginary component is negative, it will cause a decrease (see figure 13).

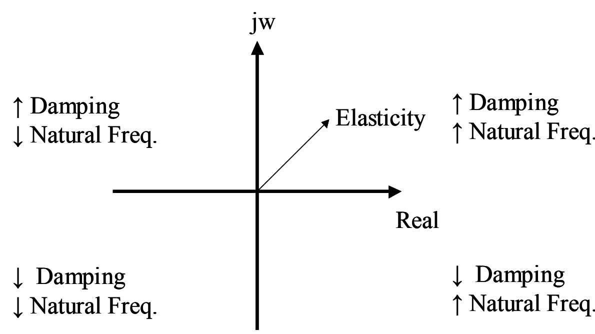

Fig. 13: The change in a complex eigenvalue resulting from a fractional increase in gain, - conditioned upon the current elasticity

Referring to figure 13, a positive imaginary component of elasticity will cause a fractional increase in the gain to induce more damping in the model. And, a negative imaginary component of elasticity will cause a fractional increase in the gain to induce less damping in the model. In general, a *larger absolute value* taken by the imaginary component, whether positive or negative, will cause a certain fractional gain increase to result in a *more* significant, marginal contribution to the damping that takes place.

Moreover, as described in section 3.1, the damping ratio has two effects on the behavior of the model, one on the contraction/expansion of the envelope within which the behavior unfolds, and one on the observed damped frequency of that behavior. Combining the findings in sections 3.1 and 3.2, therefore, we have described the impact of a fractional change in a link gain on the model behavior, conditioned upon the link elasticity.

3.3 The utilization of link elasticities in management

Managers typically face the challenge of modifying behavior by changing the link gains in a system, - here represented by link gains in the model. For that purpose, managers identify the modes that dominate the total behavior of the model and influence;

(a) the growth or decay of the monotonic behavior; or the envelope within which the oscillatory behavior unfolds; and, possibly

(b) the empirical frequency with which the behavior oscillates,

so as to obtain a more favorable overall model behavior. Our analysis in sections 3.1 and 3.2 indicates that the manager can expect a variety of different outcomes with respect to model behavior, resulting from a fractional increase in the gain of a link. This is because that outcome depends on the choice of link (and associated gain) and the combination of values taken by;

- the eigenvalues associated with the dominating modes of behavior; and
- the link elasticities associating the gains of the links in question to those eigenvalues.

For monotonic modes of behavior, characterized by real eigenvalues, a manager can utilize table 6 to identify gains that, when increased (or decreased), can be expected to yield;

(a) an increased or decreased divergence; or

(b) an increased or decreased convergence.

Depending on what is considered a favorable change of behavior, the manager can now operate through a specific subset of gains to obtain the desired modification of the current behavior.

For oscillatory modes of behavior, characterized by complex eigenvalues, a manager can utilize table 7 to identify gains that, when increased (or decreased), can be expected to yield;

(a_1) an increased (incr.) or decreased (decr.) expansion (exp.) of the envelope; or

(a_2) an increased (incr.) or decreased (decr.) contraction (contr.) of the envelope; and

(b) an increased or decreased of the natural frequency (indicated with a transparent arrow); combined with

(c) an increase or decrease in the empirical (damped) frequency, relative to the natural frequency (indicated with a regular arrow);

Again, depending on what is considered a favorable change of behavior, the manager can now operate through a carefully selected subset of gains to obtain the desired modification of behavior both with respect to expansion/contraction and frequency.

	Im{ε} > 0 Re{ε} > 0	Im{ε} > 0 Re{ε}< 0	Im{ε} < 0 Re{ε} > 0	Im{ε} < 0 Re{ε} < 0
Re{λ} > 0	Decr. Exp.	Decr. Exp.	Incr. Exp.	Incr. Exp.
	⇧↑	⇩↑	⇧↓	⇩↓
Re{λ}< 0	Incr. Contr.	Incr. Contr.	Decr. Contr.	Decr. Contr.
	⇧↓	⇩↓	⇧↑	⇩↑

Table 7: The effect of a fractional increase in a link gain on the envelope and frequency characterizing an oscillatory mode of behavior, conditioned upon the eigenvalue (λ), and the link elasticity (ε)

With respect to an increase or decrease in the expansion or contraction of the envelope, table 7 is unambiguous. With respect to the empirical frequency, table 7 is unambiguous when the empirical frequency is changed, relative to the natural frequency, in the same direction as the natural frequency (arrows in the same direction). When the two directions are in conflict (arrows in opposite directions), then the outcome with respect to the empirical frequency is ambiguous, and depends on whether the change in the natural frequency dominates the change in the relationship between the two frequencies or not. The implication is that the manager, in such a case, must rely on an empirical calculation of the net effect. In fact, as we will illustrate in section 5.5 such a calculation can be performed for all the cases represented in table 7. As we will discuss in that section, table 7 enables the manager to interpret the results of such calculations.

4 The Inventory Workforce Case

4.1 Introduction

The inventory-workforce model analyzed in this paper is taken from the CD of the Business Dynamics book (Sterman 2000). The name of the file is "laborwlo.sim" (Powersim model). The model is described in details in chapter 19 in the book. We recommend the reader to review the model, and the associated explanation. In this section, we provide an overview of the model. Basically, the model consists of two sectors, the production and inventory sector, and the labor sector. The two sectors are coupled through desired production and labor. Labor explicitly controls production in the model. In response to a demand shock, the model oscillates. According to Sterman, the research in system dynamics suggests that the business cycle is a damped oscillation originating from the interaction of inventory management with the labor supply chain. This damped oscillation is kept alive in the real world by continuous, random disturbances originating from the environment.

4.2 The production and inventory sector

Below, in figure 14, we portray the stock and flow diagram of the production and inventory sector. The equations of this sector are listed in Sterman's book (Chapter 19).

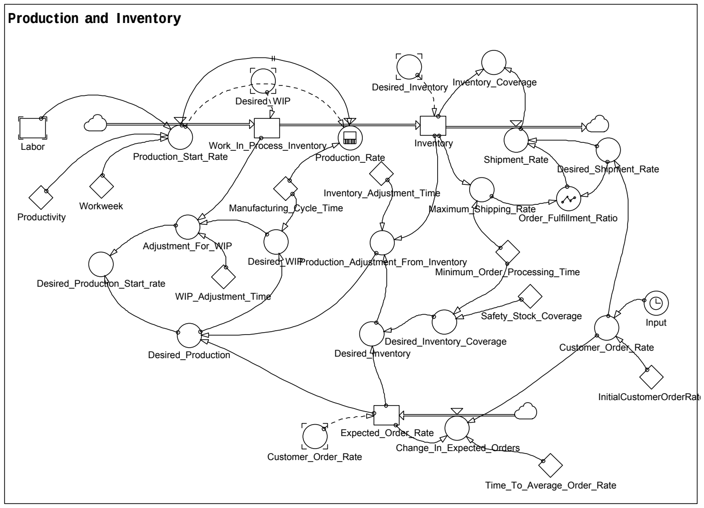

Fig. 14: Production and Inventory sector, stock and flow diagram

4.3 The labor sector

Below, in figure 15, we portray the stock and flow diagram of the labor sector. Again, the equations of this sector are listed in Sterman's book (Chapter 19).

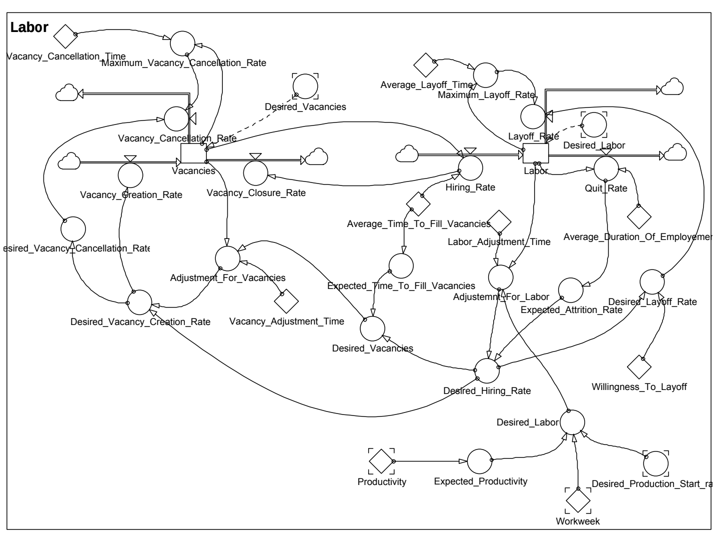

Fig. 15: Labor sector, stock and flow diagram

4.4 Model Behavior

Below, in figure 16, we portray the model response to unanticipated 20% increase in demand (customer orders) in week 5.

It is obvious that the dominant mode of behavior of the model is a mode of "damped oscillations" of period about 53 weeks (~ 1 year). This mode of behavior represents the business cycle.

In the next section - the model analysis section - we will identify the structural components of the model that dominates this mode of behavior (i.e. the damped oscillations). By structural components we mean the links in the model.

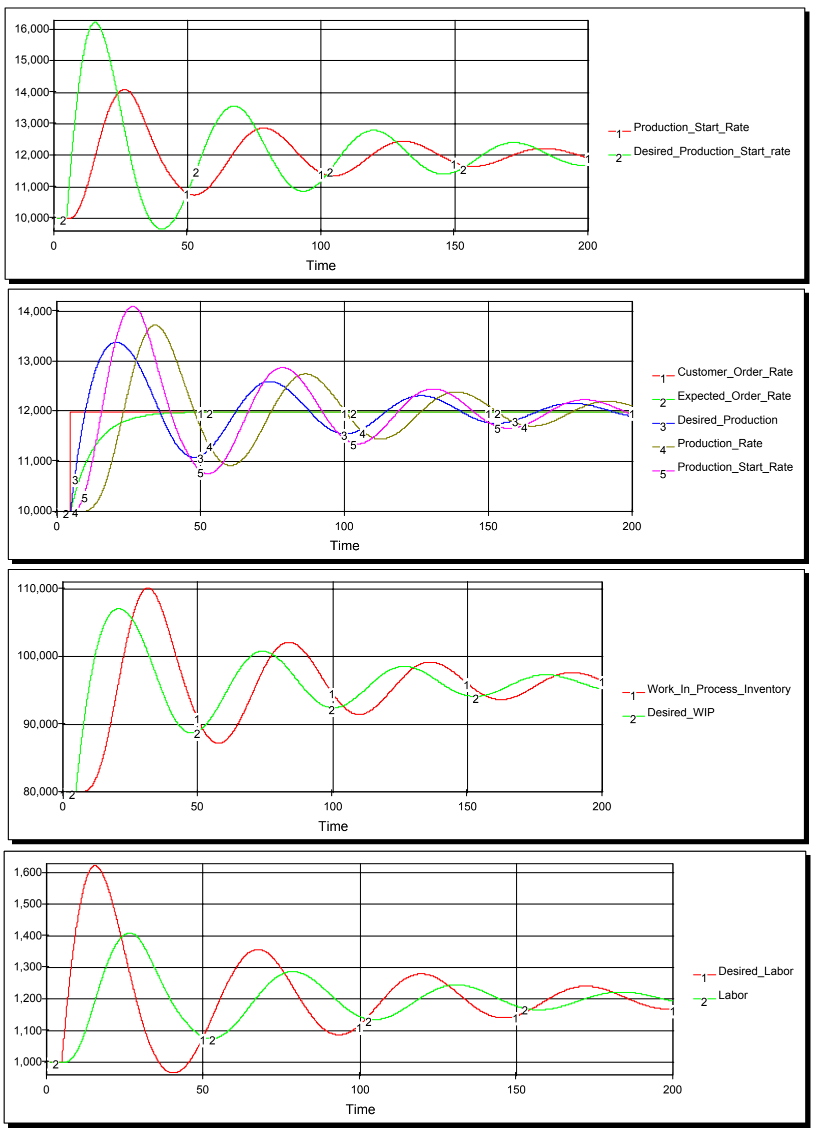

Fig. 16: Model Behavior

5 Model Analysis

5.1 Introduction

In section 5, we apply eigenvalue analysis to the inventory-workforce model, in order to identify the structural origin of the damped oscillation mode. Here are the steps of this analysis:

1. In section 5.2, we identify the state variables in the model.
2. In section 5.3, we identify the gain matrix, which characterizes the model structure.
3. In section 5.4, we identify the eigenvalues of the model, which are computed from the gain matrix.
4. In section 5.4, we also identify the dominant mode of behavior (i.e. the dominant eigenvalue).
5. Finally, in section 5.5, we compute all link elasticities associated with the dominant eigenvalue so as to identify the dominant structure that most significantly impact the dominant mode of behavior. By analyzing these elasticities, we can identify the significance of various points of intervention, i.e. link gains, with respect to model behavior and thus develop a key to the management of such behavior.

5.2 State variables

The first step in the model analysis is to identify the state variables of the model. The state variables include the stock variables in the model (the levels), in addition to the state variables hidden in the delay functions. In this model, there is only one material delay function of third order between Production_Start_Rate and the Production_ Rate. Below is a list of the state variables identified:

1.Expected_Order_Rate 2.Inventory 3.Labor 4.Production_Transit_1 (in the delay function) 5.Production_Transit_2 6.Production_Transit_3 7.Vacancies 8.Work_In_Process_Inventory

5.3 The Gain Matrix

The properties of the structure of the model that we will be considering are the gains of the links that constitute the structure of the model. In any model, we can identify the gain matrix. Each element ($\partial \dot{x}_i / \partial x_j$) in the gain matrix constitutes a compact gain; i.e. the change in the net rate (slope) of each state variable in response to a change in the level (value) of any state variable in the model.

In this model, after week 50, the gain matrix is constant -- i.e. the model is, in fact, reduced to a linear one after week 50 -- and is given as:

$$G=\begin{bmatrix} \partial\dot{x}_1/\partial x_1 & \partial\dot{x}_1/\partial x_2 & \cdot & \partial\dot{x}_1/\partial x_n \\ \partial\dot{x}_2/\partial x_1 & & & \cdot \\ \cdot & & & \cdot \\ \partial\dot{x}_n/\partial x_1 & \cdot & \cdot & \partial\dot{x}_n/\partial x_n \end{bmatrix}=\begin{bmatrix} -0.125 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0000 & 0.0 & 0.0 & 0.0 & 0.375 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.01 & 0.0 & 0.0 & 0.0 & 0.125 & 0.0 \\ 0.0 & 0.0 & 10 & -0.375 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.375 & -0.375 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.375 & -0.375 & 0.0 & 0.0 \\ 0.0718 & -0.005 & -0.201 & 0.0 & 0.0 & 0.0 & -0.375 & -0.004 \\ 0.0 & 0.0 & 10 & 0.0 & 0.0 & -0.375 & 0.0 & 0.0 \end{bmatrix}$$

Whoever is interested in the details of the computations of the elements of this gain matrix, and all the other computations associated with the (eigenvalue) analysis of this model can email the authors.

5.4 Eigenvalues in the model

One can directly compute the eigenvalues from the gain matrix, where each eigenvalue characterizes a certain mode of behavior. The dominant mode of behavior is the mode associated with the dominant eigenvalue.

In this model, the gain matrix is constant after week 50 -- i.e. the model is a linear one after week 50 -- and thus the eigenvalues will also be constant. Here is the list of the eigenvalues:

{-0.0128 + 0.1194i ; -0.0128 - 0.1194i; -0.2840 + 0.1201i; -0.2840 - 0.1201i; -0.4582 + 0.0767i; -0.4582 - 0.0767i; 0.0000; -0.1250}

As the gain matrix is constant then the dominant eigenvalue in the long run will be the eigenvalue with the greatest real part. In our case, this will be the complex conjugate pair of eigenvalues (which represents an oscillatory mode of behavior) whose real component is equal to -0.0128. We have ignored the "zero" eigenvalue, as its left eigenvector is orthogonal to the slope vector. Since the other eigenvalues have real components that are much more negative (an order of ten) than the real component of the dominant eigenvalue (i.e. -0.0128), those modes of behavior will die (dampen) more rapidly than the dominant mode of behavior.

Thus, in conclusion, the dominant mode of behavior (in the long run) will be the mode represented by the complex eigenvalue whose real component is equal to -0.0128, and its imaginary component is equal to 0.1194. This complex eigenvalue represents a mode of "damped oscillations". The period (T) of one damped cycle is given as:

$$T = 2\pi/\text{ imaginary part; I.e. } T = 2\pi/0.1194 = 52.63 \text{ month} \approx 1 \text{ year}$$

This period corresponds to (matches) the behavior generated by the model (see figure 15). Consequently, we have obtained an eigenvalue characterization of the model behavior resulting from an analysis of the underlying model structure, in particular the gains of the compact links constituting this structure.

Note that, in our case, the model happened eventually to develop into a linear model in the long run. In general, however, one cannot expect that such a linearization takes place and we need to recognize the fact that there may be shifts in mode dominance. In such a case, we will need to consider the model and its behavior over infinitely small time intervals and emulate this process by iterating the analysis procedure over a finite number of sufficiently small time intervals (Saleh & Davidsen, 2000).

5.5 Links elasticities

The link elasticities associated with the dominant eigenvalue of the model are summarized in table 8. A careful analysis of these elasticities allows us to map the effect of changing the associated link gains on model behavior. We can thus classify the various link gains with respect to their qualitative impact on that behavior and rank them with respect to their marginal effect on that behavior. We classify the impact of a fractional increase in the gain of each of the links (column 1) in the model into;

- the marginal impact on the natural frequency (column 2); and
- the marginal impact on the damping ratio (column 3).

Relationship (link)	Marginal impact on natural frequency	Marginal impact on damping
Adjustemnt_For_Labor →Desired_Hiring_Rate	+ 0.5054	+ 0.0719
Adjustment_For_Vacancies→DesiredVacancy_Creation_Rate	+ 0.0151	+ 0.1029
Adjustment_For_WIP→ Desired_Production_Start_rate	+ 0.3722	- 0.0284
Desired_Hiring_Rate →Desired_Vacancies	+ 0.3295	+ 0.0218
Desired_Hiring_Rate→ Desired_Vacancy_Creation_Rate	+ 0.1648	+ 0.0109
Desired_Labor→ Adjustemnt_For_Labor	+ 0.4199	- 0.2299
Desired_Production →Desired_Production_Start_rate	+ 0.0477	- 0.2015
Desired_Production_Start_rate → Desired_Labor	+ 0.4199	- 0.2299
Desired_Production→Desired_WIP	+ 0.0637	- 0.2686
Desired_Vacancies→ Adjustment_For_Vacancies	+ 0.3295	+ 0.0218
Desired_WIP→ Adjustment_For_WIP	+ 0.0637	- 0.2686
DesiredVacancy_Creation_Rate→Vacancy_Creation_Rate	+ 0.1798	+ 0.1138
Expected_Attrition_Rate→ Desired_Hiring_Rate	- 0.0111	- 0.0392
Hiring_Rate→Vacancy_Closure_Rate	- 0.1572	+ 0.0405
Inventory → Production_Adjustment_From_Inventory	+ 0.1114	- 0.4701
Labor→ Adjustemnt_For_Labor	+ 0.0855	+ 0.3018
Labor→ Production_Start_Rate	+ 0.4198	- 0.2299
Labor→Quit_Rate	- 0.0129	+ 0.0022
Production_Adjustment_From_Inventory → Desired_Production	+ 0.1114	- 0.4701
Production_Start_Rate—‖→Production_Rate	+ 0.0159	- 0.0672
Quit_Rate→ Expected_Attrition_Rate	- 0.0111	- 0.0392
Vacancies→ Hiring_Rate	+ 0.3370	+ 0.0732
Vacancies→Adjustment_For_Vacancies	- 0.3144	+ 0.0811
Work_In_Process_Inventory → Adjustment_For_WIP	+ 0.3085	+ 0.2402

Table 8: The effects of fractional increases in links gains on the natural frequency and damping

By sorting table 8 according to column 2, we rank the links with respect to the marginal impact of a fractional increase in the associated gains on the natural frequency of the model. Consequently, if there is a need to modify the frequency of the oscillations, then the resulting table indicates which gains have the largest marginal impact on that frequency.

By sorting table 8 according to column 3, we rank the links with respect to the marginal impact of a fractional change in the associated gains on the damping ratio of the model. Consequently, if there is a need to modify the envelope of the oscillations, then the resulting table indicates which gains have the largest marginal impact on the damping of that envelope.

In section 3.3, we presented the theoretical framework for interpreting the relationship between a change in the link gains characterizing the structure of the model and the resulting change in a mode of behavior. Referring to (Kampmann, 1996), we can, in general, calculate, at any point in time, the marginal impact of a fractional change in each such gain on the real and imaginary components of the eigenvalue characterizing such a mode of behavior. As described in section 3.1, the real component corresponds to the fractional expansion (growth) or contraction (decay) of the envelope within which the oscillatory behavior unfolds; while the imaginary component of an eigenvalue corresponds to the frequency empirically observed in a convergent or divergent oscillatory behavior. In table 9, we classify the impact of a fractional increase in the gain of each of the links (column 1) in the model into;

-

- the marginal impact on the expansion / contraction of the envelope (column 2), i.e. the marginal impact on the real component of the eigenvalue.

$$(\delta Re\{\lambda_k\} /|\lambda_k|) \quad / (\delta g_j \ / g_j)$$

- the marginal impact on the empirical (damped) frequency (column 3), i.e. the marginal impact on the imaginary component of the eigenvalue.

$$(\delta Im\{\lambda_k\} /|\lambda_k|) \quad / (\delta g_j \ / g_j)$$

By sorting table 9 according to column 2, we rank the links with respect to the marginal impact of a fractional increase in the associated gains on the expansion or contraction exhibited by the behavior envelope of the model. Consequently, if there is a need to modify that expansion or contraction, then the resulting table indicates which gains have the largest marginal impact on the envelope.

By sorting table 9 according to column 3, we rank the links with respect to the marginal impact of a fractional increase in the associated gains on the empirical (damped) frequency of the model. Consequently, if there is a need to modify the empirical frequency of the oscillations, then the resulting table indicates which gains have the largest marginal impact on that frequency.

Table 7 in section 3.3 furnishes the manager with an interpretation of the calculations resulting in the numbers in table 9. Column 2 in table 9 corresponds to the upper compartment of the cells in table 7. Column 3 in table 9 represents the net effect of the change in the natural frequency combined with the change in the relationship between the natural and empirical frequency of the model behavior, - each portrayed with arrows in the lower compartment in the cells in table 7. Thus the net effect represented in column 3 (table 9), can be decomposed in these two partial effects.

Relationship (link)	**Marginal impact on envelope**	**Marginal impact on damped frequency**
Adjustemnt_For_Labor →Desired_Hiring_Rate	-0.1254	+0.4949
Adjustment_For_Vacancies→DesiredVacancy_Creation_Rate	-0.1039	+0.0040
Adjustment_For_WIP→ Desired_Production_Start_rate	-0.0114	+0.3731
Desired_Hiring_Rate →Desired_Vacancies	-0.0568	+0.3253
Desired_Hiring_Rate→ Desired_Vacancy_Creation_Rate	-0.0284	+0.1627
Desired_Labor→ Adjustemnt_For_Labor	+0.1838	+0.4420
Desired_Production →Desired_Production_Start_rate	+0.1953	+0.0689
Desired_Production_Start_rate → Desired_Labor	+0.1838	+0.4420
Desired_Production→Desired_WIP	+0.2603	+0.0920
Desired_Vacancies→ Adjustment_For_Vacancies	-0.0568	+0.3253
Desired_WIP→ Adjustment_For_WIP	+0.2603	+0.0920
DesiredVacancy_Creation_Rate→Vacancy_Creation_Rate	-0.1323	+0.1666
Expected_Attrition_Rate→ Desired_Hiring_Rate	+0.0402	-0.0069
Hiring_Rate→Vacancy_Closure_Rate	-0.0235	-0.1606
Inventory → Production_Adjustment_From_Inventory	+0.4555	+0.1609
Labor→ Adjustemnt_For_Labor	-0.3092	+0.0528
Labor→ Production_Start_Rate	+0.1838	+0.4419
Labor→Quit_Rate	-0.0008	-0.0131
Production_Adjustment_From_Inventory → Desired_Production	+0.4555	+0.1609
Production_Start_Rate—‖→Production_Rate	+0.0651	+0.0230
Quit_Rate→ Expected_Attrition_Rate	+0.0402	-0.0069
Vacancies→ Hiring_Rate	-0.1087	+0.3273
Vacancies→Adjustment_For_Vacancies	-0.0471	-0.3213
Work_In_Process_Inventory → Adjustment_For_WIP	-0.2717	+0.2811

Table 9: The effects of fractional increases in links gains on the damped frequency and the envelope

6 Conclusion: A management perspective on complex, dynamics system

From a system dynamics perspective, model behavior can be managed through a favorable choice of gain values associated with the links of the system. We have developed a method whereby complex, dynamic systems can be investigated and managed this way. To that end, we use a system dynamics model to represent and investigate the system and to rank the links that constitute the system structure according to their impact on the behavior of the model. By controlling the gain of links of high significance, we may exercise control through relatively small gain modifications. *Whether it is practically of economically feasible to exercise such control, is another matter.*

In principle, we may therefore take into consideration the feasibility of exercising control over a particular link gain by multiplying table 9 (or table 8) by a *feasibility table* in which each cell contains a binary number indicating whether control might be exercised (1) or not (0). Moreover, one can multiply table 9 (or table 8) with a *cost table*, indicating the marginal costs of introducing a unit change in the each link gain. This enables the manager to calculate the marginal effect of an investment in control through the utilization of a particular link gain, so as to compare the resulting effects and to identify the most cost effective and efficient mode of control.

In general, a gain link is associated with all modes of behavior from which the model behavior arise. Thus there are multiple ways in which the choice of a certain link gain value may impact the behavior of a model. In practice, one will primarily consider the impact conveyed through each of the most dominating modes of behavior. Thus managing systems according to this method implies a two-step process: First, we identify the most dominant modes of behavior. Subsequently, we identify the links (and associated gains) of greatest significance to those modes of behavior.

Herein lies a managerial challenge not addressed so far in this paper: Suppose the behavior of a system is composed of a number of dominant modes of behavior. Then the modification of a particular gain, may result in multiple, partial effect on the total behavior, - each constituting the effect on a single mode of behavior. Consequently, the manager is forced to consider the combined effects of modifying each of the link gains under his/her control and carefully select and utilize a subset of such gains. Preferably, these are the gains that are expected to allow him to exercise the most precise (unambiguous) control over the total behavior of the system.

As has already been stated throughout the paper. In nonlinear systems, the eigenvalues, right eigenvectors and link elasticities are subject to changes resulting from the dynamics of the system. Consequently, the method outlined in this paper must be applied iteratively over a sequence of appropriately small time intervals.

Finally, recognizing that links form the loops that constitute the structure of feedback models, in our subsequent work, we will investigate the significance of such loops with respect to model behavior. Thus we will shift our structural unit of analysis from links to loops.

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