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Some Dynamics Balance of Production via Optimization and Simulation within System Dynamics Method

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Abstract

The purpose of this paper is to present the dynamic optimal balance of production. Two different formulations of this problem discussed here on the examples presented by authors. The constrained and unconstrained optimization are presented on the basis of many simulation experiments carried out by authors.

Keywords: System Dynamics Method, Optimization during Simulation, Simulation during Optimization, Balance of Production.

1 Introduction

The problem of optimal dynamics balance of flows is almost completly new on the area of System Dynamics method. The first attempts in this subject were undertaken by Kasperska (Kasperska 1990), then by Kasperska and Słota (Kasperska and Słota 2000) and the newest publication by Kasperska, Mateja-Losa and Słota (Kasperska et al. 2000a, Kasperska et al. 2000b). These attempts were conected with unconstrained optimization during simulation and simulation during optimization in sense of Coyle (Coyle 1996, Coyle 1998, Coyle 1999). Now authors extended these formulations to constrained versions of the problem. Many interesting experiments were taken and some conclusions are presented.

2 The constrained and unconstrained optimizations of the dynamics balance of production

To approach this problem we have to return to the example of dynamics balance of production, discussed in paper (Kasperska et al. 2000a, Kasperska et al. 2000b). In Figure 1 an example of the production system, in view of extended Łukaszewicz symbols (Łukaszewicz 1975, Łukaszewicz 1976), is presented.

The element in "double surrounding" was called "optimal balance of α, β, γ", and has more important meaning here. Now we want to concentrate on these parts of the model which are related to constrained and unconstrained optimization in sense of Coyle (Coyle 1996, Coyle 1999).

The parameters α, β, γ were optimized during simulation (and have lower and upper limits). We assume that there are six cases of balances:

I) **unconstrained balance of flows,** $\alpha + \beta + \gamma$ is optional (any), the formulation of the objective function f_{ob} is clear from the figure 1 (it consists of three elements with three weighting factors);

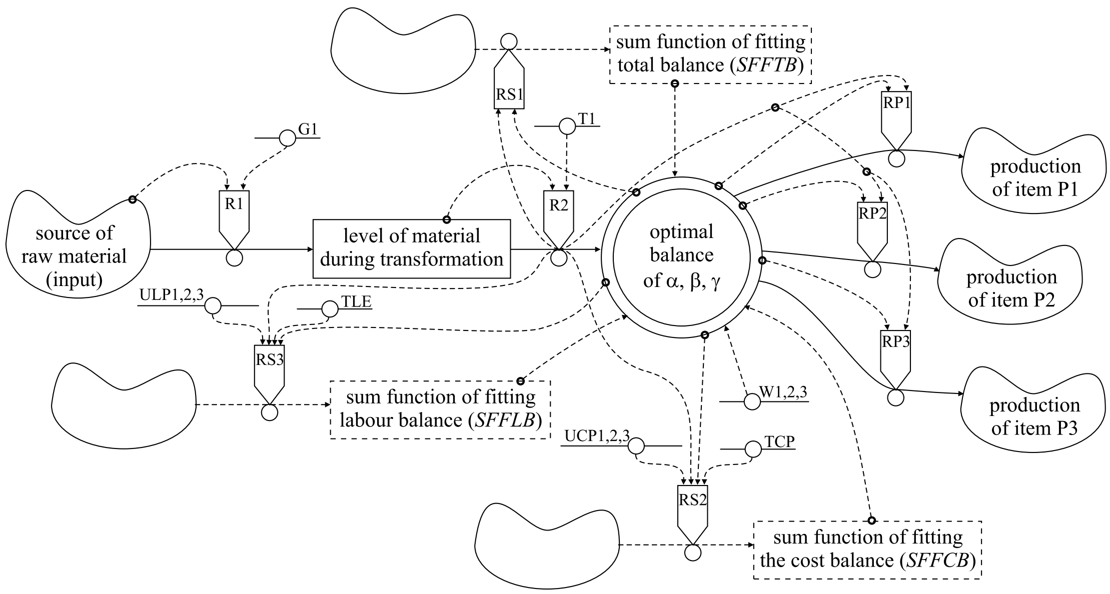

Figure 1. Optimal dynamics balance of production.

II) **constrained balance of flows,** $\alpha + \beta + \gamma = 1$, this condition denotes full accordance with the actual production of three items with optimal value of their production. Addition of penalty function was required what resulted in the occurence of discrepancies from the condition ($\alpha+\beta+\gamma = 1$). When the condition is not fulfiled ($\alpha+\beta+\gamma < 1$ or $\alpha+\beta+\gamma > 1$) the massive penalty factor named $kara$ is added to the value of the base function f_{ob}. Technically speaking it has a form of:

$$penalty = kara * \max\Big(0, \text{abs}(\alpha + \beta + \gamma - 1)\Big);$$

III) **constrained balance of flows,** $\alpha + \beta + \gamma \leqslant 1$. The condition of balance is now "not sharp". The difference between case II a this one is that now penalty function has the form of:

$$penalty = kara * \max\Big(0, \alpha + \beta + \gamma - 1\Big).$$

Only when $\alpha + \beta + \gamma > 1$ the $kara$ is added to the base function f_{ob};

IV) **constrained balance of flows,** $\alpha + \beta + \gamma < 1$. The condition of balance is now "sharp". The logistic alternative function clip (see Coyle 1977, Coyle 1996, Forrester 1961, Forrester 1972) is now added:

$$penalty = kara * \text{clip}\Big(1, 0, \alpha + \beta + \gamma, 1\Big).$$

The interesting point of view is: how to interpret such a condition $\alpha + \beta + \gamma < 1$. We think that in such a case the actual values of possible production are bigger than those required by optimal balance of production. In the affect it should cause the limitation of actual production;

V) **constrained balance of flows,** $\alpha + \beta + \gamma \geqslant 1$. The condition of balance is now "not sharp". Similarly to IV, the logistic function clip is added:

$$penalty = kara * \text{clip}\Big(0, 1, \alpha + \beta + \gamma, 1\Big).$$

We can interpret such a condition in the following way: the possibilities of production of three items are too small when compared to those suggested by optimal balance ($\alpha+\beta+\gamma > 1$) and this should cause the development of these possibilities;

VI) **constrained balance of flows,** $\alpha + \beta + \gamma > 1$. This is a "sharp" version of type V. Similarly to IV and V, we added the logic function clip:

$$penalty = kara * \text{clip}\Big(1, 0, 1, \alpha + \beta + \gamma\Big).$$

Different cases of balances (I–VI) require different base conditions of parameters α, β, γ (see tables 1–6 in the next section of this paper).

Many interesting experiments were undertaken by authors using COSMIC and COSMOS (Cosmic 1994, Coyle 1996). We have to stress that we are on the beginning of the way to better experimentation with COSMIC and COSMOS. Lots of attention should be paid to chosing the factor $kara$ and the base value of parameters.

Now we want to apply the constrained optimization to our idea presented in Lozanna (Kasperska et al. 2000a, Kasperska et al. 2000b), named "optimization during simulation". In that article we promised (in conclusions) that it would be interesting to investigate the so called "pseudosolution" of differencies $Mx - b$ (see Legras 1974) at the condition $x_i \geqslant 0$, $i = 1, 2, 3$. In such a case of optimal balance of production we have to solve the system of equations, which is created from the balance of the value of three properties of flows: mass balance ("rate of flow" in Forrester sense), cost balance and personal balance. The idea of constrained optimization ($x_i \geqslant 0$) required the extension of matrix M to the form of:

$$M = \begin{pmatrix} 1 & 1 & 1 \\ ucp_1 & ucp_2 & ucp_3 \\ ulp_1 & ulp_2 & ulp_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where: ucp_i – unit cost of the production of product p_i, $i = 1, 2, 3$, ulp_i – unit labour of the production of product p_i, $i = 1, 2, 3$. The matrix b should be extended to the form:

$$b = \Big(r_2, tcp, tle, b_4, b_5, b_6\Big)^T,$$

where: r_2 – output rate of raw material (production), tcp – total cost expenditure of production, tle – total labour expenditure of production, b_i, $i = 4, 5, 6$ – the number of large value (chosen experimentally). The solution of equation:

$$M \cdot x = b$$

has taken the form of (Legras 1974):

$$x = (M^T \cdot M)^{-1} \cdot M^T \cdot b.$$

To computer programing of this we used DYNAMO for Windows (Dynamo 1994). Authors' ideas were supported by their mathematical background and possibilities of language DYNAMO. The readers can see this in Appendix A.

The comparison of the results of both authors' investigations: optimization during simulation (in sense of solving system of balance equation during simulation) and simulation during optimization (in sense of Coyle), we can see discussing figures in the next section of the paper.

3 The results of simulation

First we want to present some of the results of the experiments that we achieved from simulation during optimization (using COSMIC and COSMOS).

Tabel 1 contains the results of the first experiment type I of optimization (described in section 2 of our paper). We can see that all of the three parameters: α, β, γ are assuming their upper limits and find the value of function f_{ob} which constitutes 34 % of its initial value.

Table 1.

Parameter	Final value	Orginal value	Lower limit	Upper limit
α	1.0	0.50	0	1
β	1.0	0.25	0	1
γ	1.0	0.25	0	1
Initial value of objectiv function f_{ob}				$0.124 \cdot 10^{10}$
Final value of objectiv function f_{ob}				$0.420 \cdot 10^{9}$
Final value of $sfftb$				$10.3 \cdot 10^{7}$
Final value of $sffcp$				$10.2 \cdot 10^{8}$
Final value of $sfflp$				$80.5 \cdot 10^{6}$

Table 2 contains the results of the experiment number 2 type I of optimization, which differ from the assumption of the experiment number 1 about the values of parameters tcp and tle (tcp differ from 7000 to new value 3500 and tle from 1800 to 1000). Parameter α assumes its lower imit, and final value of function f_{ob} has 19 % of its initial value.

Table 2.

Parameter	Final value	Orginal value	Lower limit	Upper limit
α	0.000	0.50	0	1
β	0.855	0.25	0	1
γ	0.924	0.25	0	1
Initial value of objectiv function f_{ob}				$0.203 \cdot 10^{9}$
Final value of objectiv function f_{ob}				$0.383 \cdot 10^{8}$
Final value of $sfftb$				$15.7 \cdot 10^{6}$
Final value of $sffcp$				$76.0 \cdot 10^{6}$
Final value of $sfflp$				$18.9 \cdot 10^{6}$

Table 3 contains the results of the experiment number 3 of type II of optimization ($\alpha+\beta+\gamma = 1$). Parameters α and γ assume their lower limits and parametr β assumes its upper limit. The value of objective function f_{ob} is 61 % of its initial value. The characteristic result is that $sfftb = 0$.

Table 4 contains the results of the experiment number 4 of type III of optimization ($\alpha+\beta+\gamma \leqslant 1$). The sum of values of parameters α, β and γ is considerably smaller than one, and the value of objective function f_{ob} is 84 % of its initial value.

Table 5 contains the results of the experiment number 5 of type IV of optimization ($\alpha+\beta+\gamma < 1$). The assumed values of α, β and γ are such that their sum is smaller than one. The value of objective function f_{ob} is 92 % of its initial value. Such results show that chosen initial values were very close to the optimum values.

Table 3.

Parameter	Final value	Orginal value	Lower limit	Upper limit
α	0.0	0.50	0	1
β	1.0	0.25	0	1
γ	0.0	0.25	0	1
Initial value of objectiv function f_{ob}				$0.186 \cdot 10^{10}$
Final value of objectiv function f_{ob}				$0.113 \cdot 10^{10}$
Final value of $sfftb$				0.0
Final value of $sffcp$				$20.89 \cdot 10^{8}$
Final value of $sfflp$				$71.74 \cdot 10^{6}$

Table 4.

Parameter	Final value	Orginal value	Lower limit	Upper limit
α	0.429	0.50	0	1
β	0.559	0.25	0	1
γ	0.003	0.25	0	1
Initial value of objectiv function f_{ob}				$0.124 \cdot 10^{10}$
Final value of objectiv function f_{ob}				$0.105 \cdot 10^{10}$
Final value of $sfftb$				$18.34 \cdot 10^{2}$
Final value of $sffcp$				$29.36 \cdot 10^{8}$
Final value of $sfflp$				$73.23 \cdot 10^{6}$

Table 5.

Parameter	Final value	Orginal value	Lower limit	Upper limit
α	0.365	0.30	0	1
β	0.317	0.25	0	1
γ	0.317	0.25	0	1
Initial value of objectiv function f_{ob}				$0.194 \cdot 10^{10}$
Final value of objectiv function f_{ob}				$0.176 \cdot 10^{10}$
Final value of $sfftb$				1.0314
Final value of $sffcp$				$32.87 \cdot 10^{8}$
Final value of $sfflp$				$97.95 \cdot 10^{6}$

Table 6 contains the results of the experiment number 6 of type VI of optimization ($\alpha+\beta+\gamma > 1$). The parameters α, β and γ assumed their upper limit values. The value of objective function f_{ob} is 34.3 % of its initial value.

Some of these results are presented in an illustrative form on figures 2–10.

Now we want to present some of the results of the experiments that we achieved from optimization during simulation (using DYNAMO for Windows). Some values of parameters ucp_1 (see Section 2 of this paper), tle, tcp and values of b_i, $i = 4, 5, 6$ vary in respective experiments numbered 1–5 (see program in Appendix A). The system appeared to be very sensitive to these values, what readers can see in figures 11–13.

Table 6.

Parameter	Final value	Orginal value	Lower limit	Upper limit
α	1.0	1.00	0	1
β	1.0	0.25	0	1
γ	1.0	0.25	0	1
Initial value of objectiv function f_{ob}				$0.168 \cdot 10^{10}$
Final value of objectiv function f_{ob}				$0.577 \cdot 10^{9}$
Final value of $sfftb$				$10.32 \cdot 10^{7}$
Final value of $sffcp$				$10.22 \cdot 10^{8}$
Final value of $sfflp$				$80.45 \cdot 10^{6}$

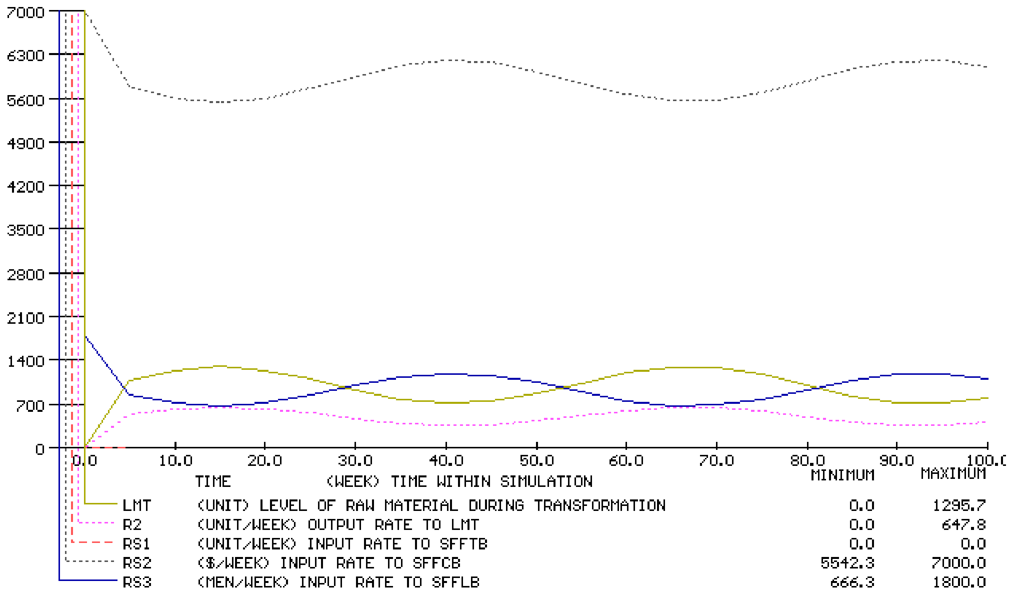

Figure 2. The results of unconstrained optimization: $\alpha + \beta + \gamma = optional$ (the dynamics of the characteristic of some variables of the production system).

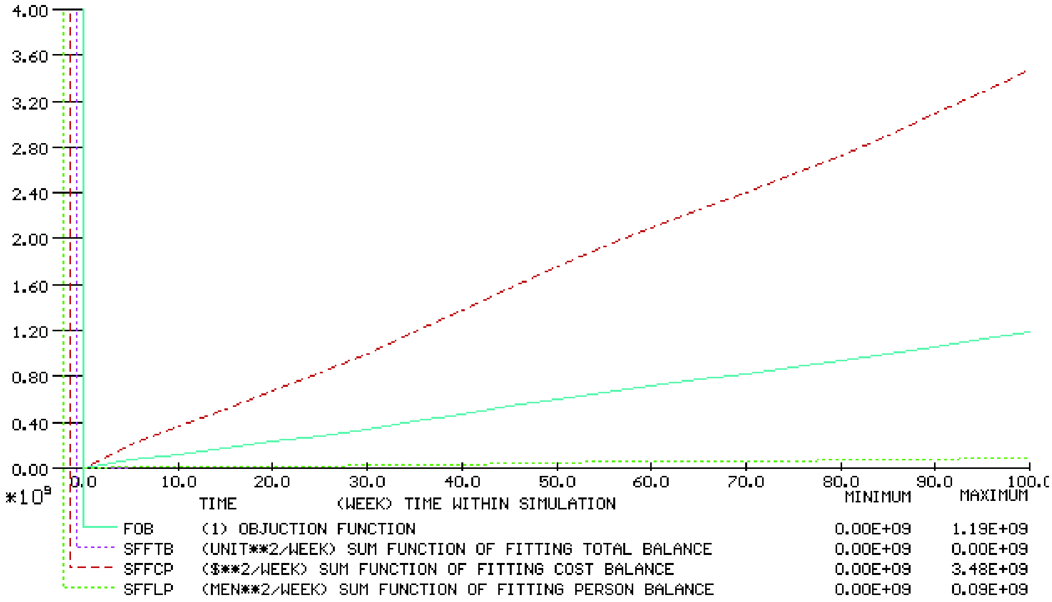

Figure 3. The results of unconstrained optimization: $\alpha+\beta+\gamma = optional$ (the dynamics of the characteristic of the objective function and its elements).

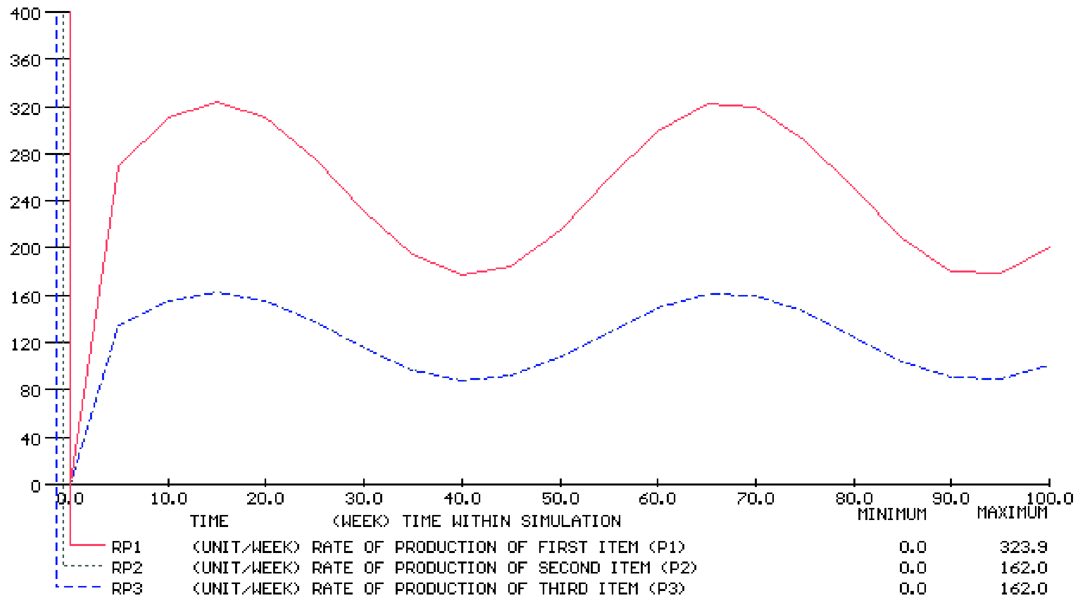

Figure 4. The results of unconstrained optimization: $\alpha+\beta+\gamma = optional$ (the dynamics of the characteristic of the production rates for three items).

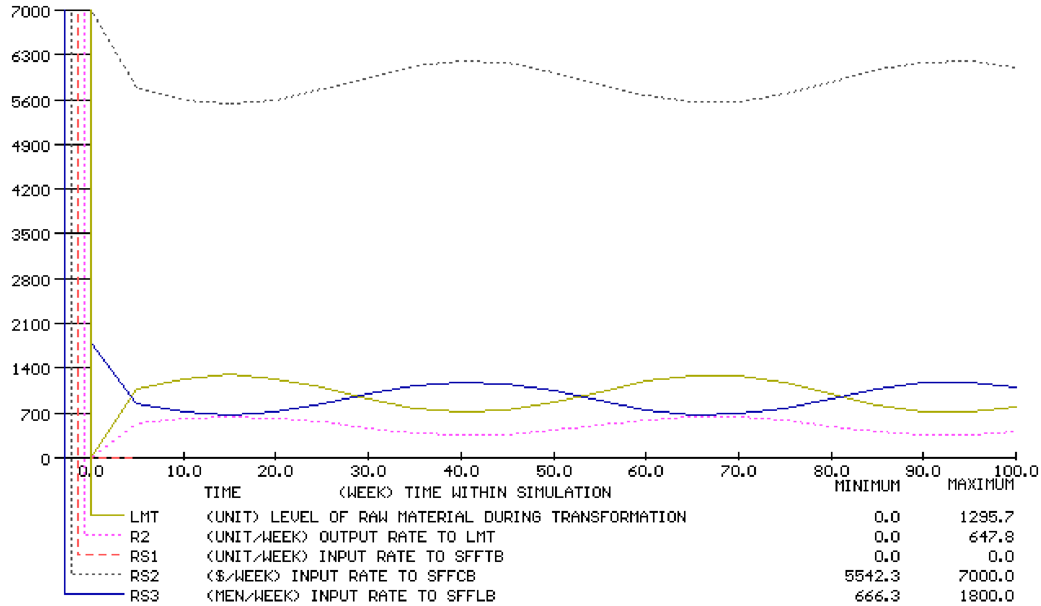

Figure 5. The results of constrained optimization: $\alpha + \beta + \gamma = 1$ (the dynamics of the characteristic of some variables of the production system).

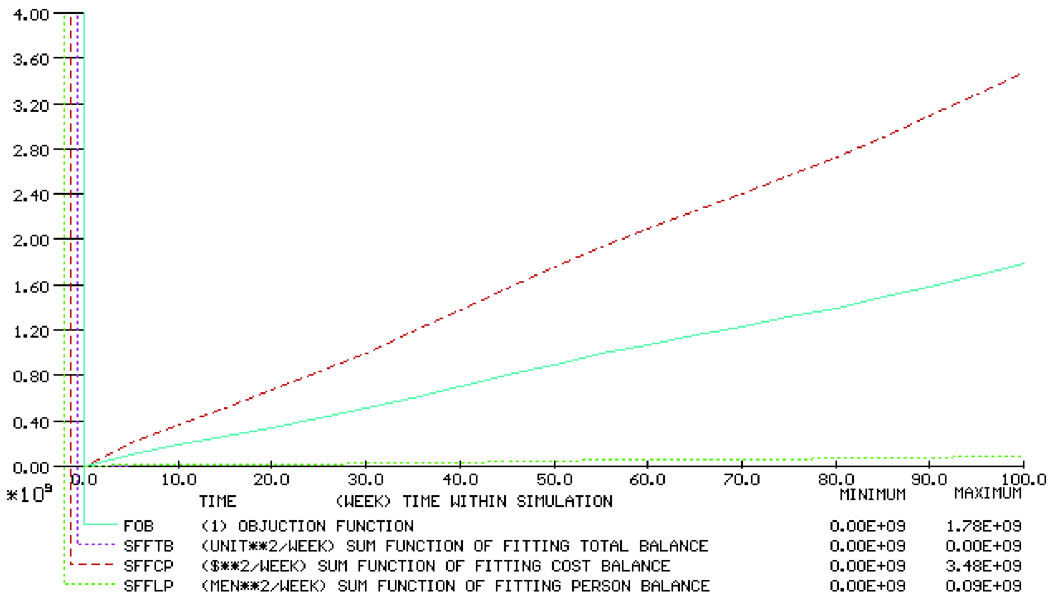

Figure 6. The results of constrained optimization: $\alpha + \beta + \gamma = 1$ (the dynamics of the characteristic of the objective function and its elements).

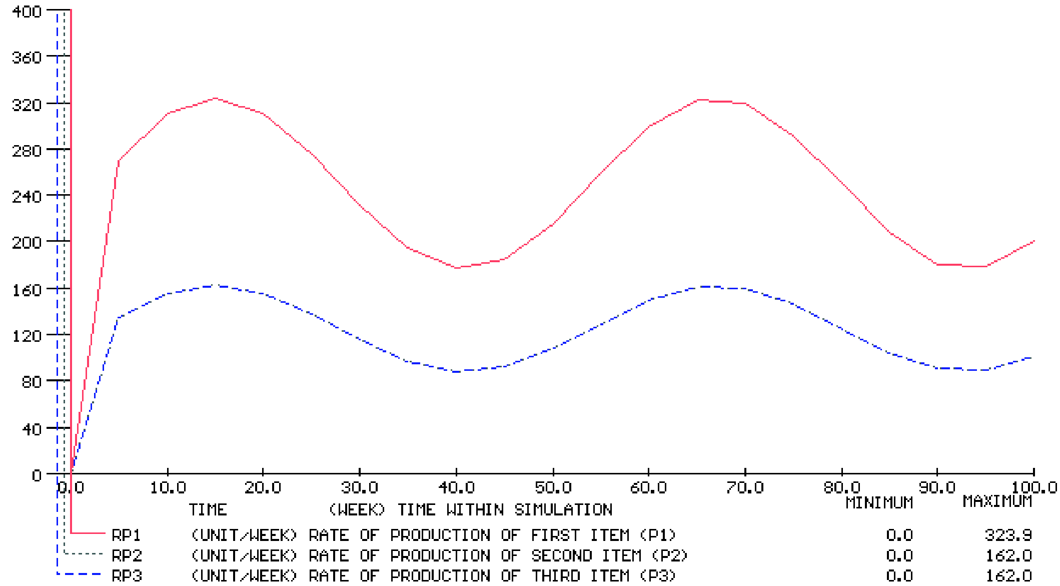

Figure 7. The results of constrained optimization: $\alpha + \beta + \gamma = 1$ (the dynamics of the characteristic of the production rates for three items).

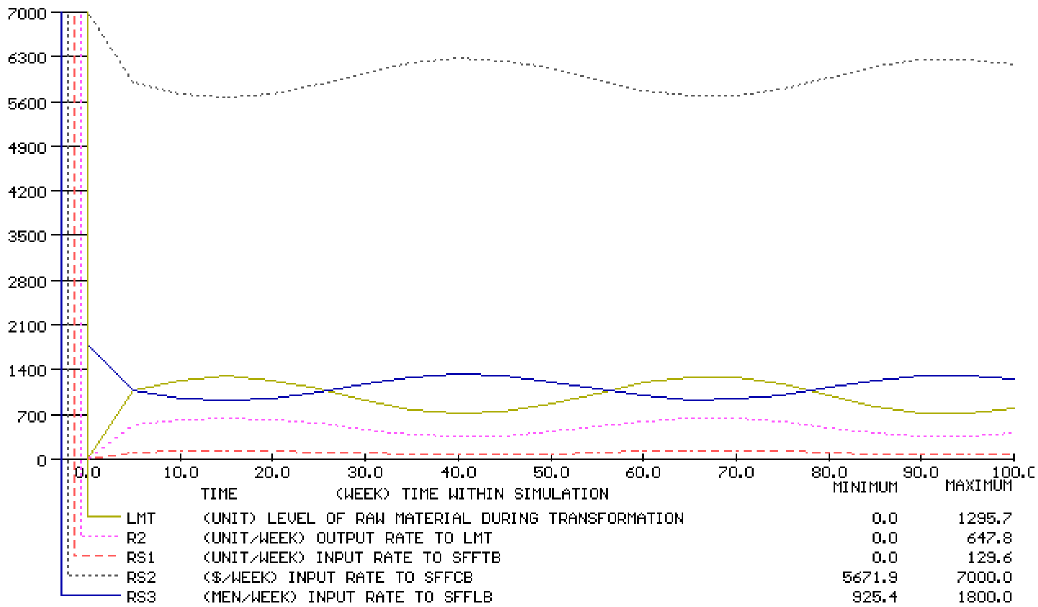

Figure 8. The results of constrained optimization: $\alpha + \beta + \gamma < 1$ (the dynamics of the characteristic of some variables of the production system).

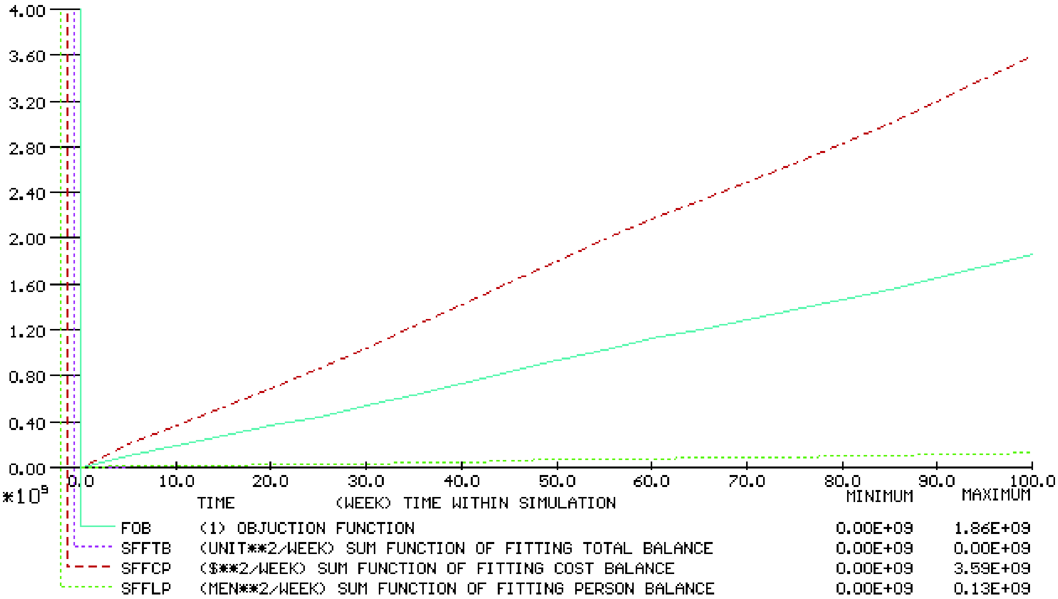

Figure 9. The results of constrained optimization: $\alpha + \beta + \gamma < 1$ (the dynamics of the characteristic of the objective function and its elements).

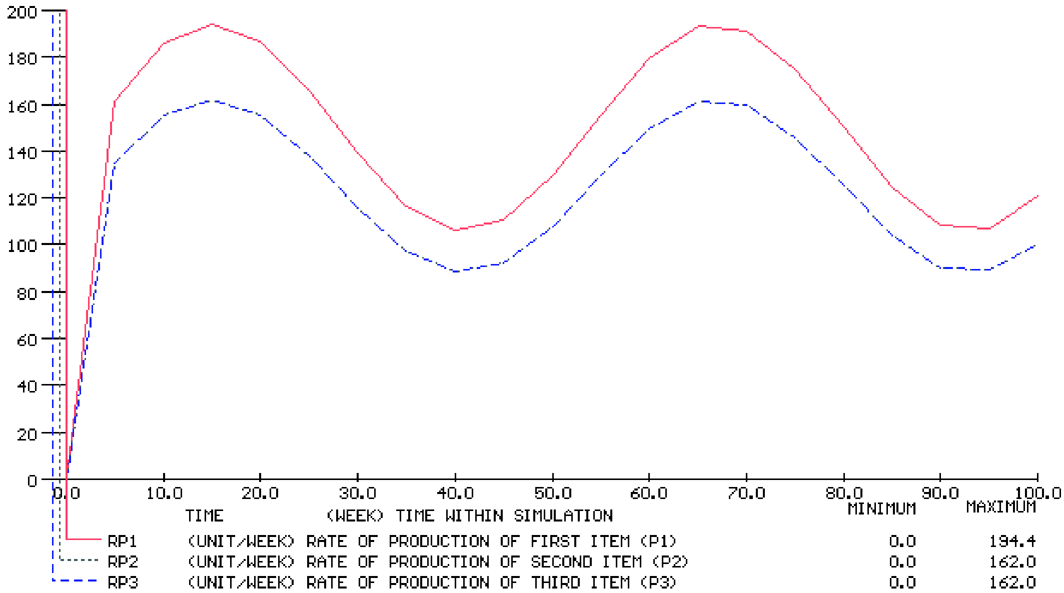

Figure 10. The results of constrained optimization: $\alpha + \beta + \gamma < 1$ (the dynamics of the characteristic of the production rates for three items).

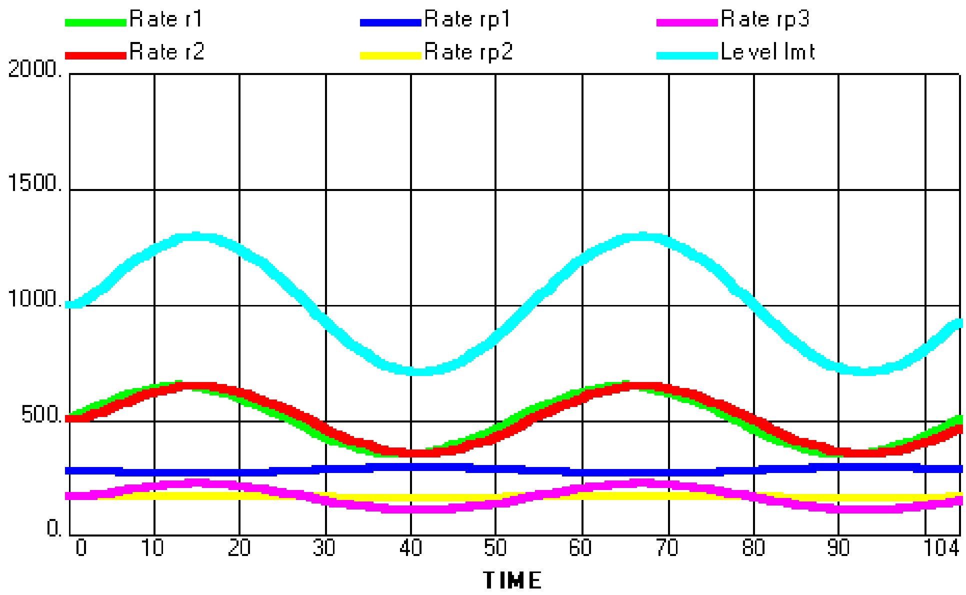

Figure 11. The dynamics of the characteristic of some variables of the model by optimization during simulation ($ucp_1 = 10$, $tcp = 4000$, $tle = 990$, $b_4 = b_5 = b_6 = 300$).

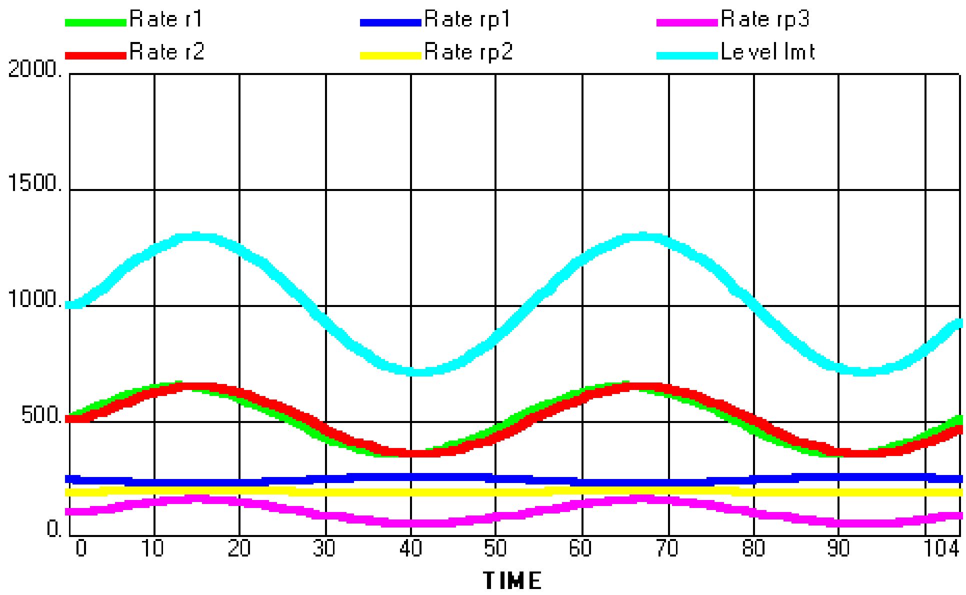

Figure 12. The dynamics of the characteristic of some variables of the model by optimization during simulation ($ucp_1 = 10$, $tcp = 3500$, $tle = 1000$, $b_4 = b_5 = b_6 = 50$).

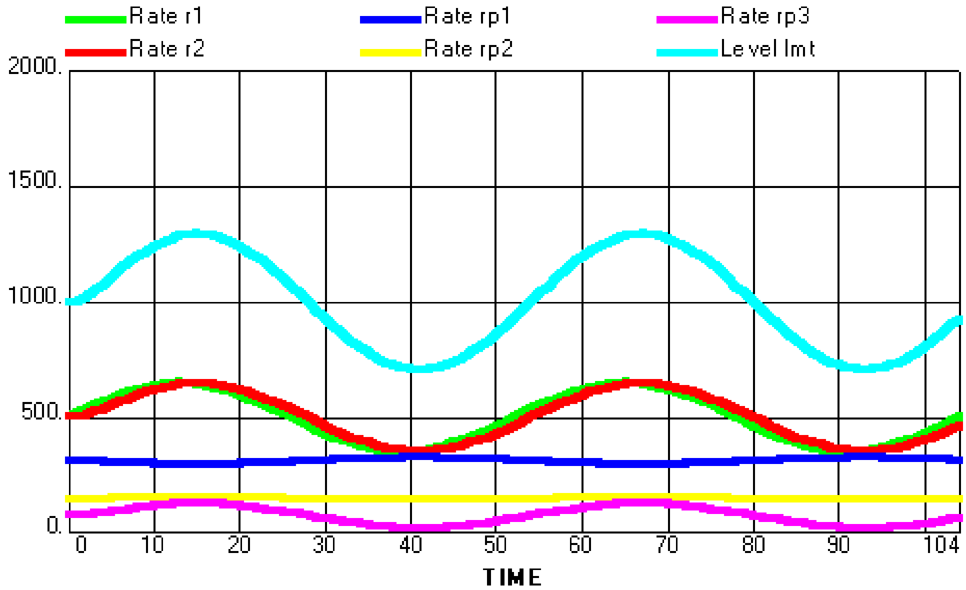

Figure 13. The dynamics of the characteristic of some variables of the model by optimization during simulation ($ucp_1 = 10$, $tcp = 4000$, $tle = 990$, $b_4 = 100$, $b_5 = b_6 = 50$).

4 Conclusion

After modelling and simulating some optimal balance of production we have come up to the following conclusions:

a) The optimization during simulation allows to achieve the optimal pseudosolutions of the underdetermined system of equations (in sense of Legras (Legras 1974)). The solutions keep limitation $x_i \geqslant 0$, $i = 1, 2, 3$, which was required by the physical sense of flows. The future experiments will go towards more precise selection of the value of "large" variables (see section 2) in matrix b, to minimize the norm of the discrepancies $(Ax_i - b_i)$, $i = 1, 2, 3$.

b) The simulation during optimization (in sense of Coyle) allows to achieve the optimal solution (the value of parameters α, β, γ) and the objective function f_{ob}. The future experiments should extend the set of optimizing parameters, for example the parameters: tle, tcp, ucp_i, $i = 1, 2, 3$. Much more attention should be paid to selecting the base value of parameters α, β, γ in different kinds of constrained optimization.

The authors are on the beginning of the way to better experimentation with COSMIC and COSMOS. We are planning to extend the optimal balancing of flows to more value of dimension and more sophisticated form of the objective function.

We want to thank to Prof. R. G. Coyle for the inspiration from his books and articles and for his COSMOS and COSMIC which are the fruitful tools for working with System Dynamics models.

References

COSMIC and COSMOS user manuals. 1994. The COSMIC Holding Co.: Shrivenham.

Coyle, R. G. 1977. Management System Dynamics. John Wiley & Sons: New York.

Coyle, R. G. 1978. System Dynamics – The state of the art. Dynamica 5: 3–23.

Coyle, R. G., Wolsterholm, E. P. 1980. Modelling discrete events in System Dynamics model. A case study. Dynamica 6: 21–27.

Coyle, R. G. 1996. System Dynamics modelling. A practical approach. Chapman & Hall: London.

Coyle, R. G. 1998. The practice of System Dynamics: milestones, lessons and ideas from 30 years experience. System Dynamics Review 14: 343–365.

Coyle, R. G. 1999. Simulation by repeated optimization. Journal of the Operational Research Society 50: 429–438.

Forrester, J. W. 1961. Industrial Dynamics, MIT Press: Massachusetts.

Forrester, J. W. 1972. Principles of Systems, Cambridge Press: Massachusetts.

Forrester, J. W. 1975. Collected papers of Jay W. Forrester. Cambridge Press: Massachusetts.

Kasperska, E. 1990. Methods of simulation of the investigation into supporting planning and organization in industry with continuous processes. Ph.D. thesis. Polish Academy of Science: Warsaw (in Polish).

Kasperska, E., Słota, D. 2000. Mathematical Method of Management in the Concept of System Dynamics. Silesian Technical University: Gliwice (in Polish).

Kasperska, E., Mateja-Losa, E., Słota, D. 2000. Some extension of System Dynamics method – theoretical aspects. In Proc. 16th IMACS World Congress. Deville M. and Owens R. (eds). IMACS: Lausanne; 718-10, 1–6.

Kasperska, E., Mateja-Losa, E., Słota, D. 2000. Some extension of System Dynamics method – practical aspects. In Proc. 16th IMACS World Congress. Deville M. and Owens R. (eds). IMACS: Lausanne; 718-11, 1–6.

Legras, J. 1974. Praktyczne metody analizy matematycznej. WNT: Warszawa (translation from French: Methodes et Technique De l'Analyse Numerique. Dunod: Paris 1971).

Łukaszewicz, R. 1975. Management System Dynamics. PWN: Warsaw (in Polish).

Łukaszewicz, R. 1976. The direct form of structure models within System Dynamics. Dynamica 2: 36–43.

Professional DYNAMO 4.0 for WINDOWS. Reference manual. 1994. Pugh-Roberts Associates: Cambridge.

Appendix A. Program in DYNAMO

```
* Balance of production of 3 items
note
note  level of raw material during transformation
note
n lmt=1000
l lmt.k=lmt.j+dt*(r1.jk-r2.jk)
note
note input rate to lmt (r1)
note
r r1.kl=g1*input.k
c g1=5
note
note input - source of raw material
note
a input.k=po+p1*sin((6.28*time.k)/perd)
c po=100
c p1=30
c perd=52
note
note output rate from lmt (r2)
note
r r2.kl=lmt.k/t1
c t1=2
note
note unit cost of production of 1-st item (ucp1)
note unit cost of production of 2-st item (ucp2)
note unit cost of production of 3-st item (ucp3)
note
c ucp1=10
c ucp2=5
c ucp3=2
note
note unit labour of production of 1-st item (ulp1)
note unit labour of production of 2-st item (ulp2)
note unit labour of production of 3-st item (ulp3)
note
c ulp1=2
c ulp2=2
c ulp3=1
note
note total cost of expediture of production (tcp)
note total labour of expediture of production (tlc)
note
c tcp=4000
c tle=990
note
note vector b
note
a b1.k=r2.kl
```

```
a b2.k=tcp
a b3.k=tle
a b4.k=100
a b5.k=50
a b6.k=50
note
note vector at.b
note
a bb1.k=b1.k+b4.k+b2.k*ucp1+b3.k*ulp1
a bb2.k=b1.k+b5.k+b2.k*ucp2+b3.k*ulp2
a bb3.k=b1.k+b6.k+b2.k*ucp3+b3.k*ulp3
note
note matrix c=at.a
note
a c11.k=2+ucp1*ucp1+ulp1*ulp1
a c12.k=1+ucp1*ucp2+ulp1*ulp2
a c13.k=1+ucp1*ucp3+ulp1*ulp3
a c21.k=1+ucp1*ucp2+ulp1*ulp2
a c22.k=2+ucp2*ucp2+ulp2*ulp2
a c23.k=1+ucp2*ucp3+ulp2*ulp3
a c31.k=1+ucp1*ucp3+ulp1*ulp3
a c32.k=1+ucp2*ucp3+ulp2*ulp3
a c33.k=2+ucp3*ucp3+ulp3*ulp3
note
note  determinant of matrix c=at.a
note
a detc.k=-c13.k*c22.k*c31.k+c12.k*c23.k*c31.k+c13.k*c21.k*c32.k ^
         -c11.k*c23.k*c32.k-c12.k*c21.k*c33.k+c11.k*c22.k*c33.k
note
note matrix d=Det[c]*Inverse[c]
note
a d11.k=c22.k*c33.k-c23.k*c32.k
a d12.k=c13.k*c32.k-c12.k*c33.k
a d13.k=-c13.k*c22.k+c12.k*c23.k
a d21.k=-c21.k*c33.k+c31.k*c23.k
a d22.k=-c13.k*c31.k+c11.k*c33.k
a d23.k=c13.k*c21.k-c11.k*c23.k
a d31.k=-c22.k*c31.k+c21.k*c32.k
a d32.k=c12.k*c31.k-c11.k*c32.k
a d33.k=-c12.k*c21.k+c11.k*c22.k
note
note rate of production of 1-st item (rp1)
note rate of production of 2-st item (rp2)
note rate of production of 3-st item (rp3)
note
a rp1.k=(bb1.k*d11.k+bb2.k*d12.k+bb3.k*d13.k)/detc.k
a rp2.k=(bb1.k*d21.k+bb2.k*d22.k+bb3.k*d23.k)/detc.k
a rp3.k=(bb1.k*d31.k+bb2.k*d32.k+bb3.k*d33.k)/detc.k
note
note
note
```

```
a bl1.k=(b1.k-rp1.k-rp2.k-rp3.k)*(b1.k-rp1.k-rp2.k-rp3.k)
a bl2.k=(ucp1*rp1.k+ucp2*rp2.k+ucp3*rp3.k-b2.k)
x *(ucp1*rp1.k+ucp2*rp2.k+ucp3*rp3.k-b2.k)
a bl3.k=(ulp1*rp1.k+ulp2*rp2.k+ulp3*rp3.k-b3.k)
x *(ulp1*rp1.k+ulp2*rp2.k+ulp3*rp3.k-b3.k)
a bl4.k=(rp1.k-b4.k)*(rp1.k-b4.k)
a bl5.k=(rp2.k-b5.k)*(rp2.k-b5.k)
a bl6.k=(rp3.k-b6.k)*(rp3.k-b6.k)
a functio.k=bl1.k+bl2.k+bl3.k+bl4.k+bl5.k+bl6.k
note
note  parameters of simulation
note
spec length=104/dt=1/savper=1
save rp1,rp2,rp3,r1,r2,lmt,detc,functio
```

Appendix B. Program in COSMIC

```
note Balance of production of 3 items
l lmt.k=lmt.j+dt*(r1.jk-r2.jk)
n lmt=0
c g1=5
c po=100
c p1=30
c perd=52
c t1=2
r r1.kl=input.k*g1
c tcp=7000
c ucp1=10
c ucp2=5
c ucp3=2
c alfa=0.5
c beta=0.25
c gamma=0.25
c tle=1800
c ulp1=2
c ulp2=2
c ulp3=1
c w1=10
c w2=2
c w3=2
r r2.kl=lmt.k/t1
a input.k=po+p1*SIN(6.283*time.k/perd)
a fob.k=w1*sfftb.k+w2*sffcp.k+w3*sfflp.k
r rs1.kl=(1-alfa-beta-gamma)*ar2.k
a ar2.k=lmt.k/t1
r rp1.kl=alfa*ar2.k
r rp2.kl=beta*ar2.k
r rp3.kl=gamma*ar2.k
r rs2.kl=tcp-rp1.kl*ucp1-rp2.kl*ucp2-rp3.kl*ucp3
r rs3.kl=tle-rp1.kl*ulp1-rp2.kl*ulp2-rp3.kl*ulp3
l sfftb.k=sfftb.j+dt*((rs1.jk)**INT(2))
```

```
n sfftb=wp1
l sffcp.k=sffcp.j+dt*((rs2.jk)**INT(2))
n sffcp=wp2
l sfflp.k=sfflp.j+dt*((rs3.jk)**INT(2))
n sfflp=wp3
c wp1=0
c wp2=0
c wp3=0
note
note output and control sector
note
c dt=1
c length=104
c prtper=5
c pltper=5
print 1)rp1,rp2,rp3
plot rp1=1,rp2=2,rp3=3
note
note simulation experiments
note
run basic model of balance
note
note definitions of variables
note
d alfa=(1) fraction of production ar2 (of item P1)
d ar2=(unit/week) transformation of raw material (production)
d beta=(1) fraction of production ar2 (of item P2)
d dt=(week) solution interval
d fob=(1) objuction function
d g1=(1) gain between input and output rate r2
d gamma=(1) fraction of production ar2 (of item P3)
d input=(unit/week) source of raw material
d length=(week) simulated period
d lmt=(unit) level of raw material during transformation
d p1=(1) parameter of amplitude of sinusoidal input
d perd=(week) parameter, period of input
d pltper=(week) graph plotting interval
d po=(1) parameter step of input
d prtper=(week) table printing interval
d r1=(unit/week) input rate to lmt
d r2=(unit/week) output rate to lmt
d rp1=(unit/week) rate of production of first item (p1)
d rp2=(unit/week) rate of production of second item (P2)
d rp3=(unit/week) rate of production of third item (P3)
d rs1=(unit/week) input rate to sfftb
d rs2=(dolar/week) input rate to sffcb
d rs3=(men/week) input rate to sfflb
d sffcp=($**2/week) sum function of fitting cost balance
d sfflp=(men**2/week) sum function of fitting person balance
d sfftb=(unit**2/week) sum function of fitting total balance
d t1=(week) average time of production
```

```
d tcp=($/week) total cost expenditure of production (rate)
d time=(week) time within simulation
d tle=(men/week) total labour expenditure of production (rate)
d ucp1=($/unit) unit cost of production of product P1
d ucp2=($/unit) unit cost of production of product P2
d ucp3=($/unit) unit cost of production of product P3
d ulp1=(men/unit) unit labour of production of product P1
d ulp2=(men/unit) unit labour of production of product P2
d ulp3=(men/unit) unit labour of production of product P3
d w1=(1) weighting factor for sfftb
d w2=(1) weigting factor for sffcb
d w3=(1) weighting factor for sfflb
d wp1=(unit**2/week) initial condition for sfftb
d wp2=($**2/week) initial condition for sffcb
d wp3=(men**2/week) initial condition for sfflb
```