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TESTING THE DECISION RULES USED IN STOCK MANAGEMENT MODELS

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Abstract

This paper evaluates the basic decision rules typically used in dynamic decision-making modeling. The plausibility and consistency of each rule is evaluated first. Then, the adequacy of these rules is tested empirically by comparing them against the performances of different subjects (players) in an experimental stock management game. For this purpose, the generic stock (inventory) management problem, one of the most common dynamic decision problems, is chosen as the interactive gaming environment. Experiments are designed to test the effects of three factors on decision making behavior: two different patterns of customer demand, minimum possible decision interval and finally representation of receiving delays. The performances of subjects are compared with the simulated results obtained using the common linear "Anchoring and Adjustment Rule." Next, several alternative (non-linear) rules are formulated and tested. Finally, some typical inventory control rules (such as (s, Q)) from the standard inventory management literature are tested. These rules, compared to the linear stock adjustment rule, are found to be more representative of the subjects' decisions in certain cases. Another major finding is the fact that the well-documented oscillatory dynamic behavior of the inventory is true not just with the linear anchor-and-adjust rule but also with the non-linear rules as well as the standard inventory management rules.

1. Stock Management Game

The objective of the game is defined as "keeping the inventory level as low as possible while avoiding the backorders." If there is not enough goods in the inventory at any time, customer orders are entered as backorders to be supplied later. This situation, is modeled very simply, by letting the inventory become negative until enough goods are eventually received. "Order decisions" are the only means of controlling the inventory level. The general structure of the stock management problem is illustrated in Figure 1.1. The three empty boxes: Expectation Formation, Goal Formation and Decision Rule are deliberately left blank, as they are unknown, since they take place in the "minds" of the players. (Later, in the simulation version of the game, these three boxes will have to be specified. For instance, the expectation formation will be formulated by exponential smoothing; inventory goal will be set to inventory coverage times expected demand and supply line goal will be order delay times expected demand. As for the decision rule, different formulations will be tried: linear stock adjustment rule, three different non-linear adjustment rules and finally various standard discrete inventory control rules.)

While playing the game, subjects can monitor the system from information displays showing their inventory, supply line levels and customer demand. As the game progresses, they can also see the dynamics of these variables plotted on graphs. Neither costs associated with high inventories nor costs resulting from backorders are accounted for explicitly in the simulation games. However, the relation between keeping these costs at a minimum and the objective of the game is stated in the instruction given to subjects. Before beginning the game, each subject is given a written instruction presenting the problem and their task. Time available to accomplish the task is not limited. No explicit reward is used to motivate the subjects.

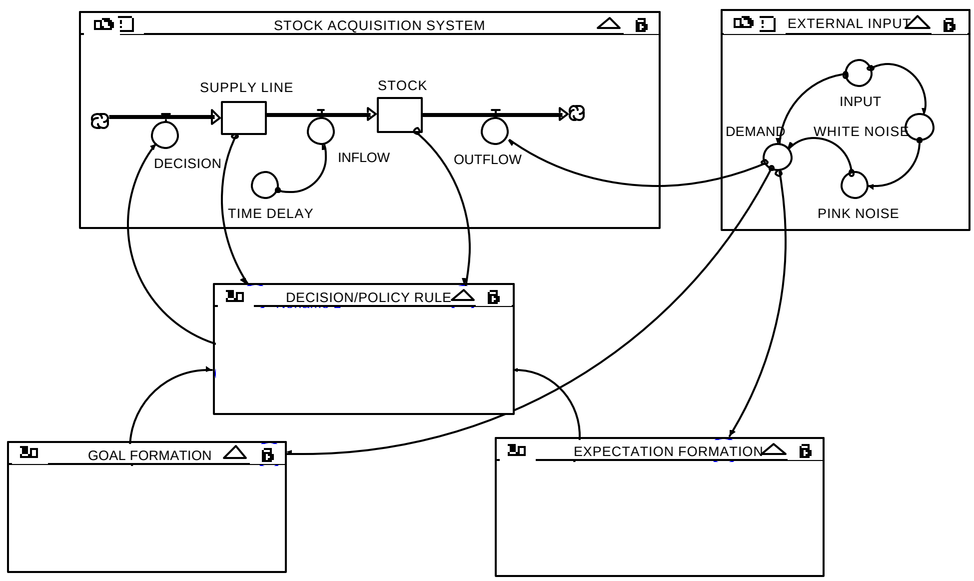

FIGURE 1.1. Stock Management Problem

2. Experimental Design

We design our experiments for testing the effects of three factors on the decision making behavior of the subjects:

(a) minimum possible decision interval;

(b) representation of the receiving delays;

(c) two different patterns of customer demand

2.1. Decision Interval

Decision-makers were allowed to order at "each period" in the first group of experiments (Short Game), whereas they were allowed to order "once every five periods" in the second group of experiments (Long Game). Short Games are simulated for 100 time periods whereas Long Games are simulated for 250 time periods (50 decision intervals).

2.2. Receiving Delay

The average receiving delay is set as four and ten in the Short and Long Games respectively. The other factor is the nature of the delay. In different inventory acquisition systems, continuous first-order exponential delays or discrete delay representations may be appropriate. Since "Receiving" is the inflow to inventory, its transient behavior may influence decision-maker's interpretation of the results of his/her own order decisions. Therefore, the effects of time delay representation is tested in the experiments. The two extremes of the exponential delay family, namely the discrete delay and first order exponential delay are chosen as high and low levels of delay representation factor respectively.

2.3. Patterns of Customer Demand

Until the fifth decision interval, average customer demand remains constant at 20. At the beginning of fifth decision interval (at the fifth period in the Short Game and at the 25'th period in Long Game) an unannounced, one-time increase of 20 units occurs in demand in the customer demand patterns used in the experiments. The increase in demand facilitates the analysis since the subjects must react to disequilibrium created by the disturbance. After the fifth decision interval, demand remains constant at 40 in the first type of customer demand pattern which we call "*step up in customer demand*".

In the second type of demand pattern, "*step up and down in customer demand*," a second disturbance, one-time decrease in demand follows the first increase, after some time interval, reducing the demand back to its original level of 20. The time interval between the step up and down in customer demand is chosen as roughly half of the natural periodicity of the model.

"Pink noise" (auto-correlated noise) is added to the patterns of average customer demand described above to obtain more realistic demand patterns. The standard deviation of the white noise is set to be 15 percent of average customer demand. The delay constant of the exponential smoothing (the correlation time used to create pink noise) is taken as two time units.

TABLE 2.1. Design of Experiments

Runs	Minimum Ordering Interval		Receiving Delay		Pattern of Customer Demand	
	1	5	Exponential	Discrete	Step-Up Customer Demand	Step-Up-and-Down Customer Demand
1	X		X		X	
2	X		X			X
3	X			X	X	
4	X			X		X
5		X	X		X	
6		X	X			X
7		X		X	X	
8		X		X		X

(X) indicates the selected level of factors for the corresponding runs.

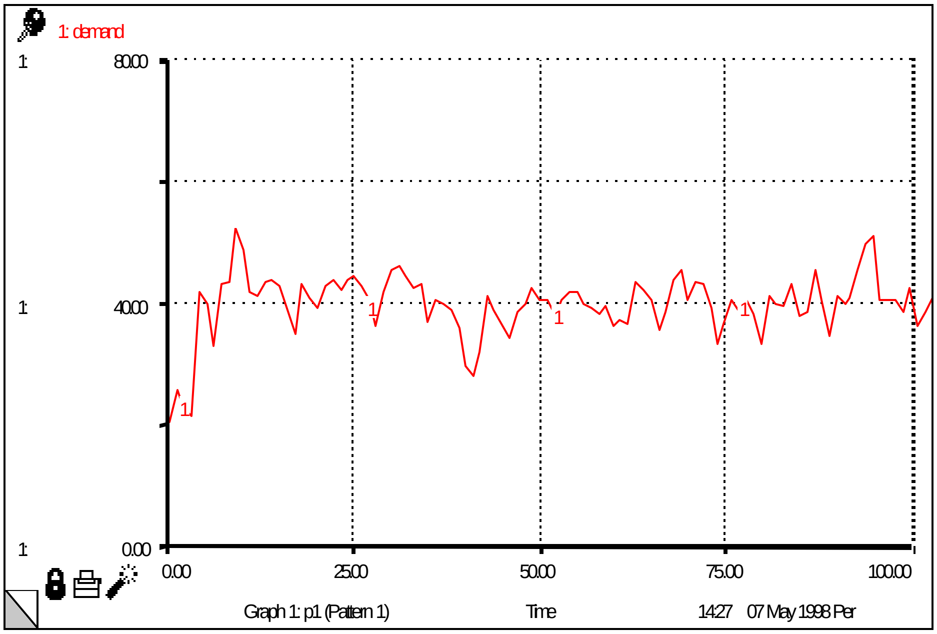

FIGURE 2.3.1. Step Up Customer Demand for Short Games

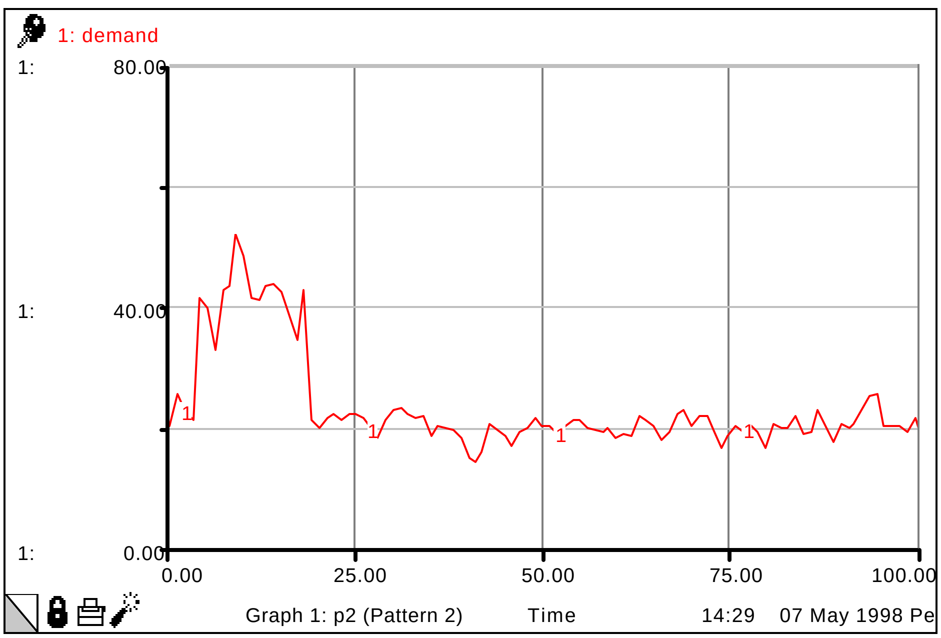

FIGURE 2.3.2. Step Up and Down Customer Demand for Short Games

There are eight combinations of the above three factors across the two levels of each. Each condition is played six times (random replications) yielding a total of 48 runs. Since the demand pattern is discovered by the subjects once the game is played and because they can improve their performance by practice, to obtain unbiased results, the subjects never played two Short Games or two Long Games. However, some of the subjects were allowed to play one Short Game and one Long Game, since transferring experience between Short and Long Games is not easy.

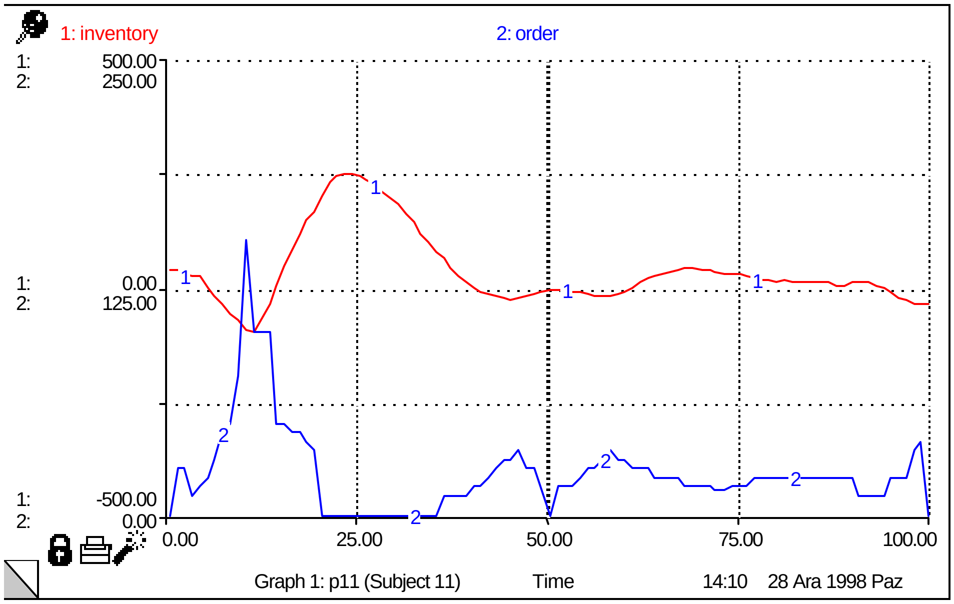

FIGURE 3.1.1. Performance of a Player in Game 11 (Short Game with Orders Each Period, Step Up and Down in Customer Demand, Exponential Delay)

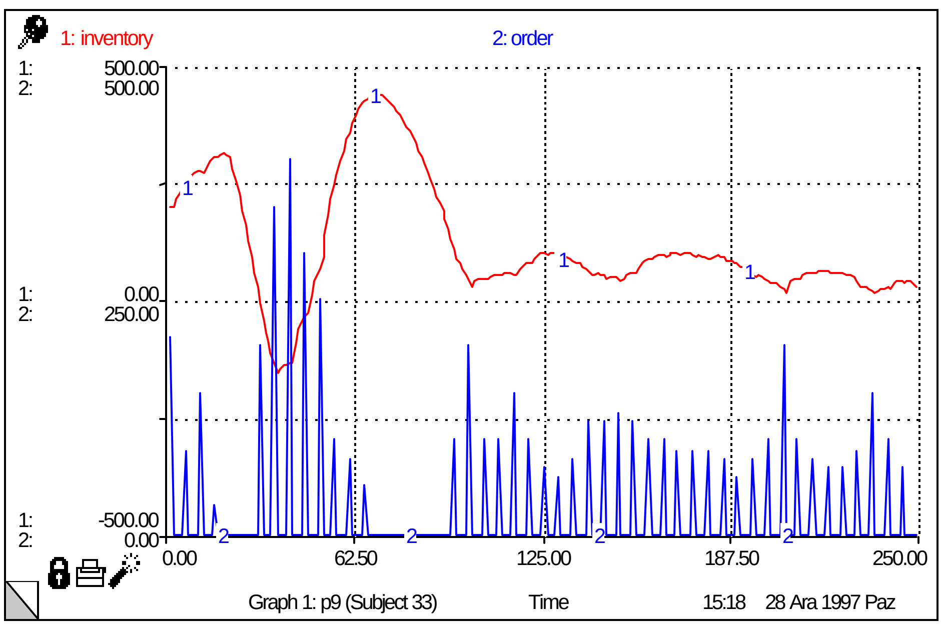

FIGURE 3.1.2. Performance of a Player in Game 33 (Long Game with Orders Once Every Five Periods, Step Up and Down in Customer Demand, Exponential Delay)

2.4. Initial Conditions

Games start at equilibrium. Supply line level is initially set (80 in Short Games and 200 in Long Games) such that no backordering occurs even when the decision-maker does not order goods during the first four decision intervals, until disturbance in customer demand creates disequilibria. Inventory levels are initially set arbitrarily (40 and 200 respectively) so as to satisfy the average initial customer demand for the first two decision intervals (two time periods for Short Games and ten time periods for Long Games).

3. Analysis Of Experiments

The fundamental behavior pattern of inventory in most games is one of oscillations. (See Figures 3.1.1., 3.1.2. and most other games illustrated in the sections to follow). This finding is consistent with overwhelming evidence on oscillating inventories in system dynamics literature and elsewhere (for instance, Sterman 1987, 1989 and Lee et al 1997).

3.1. Effect of Different Patterns of Customer Demand

Experiments show that the "demand pattern" (step up only or step up and down) does not have a major effect on the ordering behavior of the subjects. There are some numerical differences but no major qualitative differences. (See Table 3.1, below).

3.2. Effect of the Length of Decision Intervals

When subjects can order “every period,” they prefer to order intermittently. Consequently, inventory levels are noisy. When subjects can only order "every five periods" (instead of each period), inventory oscillations become less “noisy”, having longer and smoother cycles. In these games, since subjects order larger amounts for longer periods, they order more consistently. However, fundamental behavior patterns of the inventory do not seem to be affected by the length of decision interval.

3.3. Effect of Different Representations of Receiving Delays

The "type of receiving delay" seems to affect the difficulty level of the game. With continuous exponential delay, subjects are able to manage the inventories in a relatively more stable way. It seems that when the goods ordered arrive gradually over some period of time, it prevents the players from over-ordering or under-ordering excessively. In contrast, discrete delay representation affects their performance very badly by causing fluctuations and noisy orders. When the receiving delay is discrete, subjects fail to bring the oscillations under control in most cases. Subjects seem to have difficulties in accounting for the effects of sudden receiving. They generally overreact and their reactions are destabilizing in these games (Figures 3.3.1.and 3.3.2). The above results are consistent with research evidence on the effect of delays on dynamic decision making performance, in system dynamics literature (Sterman 1989), as well as in experimental psychological research (Brehmer 1989).

3.4. Summary of Experiments

Some numerical characteristics of orders and inventory level obtained from the experiments are summarized in Table 3.1. The ten characteristics are tabulated for each of the 48 games. The averages of each characteristic for each of the six runs are also

TABLE 3.1. Selected Characteristics from Game Results

Experiment	Min. Order	Max. Order	Range Of Orders	Min. Inven-tory	Max. Inven-tory	Range Of Inven-tory	Initial Back-order Time	Final Back-order Time	Dura-tion Of Back-orders	Oscil-lation Period
1	0	100	100	-125	50	175	4	38	34	N/A
2	0	100	100	-125	150	275	4	33	29	31
3	0	100	100	-175	150	325	4	18	14	29
4	0	200	200	-75	100	175	6	16	10	25
5	0	300	300	-225	100	325	5	43	38	N/A
6	20	60	40	-60	40	100	6	37	31	N/A
Avg. Of Run 1	**3,3**	**143**	**140**	**-131**	**98**	**229**	**4.8**	**31**	**26**	**28.3**
7	0	100	100	-60	115	175	10	21	11	30
8	0	100	100	-60	250	310	5	13	8	21
9	0	60	60	-60	60	120	7	20	13	N/A
10	0	400	400	-80	180	260	6	21	15	39
11	0	150	150	-100	250	350	6	14	8	28
12	40	70	30	-60	150	210	6	18	12	N/A
Avg. Of Run 2	**6,7**	**146,6**	**140**	**-70**	**167,5**	**237,5**	**6,7**	**17,8**	**11,2**	**29.5**
13	0	100	100	-225	300	525	5	34	29	43
14	0	220	220	-225	225	450	5	35	30	45
15	0	150	150	-75	160	235	14	25	11	N/A
16	0	60	60	-15	190	215	N/A	N/A	N/A	25
17	0	100	100	-125	250	375	5	38	33	N/A
18	0	300	300	-260	180	440	5	18	13	27
Avg. Of Run 3	**0**	**155**	**155**	**-154**	**218**	**372**	**6,8**	**30**	**23,2**	**35**
19	0	150	150	-25	150	175	N/A	N/A	N/A	N/A
20	0	150	150	-100	290	390	6	21	15	N/A
21	0	80	80	-145	80	225	5	24	19	30
22	0	100	100	-40	350	390	12	18	4	32
23	0	100	100	-250	210	460	6	27	21	45
24	0	200	200	-150	250	400	5	22	17	N/A
Avg. Of Run 4	**0**	**130**	**130**	**-118**	**222**	**340**	**6,8**	**22,4**	**15,2**	**35,7**
25	0	600	600	-260	425	685	20	30	10	44
26	0	400	400	-350	200	550	25	85	60	N/A
27	0	300	300	-280	200	480	20	85	65	N/A
28	0	400	400	-300	200	500	25	70	45	63
29	0	560	560	-150	370	520	20	40	15	39
30	0	250	250	-20	425	445	35	45	10	73
Avg. Of Run 5	**0**	**418**	**418**	**-227**	**303**	**530**	**24**	**59**	**34**	**54,8**
31	40	375	335	-200	250	450	20	40	20	N/A
32	0	250	250	-100	200	300	25	55	30	N/A
33	0	400	400	-150	475	625	30	45	15	73
34	50	250	200	-60	200	260	30	55	25	66
35	50	300	250	0	275	275	N/A	N/A	N/A	N/A
36	100	250	150	-160	250	410	25	55	30	73
Avg. Of Run 6	**40**	**304**	**264**	**-112**	**275**	**387**	**26**	**50**	**24**	**70,7**
37	100	270	170	-100	500	600	20	40	20	85
38	20	500	480	-500	300	800	25	90	65	102
39	80	250	170	-40	450	490	N/A	N/A	N/A	N/A
40	50	250	200	-200	425	625	30	40	10	75
41	0	1000	1000	-1000	2000	3000	20	100	80	133
42	0	450	450	-375	420	795	35	60	25	52
Avg. Of Run 7	**42**	**453**	**412**	**-369**	**683**	**1052**	**26**	**66**	**40**	**52**
43	0	250	250	-125	400	525	N/A	N/A	N/A	75
44	0	450	450	-500	200	700	20	60	40	N/A
45	0	375	375	-750	200	950	20	70	50	N/A
46	20	300	280	-330	330	660	25	55	30	55
47	50	250	200	-180	300	480	30	45	15	114
48	0	300	300	-375	350	725	30	60	30	N/A
Avg. Of Run 8	**12**	**321**	**309**	**-377**	**297**	**673**	**25**	**58**	**33**	**78,5**

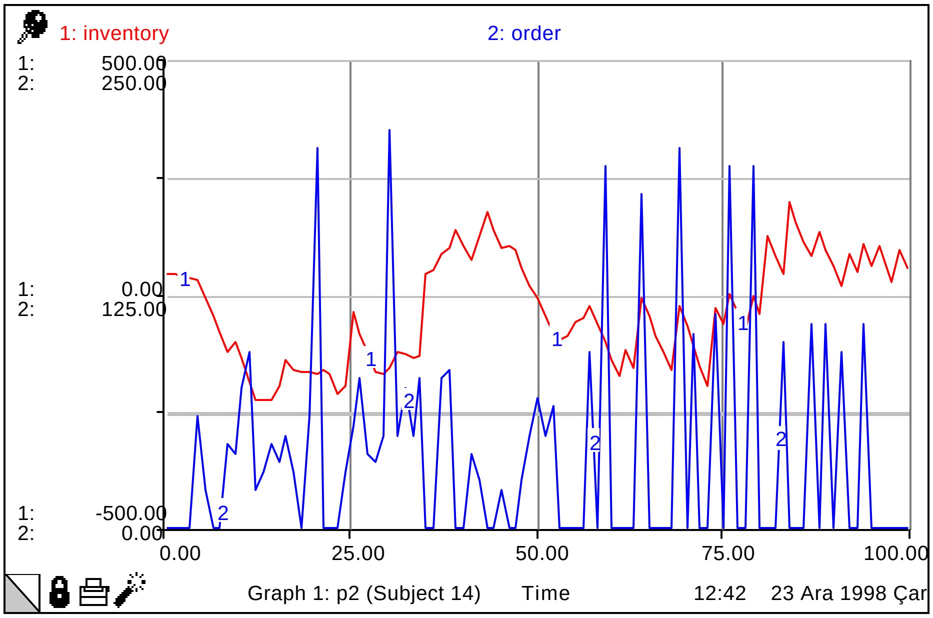

FIGURE 3.3.1. Performance of a Player in Game 14 (Short Game with Orders Each Period, Step Up in Customer Demand, Discrete Delay)

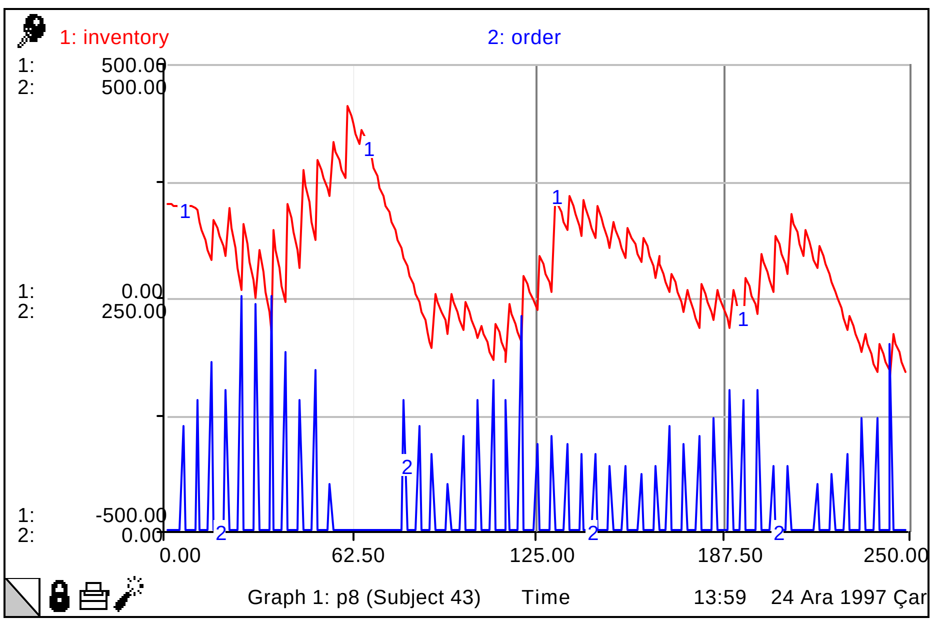

FIGURE 3.3.2. Performance of a Player in Game 43 (Long Game with Orders Once Every Five Periods, Step Up and Down in Customer Demand, Discrete Delay)

calculated. From these averages, it is possible to observe the effects of the game parameter settings on some performance characteristics. The averages of the runs obtained from to the low and high settings of each of the three experimental factors are also statistically compared pair-wise (interactions ignored), using the t-test. None of the differences between the high and low settings of a given factor are found to be statistically significant. Thus, we can say that the high and low settings of each of the three game factors do not have a statistically significant effect on the selected output characteristics.

4. Linear Anchoring and Adjustment Rule

Linear Anchoring and Adjustment Rule is frequently used to model decision-making behavior in System Dynamics models (Sterman, 1987). In making certain decisions, one can start from an initial point, called the anchor, and then make some adjustments to come up with the final decision. In the context of inventory management, a plausible starting point for order decisions is the expected customer demand. (When the inventory manager can only order once every five periods, the anchor should be the total of expected customer demand for five periods between subsequent decisions.) When changes in either customer demand or receiving delay cause discrepancies between desired and actual inventory levels and/or between desired and actual supply line, adequate adjustments are made so as to bring the inventory and the supply line back to desired levels. Therefore, the order equation based on Anchoring and Adjustment heuristic is formulated as:

$$O_t = E_t + ? *(I_t^* - I_t) + ? *(SL_t^* - SL_t) \quad (4.1)$$

When orders can be given once every five periods, then the anchor of the rule is modified as follows, the adjustment terms being as before:

$$O_t = 5*E_t + ? *(I_t^* - I_t) + ? *(SL_t^* - SL_t) \quad (4.2)$$

where E_t represents customer demand expectations, I_t^* represents the desired inventory, I_t the inventory, SL_t^* the desired supply line and SL_t the supply line. ? and ? are the adjustment fractions.

In real life, safety stocks are determined by considering the inventory and backordering costs. Although, an optimum inventory level minimizing these costs may be found mathematically, more often safety stocks are set approximately. The desired inventory I_t^* is thus modeled as proportional to customer demand to allow adjustments in safety stocks when changes in customer demand occur.

$$I_t^* = k*E_t \quad (4.3)$$

To maintain a receiving rate consistent with receiving delay and customer demand, SL_t^* is formulated as a function of receiving delay ? and expected customer demand E_t.

$$SL_t^* = ?*E_t \quad (4.4)$$

4.1. Comparison of Anchoring and Adjustment Rule with Experiments

The rule can mimic the subjects' performances adequately in experiments where they tend to order continuously (Figure 4.1.1.), which is especially the case when orders are placed every five periods. (See Özevin 1999 for more illustrations). However, when they can order each period, most subjects tend to order intermittently especially when the receiving delay representation is discrete. Their orders result in zigzagging inventory patterns. Furthermore, some subjects tend to order suddenly very large quantities, after some period of zero or negligible ordering (Figure 3.3.1.).The linear "Anchoring and Adjustment Rule" can not yield such intermittent or infrequent large orders, hence does not represent well the subjects' performances in these cases. Such cases will be addressed below in two separate sections: Non-linear adjustment rules and standard inventory control rules.

5. Rules with Nonlinear Adjustment

The linear adjustment rule makes adjustments in the orders proportional to the discrepancy between the desired and observed stock levels. Some orders are placed regularly each period, the quantity of which depending on the discrepancy between the desired and observed stock levels. However, some decision-makers do not seem to place such smooth orders. They cease ordering when the inventory is around the desired level and give rather large orders as the discrepancy between the desired and actual inventory becomes larger. In this section, three alternative decision rules, which may represent this "nonlinear" adjustment aspect of subjects' ordering, are tested.

5.1. Cubic Adjustment Rules

Similar to the Linear Anchoring and Adjustment rule, Cubic Adjustment rules also start with customer demand expectations as an initial point but the adjustments are not formulated as linear. One or both of the adjustment terms may be cubic in discrepancies (in inventory and/or in supply line). Alternative order equations can be mathematically expressed as:

$$O_t = E_t + ?*(I_t^* - I_t)^3 \quad (5.1)$$

where only the discrepancy in inventory is taken into account,

$$O_t = E_t + ?*(I_t^* - I_t)^3 + ?*(SL_t^* - SL_t) \quad (5.2)$$

$$O_t = E_t + ?*(I_t^* - I_t) + ?*(SL_t^* - SL_t)^3 \quad (5.3)$$

where one of the adjustments is made cubic and the others are linear and finally,

$$O_t = E_t + ?*(I_t^* - I_t)^3 + ?*(SL_t^* - SL_t)^3 \quad (5.4)$$

where both of the adjustments are formulated as cubic. In the equations above, E_t represents customer demand expectations, I_t^* and SL_t^* the desired inventory and supply line levels, I_t and SL_t the actual inventory and supply line. ? and ? are the fraction of the discrepancy corrected by the decision-maker at each period. The internal consistency of the rules can be shown mathematically (See Özevin 1999). When orders can be given once every five periods, the adjustments are as before, however, the anchor is increased to five times the expected customer demand.

Although choosing stable values for the adjustment fractions is not always easy with Cubic Adjustment rules, it is possible to generate ordering patterns consisting of intermittent orders and sudden large adjustments within quiet ordering periods, as observed in some subjects. Dynamics of the inventory levels corresponding to these orders are very similar with the ones observed in some of the graphs obtained in the games (Figures 5.1.1.and 5.1.2.).

Most of the time, when orders are given once every five periods, the cubic adjustments rules fail to generate stable orders. When simulated with exponential delay representation, the orders are either unstable or when stable, they become very similar with the ones obtained by simulating the Anchoring and Adjustment rule. In other words, the performance of the rule is too sensitive to the adjustment parameter values.

Anchoring and Adjustment rules are not suitable to generate certain fluctuating inventory dynamics observed with discrete delays when orders are given once every five periods. Although the rules can potentially yield such fluctuating behavior with discrete delays, the range in which they can yield fluctuating yet stable dynamics is too narrow to offer a rich repertoire of oscillations. (See Özevin 1999).

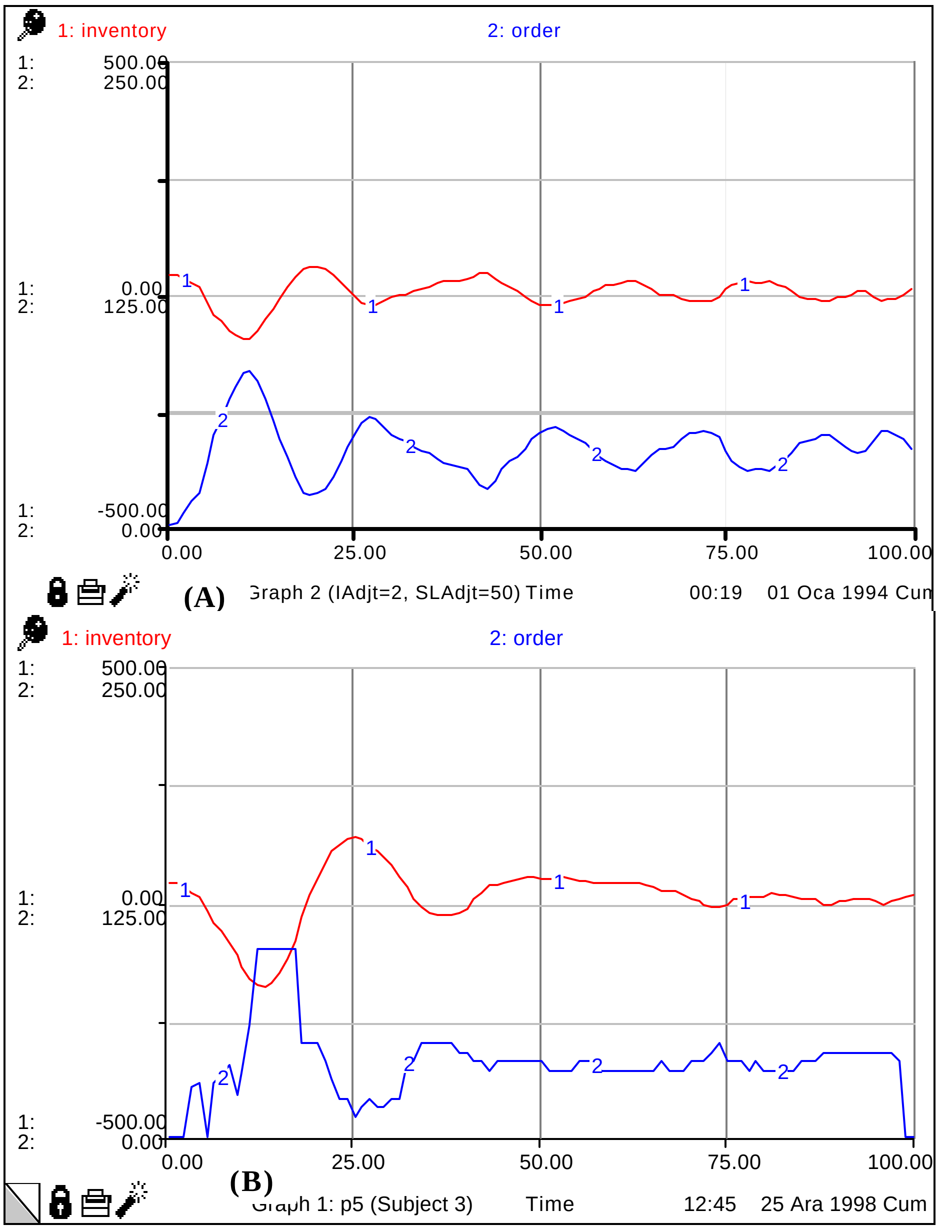

FIGURE 4.1.1. Comparison of (A) the Anchoring and Adjustment Rule (? = 0.5, ? = 0.02), with (B) the Performance of a Player in Game 3 (Short Game with Orders Each Period, Step Up in Customer Demand, Exponential Delay)

5.2. Variable Adjustment Fraction Rule

Analogous to the Anchoring and Adjustment Rule and the Cubic Adjustment Rules, Variable Adjustment Fraction Rule also starts with expectations about customer demand. However, the adjustments are increased sharply (nonlinearly) when the discrepancy in inventory increases. The order equation of the rule can be mathematically expressed as

$$O_t = E_t + ?\ (I_t^{*} - I_t) \qquad (5.5)$$

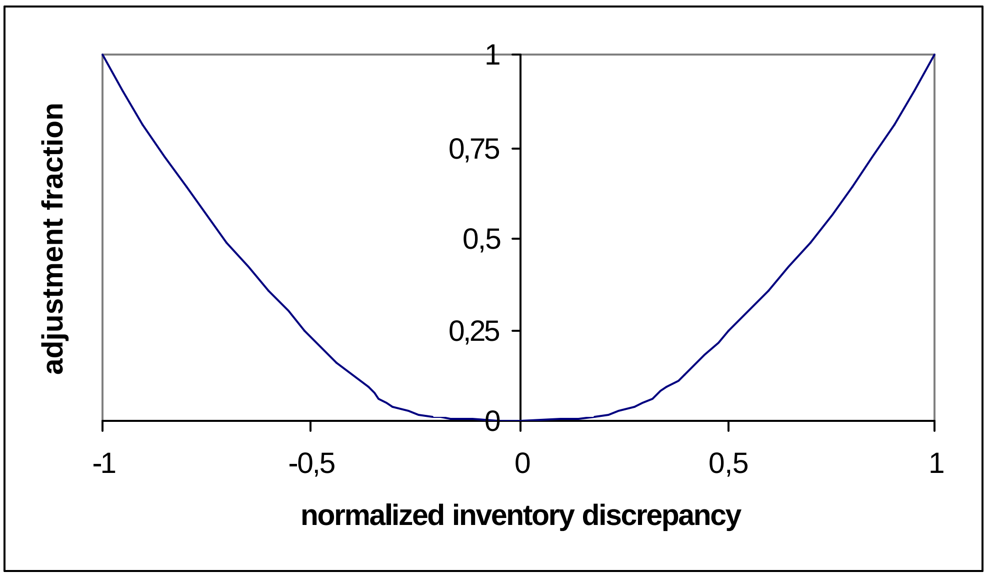

FIGURE 5.2.1. Graphical Adjustment Fraction Function

where the variable fraction ? is a function of the discrepancy in inventory, normalized by the desired inventory. The shape of the function yields increased adjustments when the discrepancy in inventory is increased. Normalized inventory discrepancy ? is defined as

$$? \ ? \ \frac{I^{*} \ ? \ I}{I^{*}} \qquad (5.6)$$

The rule is not mathematically unbiased in the ideal known demand case; there will be some small, deliberate steady state discrepancy between the inventory and its desired level. But this may well be a "realistic" bias in order to be able to obtain a non-linear ordering behavior similar to some subjects. The rule performs quite realistically in the "noisy" demand case, in which case the steady state bias is negligible anyway and may be irrelevant in real life.

Orders generated by the Variable Adjustment Fraction rule display very large variability, the resulting inventory patterns being much smoother. Therefore, they may be used to represent subjects' behavior especially when orders are intermittent and corresponding inventory patterns are continuous, as an alternative to Anchoring and Adjustment rules (Figure 5.2.2.).

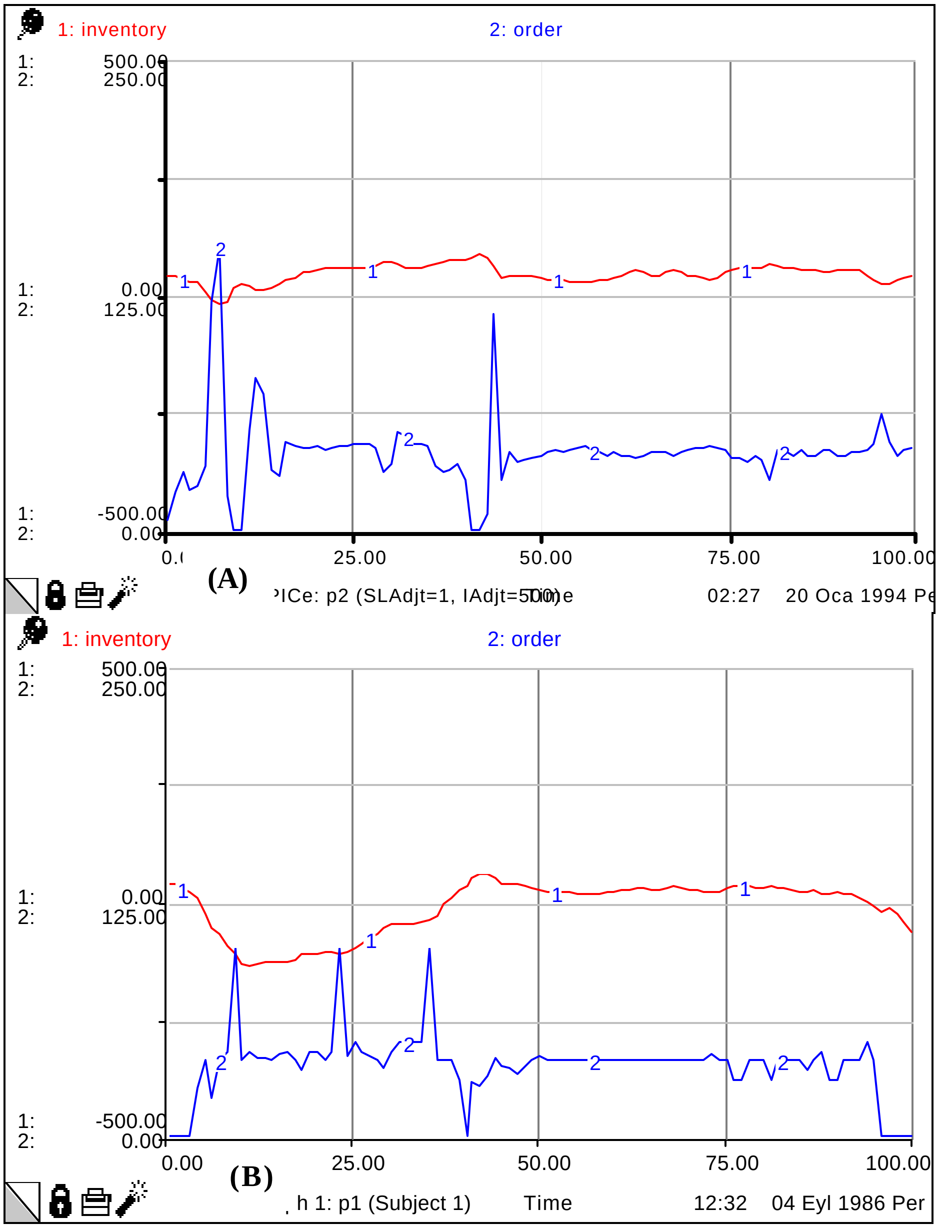

FIGURE 5.1.1. Comparison of (A) Performance of "Linear Supply Line and Cubic Inventory Adjustment Rule" (? =1/500, ? =1) with (B) the Performance of a Player in Game 1 (Short Game with Orders Each Period, Step Up in Customer Demand, Exponential Delay)

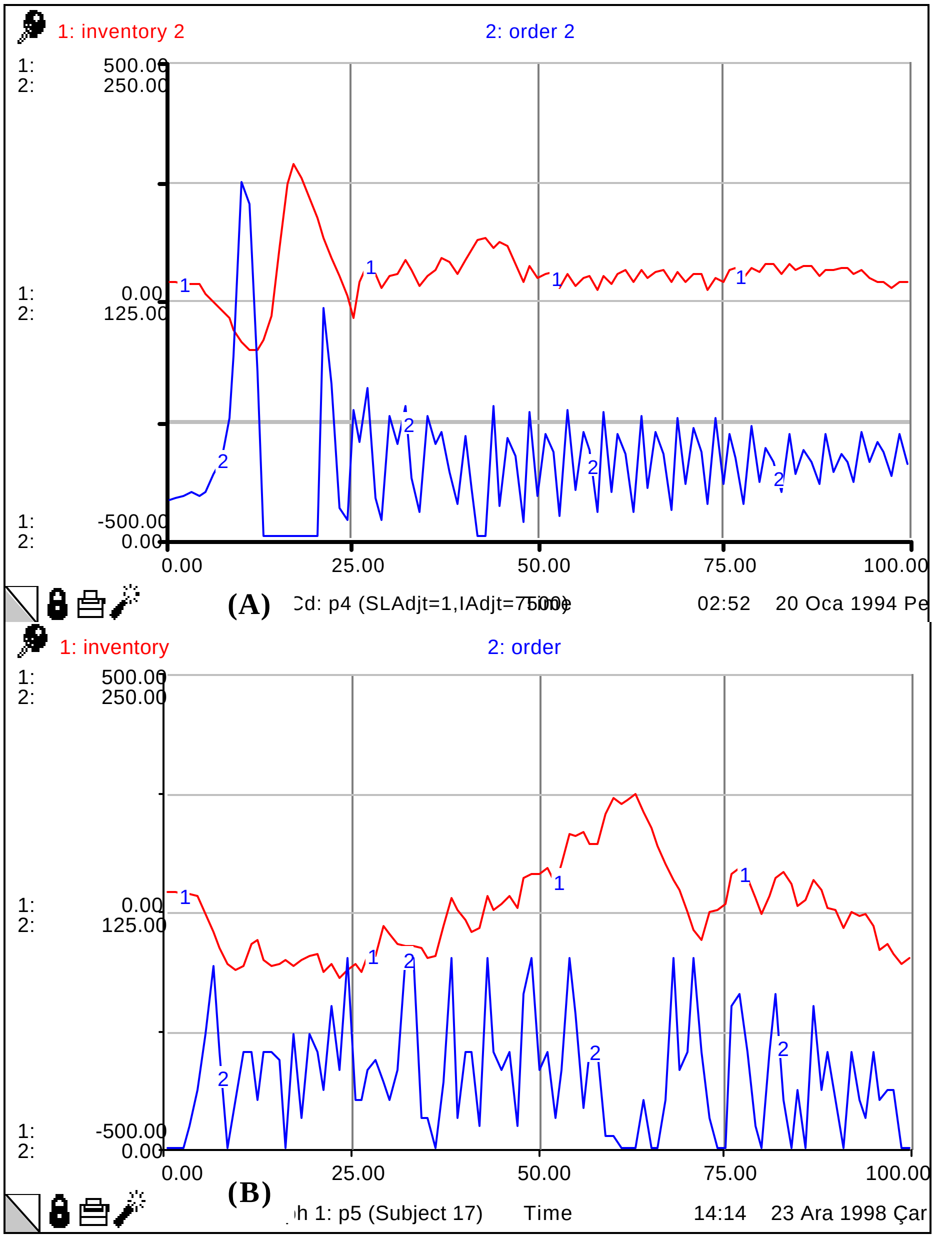

FIGURE 5.1.2. Comparison of (A) Performance of “Linear Supply Line and Cubic Inventory Adjustment Rule” (? =1/7500, ? =1) with (B) Performance of a Player in Game 17 (Short Game with Orders Each Period, Step Up in Customer Demand, Discrete Delay)

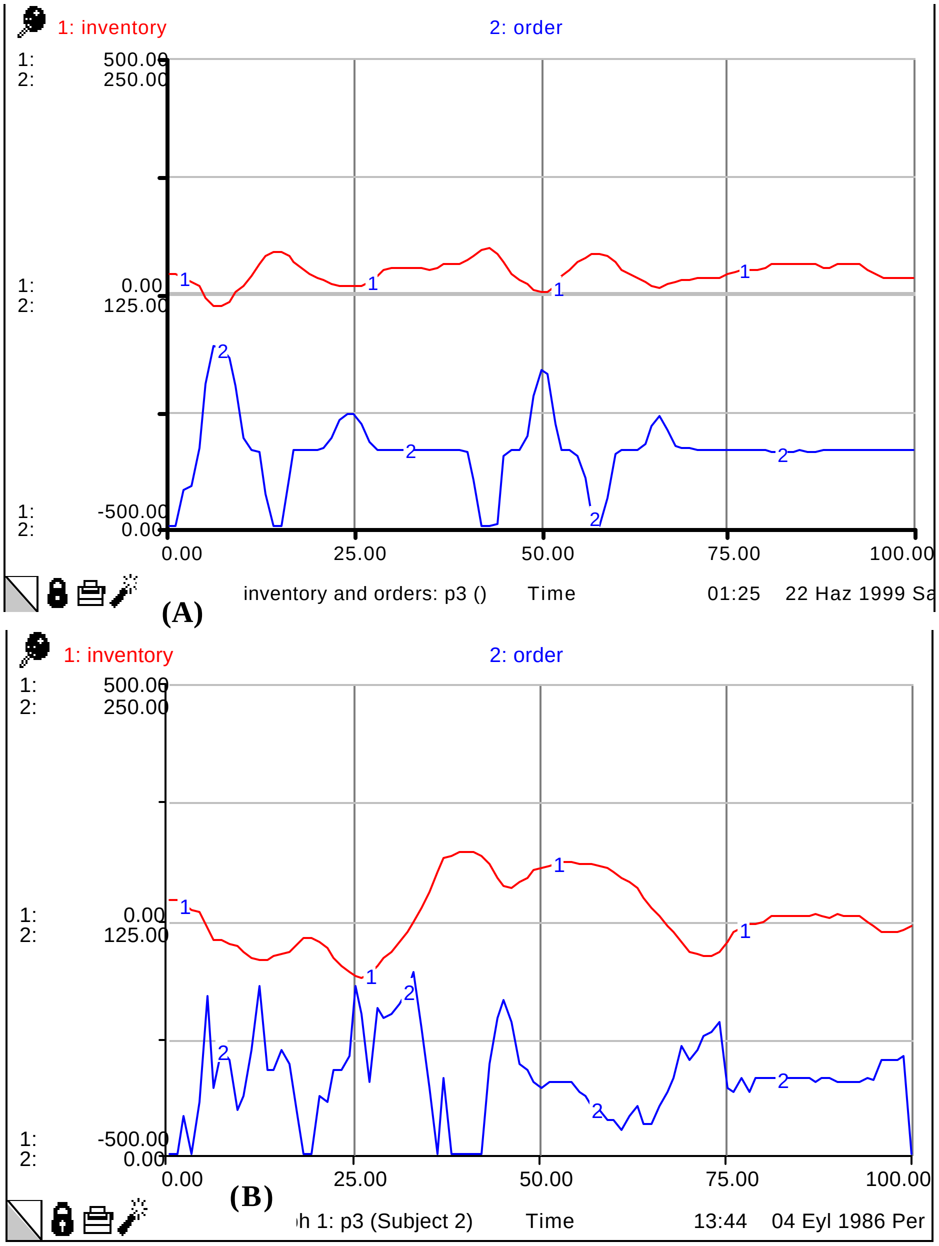

FIGURE 5.22. Comparison of (A) Variable Adjustment Fraction Rule with (B) the Performance of a Player in Game 2 (Short Game with Orders Each Period, Step Up in Customer Demand, Exponential Delay)

5.3. Nonlinear Expectation Adjustment Rule

The order equation of the expectation adjustment rule can be mathematically expressed as

$$O_t = ? * E_t \quad (5.7)$$

where the variable adjustment coefficient ? is a function of discrepancy in inventory normalized by desired inventory and E_t represents the customer demand expectations. ? is equal to one when the inventory is at the desired level since adjustments are not necessary when the system is in equilibrium. The shape of the ? function causes increasing upward adjustments in orders when the inventory level is below the desired level and it causes reductions in orders when the inventory is above its desired level.

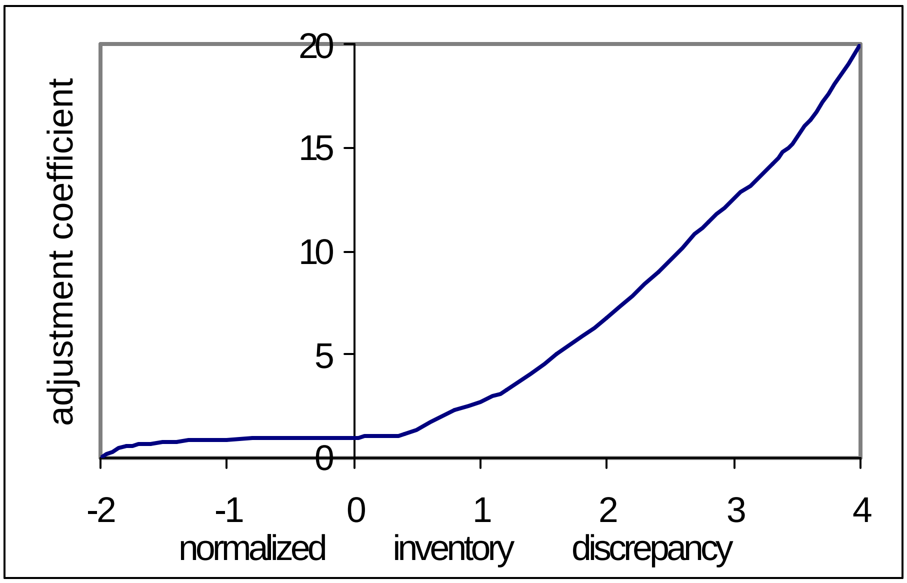

FIGURE 5.3.1. Graphical Adjustment Coefficient Function

The rule is also simulated with discrete delays and similar ordering and inventory patterns are obtained. With discrete delays, orders given previously are received all at once. Therefore, in the discrete case, although the shape and the range of the function is kept as before, adjustment values corresponding to positive normalized inventory discrepancies are decreased to keep orders stable.

Similar with the Variable Adjustment rule, Nonlinear Expectations Rule can yield intermittent orders yet continuous inventory patterns. Therefore, it may provide an alternative to Anchoring and Adjustment rule, when subjects' orders are intermittent and corresponding inventory patterns are continuous. (Figures 5.3.2 and 5.3.3).

There is another important general finding in this section: the well-documented oscillatory dynamic behavior of the inventory is true not just with the linear anchor-and-adjust rule but also with the non-linear rules seen above. (Figures 5.1.1 through 5.3.3).

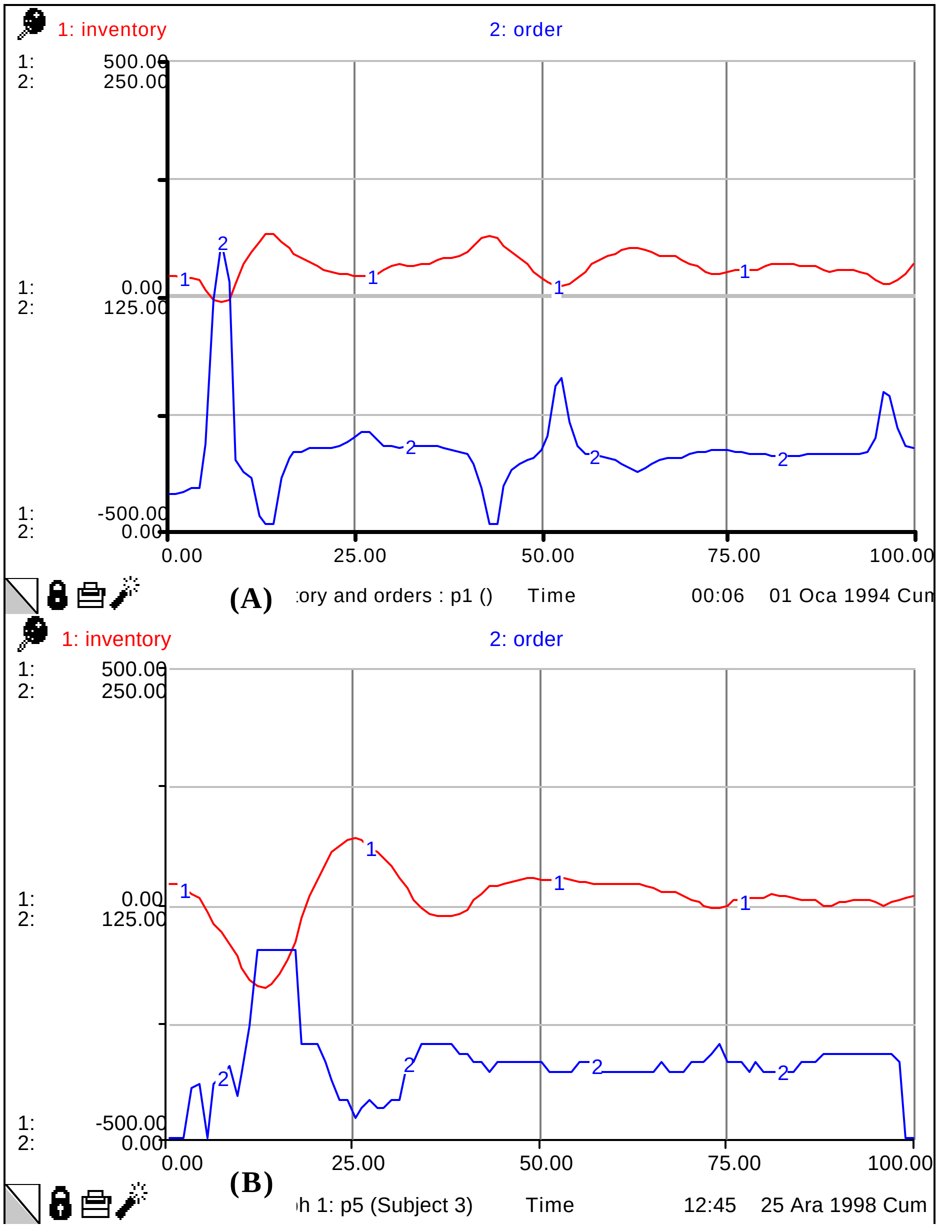

FIGURE 5.3.2. Comparison of (A) Nonlinear Expectation Rule with (B) the Performance of a Player in Game 3 (Short Game with Orders Each Period, Step Up in Customer Demand, Exponential Delay).

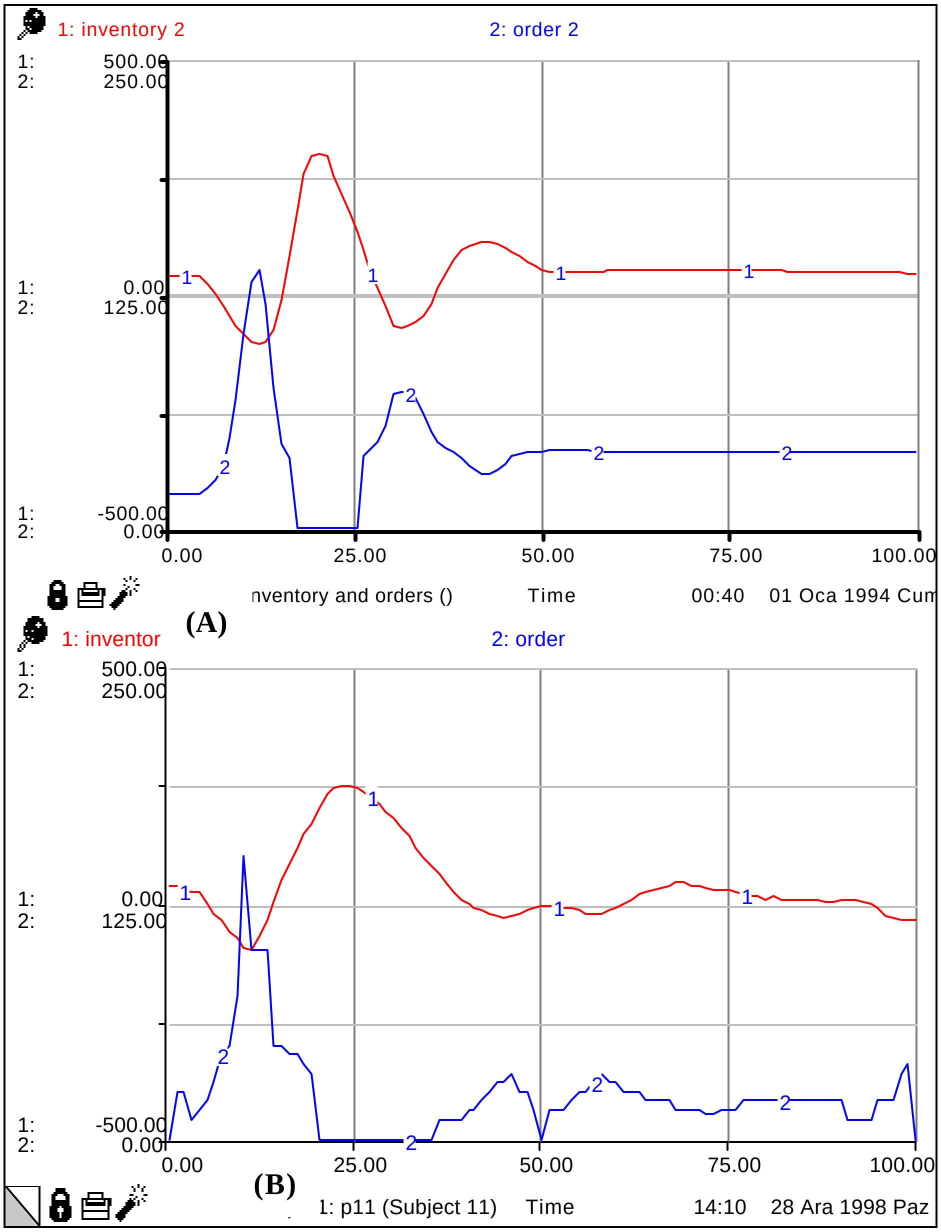

FIGURE 5.3.3. Comparison of (A) Nonlinear Expectation Rule with (B) the Performance of a Player in Game 11 (Short Game with Orders Each Period, Step Up and Down in Customer Demand, Exponential Delay).

6. Standard Inventory Control Rules

Intermittent ordering of subjects and the resulting zigzagging inventory patterns suggest that the discrete inventory control rules used in inventory management may be suitable. The following four policies are most frequently used in inventory management literature:

Order Point, Order Quantity (s, Q) Rule;
Order Point, Order Up to Level (s, S) Rule;
Review Period, Order Up to Level (R, S) Rule;

(R, s, S) Rule.

Two fundamental questions to be answered by any inventory control system are "how many" and "when" (or "how often") to order. "Order-Point" systems determine how many to order in contrast to "Periodic-Review" systems, which determine how often to order (Silver and Peterson, 1985), (Tersine, 1994). When subjects can order every period, they are free to order at any time they desire. Therefore, order-point systems, rather than periodic review systems, are more appropriate to represent the ordering behavior in these situations. In contrast, when subjects can order only once every five periods, periodic-review systems with five as review period may be more appropriate as decision rules. These inventory control rules implicitly assume that time is discrete. Therefore, these rules should be tested only with discrete delays.

6.1. Order Point-Order Quantity (s, Q) Rule:

Order Point-Order Quantity (s, Q) rule can be mathematically expressed as

$$O_t = Q, \quad \text{if } EI_t \text{ ? } s$$
$$0, \quad \text{otherwise} \qquad (6.1)$$

where EI_t represents the effective inventory and s the "order point". Effective inventory and order point are calculated as follows:

$$EI_t = I_t + SL_t \qquad (6.2)$$
$$s = DAVGSL_t + DMINI_t \qquad (6.3)$$

I_t and SL_t represent goods in inventory and in supply line respectively. $DAVGSL_t$ refers to the desired average supply line and $DMINI_t$ to desired minimum inventory. Desired average supply line and desired minimum inventory are calculated as

$$DAVGSL_t = ?*E_t \qquad (6.4)$$
$$DMINI_t = E_t + SS \qquad (6.5)$$

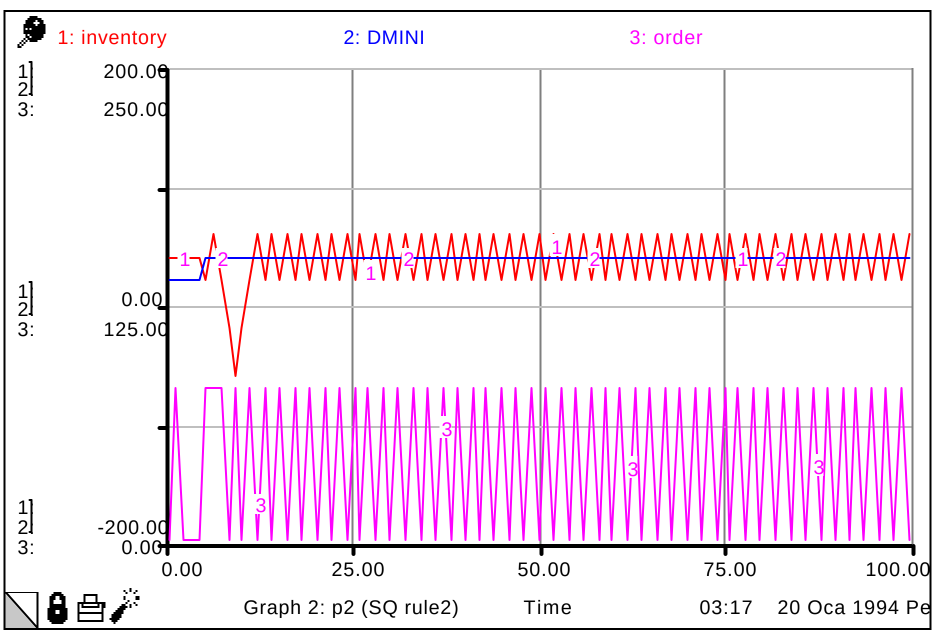

FIGURE 6.1.1. Performance of Order Point-Order Quantity Rule ($Q=4*D_{t0}$, $DAVGSL_t = 4*D_t$, DMINI = 0) in Short Game with Orders Each Period, Known Step Up in Customer Demand, Discrete Delay.

in terms of receiving delay ?, expected demand E_t and safety stocks SS. Order quantity Q and safety stock SS are fixed arbitrarily as constants in this research. Desired minimum inventory $DMINI_t$ is defined as the sum of a constant safety stock and demand expectation. With such a definition, desired minimum inventory can be adapted to variations in customer demand. The performance of this rule with deterministic demand is seen in Figure 6.1.1. Observe that the Order Point-Order Quantity (s, Q) rule can not prevent the inventory falling below the desired minimum inventory when demand is not constant, even when no noise is added to the demand. This particular rule is therefore not suitable for our purpose (i.e. comparative evaluation against the continuous stock adjustment rules).

6.2. Order Point-Order Up to Level (s, S) Rule

Order Point-Order Up to Level (s, S) rule can be mathematically expressed as

$$O_t = \begin{cases} S-s, & \text{if } EI_t \ ? \ s \\ 0, & \text{otherwise} \end{cases} \qquad (6.6)$$

where EI_t represents the effective inventory, s the order point and S the upper level of inventory. Effective inventory, order point and upper level S of inventory are calculated as follows

$$EI_t = I_t + SL_t \qquad (6.7)$$

$$s = DAVGSL_t + DMINI_t \qquad (6.8)$$

$$S = s + Q \qquad (6.9)$$

I_t and SL_t represent goods in inventory and in supply line respectively. $DAVGSL_t$ refers to the desired average supply line and $DMINI_t$ to desired minimum inventory. Desired average supply line and desired minimum inventory are calculated as

$$DAVGSL_t = ?*E_t \qquad (6.10)$$

$$DMINI_t = E_t + SS \qquad (6.11)$$

in terms of receiving delay ?, expected demand E_t and safety stocks SS. Order size Q and safety stock SS are initially set as constants. But note that the actual order quantity O_t (eq. 6.6.) is a variable in this rule. Desired minimum inventory $DMINI_t$ is defined as the sum of expectations and safety stocks. As such, desired minimum inventory $DMINI_t$ may be adapted to variations in customer demand. This rule can be shown to be unbiased and mathematically consistent in the deterministic case. (See Özevin 1999). The inventory reaches equilibrium at the desired minimum inventory level, but in the “noisy” case it may fall below the desired minimum due to discrepancies between "expected" and "actual" customer demand (Figures 6.2.1. and 6.2.2.).

6.3. Review Period, Order Up to Level (R,S) Rule

Order Point-Order Up to Level rule can be mathematically expressed as:

$$O_t = \begin{cases} S-EI_t & \text{if } t = R*k \\ 0 & \text{otherwise} \end{cases} \qquad (6.12)$$

where EI_t represents the effective inventory, t the time, S the upper level of inventory. k is an integer and R is the review period. Effective inventory, order point and the upper level S of inventory are calculated as follows

$$EI_t = I_t + SL_t \qquad (6.13)$$

$$S = DAVGSL_t + DMINI_t + R*E_t \qquad (6.14)$$

I_t and SL_t represent goods in inventory and in supply line respectively. Review period R

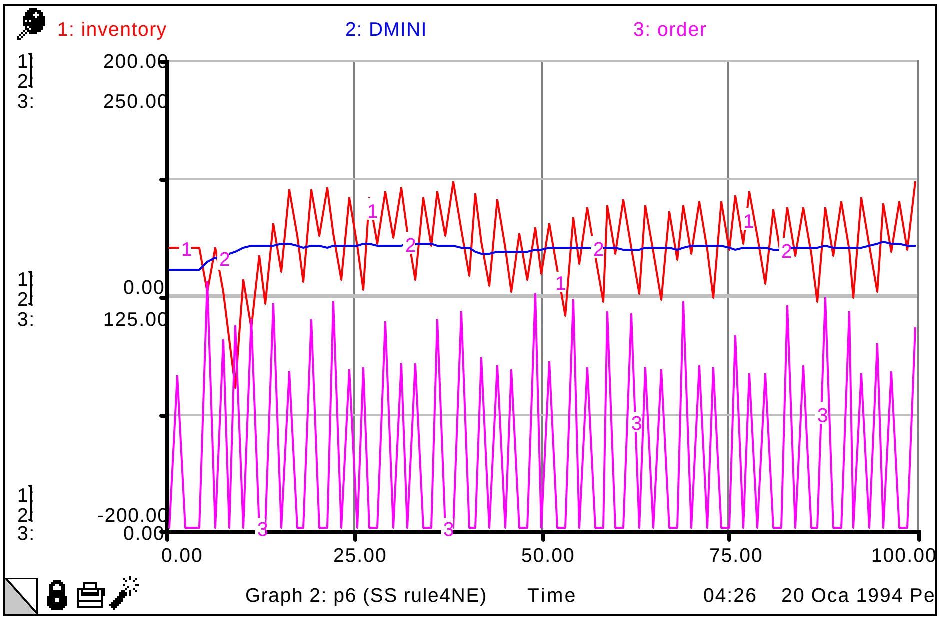

FIGURE 6.2.1. Performance of Order Point-Order Up to Level (s, S) Rule ($DAVGSL_t = 4*E_t$, $DMINI = E_t$) in Short Game with Orders Each Period, Step Up in Customer Demand, Discrete Delay.

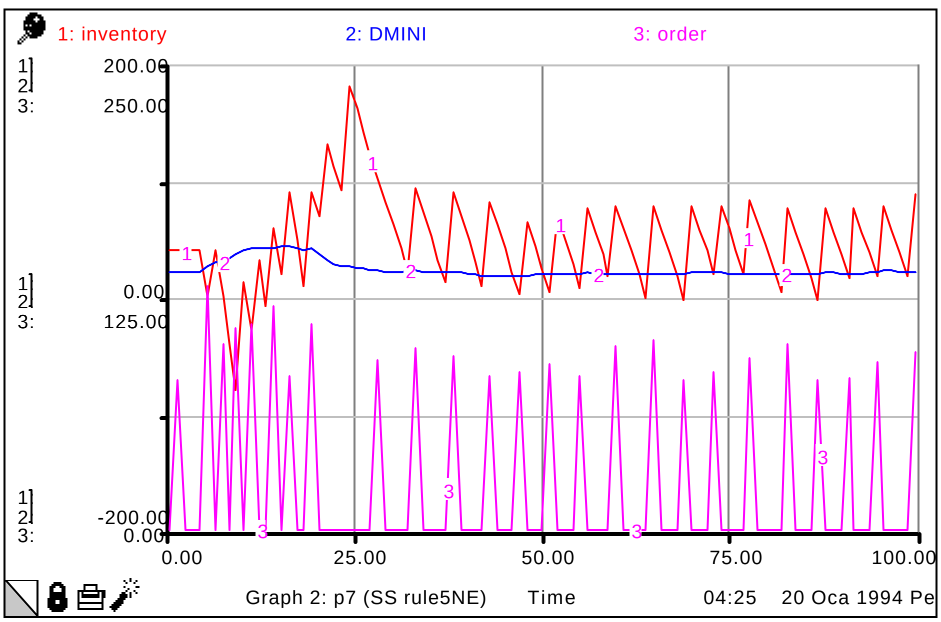

FIGURE 6.2.2. Performance of Order Point-Order Up to Level Rule ($DAVGSL_t = 4*E_t$, $DMINI = E_t$) in Short Game with Orders Each Period, Step Up and Down in Customer Demand, Discrete Delay.

is five. $DAVGSL_t$ refers to the desired average supply line and $DMINI_t$ to desired minimum inventory. Desired average supply line is calculated as

$$DAVGSL_t = ?*E_t \tag{6.15}$$

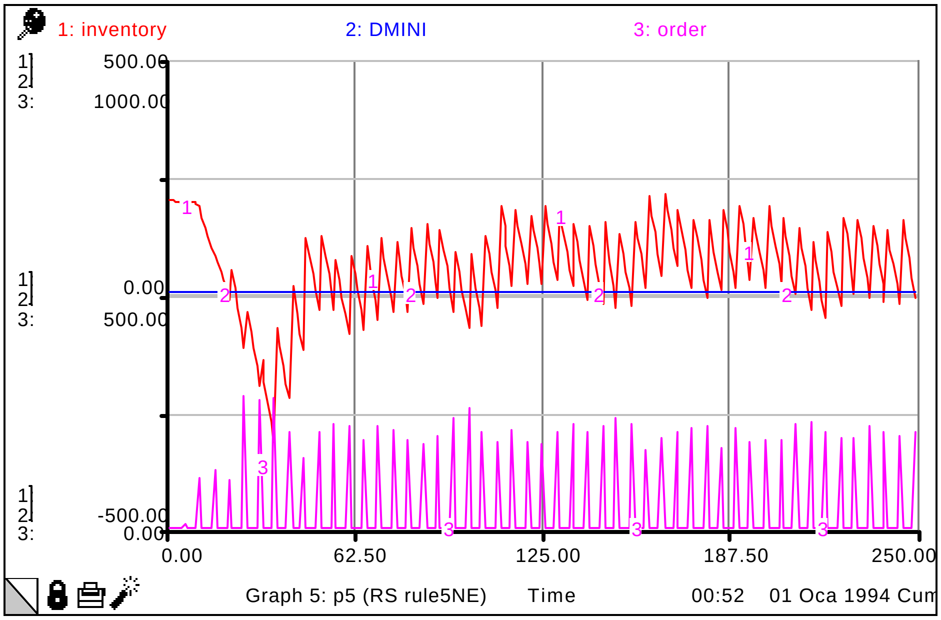

FIGURE 6.3.1. Performance of Review Period-Order Up to Level Rule ($DAVGSL_t$ = $10*E_t$, DMINI = 0) in Long Game with Orders Every Five Periods, Step Up in Customer Demand, Discrete Delay.

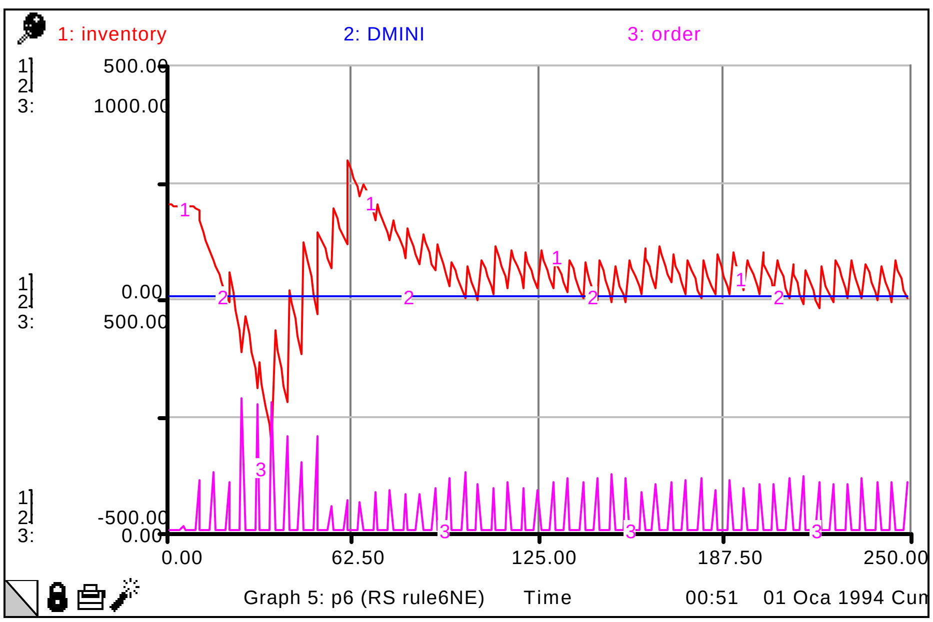

FIGURE 6.3.2. Performance of Review Period-Order Up to Level Rule ($DAVGSL_t$ = $10*E_t$, DMINI = 0) in Long Game with Orders Every Five Periods, Step Up and Down in Customer Demand, Discrete Delay.

in terms of receiving delay ?, expected demand E_t. Desired minimum inventory $DMINI_t$ corresponds to the safety stock. $DMINI_t$ is arbitrarily fixed as constant. This rule can be shown to be unbiased and mathematically consistent in the deterministic case. (See

Özevin 1999). The inventory reaches equilibrium at the desired minimum inventory level, but in the "noisy" case it may fall below the desired minimum due to noise effects. (Figures 6.3.1. and 6.3.2.).

6.4. (R, s, S) Rule:

(R, s, S) rule can be mathematically expressed as

$$O_t = S - EI_t \quad \text{if } t = k*R \text{ and } EI \,?\, s$$
$$0 \quad \text{otherwise} \tag{6.16}$$

where S represents the upper level of inventory, EI_t the effective inventory, R the review period, t the time and s the order point. Review period R is five. Effective inventory, order point, upper level S of inventory and safety stock SS are calculated as follows

$$EI_t = I_t + SL_t \tag{6.17}$$
$$SS = R*E_t + DMINI_t \tag{6.18}$$
$$s = DAVGSL_t + SS_t \tag{6.19}$$
$$S = s + R*E_t \tag{6.20}$$

I_t and SL_t represent goods in inventory and in supply line respectively. $DAVGSL_t$ refers to the desired average supply line, $DMINI_t$ to desired minimum inventory, SS to safety stock, E_t to expected demand. Desired average supply line is calculated as:

$$DAVGSL_t = \,?*E_t \tag{6.21}$$

in terms of receiving delay ?, expected demand E_t. $DMINI_t$ is determined as a constant. This rule can be shown to be unbiased and mathematically consistent in the deterministic case (See Özevin 1999). Due to the difference between the expected and actual customer

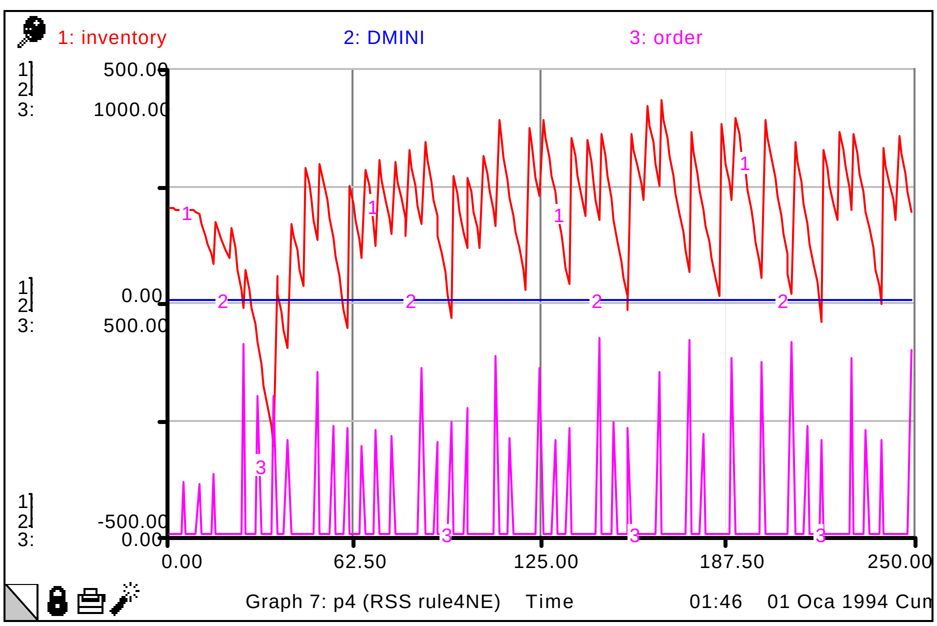

FIGURE 6.4.1 Performance of (R, s, S) Rule ($DAVGSL_t = 10*E_t$, DMINI = 0) in Long Game with Orders Once Every Five Periods, Step Up in Customer Demand, Discrete Delay.

demand in the noisy case, (R, s, S) rule may not result in ordering each time the effective inventory falls to or below the order point; orders are sometimes delayed until the following period. But the inventory never drops below the desired minimum level as can be seen in Figures 6.4.1 and 6.4.2.

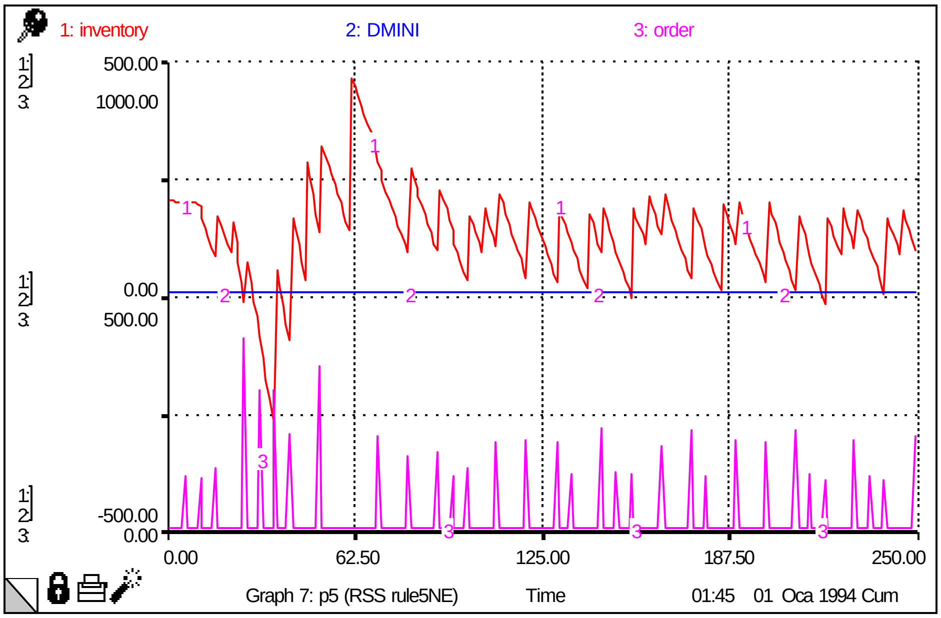

FIGURE 6.4.2. Performance of (R,s, S) rule (Divest = 10*E_t, DMINI = 0) in Long Game with Orders Once Every Five Periods, Step Up and Down in Customer Demand, Discrete Delay.

6.5. Comparison of the Standard Inventory Rules with Experiments

Order point, Order Quantity (s, Q) rule is not a plausible decision rule formulation when demand is not constant, as seen above. Order Point, Order Up to Level (s, S) rule can provide an adequate representation of subjects' performance in some cases where the continuous Anchoring and Adjustment Rule is inadequate. One such case is depicted in Figures 6.5.1. and 6.5.2.

The orders patterns generated by the Review Period, Order Up to Level (R, S) rule are in general very similar to the ones generated by the Anchoring and Adjustment rule. Therefore, (R, S) rule does not provide any novel behavior patterns which Linear Anchoring and Adjustment rule fails to represent. On the other hand, when orders are given every five periods, (R, s, S) rule can represent subjects' decisions in some cases where Anchoring and Adjustment rule fails. One such case is depicted in Figures 6.5.3. and 6.5.4. Finally, it is important to observe that the well known oscillatory dynamic behavior of the inventory is true not just with the linear anchor-and-adjust rule, but also with the standard inventory management rules. (Figures 6.2.1 through 6.5.3).

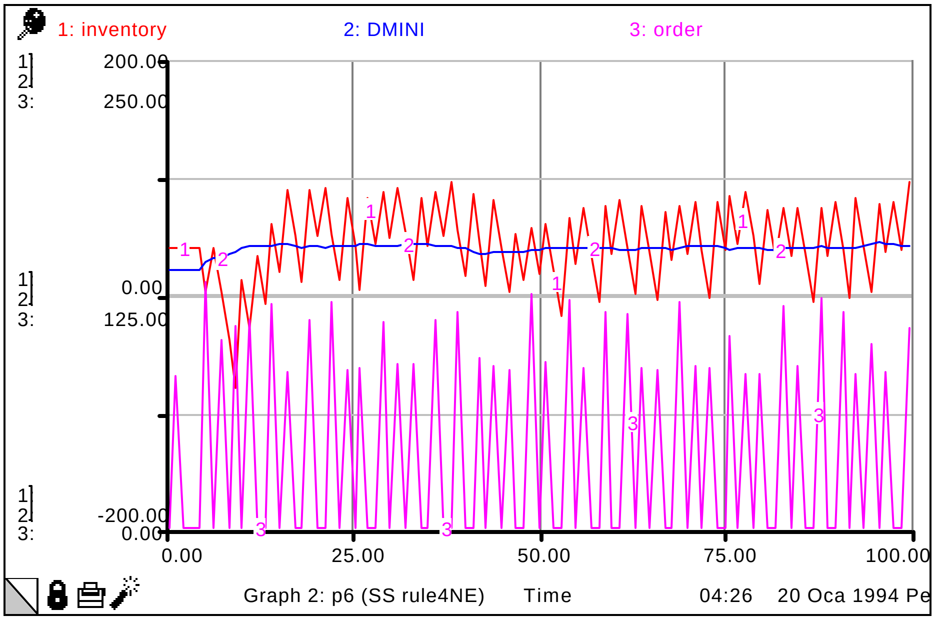

FIGURE 6.5.1. Performance of Order Point-Order Up to Level (s, S) Rule ($DAVGSL_t = 4*E_t$, $DMINI = E_t$) in Short Game with Orders Each Period, Step Up in Customer Demand, Discrete Delay.

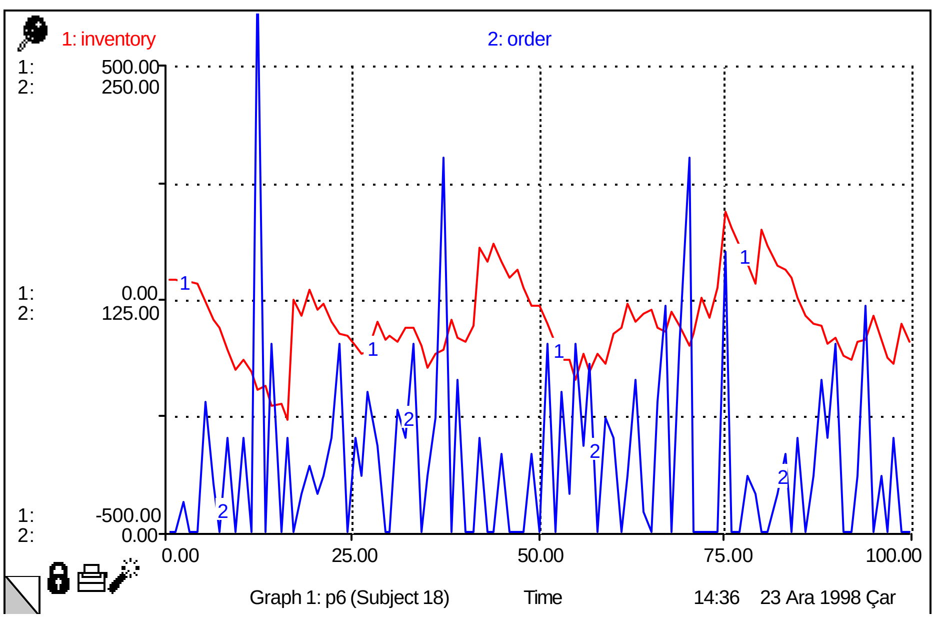

FIGURE 6.5.2. Performance of a Player in Game 24 (Short Game with Orders Each Period, Step Up in Customer Demand, Discrete Delay).

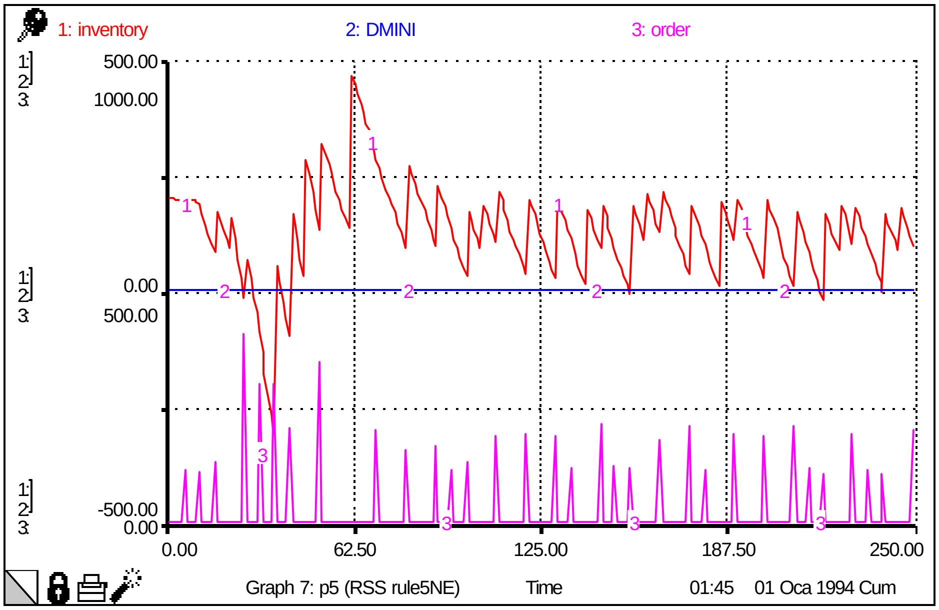

FIGURE 6.5.3. Performance of (R, s, S) rule ($DAVGSL_t = 10E_t$, DMINI = 0) in Long Game with Orders Once Every Five Periods, Step Up and Down in Customer Demand, Discrete Delay.

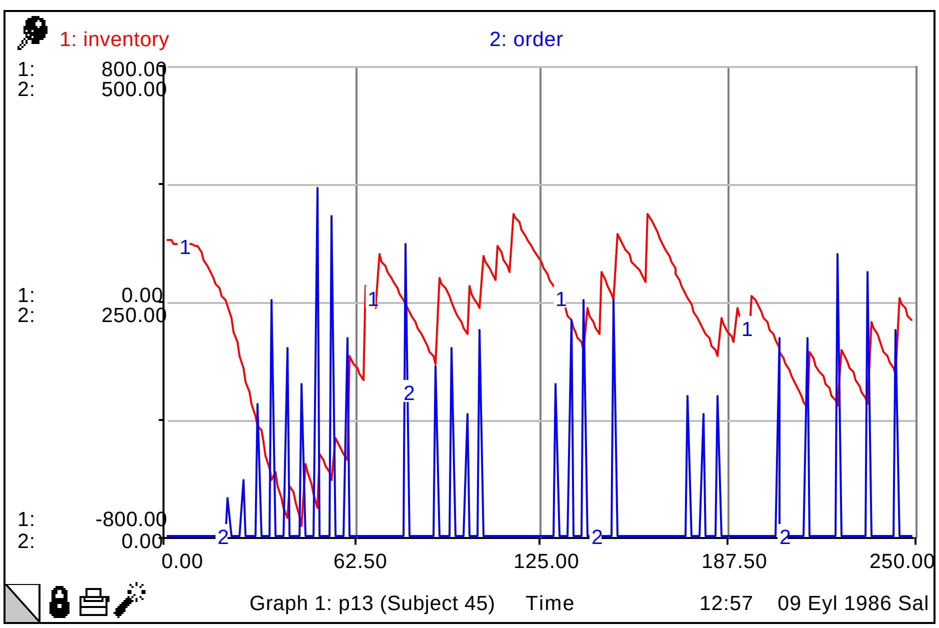

FIGURE 6.5.4. Performance of a Player in Game 14 (Long Game with Orders Once Every Five Periods, Step Up and Down in Customer Demand, Discrete Delay).

7. Conclusion

In this research, performances of subjects in an experimental stock management game are compared with simulated results obtained using some typical stock management rules. The research shows that the common linear anchoring and adjustment rule is not always adequate in representing decision-making behavior of subjects in a dynamic stock control environment. The rule can mimic the subjects performances' when they tend to order continuously. However, most people tend to order intermittently when they are allowed to order each period and/or the receiving delays are discrete in nature, causing sharp inventory oscillations. To address this particular behavior, three different types of nonlinear adjustment formulations have been designed and tested. Some "nonlinear" adjustment rules have been found to be more representative of subjects' decisions in many cases. Finally, some standard inventory management rules have been tested and in some cases, the classical rules such as (s, S) and (R, s, S) are found to represent better the players' ordering behavior. Another major finding is the fact that the well-documented oscillatory dynamic behavior of the inventory is true not just with the linear anchor-and-adjust rule, but also with the non-linear rules, as well as the standard inventory management rules. More research is needed to formulate and test other non-linear formulations. It would also be interesting to test these rules in more complex and realistic game environments (such as multi-player supply chains).

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