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The Fundamental Equation of Neo-classical Economic Growth Reconsidered

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Abstract

This paper offers an original model that enables to reconcile growth and long cycles as well as growth and sustainable development. The theoretical equations generalise *the fundamental equation of neo-classical economic growth* (FENEG) corresponding to the equation (6) of Solow's (1956) original paper taking account of unemployment, resource rent and natural capital in a closed economy. This model grasps the *hypothetical law of motion* of advancing capitalism that subsumes multiple historical patterns and enable to project future events. The state variables are relative wage, employment ratio, man-made capital-output ratio, the share of environmental investments in net output, real natural capital-output ratio and desired natural capital-output ratio. This law generates endogenously the long waves (Kondratiev's cycles, in particular). Stylised facts and simulation experiments by POWERSIM support the given analytic presentation.

1. A Generalisation of the FENEG

1.1 Basic Elements of Solow's Model of Long-Run Growth

The Solow paper starts with pointing out that in the Harrod -- Domar model (HDM) even for the long run the economic system is at best on a knife-edge of equilibrium growth. Were the magnitudes of key parameters -- the saving ratio, the capital-output ratio, the rate of increase of labour force -- to slip ever so slightly from dead centre, the consequence would be either growing unemployment or prolonged inflation.

The paper argues that this fundamental opposition of warranted and natural rates turns out in the end to follow from the critical assumption that production takes place under conditions of fixed proportions. There is no possibility of substituting labour for capital in production in the HDM. If this assumption is abandoned, the knife-edge notion of unstable balance seems to go with it.

A closed economy produces net output designated by P. Part of it is consumed and the rest, qP, is saved and invested without material delay. The national stock of capital K takes the form of the composite commodity. Net investment and the rate of increase of this capital stock are identical (1.1). Here and below time derivatives are denoted by a dot, while growth rates are indicated by a hat. Two factors of production, fixed capital and labour, are used.

$$\dot{K} = qP \qquad (1.1)$$

Aggregate savings are independent of the functional distribution of income between wages and profits; savings are smoothly transformed into investment via an appropriate interest rate, quite independently of the going profit rate. The rate of labour input is L. Technological possibilities are represented by a production function (1.2). It shows constant returns to scale (homogeneity of first degree).

$$P = F(K,L). \qquad (1.2)$$

Inserting (1.2) in (1.1) we get

$$\dot{K} = qF(K,P)\,. \qquad (1.3)$$

It is assumed (1.4) that the labour force increases at a constant relative rate n as a result of exogenous population growth.

$$L(t) = L_0 e^{nt}\,. \qquad (1.4)$$

Full employment of labour and capital is perpetually maintained. Therefore it is possible to insert (1.4) in (1.3) to get

$$\dot{K} = qF(K, L_0 e^{nt}). \qquad (1.5)$$

The marginal productivity equation determines the wage rate which will actually rule: $\frac{\partial F(K,L)}{\partial L} = w$. The complete set includes the latter equation together with (1.3) and (1.4). A similar marginal productivity equation for capital determines the real rental per unit of time for the services of capital stock. Once we know the time path of capital stock and that of labour force, we can compute from production function the corresponding time path of real output.

1.2. Possible Growth Patterns

A new variable (capital-labour ratio, or capital intensity) is introduced $r = K/L$. Hence we have $K = rL = rL_0 e^{nt}$. After differentiating with respect to time and substituting in (1.5) we get

$$\dot{K} = L_0 e^{nt}(\dot{r} + nr) = qF(K, L_0 e^{nt}).$$

Due to constant returns to scale we can divide both variables in F by $L = L_0 e^{nt}$ provided we multiply F by the same factor. Thus

$$L_0 e^{nt}(\dot{r} + nr) = qL_0 e^{nt} F(\frac{K}{L_0 e^{nt}}, 1)$$

and dividing out the common factor we arrive finally at

$$\dot{r} = qF(r, 1) - nr. \qquad (1.6)$$

Here we have a differential equation involving the capital-labour ratio alone. In the subsequent literature, (1.6) is called the FENEG (Jones: 75).

The rate of change of the capital-labour ratio, r, is determined by the difference between the amount of saving (and investment) per worker and the amount required to keep the capital-labour ratio constant as the labour force grows. When $\dot{r} = 0$, the capital-labour ratio is a constant, and the capital must be expanded at the same rate as the labour force, namely n. The reader may notice that there is no equilibrium for n = 0, q > 0.

With constant returns to scale marginal productivities depend only on the capital-output ratio r, and not on any scale quantities. The factor markets in the Solow model work perfectly since the wage rate and profit adjust smoothly and *instantaneously* to changing circumstances. The rate of profit, being a reflection of how scarce capital in relation to the labour force, has not any independent significance for the growth rate. If the real wage is held at some arbitrary level $(\frac{\overline{w}}{p})$ by an exogenous force, the employment must be such as to keep the marginal product of labour at this level. Since the marginal productivities depend only on the capital-labour ratio, it follows that fixing the real wage fixes r at, say, $\bar{r}$. Thus $K/L = \bar{r}$.

Eq. (1.3) becomes $\bar{r}\dot{L} = qF(\bar{r}L, L)$, or $\hat{L} = \frac{\dot{L}}{L} = \frac{s}{\bar{r}}F(\bar{r},1)$. This says that *employment* will increase exponentially at the rate $\frac{s}{\bar{r}}F(\bar{r},1)$. If this rate falls short of n, unemployment will develop and increase. If $\frac{s}{\bar{r}}F(\bar{r},1) > n$, labour shortage will be outcome and presumably the real wage will eventually become flexible upward. Further adjustments and feed-back loops are not considered explicitly in this neo-classical model.

1.3. Examples

The Cobb-Douglas Function

Let $P = K^{\alpha}L^{\beta}$, where $0 < \alpha < 1$, $\beta = 1 - \alpha$. The asymptotic behaviour is always balanced growth at the natural rate (see Figures I and II). The following magnitudes are set: K_0 = 3.80, L_0 = 1.20, α= 0.4, n = 0.02, $q \approx 0.06$.

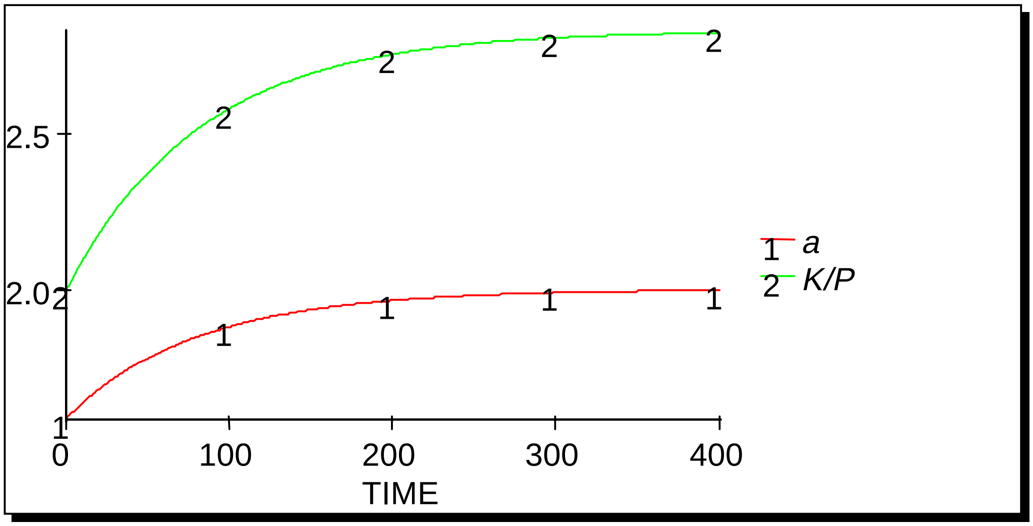

Figure I

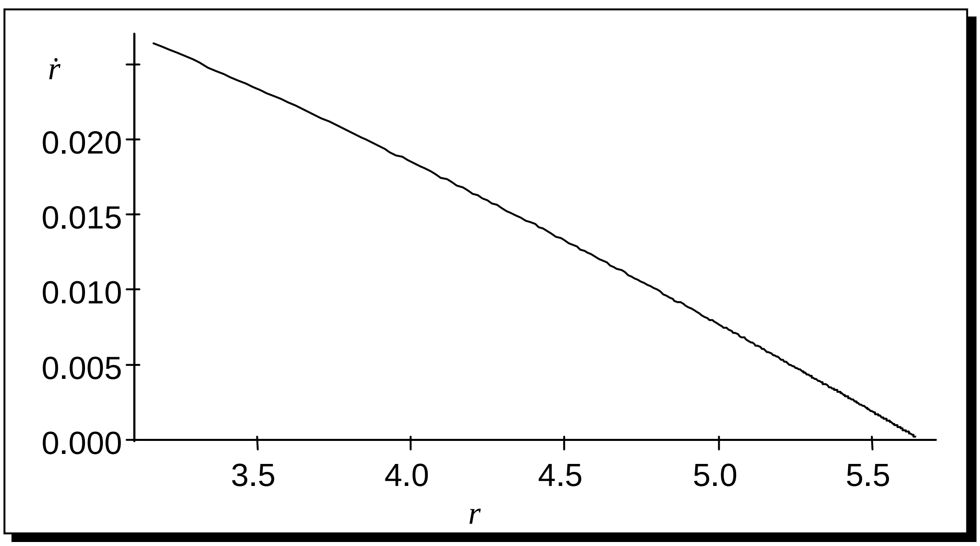

Figure II

Neutral Technological Change

We alter (1.2) to get:

$$P = A(t)F(K,L)$$

Take $A(t) = e^{\gamma t}$ and the Cobb-Douglas case. The special property of the Cobb-Douglas function is that the relative share of labour is constant at $1 - \alpha$. Then in the long run the capital stock increases at the relative rate $n + \gamma/\beta$ compared with n in the case of no technical change. The eventual rate of increase of real output is not $n + \alpha\gamma/\beta$, as given in the original text (Solow: 85), but $n + \gamma/\beta$. Consequently the capital coefficient grows eventually at rate $n + \gamma/\beta - (n + \gamma/\beta) = 0$. The rate of growth, warranted by the appropriate return to capital, asymptotically equals the natural rate unlike the conclusion (Solow: 86). Our Figures III and IV support this refinement. The following constellation is chosen: $K_0 = 4.17$, $L_0 = 1$, $\alpha = 0.5$, $\gamma = 0.02$, $n = 0.02$, $q = 0.25$.

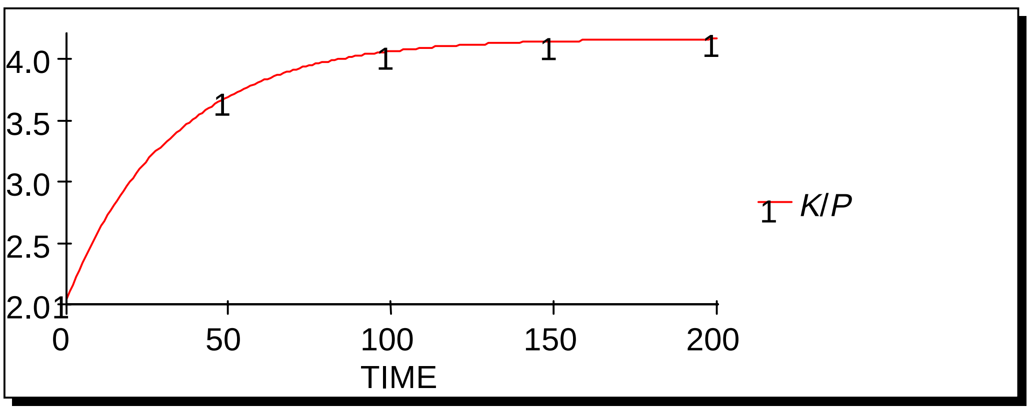

Figure III

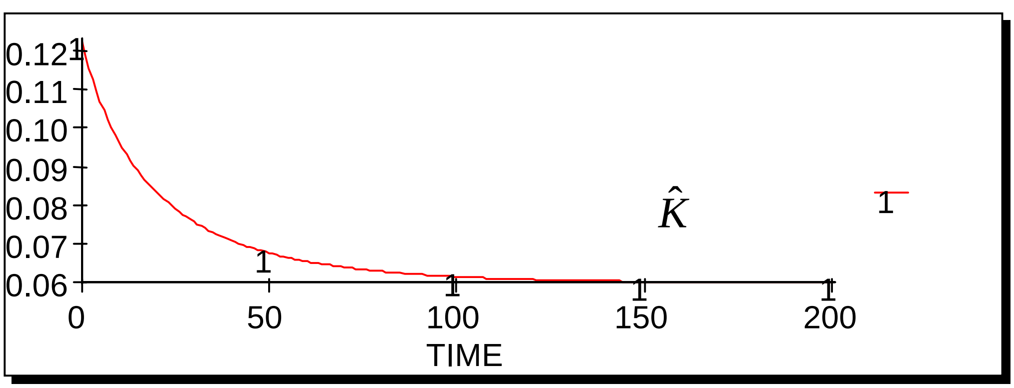

Figure IV

Both labour force and the existing capital stock are thrown on the market inelastically, with real wage and real rental of capital adjusting instantaneously so as to clear the market. Price variables can be calculated once the particular growth path is known. This kind of market behaviour causes the model economy to follow eventually the path of equilibrium growth.

This neo-classical model does allow a substitution between capital and labour and a varying capital-output ratio. A trendless capital-output ratio, trendless factors shares, increasing productivity and real wages correspond to the well-known stylised facts. Still immediate clearing of labour market, the balanced rate of employment, equalisation of warranted and natural rate without cycles are its the most important characteristics. A DIW expert writes (Horn: 6-7): "The empirical evidence contradicts the theoretical view that imperfections on the labour market are the central cause of unemployment."

1.4. A Generalisation of the Fundamental Neo-classical Equation

I preserve Solow's assumptions are with the following exceptions: the absence of technical change, the constant returns to scale, instantaneous adjustment of the real wage and clearing of the labour market, simple reproduction are not required. Transformations come next from the identity $K\hat{/}L = \hat{K} - \hat{L}$ and (1.3)

$$K\dot{/}L = \hat{K}(K/L) - \hat{L}(K/L)$$

$$= \dot{K}/L - \hat{L}(K/L)$$

$$= qP/L - \hat{L}(K/L)\ .$$

Let labour productivity $a = P/L$. We have finally

$$\dot{r} = qa - \hat{L}r. \qquad (1.7)$$

This equation is the generalisation we have looked for. In particular, the FENEG (1.6) is valid for $\hat{L} = n, a = F(r,1).$ Two other specific forms of (1.7) are presented below.

2. The Basal Model of a Closed Capitalist Economy

We portray capital accumulation as an evolving system of coupled non-linear feedback loops from the system dynamics perspective. Levels and rates are the fundamental variable types within a feedback causal loop. A discrepancy between a goal and an apparent condition is a component of a rate or policy.

2.1. The Premises of the Basal Model

The closed capitalist economy is not restricted by natural resources (a *cowboy economy*). It is assumed that a change in capital intensity and technical progress are not separable due to a flow of invention and innovation over time; a qualification of the labour force corresponds to technological requirements; fixed assets and labour are essentially complementary to each other and are also substitutes to some degree depending on relative price changes.

The other most important premises are such:

(1) two social classes (capitalists and workers); the State enforces the property rights, yet costs of such an enforcement are not treated explicitly;

(2) only two factors of production, labour force and means of production, both homogenous and non-specific;

(3) only one good is produced for consumption, investment and circulation purposes, its price is identically one;

(4) production (supply) equals effective demand;

(5) all productive capacities are operated;

(6) all wages consumed, all profits saved and invested;

(7) steady growth in the labour force that is not necessarily fully employed;

(8) a growth rate of a unit real wage rises in the neighbourhood of full employment;

(9) a change in capital intensity and technical progress are not separable due to a flow of invention and innovation over time;

(10) total wage paid during a period of time equals capital outlay for labour power multiplied by a number of turnovers of variable capital (n_v) during this period; for simplicity n_v equals one;

(11) a qualification of the labour force corresponds to technological requirements;

(12) fixed assets and labour are essentially complementary to each other and are also substitutes to some degree depending on relative price changes. Mechanisation is encouraged by a high wage share, i.e., high labour costs per unit of net product.

The product-money identity and the supply-demand equivalence stated in the third and fourth assumptions do not contradict the two-fold character of labour embodied in commodities. This model mirrors the twofold nature of labour power, the unity and contradiction of its value and use-value. The creative functions of labour market as an instrument for transmitting impulses to economic change are the focal point.

The model does not describe the formation of real income of the unemployed persons. It is assumed that a part of wages and salaries covers indirectly the needs of the unemployed. The latter do not play an active role in the model economy. Social security contributions and benefits are not explicitly shown.

The model assumes supremacy of production over final demand. This assumption abstracts from the relative independence of final demand and changes in a product mix. It is more acceptable for the long-run as for the short-run: although in the shorter run aggregate demand influences output, in the very long run output dominates over demand. Capital adapts the output to the scale of production.

The model abstracts from over-production of commodities inherent in over-production of capital during certain phases of industrial cycles; it neglects the changes in the intensity of labour as well. The assumption (10) not only simplifies definition of the profit rate. It may be a key to explanation of the fact that the rate of profit on capital of order of 15 or 20 per cent per annum is compatible with a rate of economic growth of two or three per cent per annum (if $n_v \geq 1$).

The assumption (5) is a strong ameliorating idealisation excluding excessive productive capacities in such forms of productive capital as machines, buildings, etc. The assumption (7) means that the labour force grows exponentially over time. This assumption may be substituted by an assumption of an asymptotic growth or another hypothesis. The assumption (6) corresponds to the immediate aim of capitalist production. Capital produces surplus product and profit as a monetary form of surplus-value.

2.2. The Basal Model Equations

The simplified version of the basal model consists of the following equations:

$$P = K/s \tag{2.1}$$

$$a = P/L \tag{2.2}$$

$$u = w/a \tag{2.3}$$

$$\hat{a} = m_1 + m_2(\widehat{K/L}), \; m_1 \geq 0, \; 1 \geq m_2 \geq 0 \tag{2.4}$$

$$(\widehat{K/L}) = n_1 + n_2 u, \quad n_2 \geq 0 \tag{2.5}$$

$$v = L/N \tag{2.6}$$

$$N = N_0 e^{nt}, \; n = const \geq 0, \; N_0 > 0 \tag{2.7}$$

$$\hat{w} = -g_1 + rv, \; g_1 \geq 0, \; r > 0 \tag{2.8}$$

$$M = (1 - w/a)P = (1 - u)P \tag{2.9}$$

$$\dot{K} = (1 - u)P \; or \; P = wL + \dot{K}. \tag{2.10}$$

Eq. (2.1) postulates a technical relation between the capital stock (K) and net output (P). The variable s is called capital-output ratio. Eq. (2.2) relates labour productivity (a), net output (P) and labour input or employment (L). Eq. (2.3) describes the shares of labour in national income

(u). Eq. (2.6) outlines the rate of employment (v) as a result of the buying and selling of labour-power. Eqs. (9) and (10) reflect production of surplus product and its conversion into capital. They show that profit (M), savings, investment and incremental capital ($\dot{K}$) are equal. Workers do not save at all.

Eq. (2.10) is also the balance between the net output P, on the left side, and the sum of the workmen's consumption wL and net capital accumulation $\dot{K}$, on the right side. An immediate effect of an increase in relative wage is depressive for investment. Still such an increase induces labour-saving technical change.

Eq. (2.7) defines the exponential growth of the labour supply (N) with the rate n. The employment ratio v is such that usually $0 < v < 1$. Demand for labour power does not necessarily keep pace with accumulation of capital, so the unemployment ratio $(1 - v)$ may grow.

Eq. (2.4) is a linear form of Kaldor's technical progress function: the growth rate of labour productivity is assumed to depend linearly on the growth rate of capital intensity. In a more sophisticated version of the model displayed below, increases of the employment ratio facilitate labour productivity gains additionally. This factor destabilises cyclical growth while an "intra-specific" competition among employees is a balancing factor.

Eq. (2.8) represents the linear approximation of the real Phillips curve. In this equation, g_1 and r are the intercept and slope, respectively: the first reflects the tendency of capitalist production to push the value of labour power more or less to its minimum level, the second represents working men's bargaining power. A rising rate of employment is assumed to affect wage increases (in real terms). There is no money illusion.

Instead of assuming, as in the usual Phillips relation, that the rate of change of the wage rate (w) depends only on the employment rate (v), let this rate be additionally influenced by the rate of change of capital intensity (K/L):

$$\hat{w} = -g_1 + rv + g_2 + b(\widehat{K/L}) = -g + rv + b(\widehat{K/L}), \qquad (2.11)$$

where $g = g_1 - g_2 \geq 0$. It is assumed additionally that $b < m_2$ and $b \geq 0$ in the modern epoch.

The higher the qualification, the higher is the capital intensity, and vice versa. The capital intensity may be used as the indicator of qualification in dynamics as well. This modification also takes into consideration the historical or moral element in the value of labour power. It may be helpful for explaining the downward rigidity of the real wage.

Ricardo and Marx wrote that machinery is in constant competition with labour and can often be introduced when price of labour has reached a certain height. A mechanisation function, which follows from assumption (12), is introduced in the equation (2.5).

The next peculiarity of the model is that it has only implicit delays. Due to them, the model gets rid of the instantaneous adjustment to an equilibrium with full employment of labour force used by the earlier neo-classical theories of economic growth. An explicit investment delay is still set aside.

2.3. Productivity, Profitability and the Scale Effect

In the model economy, technical progress and growth of labour force tend to result in steady economic growth, while the long waves could represent important fluctuations about this trend, lasting some 60-65 years. Such a long-wave pattern of economic growth is determined by the internal structure of capital accumulation displayed above within the very stylised institutional setting. The basal model with endogenous technological progress generates long waves via the Andronov – Hopf bifurcation.

In this model, when the profitability (warranted rate of economic growth) is lowest at the end of the recession phase, the rate of change of labour productivity ($\hat{a}$) and natural growth rate ($\hat{a} + n$) are highest (Figure V) . How to explain it?

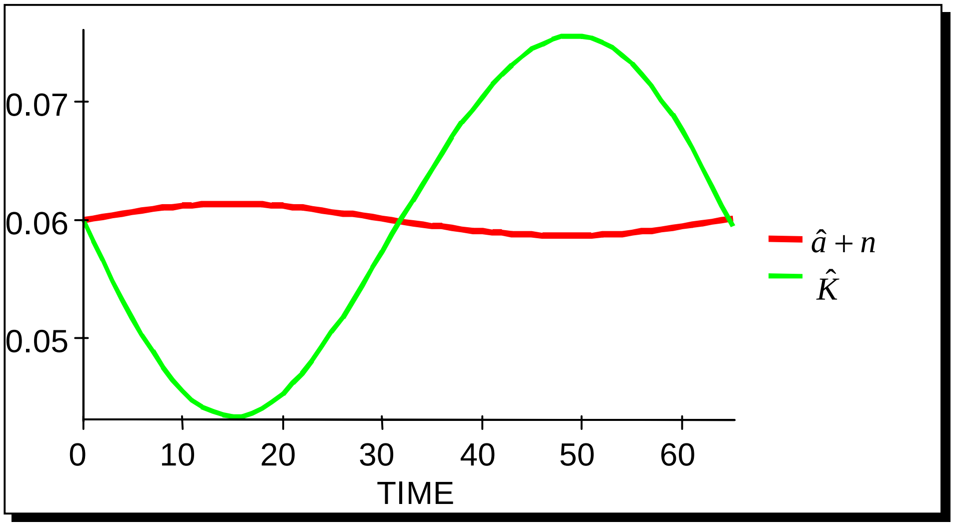

Figure V

The following interpretation suggests itself ((Ryzhenkov 2000: 100-105). The innovations induced by income distribution then exert a strong positive influence on profitability as if they occurred in clusters. Thus the technological breakthrough in the model is the reaction of entrepreneurs to the down-turn. If it is true, then this property of the outlined theory incorporates the Schumpeter and G. Mensch view that the clustering of innovations causes the Kondratiev cycles. The difference is that such a clustering becomes endogenous instead of being exogenous.

Still this interpretation in favour of the model is not yet indisputable. Innovations should, of course, materialise in new investments before they can effect the growth of labour productivity. It first requires investment in capital goods industries, a process that can feed on itself for decades. It is logically to expect that a maximal growth rate of labour productivity happens later than it is predicted by the basal model. This delay does appear after an introduction of a scale effect together with a new element of competition between workers below.

The long term dynamics of capital goods industries, excess capacities and self-ordering of durable assets have not yet been treated explicitly in the above model of the long wave. The motion of labour productivity and capital intensity is synchronous in the model with that of relative wage, since the relationships between $\hat{a}$, $\widehat{K/L}$ and u are linear and positive. This rather simple and doubtful pattern of motion is an additional indirect evidence that there exist factors which affect these variables but not yet have been included in the model.

It follows from the equations (2.4) and (2.5) that the growth rate of labour productivity changes according to the formula

$$\hat{a} = m_1 + m_2(n_1 + n_2 u).$$

It is clear that the higher u, the greater is $\hat{a}$. So the model generates an accelerated productivity growth during the boom and recession of the long wave. This property is hardly empirically valid. There is an argument against trying to preserve jobs by curbing productivity growth: “The reason for this is that rapid productivity growth tends to go hand in hand with rapid output growth. In 1960s, when productivity in OECD economies grew more than twice as fast as it has over the past decade, unemployment remained low. Only in 1970s, when the growth in productivity (and in output) slumped, did unemployment rise” (The Economist, 1995, 337 (7942): 21-22).

Take into account additionally that a growth of labour productivity is retarded by a slowdown in output because of the scale effect. This consideration could solve at least partially problem of disparity between the pattern of behaviour generated by the model and the apparent development. A modified technical progress function is now:

$$\hat{a} = m_1 + m_2(\widehat{K/L}) + m_3\hat{v},\ m_1 \geq 0,\ 1 \geq m_2 \geq 0,\ m_3 \geq 0. \qquad (2.4')$$

Increases of the employment ratio facilitate labour productivity gains in this equation additionally.

This augmentation - positive feedback between a production (demand) gain and increasing labour productivity - destabilises cyclical growth. Simulations which are skipped here have demonstrated diverging fluctuation in the phase space for different reasonable constellations of parameters.

In fact, the "intra-specific" competition among employees is a balancing factor. It is already reflected in the equation for the relative wage but not in the equation for the employment ratio in the basal model. The employment ratio effects capital intensity positively in an extended mechanisation (automation) function:

$$(\hat{K / L}) = n_4 + n_2 u + n_3 v =$$

$$n_1 + n_2 u + n_3 v - n_3 v_2 = n_1 + n_2 u + n_3(v - v_2), \quad (2.5')$$

where $n_4 = n_1 - n_3 v_2$, $n_2 \geq 0$, $n_3 \geq 0$.

These modifications are helpful for explaining why the labour productivity slowdown happens during the boom and recession phases of the long wave: although a growing relative wage promotes productivity growth as it was in the basal model, the scale effect outbalances this positive influence. This association reminds us the real productivity slowdown in the USA that started when there still was the boom of the "golden age".

An accelerating of the growth of labour productivity during the late depression brings about a delayed increment of the employment ratio above the minimum during the recovery in the extended model whereas in the basal model a similar increase of employment ratio is accompanied by decelerating growth of labour productivity.

The inclusion of the scale and competition effects has modified the connection between the growth rate of labour productivity and profitability: they do not move in opposite phases any more; improvements in profitability are the key for accelerating increases of labour productivity, worsening profitability paves the way to the slowdown of productivity growth. It appears to be realistic for the fourth Kondratiev cycle, indeed.

A study of the investment trends suggested that declining profitability accounted for a major part of the investment slowdown in the major OECD countries after 1973 (Bhaskar and Glyn). Profitability recovery was strongest in the 1980s in those countries where unemployment rose most and labour cost competitiveness improved. The crucial difference for profitability between the 1980s and 1973-79 lay in the slower growth of real wages. The cut of two-thirds in real wage growth more than accounts for the reversal in the trend of the wage share from increase to decrease.

Glyn concludes (Glyn: 608): "the fairly robust correlation between rising unemployment and restored manufacturing profitability confirms that differences in the unemployment rate across countries do indicate the pressures exerted in labour market by the reserve army of labour. These evidently have the impact on the profit rate described by the classical economists."

The simulated rates of profit and rates of change of productivity do not exhibit still the same variability as known from the real data in an advanced capitalist economy. This implies a search for more intricate non-linearity.

The exposition has already taken into account that the economy achieves substantial productivity gains in periods when demand for labour power expands especially rapidly. Still there is likely a weakening influence of each additional infinitesimal increment of the rate growth of the employment ratio on the growth rate of labour productivity.

In reality, major technological and productivity improvements take place during expansionary phases of a long wave. A slower expansion of employment, first, and decelerated productivity growth, second, indicates a maturing of a national technological system during a boom. A growth of productivity starts to decelerate when capital plant reaches the point of diminishing returns and this deceleration can become especially severe during a recession.

For presenting this stylised behavioural pattern, a particular functional form of a technical progress function is chosen:

$$\hat{a} = m_1 + m_2(\widehat{K/L}) + m_3\mathrm{SIGN}(\hat{v})\mathrm{ABS}(\hat{v})\hat{}\, j, \qquad (2.4'')$$

where $m_1 \geq 0$, $1 \geq m_2 \geq 0$, $m_3 \geq 0$, $0 < j \leq 1$.

The POWERSIM built-in functions are used in this definition: ABS(x) is absolute value of x that is non-negative, x^j is x raised to the j-th power, SIGN(x) is a sign of x. This modification produces no influence on the steady state $E_2 = (s_2, v_2, u_2)$ defined in the section 1.3. The equation (2.4'') generalises the equations (2.4') and (2.4). Notice that the power j is the new control parameter of the model economy.

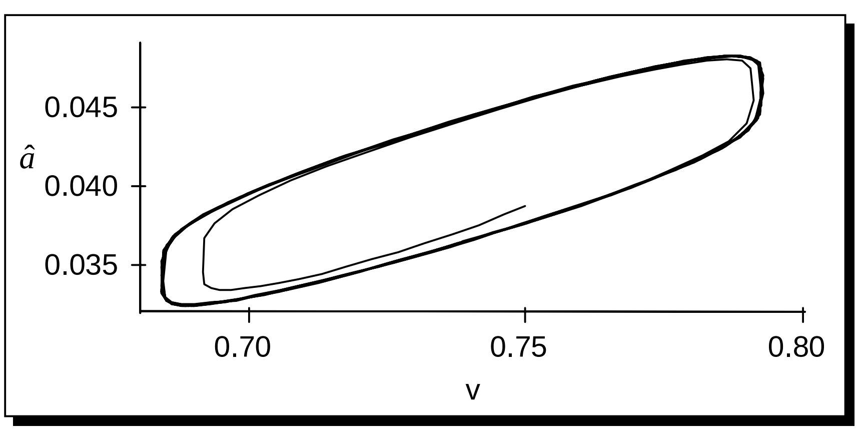

Figure VI

The simulation run have used such initial values of the variables and parameters: s_0 = 4.16667, u_0 = 0.75, v_0 = 0.75; b = 0.357, g= 0.02, m_1= 0.02, m_2 = 0.5, m_3 = 0.05, n = 0.02, n_1 = 0.01, n_2 = 0.04, n_3 = 0.25, r = 0.062, j = 0.5. For this constellation, the numerical experiment generates a seemingly stable limit cycle with a period about 68.5 years (see Figure VI).

The maximal (minimal) profitability is correlated now with the fastest (slowest) growth of labour productivity. For the same profitability, the rate of change of labour productivity is higher when the profitability is growing than when profitability is declining.

It follows from (2.10) that $\hat{K} = (1-u)P/K$ and hence $(K\hat{/}L) = (1-u)P/K - \hat{L}$ or

$$K\dot{/}L = (1-u)a - \hat{L}(K/L)\,, \qquad (2.12)$$

where (1 - u) is the saving ratio. This equation is the second particular case of the equation (1.7). The growth rate of labour force is equal to the growth rate of employment in the neo-classical growth theory, but these rates usually differ in our post-Marxian model. They are equal at the steady state, in particular. Unlike neo-classical Golden Ages, there is a persistent unemployment at the steady state in our model that is more realistic.

3. The Model of Economic-Ecological Reproduction on the Increasing Scale

The world economy has been driven to a level of throughput that exceeds the environment's carrying capacity. The latter cannot withstand a systematic increase of the throughput of matter-energy, but it can support exponential increases of information and knowledge. The developed capitalist countries are entering a new phase of development, that has been termed *eco-capitalism*, or *natural capitalism*. This transition is to occur within the fifth Kondratiev cycle -- the *eco-wave*.

Produced capital is an embodiment of knowledge and, similarly, natural capital is a stock of information. Some conversion factors are needed for aggregating information content of different constituents. The ecologically extended model reflects the impact of economic activities upon natural environmental conditions. These conditions, in their turn, influence the growth rates of labour productivity and capital intensity. A policy, based on a perception of resource scarcity and pollution levels, is also included in this model.

The following equations are used

$$P = K/s \tag{3.1}$$

$$a = P/L \tag{3.2}$$

$$u = w/a \tag{3.3}$$

$$\hat{a} = m_1 + m_2(K\hat{/}L) + m_3\mathrm{SIGN}(\hat{v})\mathrm{ABS}(\hat{v})\wedge j + m_5 F\hat{/}L, \tag{3.4}$$

$$m_1 \geq 0,\ 1 \geq m_2 \geq 0,\ m_3 \geq 0,\ m_5 \geq 0,\ 0 < \mathrm{j} \leq 1$$

$$(K\hat{/}L) = n_4 + n_2 u + n_3 v + n_5(Z/P) \tag{3.5}$$

$$= n_1 + n_2 u + n_3(v - v_b) + n_5(Z/P),$$

$$n_4 = n_1 - n_3 v_b,\ n_2 \geq 0,\ n_3 > 0,\ n_5 \geq 0$$

$$v = L/N \tag{3.6}$$

$$N = N_0 e^{nt},\ n = const \geq 0,\ N_0 > 0 \tag{3.7}$$

$$\hat{w} = -g + rv + b(K\hat{/}L),\ g \geq 0,\ r > 0 \tag{3.8}$$

$$P = C + M + Y \tag{3.9}$$

$$\dot{F} = Y - Z \tag{3.10}$$

$$Z = eP, \qquad 0 < e = const < 1 \tag{3.11}$$

$$\hat{y} = o_1(h - f) + o_2\hat{f},\ o_1 \geq 0, o_2 \leq 0,\ y = Y/P \geq 0 \tag{3.12}$$

$$\hat{X} = d, d \geq 0 \tag{3.13}$$

$$f = F/P \tag{3.14}$$

$$c = X/P \tag{3.15}$$

$$\dot{K} = M = (1 - w/a)P - Y = (1 - u)P - Y. \tag{3.16}$$

Equation (3.1) postulates a technical relation between the capital stock (K) and net output (P). The variable s is called capital-output ratio. Equation (3.2) relates labour productivity (a), net output (P) and labour input or employment (L). Equation (3.3) describes the shares of labour in national income (u). Equation (3.6) outlines the rate of employment (v) as a result of the buying and selling of labour-power. Labour force grows exponentially in (3.7). In the equation (3.9), C is the final consumption, $C = wL = uP$, M is the net formation of produced fixed capital, $M = \dot{K}$, where K is man-made fixed assets, Y is the accumulation of developed natural assets. Equations

(3.9) and (3.16) show that profit (M) and incremental man-made capital ($\dot{K}$) are equal. Workers do not save at all. In the equation (3.8), the rate of change of the wage rate (w) depends on the employment rate (v), as in the usual Phillips relation, and on the rate of change of capital intensity (K/L), additionally.

In the equation (3.10), $\dot{F}$ is a net accumulation (loss) of the natural capital (F). It is assumed that investments are allocated firstly in natural capital because of a poor state of the natural environment (the equation (3.16)). This means that all resource rents are saved and invested like the all profits. These investments are made by the State and private enterprises. The accumulation of the developed natural assets (Y) includes

– additions to their value (in practice, these consist of restoration of the quality and improvements to land, other natural assets and mineral exploration);

– the change in the stock as a result of the transfer of environmental assets to economic uses (net additions to proven reserves of subsoil assets, bringing land and other environmental assets under the direct control, responsibility and management of institutional units: for example, the conversion of wild forests to timber tracts or agricultural land);

– investments for pollution abatement and control to improve the quality and waste disposable capacity of the air and water, or at least to offset the degradation/depletion occurring in the current period.

Z is the net environmental damage in the equation (3.11), i.e., depletion and degradation of non-produced natural assets (land, soil, landscape, ecosystems) due to economic uses above the regeneration rate (cf. Commission of the European Communities – Eurostat et al.: 510-511).[1] A key suggestion is that resource use or pollution has a fixed relationship to output (the linearity of this relationship constitutes a particular case). The desired developed natural capital, X, increases exponentially in the equation (3.13). The equation (3.12) defines investment policy that is aimed to develop the natural capital in accordance with the desired developed natural capital. A

[1] The rate of regeneration is given by a function $Q(F, Y)$, satisfying $Q(0, Y) = 0$, $\partial Q/\partial Y > 0$ (at least for F above a certain minimal level of F) in a more detailed model of sustainable development. There is a perceived social need of directing technological progress to the development of material resources with a shorter regeneration time after the epoch of the increasing aggregate regeneration time of the resource package in use (see Saeed: 124-130). These aspects are skipped in this paper.

combination of proportional and derivative control over the investment in developed natural assets is used hereby. The stock of environmental assets is not treated explicitly in this model.

The environmentally adjusted net domestic product (EDP) equals *P* - *Z* in this model abstracting from entries not related to accumulation (the natural growth of non-cultivated biological resources, catastrophic losses, etc.). The natural capital-output ratios -- real, *f*, and desired, *c*, in the equations (3.14) and (3.15) -- are the new state variables of the model; the share of the investment in the developed natural assets is an auxiliary variable $y = Y/P \geq 0$.

In this model, the analogue of the FENEG (1.6) is

$$(K\hat{/}L) = (1-u-y)P/K - \hat{L} \text{ or}$$

$$K\dot{/}L = (1-u-y)a - \hat{L}(K/L). \qquad (3.17)$$

This equation is the third special case of (1.7).

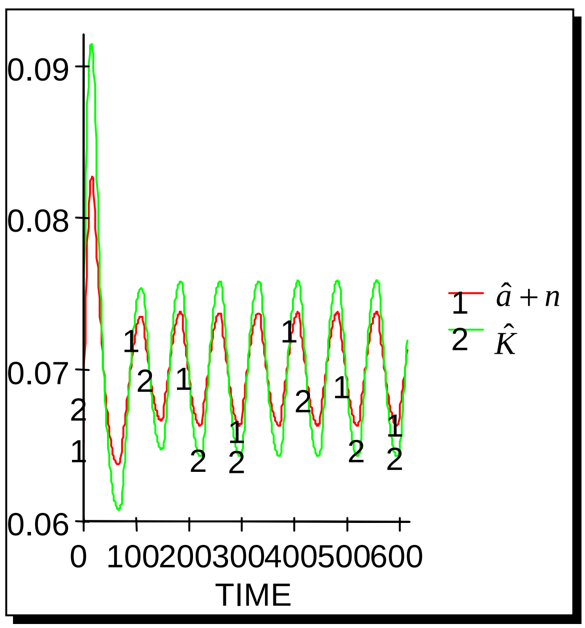

Figure VII

Figure VII displays synchronous dynamics of the warranted and natural rates of growth for sustainable development of the capitalist economy. The trajectory of the warranted growth rate has a higher amplitude of fluctuations than the trajectory of the natural growth rate.

The following initial values and parameters are used in a simulation run: d = 0.07, e = 0.025, m_1 = 0.02, m_2 = 0.5, m_3 = 0.05, m_5 = 0.1, n_1 = 0.01, n_2 = 0.04, n_3 = 0.25, n_5 = 0.48, n = 0.02, r = 0.062, b = 0.5 - 1/7, g = 0.02, o_1 = 0.03, o_2 = -0.006, s_a = 3.235, $v_a \approx 0.841$, u_a = 0.7, $f_a = c_a \approx 0.694$, $y_a \approx 0.0736$, $s_0 = s_a$ = 3.235, v_0 = 0.75, $u_0 = u_a$, $f_0 = 0.8f_a$, $c_0 = f_a$, y_0 = 0.0804, j = 0.56. The amplitude of oscillations is smaller but the period (about 75 years) is longer than in the simpler case from section 2.3 (68.5 years). It may be not true for other magnitudes of the control parameters and initial values.

Conclusion

The fixed points in the post-Marxian models of long waves and sustainable development are either not locally stable or lose their local stability if a specific control parameter crosses the critical magnitude. In the latter case, the solutions bifurcate into apparently stable closed orbits. Additional efforts are required to investigate their topological properties deeper.

The warranted rate of growth is out of phase with natural rate of growth in the basal model because of neglect of the scale effects and competition for jobs. After inclusion of these factors, these growth rates show greater affinity in the simulation runs.

The paper (Solow 1956) has offered the unsatisfactory solution of the knife-edge property of the HDM equilibrium growth path. It overstates the role of negative (control) feed-back loops and understates the importance of informational delays. We can only agree with the statement (Solow: 65): "When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect." The FENEG is generalised in the presented system dynamics framework. The further relaxation of the initial premises will be done in future research.

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