A Period -- Stability Trade-off in a Model of the Kondratiev Cycle

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Abstract

This paper reconvenes a generic structure generating the long-period fluctuations and growth. Forms of technological progress that stabilise cyclical growth in vicinity of a steady-state are explicitly formulated.

Capital intensity is used as a proxy for qualification of labour force. If dependence of a rate of growth of real wage upon qualification becomes stronger than critical, then there occurs the Andronov -- Hopf bifurcation of the fixed point into a closed orbit.

A first order approximation of a period of the Kondratiev cycle is derived. This period is shorter, the higher are the rate of technological progress and rate of growth of the labour force. It is the longer, the higher is the rate of growth of capital intensity (as a function of two structural coefficients). If productivity growth remains laggard, stabilising the world economy by accelerated capital deepening may lengthen the depression phase of the current Kondratiev cycle globally.

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1. The Premises of the Abstract Model

The closed capitalist economy is not restricted by natural resources. This assumption is relaxed in (Ryzhenkov 1999). The other most important premises are such:
(1) two social classes (capitalists and workers); the State enforces the property rights, yet costs of such an enforcement are not treated explicitly;
(2) only two factors of production, labour force and means of production, both homogenous and non-specific;
(3) only one good is produced for consumption, investment and circulation purposes, its price is identically one;
(4) production (supply) equals effective demand;
(5) all productive capacities are operated;
(6) all wages consumed, all profits saved and invested;
(7) steady growth in the labour force that is not necessarily fully employed;

(8) a growth rate of a unit real wage rises in the neighbourhood of full employment;
(9) a change in capital intensity and technical progress are not separable due to a flow of invention and innovation over time;
(10) total wage paid during a period of time equals capital outlay for labour power multiplied by a number of turnovers of variable capital (n_v) during this period; for simplicity n_v equals one;
(11) a qualification of the labour force corresponds to technological requirements;
(12) fixed assets and labour are essentially complementary to each other and are also substitutes to some degree depending on relative price changes. “Mechanisation is encouraged by a high wage share, i.e., high labour costs per unit of net product” (Glombowski and Krüger 1984: 265).

The product-money identity and the supply-demand equivalence stated in the third and fourth assumptions do not mean that we abstract from the two-fold character of labour embodied in commodities entirely. This Goodwin-like model mirrors the two-fold nature of labour power, the unity and contradiction of its value and use-value. The creative functions of labour market as an instrument for transmitting impulses to economic change are the focal point of this model.

The model does not describe the formation of real income of the unemployed persons. I assume that a part of wages and salaries covers indirectly the needs of the unemployed. The latter do not play an active role in the model economy. Social security contributions and benefits are not explicitly shown.

The model omits Goodwin’s assumption of constant capital-output ratio, but preserves his premise of the supremacy of production over final demand. This assumption abstracts from the relative independence of final demand and changes in a product mix. It is more acceptable for the long-run as for the short-run: although in the shorter run aggregate demand influences output, in the very long run output dominates over demand. Capital adapts the output to the scale of production.

We abstract from over-production of commodities inherent in over-production of capital during certain phases of industrial cycles. We neglect the changes in the intensity of labour as well. The assumption (10) not only simplifies definition of the profit rate. It may be a key to explanation of the fact that the rate of profit on capital of order of 15 or 20 per cent per annum is compatible with a rate of economic growth of two or three per cent per annum (if $n_v \geq 1$).

The assumption (5) is a strong ameliorating idealisation excluding excessive productive capacities in such forms of productive capital as machines, buildings, etc. The assumption (7) means that the labour force grows exponentially over time. This assumption may be substituted by an assumption of an asymptotic growth or another hypothesis. The assumption (6) corresponds to the immediate aim of capitalist production. Capital produces surplus product and profit as a monetary form of surplus-value.

2. The Model Equations

The model is formulated in continuous time. Time derivatives are denoted by a dot, while growth rates are indicated by a hat. The simplified version of the model consists of the following equations:

$$P = K/s \tag{1}$$

$$a = P/L \quad (2)$$

$$u = w/a \quad (3)$$

$$\hat{a} = m_1 + m_2(\hat{K/L}),\ m_1 \geq 0,\ 1 > m_2 > 0 \quad (4)$$

$$(\hat{K/L}) = n_1 + n_2 u,\ \ n_2 \geq 0 \quad (5)$$

$$v = L/N \quad (6)$$

$$N = N_0 e^{nt},\ \ n = const \geq 0,\ N_0 > 0 \quad (7)$$

$$\hat{w} = -g_1 + rv,\ \ g_1 \geq 0,\ r > 0 \quad (8)$$

$$M = (1 - w/a)P = (1 - u)P \quad (9)$$

$$\dot{K} = (1 - u)P \ or\ P = wL + \dot{K}. \quad (10)$$

Eq. (1) postulates a technical relation between the capital stock (K) and net output (P). The variable s is called capital-output ratio. Eq. (2) relates labour productivity (a), net output (P) and labour input or employment (L). Eq. (3) describes the shares of labour in national income (u). Eq. (6) outlines the rate of employment (v) as a result of the buying and selling of labour-power. Eqs. (9) and (10) reflect production of surplus product and its conversion into capital. They show that profit (M), savings, investment and incremental capital ($\dot{K}$) are equal. Workers do not save at all.

Eq. (10) is also the balance between the net output P, on the left side, and the sum of the workmen's consumption wL and net capital accumulation $\dot{K}$, on the right side. An immediate effect of an increase in relative wage is depressive for investment. Still such an increase induces labour-saving technical change.

Eq. (7) defines the exponential growth of the labour supply (N) with the rate n. The employment ratio v is such that usually $0 < v < 1$. Demand for labour power does not necessarily keep pace with accumulation of capital, so the unemployment ratio $(1 - v)$ may grow.

Eq. (4) is a linear form of Kaldor's technical progress function: the growth rate of labour productivity is assumed to depend linearly on the growth rate of capital intensity. In a more sophisticated version of the model not displayed in this paper, increases of the employment ratio facilitate labour productivity gains additionally. This factor destabilises cyclical growth while an "intra-specific" competition among employees is a balancing factor.

Eq. (8) represents the linear approximation of the real Phillips curve. In this equation, g_1 and r are the intercept and slope, respectively: the first reflects the tendency of capitalist production to push the value of labour power more or less to its minimum level, the second represents working men's bargaining power. A rising rate of employment is assumed to affect wage increases (in real terms). There is no money illusion.

Instead of assuming, as in the usual Phillips relation, that the rate of change of the wage rate (w) depends only on the employment rate (v), let this rate be additionally influenced by the rate of change of capital intensity (K/L):

$$\hat{w} = -g_1 + rv + g_2 + b(\widehat{K/L}) = -g + rv + b(\widehat{K/L}), \qquad (11)$$

where $g = g_1 - g_2 \geq 0$. It is assumed additionally that $b < m_2$ and $b \geq 0$ in the modern epoch.

The higher the qualification, the higher is the capital intensity, and vice versa. In my opinion, the capital intensity may be used as the indicator of qualification in dynamics as well. This modification also takes into consideration the historical or moral element in the value of labour power. It may be helpful for explaining the downward rigidity of the real wage.

Ricardo and Marx wrote that machinery is in constant competition with labour and can often be introduced when price of labour has reached a certain height. A mechanisation function, which follows from assumption (12), is introduced in (5).

The next peculiarity of our model is that it has only implicit delays. Due to them, we get rid of the instantaneous adjustment to an equilibrium with full employment of labour force used by the earlier neo-classical theories of economic growth. An explicit investment delay is still set aside.

3. A Hypothetical Law of Motion and Steady State

The central variables of our basal model are the employment rate (v), the labour bill share (u) and the capital coefficient (s). To be economically meaningful, they should be strictly positive. Moreover, the rates of growth are not defined for (0,0,0).

The hypothetical law of motion of the model economy is given by the following system of non-linear ordinary differential equations derived from the Eqs. (1) -- (11):

$$\dot{s} = -(m_1 + (m_2 - 1)(n_1 + n_2 u))s \qquad (12)$$

$$\dot{v} = ((1 - u)/s - (n_1 + n_2 u) - n)v \qquad (13)$$

$$\dot{u} = (-g + rv - m_1 + (b - m_2)(n_1 + n_2 u))u. \qquad (14)$$

We will use at first the simplest notion of equilibrium (a fixed point in a phase space). A nontrivial equilibrium for $m_2 \neq 1$, $n_2 > 0$ is given by:

$E_2 = (s_2, v_2, u_2)$, where $n_1 + n_2 > \tilde{w} > n_1$, $r \geq g + m_1 + (m_2 - b)\tilde{w} > 0$,

$u_2 = (\tilde{w} - n_1)/n_2$, $s_2 = (n_1 + n_2 - \tilde{w})/(n_2(\tilde{w} + n))$,

$v_2 = (g + m_1 + (m_2 - b)\tilde{w})/r$,

while the equilibrium growth rate of labour productivity and unit real wage $\tilde{w} = n_1 + n_2 u_2 = m_1/(1 - m_2)$.

The labour income share, the rate of employment, capital-output ratio and profit rate are constant at the steady state. In our model like neo-classical models, the higher the equilibrium rate of productivity growth, the greater are the employment ratio and profit rate. The latter equals the rate of investment and the rate of economic growth: $\hat{K}_2 = \hat{P}_2$, like at a neo-classical golden-age growth path (Phelps 1961).

The constancy of the labour share over the long period is known as Bowley's law. The properties of the steady-state growth in our model correspond to Kaldor's five stylised facts:

1. The aggregate volume of production and output per worker show continuing growth at a steady rate with no tendency for a falling rate of growth of productivity.
2. Capital per worker shows continuing growth.
3. The rate of profit on capital is steady at least in the developed capitalist societies.
4. The capital-output ratio is steady over long periods, hence the aggregate volume of production and fixed capital tend to grow at the same rate.
5. Labour and capital receive constant shares of total income. The shares of profit in national income and the share of investment in net output are closely (positively) correlated.

A Jacobian of the three-dimensional system (12) -- (14) evaluated at the nontrivial equilibrium E_2 is given by

$$J = \begin{vmatrix} 0 & 0 & n_2(1 - m_2)s_2 \\ -v_2(1 - u_2)/s_2^2 & 0 & -v_2/s_2 - n_2v_2 \\ 0 & ru_2 & (b - m_2)n_2u_2 \end{vmatrix}.$$

A characteristic polynomial is $\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$, where

$$a_0 = -det(J) = (v_2(1 - u_2)ru_2n_2(1 - m_2))/s_2 > 0, \text{ if } m_2 < 1,$$

$$a_1 = ru_2v_2(1/s_2 + n_2) > 0$$

$$a_2 = -trace(J) = (m_2 - b)n_2u_2 > 0, \text{ if } m_2 > b.$$

The Routh-Hourwitz conditions are necessary and sufficient for local stability and require that $a_0 > 0$, $a_1 > 0$ and $a_1a_2 > a_0$. The first and second inequalities are satisfied, whereas the third inequality corresponds to

$$(n_2s_2 + 1)(m_2 - b) > ((1 - u_2)/u_2)(1 - m_2) > 0. \qquad (15)$$

Provided that the inequality (15) holds, the dynamics of the model (12) -- (14), in the neighbourhood of its equilibrium, are Poincaré (locally) stable. Then the eigenvalues have negative real parts. The inequality (15) is not true, if $m_2 < b$. So $m_2 > b$ is a necessary condition of the local stability. The presence of coefficient $n_2 > 0$ on the left side of (15) shows that a distribution-induced change in the speed of technological progress produces a stabilising influence (if n_2 is less than its critical value).

4. Converging Fluctuations

I use vaguely plausible equilibrium values of the main variables relying on international statistics and our predecessors. It is not implied yet that this illustrative constellation is in fact empirically accurate.

Let us chose an initial magnitude of the rate of employment $v_0 = 0.89 \neq v_2$ without an initial displacement of other variables from their equilibrium values ($u_0 = u_2$, $s_0 = s_2$). Then a damping cyclical motion is obtained, the length of the cycle being approximately 60 years. Because the real parts of the eigenvalues are all negative, the fixed point E_2 is a sink. Moreover, it is locally asymptotically stable (see Table 1).

Table 1. A converging long wave

Steady state growth rate of labour productivity ($\hat{a}_2$)	Real eigenvalue λ	Conjugate pair of eigenvalues $\alpha \pm \beta i$	Period of fluctuations in the linearised model (years)
0.04	-0.0043	-0.0039± 0.1077i	58.3

Note: the parameters: $m_1 = 0.02$, $m_2 = 0.5$, $n_1 = 0.01$, $n_2 = 0.04$, $r = 0.062$, $b = 0.1$, $g = 0.02$, $n = 0.02$, the initial vector $(s_0, u_0, v_0) = (s_2, u_2, v_2 - 0.01) \approx (4.17, 0.75, 0.89)$.

The fluctuations are not strictly periodic. The amplitude and phasing of each variable are determined structurally. In a linear case all variables oscillate with the same frequency and damping, only their amplitudes and phasing differ, these being parameters fixed for each variable separately by extraneous factors or initial conditions.

5. Free Oscillations Resulting from a Structural Change

Our non-linear model can generate self-sustained recurrent fluctuations without exogenous shocks. The Hopf theorem may constitute the only tool to establish the existence of closed orbits. In this study of the cyclical motion we choose b (see Eq. (2)) as a bifurcation (control) parameter, although it is possible to select either.

The inequality (15) turns into equality if

$$m_2 - b_0 = (1 - u_2)(1 - m_2)/(u_2 (n_2 s_2 + 1)), \qquad (16)$$

where $1 < u_2$ and $b_0 < m_2 < 1$. By rearranging and substituting, we get

$$b_0 = m_2 - (n_1 + n_2 - \tilde{w})(1 - m_2)(\tilde{w} + n)/[(\tilde{w} - n_1)(n_1 + n_2 + n)]. \qquad (17)$$

The Hopf theorem establishes only the existence of closed orbits in a neighbourhood of x^* at $b = b_0$, and it does not clarify the stability of orbits, which may arise on either side of b_0.

Consider the equilibrium of the system (12) -- (14) as dependent on b:

$$\dot{x} = 0 = f(x, b). \qquad (18)$$

The determinant of the Jacobian matrix (J) differs from zero in our case for any possible equilibrium (x, b) if $v_2 r u_2 n_2 s_2 (m_2 - 1) \neq 0$. This requirement is typically satisfied.

The implicit function theorem ensures that for every b in a neighbourhood $B_r(b_0) \in R$ of the parameter value b_0 there exists a unique equilibrium x^*. Changes of b do not affect s_2 and u_2, whereas v_2 diminishes if b grows.

We assume that this equilibrium is stable for small values of the parameter b and the other properties are satisfied:

i) the Jacobian has a pair of pure imaginary eigenvalues and no other eigenvalues with zero real parts,

ii) the derivative $\dfrac{d(Re\,\lambda(b))}{db} > 0$ for $b = b_0$.

Then there exists some periodic solution bifurcating from $x^*(b_0)$ at $b = b_0$ and the period of fluctuations is near $2\pi/\beta_0$ ($\beta_0 = \lambda(b_0)/i$). If a closed orbit is an attractor, it is usually called a *limit cycle*.

The characteristic polynomial for $b = b_0$ is

$$\lambda^3 + a_2\lambda^2 + a_1\lambda + a_1 a_2 = \lambda^2(\lambda + a_2) + a_1(\lambda + a_2) =$$

$$(\lambda + a_2)(\lambda^2 + a_1) = 0. \qquad (19)$$

It has the following roots:

$$\lambda_1 = -a_2 = -(m_2 - b)n_2 u_2 < 0, \text{ if } m_2 > b; \qquad (20)$$

$$\lambda_{2,3} = \pm i\sqrt{-a_1} = \pm i\sqrt{r u_2 v_2 (1/s_2 + n_2)} =$$

$$\pm i\sqrt{[(\tilde{w} - n_1)(g + m_1)(1 + \frac{\tilde{w} + n}{n_1 + n_2 - \tilde{w}}) + m_1(\tilde{w} + n)]}. \qquad (21)$$

The period of oscillation near $x^*(b_0)$ is about $2\pi/\sqrt{-a_1}$ (years). One should not overlook that the approximated period of fluctuations near E_2 is independent of b in our model. Assuming $g + m_1 > 0$, the higher the rate of technological progress and rate of growth of the labour force, the shorter is this period; it is the longer, the higher are coefficients n_1 and n_2 in the mechanisation function that together with the relative wage determine the rate of capital deepening.

The critical $b_0 \approx 0.357 < m_2 = 0.5$ for the above constellation of the other coefficients in Table 1. The new nontrivial fixed point corresponding to this critical magnitude (x^*) equals approximately (4.17, 0.75, 0.74). When b is increased from $b < b_0$ to $b > b_0$, the system (12) - (14) loses its local stability at x^* because the real part of the complex conjugate eigenvalues becomes positive (see Table 2). According to Hopf bifurcation theorem (an existence part), there exist periodic solutions bifurcating from the new locally unstable fixed point at $b = b_0$.

Table 2. The self-sustained long wave

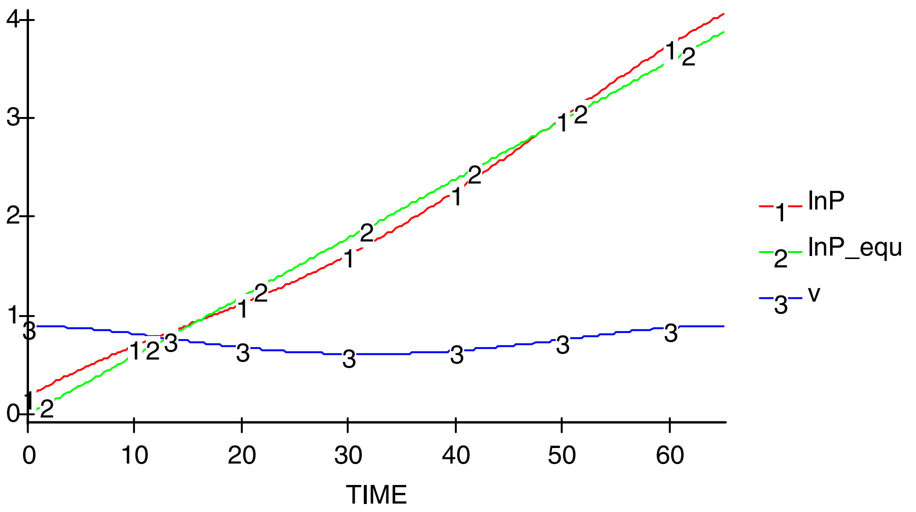
The critical value of the control parameter b_0	The steady state growth rate ($\hat{P}_2$)	The real eigenvalue λ_3	The conjugate pair of eigenvalues λ_1 and λ_2 $\alpha \pm \beta i$	The derivative $\frac{d(\mathrm{Re}\,\lambda_{1,2}(b))}{db}$ for $b = b_0$	The period of fluctuations in the linearised model (years)
0.357143	0.06	-0.004	0± 0.098i	0.075	64.127

I have simulated limit cycles in the phase space, which show the other possible pattern of long waves about the trend, using the software *Powersim*. In agreement with the Hopf theorem, numerical calculations do depict attraction of trajectories starting in the neighbourhood of x^* (at $b = b_0 \approx 0.357$) by different limit cycles (see Figure 1). A picking up of a periodic attractor in a given time integration depends on the starting conditions (the initial values of s, v, u). In each such case the oscillations increase in amplitude until limited by non-linearity in the system and persist within certain limits. Notice that a similar multiplicity of alternative stable attracting solutions dependent on the starting conditions is typical of non-linear driven oscillators.

The upswing and downswing phases of the long cycle are determined relative to the steady state for relative quantities and in relation to the net output trend. The upswing consists of recovery and prosperity, downswing embraces recession and depression. These periods are delineated based on movements of the employment ratio that mirrors very closely fluctuations of net output around the trend.

Diverging fluctuations due to an escalating class conflict over income distribution and employment can also arise in the model economy if the parameters and initial values are outside the stability range. In particular, excessive bargaining delays are detrimental (Ryzhenkov 1994).

Figure 1. Synchronous fluctuations of the employment ratio (v) and net output (lnP) around the trend (lnP_equ)



6. A Comment on the Neo-classical Growth Theory

Bounded rationality of economic behaviour and the ability of economic agents to learn are amongst premises of my model. I do not use the profit-maximising hypothesis applied by neo-classical economists for its logical weakness and lack of substantial empirical evidence in its favour.

It follows from (10) that $\hat{K} = (1-u)P/K$ and hence $(\widehat{K/L}) = (1-u)P/K - \hat{L}$ or $\dot{K/L} = (1-u)a - \hat{L}(K/L)$. This equation generalises *the fundamental equation of neo-classical economic growth* corresponding to Eq. (6) of Solow's original paper (Solow 1956): $\dot{K/L} = (1-u)a - n(K/L)$, where (1 - *u*) is the saving ratio.

The reader see that the growth rate of labour force is equal to the growth rate of employment in the neo-classical growth theory, but these rates usually differ in our model. They are equal at the steady state, in particular. Unlike neo-classical Golden Ages, there is a persistent unemployment at the steady state in our model that is more realistic.

The elasticity of substitution measures *the responsiveness* of the capital intensity to the ratio of profit rate to unit real wage (Jones 1976: 34). We define it quite independently of unpractical marginal products as

$$sigma = -\frac{\widehat{K/L}}{\widehat{K/w}} = -\frac{\widehat{K/L}}{\hat{K} - \hat{w}} = -\frac{\widehat{K/L}}{-\dfrac{\dot{u}}{1-u} - \hat{s} - \hat{w}} = -\frac{\widehat{K/L}}{-\dfrac{\dot{u}}{1-u} - \hat{s} - \hat{u} - \hat{a}} =$$

$$-\frac{K\hat{/}L}{-\frac{\dot{u}}{1-u}-\hat{u}-K\hat{/}L}=\frac{K\hat{/}L}{\frac{\hat{u}}{1-u}+K\hat{/}L}.=\frac{n_1+n_2u}{\frac{\hat{u}}{1-u}+n_1+n_2u}.$$

The equilibrium marginal rate of substitution $sigma_2 = 1$.

The neo-classical growth theory represents a ratio of profit rate to a unit real wage as a function of a capital intensity. Let us take a case without technical progress for shortness. Its graphic on a plane is represented by a curve that is sloping down (Jones 1976: 34-35, 174-176). In my model, this connection is only typical for the recession phase of the long wave.

To be specific, we assume the following magnitudes for the Andronov -- Hopf bifurcation: $m_1 = 0$, $m_2 = 0$, $n_1 = -0.01$, $n_2 = 0.0106$, $r = 0.02$, $b_0 \approx -0.0582524$, $g = 0.018$, $n = 0.02$; the equilibrium vector $(s_2, u_2, v_2) \approx (3, 0.94, 0.9)$, the initial vector $(s_0, u_0, v_0) = (3, 0.94, 0.91)$. The period is about 80 years (longer than Kondratiev's).

The careful examination of computer simulations shows that the outlined neo-classical connection is observed only during the recession phase of the long wave (see Figures 2 and 3). So our model generalises the early neo-classical presentation.

An advanced neo-classical model enriches the earlier Solow model (Solow 1956) by having endogenous technical progress and non-instantaneous clearing of the labour market (Van der Ploeg 1983, Zhang 1988). It also contains a Kaldorian technical progress function and a Phillips equation. A system of three ordinary differential equations with non-linearity and implicit delays represents the compact form of this model.

In the original Goodwin model without a mechanisation function, the higher is the exogenous rate of growth of labour productivity, the lower is on the average the relative wage. In our model, the higher the equilibrium rate of productivity growth, the higher is the equilibrium share of wages in the national income and the lower is the capital-output ratio. In the advanced neo-classical model, the higher the equilibrium rate of productivity growth due to non-autonomous technical progress, the lower is the equilibrium share of wages in the national income and – for a definite range of the parameters – the higher is the capital-output ratio.

The coefficients m_2 and n_2 are mostly important for a compromise between a speedy response and stability in our model economy (hence a trade-off between a period of a cycle and stability). The equilibrium labour bill share u_2 and capital-output ratio s_2 depend upon the compound coefficient $n_2(1- m_2)$.

This very important interconnection is absent in the advanced neo-classical model of cyclical growth. It uses a hypothesis that capitalists recruit labour and scrap machinery until the marginal productivity of labour ($\partial P/\partial L$) equals the real wage (w), so the equation for the capital-output ratio becomes (in my notations for convenience of the reader):

$$\dot{s} = \xi\,[(\frac{u}{1-u_2})^{\frac{1-u_2}{u_2}} - 1]s,$$

where ξ is the exogenous speed of adjustment of the capital-output ratio to its equilibrium, ξ ≥ 0; the magnitude $(1 - m_2)$ is the equilibrium relative wage (Zhang 1988: 162-163). Parameters ξ and m_2 are postulated to be independent, although this conjecture is not justified. An analogue of ξ in my model is $m_1 - (1 - m_2)n_1$ (based on Eq. 12).

Figure 2. The responsiveness of the profit rate -- unit real wage ratio (K_hat_dev_by_w) to changes of capital intensity (K_dev_by_L). Counter clockwise

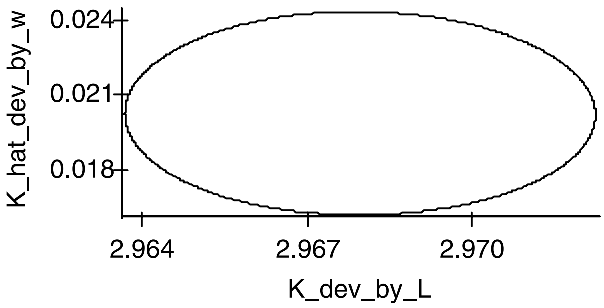
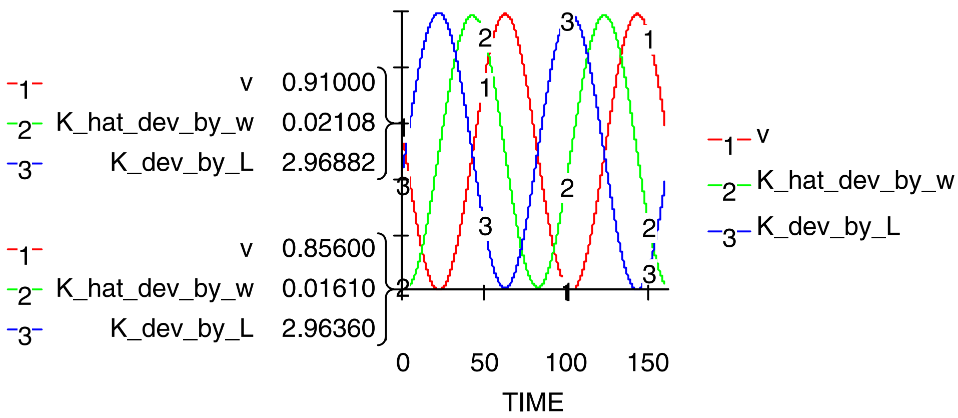


Figure 3. The phasing of the profit rate -- unit real wage ratio (K_dev_by_w), the employment ratio (v) and capital intensity (K_dev_by_L). The normalised view



Conclusion

The original model has been tested against stylised facts and undergone numerous laboratory experiments. Still a detailed statistical verification and calibration of this and more sophisticated models remains to be done.

The main model variables (the relative wage, employment and capital output ratios) have no trend. The determination of a secular trend in economic activity, i.e., a general tendency in a specific direction, is a by-product of obtaining the equations of motion for these variables. It is shown that even more lengthy long-term economic fluctuations than Kondratiev's cycles could exist on the earlier phases of emerging capitalism (before the Industrial Revolution unfolded at the end of the XVIII century in England). This approach to the model economy is free from the perspectivistic distortion in empirical studies resulting from a mechanistic (non-dialectic) separation of the trend and the long waves in statistical data (see Reijnders 1990).

The forms of feedback control, used in the model economy, are not sufficient to eliminate deviations from the steady state and tend to cause cyclical fluctuations. However, stabilisation is not the only purpose of the control. The other is to extend the scale of production. The presence of different kinds of cycles, noise and perturbations makes the control problem harder in reality. Combining algorithmic and probabilistic information is the subject for a future research of the control.

The trade-off between stability and the period, revealed above, is a significant issue for economic policy-making. If the technological advance and productivity growth remain laggard, stabilising the world economy by the accelerated capital deepening may lengthen the depression phase of the current Kondratiev cycle globally that is a destabilising factor itself. The more so since "the threat of disruption is always present in a dynamic market economy" (Economic Report of the President 1999: 20).

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