Competition in the Spanish telecommunications market: a co-evolutionary, differential game approach

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Abstract

In this paper, the problem of finding the optimal behavior of the Spanish telecommunications firms is addressed. This market is in a very special situation because is suffering a deregulation process from a monopolistic market to a competitive one with several firms. To look for the optimal behavior of the different firms a differential game is formulated. which is solved with the use of a computer tool which uses co-evolutionary algorithms. Co-evolutionary algorithms are a variant of standard genetic algorithms in which two or more genetic populations evolve interacting each other. The tool can be applied to any differential game formulated with Vensim.

1 Introduction

In this paper, the study of competition in telecommunications markets is addressed by means of differential games and co-evolutionary algorithms. Differential game theory is an extension of traditional game theory which considers problems involving several players with conflicting interests (Isaacs, 1965). While traditional games are static, differential games consider dynamical systems as the arena of the game.

This paper presents how genetic algorithms in its co-evolutionary variant can be used to solve this kind of problems when dealing with a large-scale dynamical system. Firstly, the Mistela model which simulates the Spanish telecommunications system is described briefly. Then, a computer tool for solving differential games is presented. With this tool the problem can be formulated with Vensim. Another application of the tool can be found in (Gordillo et al., 1999). Finally, the application of the tool to the Mistela model --which is written with Vensim-- is presented.

The paper is organized as follows. Section 2 describes briefly the Mistela model with special emphasis in the aspects related with the presented application. Sections 3 expounds the basis of differential games. Sect. 4 presents the application to the Mistela model. The paper ends with a section of conclusions.

2 Mistela: an Integral Simulation Model for a Telecommunications Company

In this section the Mistela model is briefly described. More detailed description of the model can be found in (Gala and Martínez, 1996; González et al., 1994; González Rodríguez and Martínez, 1995; González Rodríguez and Martínez, 1996; Martínez and Gala, 1993). Mistela model is a simulation tool used to establish the general quantitative frame for strategic planning in a telecommunications global operator. Strategic planning has been, by its own definition, a wide timing process. However, the high dynamic behavior that this sector has lately presented, has moved the horizon close to three or four years. The complexity of building such a model is settled in the need of giving figures to the different interrelations appearing between the planning items. Likewise, the election of the methodology to build a quantitative model, in this scenario, has to take into account other factors as historical data availability, work team experience in modelling, a friendly software, etc. In this case, System Dynamics methodology has been chosen due to its feasibility to model the inherent simultaneity that exists into the studied system. This methodology has been applied in so wide fields like industry, agriculture or services, because, in all of them, it is possible to find elements with dynamic qualities into its evolution.

Model structure, which appears in Fig. 1, shows a very simplified picture of the system. Different boxes define different sub-models in which the global model could be desegregated, and all of them are interrelated. This leads to the appearance of lots of feedback loops and a strong simultaneity. This picture tries to reflect the different kinds of activities developed into the company as monetary flows, plant units, investment decisions, macro-economic reality connection, etc. Furthermore, the model plays with the needed diversification degree in every sub-model, that is to say, prices desegregation, demand, incomes and profits by every kind of service, or cost-investments by networks and many more.

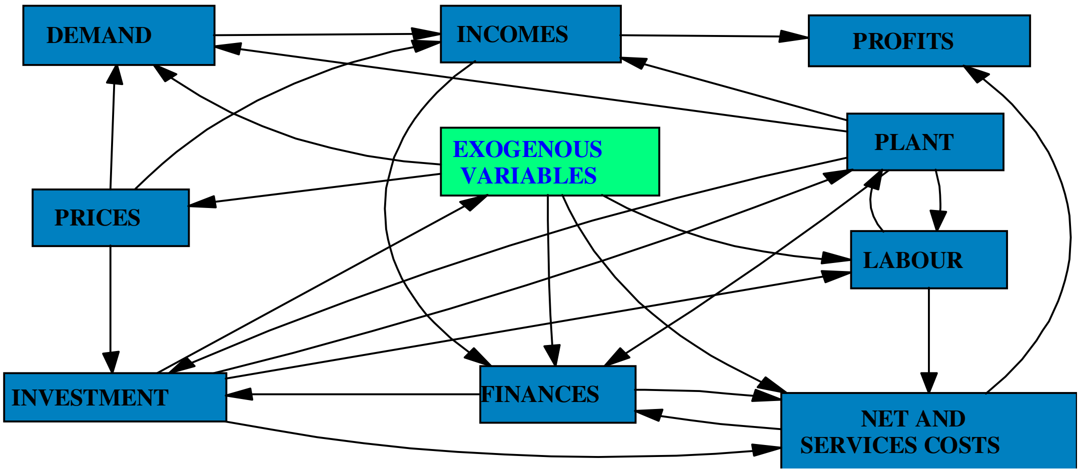

Figure 1: Model Structure

The first version of the Mistela model was created initially to provide a tool for integral planning of a telecommunications firm in a monopolistic market. In fact, only a few of the services it provided were sold under competitive conditions, and these services amounted to only a small part of the company's returns (equipment under the customers' premises and data transmission, among a few others). Voice telephony for both the fixed and mobile network were offered under monopolistic conditions. Starting in early 1996 competition for mobile telephony arises in the form of a second competitor and, since December 1998 also the fixed telephony's sector includes competition. The arrival of rivals forces to include in the Mistela model some procedure to endogeneize market shares for each of the services provided by the incumbent company. Assuming that market shares depend on price differentials among the competing firms, it becomes necessary to establish the strategies of the entering firms, particularly with respect to prices.

Two different approaches can be considered regarding the entering firm's behavior. The first one assumes that its price schedule is exogenous to our model. The second approach assumes that the entering firm will set its prices according to some procedure that links the price schedule of the competing firms. Oligopoly theory, and more specifically Bertrand's model of price competition among oligopolists as well as the model of the Kinked Demand Curve, support the latter assumptions. In fact, their key assumption is that one rival's decrease in prices is immediately matched by an identical response of all others. However, this approach is far more complex and, in some cases, fails to provide a unique answer. The interdependence of the competing firms' behavior is included in the more general planning model through to the following equations:

$$\text{Market\_Demand} = f(\text{Market\_Price})$$
$$\text{Incumbent\_Market\_Quota} = g(\text{Incumbent\_Price}, \text{Entering\_Price})$$
$$\text{Market\_Price} = h(\text{Incumbent\_Price}, \text{Entering\_Price}, \text{quotas})$$
$$\text{Incumbent\_Income} = \text{Demand} \times \text{Incumbent\_Quota} \times \text{Incumbent\_price}$$
$$\text{Entering\_Income} = \text{Demand} \times (1 - \text{Incumbent\_Quota}) \times \text{Entering\_Price}$$

Traditional models of oligopoly theory assume that the firms' goal is to maximize profits accumulated over time. However, it is reasonable to assume that in the short run, both the entering and the incumbent firms might respond to a different set of goals after the opening of the market. For example, it is conceivable that the entering firm's goal is to maximize its returns (or market share) rather than profits, and that the incumbent's goal is to reduce costs. Furthermore, price schedules are usually not set freely. It seems more likely to assume that they should meet some restrictions such as:

$$\frac{d(\text{Price})}{dt} \leq 0 \quad \forall t \tag{1}$$

$$\text{Entering\_Price}(t) \leq \text{Incumbent\_Price}(t) \quad \forall t \tag{2}$$

Thus, it is important that these restrictions are met by the procedure implemented for finding the optimum price schedules.

This theoretical problem about how different competitors can establish its own price decisions, and at the same time interrelated decisions, had to be solved into the Mistela model. This model is continuously being updated, but even the tools used until that moment could not solve this optimization problem. That is why the next step was to implement a new algorithm which could work with all the model information and, with a bigger power of calculation, could offer the optimal evolution of both, incumbent and competitor, prices for every service in competition. This new approach consists in formulating a differential game which represents the competitive environment. Due to the large dimension of the Mistela model, analytical or, even, numerical techniques are not suitable for solving this problem. Thus, co-evolutionary algorithms has been used as it is shown in the next sections.

3 Differential Games

A differential game with two players can be formulated as follows. Given a dynamical system

$$\dot{x} = f(x, u_1(t), u_2(t), t), \qquad x(t_0) = x_0 \tag{3}$$

where $x \in \Re^n$, $u_1, u_2 \in R$ and $\dot{x}$ means derivative of x with respect time. The signals $u_1(t)$ and $u_2(t)$ can be considered as the control signals (the actions) which two players can perform to modify the system behavior.

Consider also two performance criteria

$$J_1 = S_1(x(t_f), t_f) + \int_{t_0}^{t_f} L_1(x, u, v, t)\, dt \qquad (4)$$

$$J_2 = S_2(x(t_f), t_f) + \int_{t_0}^{t_f} L_2(x, u, v, t)\, dt \qquad (5)$$

The objective of player 1 is to find $u_1(t)$ such that J_1 is minimum. Likewise, player 2 must select $u_2(t)$ such that J_2 is minimum.

In fact, other kind of criteria can be considered but this formulation is the usual for analytically solvable problems. Likewise, differential games with more than two players can also be formulated.

The solution of the problem is known in some cases. In particular, when the problem is zero-sum, that is, J_1=-J_2 the ordinary differential equations which define the solution are well known (Isaacs, 1965). Nevertheless, the class of problems considered in this paper need not to fulfill this condition. Furthermore, when the system equation is other than some very simple examples, the resolution of the differential equations is not a trivial task.

Differential games have application in many diverse fields, such as missile guidance-avoidance (Isaacs, 1965), arms-race problems (Moriarti, 1984) and control theory (Vincent and Grantham, 1997).

4 Application to the Mistela model

This section presents an example of the application of the tool described in (Gordillo et al., 1999) the Spanish telecommunications market using the Mistela model. The tool solves differential games by means of co-evolutionary algorithms. Co-evolutionary algorithms are a variant of standard genetic algorithms in which two or more genetic populations evolve interacting each other. In other words, each population tries to optimize its own fitness function but this fitness function depends on the state of the rest of the populations. The tool can be applied to any differential game formulated with Vensim.

An example of using co-evolutionary algorithm with Mistela model leads to an intensive study of inputs and outputs. The link between the algorithm and the model needs consistency about what the model offers, in terms of endogenous variables used by the algorithm to be optimised, and what would the model obtain as a result of the optimisation, like trends of pricing for the incumbent and the competitors.

This example shows how to use co-evolutionary algorithms to solve the problem of pricing into the integral model of telecommunications. The picture that the model will bee able to show about the future of the market, depends on how the incumbent and the competitors would establish their own pricing policy. Inside Mistela model, due to its length, all competitors are seen as just one, which behaviour is the average competitor.

In this exercise both, incumbent and competitor, try to maximise their benefit into the horizon of the year 2001. Restrictions about the prices are just that they have to belong to the domain established as coherent. The algorithm runs with the model showing the following results.

- Year 1999:
  - The incumbent has to establish lower prices for the first year (1999), discount policy
  - The competitor follows the discount policy setting its price below the incumbent
- Year 2000:
  - Same policy for the incumbent
  - Same policy for competitor
- Year 2001:
  - The incumbent throws its prices 50% down
  - The competitor has to follow

The results can be appreciated in Fig. 2. Once the algorithm has worked and the model use the output, the benefit for the competitor is better for the first two years, but sink last year. These lowest prices could throw competitors out of the market.

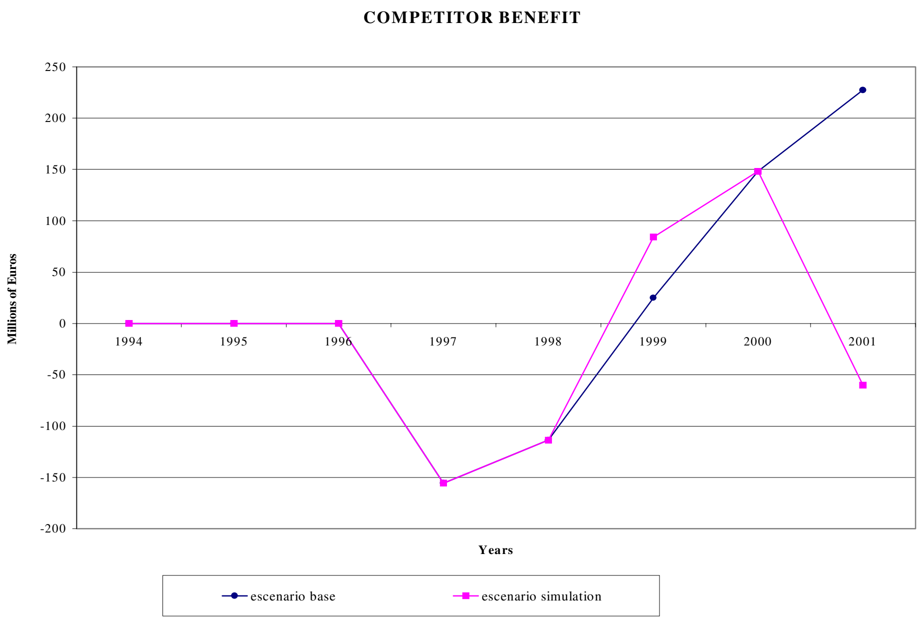

Figure 2: Competitor Benefit

5 Conclusions

Competition in telecommunications markets have been formulated as a differential game. A method based in co-evolutionary algorithms has been proposed to solve the differential game. The method has been applied to the Spanish case using the large-scale model Mistela. This is an example of using an academic approach to a real problem: the optimal price policy in a competitive telecommunications market.

Acknowledgments

This work has been supported by the Spanish Ministry of Education and Culture under grant CICYT TAP 97-0553 and by the "Departamento de Métodos Cuantitativos" of Telefónica, S.A.

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