A Case of Interaction between Systems Dynamics and Linear Programming: The Rapim-Pirenaica Model[1]

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Abstract

This objective of this paper is to reproduce the functioning of a farm which mixes agriculture with livestock-raising. Furthermore, it attempts to integrate, within the framework of a dynamic simulation model, the resolution of a problem of linear programming that yields the optimum mix of crops and livestock that satisfies the given set of restrictions. The aim of this exercise is to maximize the total value of the output of a farm that produces 16 different economic goods. This exercise considers 10 constraints including, among others, the following: the initial herd size for each type of animal; the amount of land devoted respectively to cultivation, forest, and pasture; the average returns of each crop; the restrictions on crop rotation; and the available amount of familial and extra-familial labor. The physical returns of the crops, the sale price of each crop and each type of animal, and wages, are exogenous data. Since this model is written in the program Vensim, the above-mentioned optimization model will be dealt with utilizing Vensim's external functions, calling for a function C of lineal programming problems.

The objective of the Rapim-Pirenaica model is twofold. First, it tries to reproduce the behavior and relations of the different variables that intervene in the functioning of a farm that produces both crops and livestock. Second, it aims to specify the optimum combination of crops and livestock (i.e. the combination that yields the highest return) compatible with the socioeconomic characteristics and limitations of the farm under study. The construction of such a model brings together systems dynamics and linear programming techniques. It moreover requires information regarding the following:

1. The physical characteristics of the farm, that is, the amount of land used for crop production, land that is not used for crop production, initial size of the livestock herd, and of the labor force (which presumably could include both family and non-family labor);

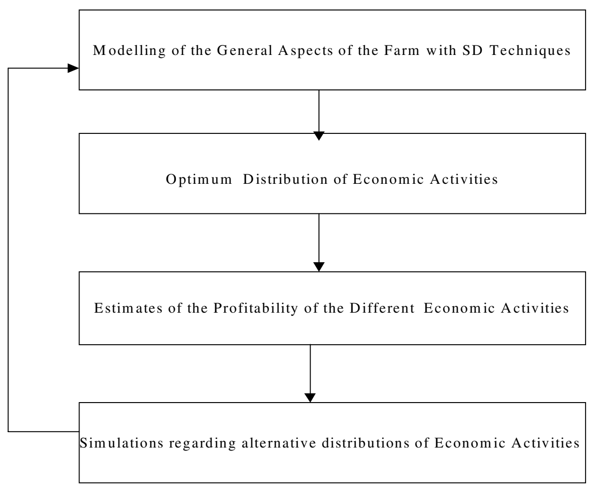
2. Initial prices of the different types of economic activities and their evolution over time; and
3. Average productivities of the various economic activities, i.e. Kg/Ha. of the different types of crops, liters of milk/head per annum, and so on.

The economic activities included are 16 (X1 to X16) and include the following:

Activity			**Type**	**Productivity**	**Units**	**Labor Req.**		**Price**		**Grow**
								Initial		**Rate**
X1	=	Potatoes	Crop	17200	Kg./Ha	50	Ha	27,4	ptas./kg	1,01
X2	=	Wheat	Crop	1800	Kg./Ha	4	Ha	21,9	ptas./kg	1,025
X3	=	Oats	Crop	1800	Kg./Ha	4	Ha	19,6	ptas./kg	1,02
X4	=	Straw	Subp.wheat and oats	1800 (x2+x3)	Kg./Ha	0,0001	Kg	4,9	ptas./kg	1,03
X5	=	Forrage crops	Crop	27000	Kg./Ha	6	Ha	7,1	ptas./kg	1,01
X6	=	Artificial meadow	Meadow	4900	Kg./Ha	2	Ha	7,1	ptas./kg	1,01
X7	=	Natural meadow	Meadow	3600	Kg./Ha	2	Ha	8	ptas./kg	1,01
X8	=	Other crops	Crop	3300	Kg./Ha	4	Ha	10	ptas./kg	1,01
X9	=	Fallow	Crop	800	Kg./Ha	2	Ha	1	ptas./kg	1,05
X10	=	Forrested land	Forrest	1600	Kg./Ha	1,1	Ha	1,2	ptas./kg	1,01
X11	=	Lambs	Livestock	1	Head	8	Head	5000	ptas/head	1,02
X12	=	Calves	Livestock	1	Head	15	Head	75000	ptas/head	1,02
X13	=	Sheep	Livestock	1	Head	7	Head	3000	ptas/head	1,01
X14	=	Cattle	Livestock	1	Head	9	Head	98000	ptas/head	1,01
X15	=	Milk	Subproduct cattle	27* X14	Liters	0,001	Liters	30	ptas/litro	1,02
X16	=	Manure	Subproduct livestock	150*X13+300*X14	Kgs.	0,0005	Kgs.	5,6	ptas./kg.	1,01

The methodological scheme of the Rapim-Pirenaica model is illustrated by the diagram below:

Methodological Scheme of the Rapim-Pirenaica Model



In his attempt to maximize the economic returns of the farm under study, the farmer runs into a number of restrictions, which are the following:

C1: The production of potatoes, wheat, oats, forage and other crops, and fallow allows some crop rotation. Thus, the sum of land under tillage every year can be up to 1.5 times the amount of land available at any one time for production, thanks to crop rotation.
C2: The sum of prairies, both natural and artificial, and forested areas should be equal to the amount of land not used for crop production.
C3: The requirements of labor needed to produce crops and livestock should not exceed the amount of labor available at any given moment of time.
C4: The amount of straw that can be sold should be equal or less than the average productivity multiplied by the number of hectares in wheat and oat.
C5: The amount of manure that can be sold should be equal or less than the average productivity per cow and sheep multiplied by the number of cows and sheep.
C6: The number of lambs that can be sold should be equal or less than the growth rate of lambs after substracting the replacement rate.
C7: The number of calves that can be sold cannot exceed the growth rate after substracting the replacement rate.
C8: The number of cows to be sold cannot exceed the rate of growth.
C9: The number of sheep to be sold cannot exceed the rate of growth.
C10: Total amount of milk to be sold should be equal or less than the average productivity per cow multiplied by the number of cows.

This attempt to maximize economic returns subject to linear constraints can be formulated as a linear programming problem that can be solved by means of the well-known Simplex method (Dantzig, 1963). In the present case, some implementation difficulties appear due to the fact that the problem is defined inside a dynamic model. These difficulties can be overcome using the external function capability of some System Dynamics languages. The Rapim-Pirenaica model is written in Vensim which allows the use of external functions. A new external function which implements the Simplex method based on the version of (Press et al., 1992) has been programmed. In this way a new function is added to the list of Vensim functions: the Simplex function that can be called from any model to solve a linear programming problem in each time step . Notice that this function is different from the Vensim MARKETP function and that the problem can not be formulated by means of the optimization capabilities of Vensim which cover the whole simulation.

The input data of the Simplex function is a matrix in which the first row contains the coefficients of the objective function (in the present case, the sum of the profitability of each economic activity $R_i = \text{Productivity}_i * \text{Price}_i$) and the rest of rows contain the coefficients of the constraints (C1-C10). The function returns a vector with the optimal values of the unknowns (crops and stock., in the current problem).

The constraint block of the input matrix takes the following form:

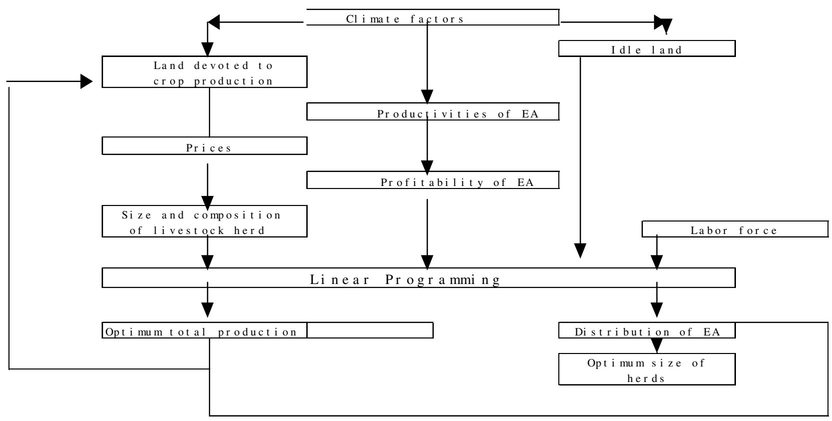
		R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16
C1	>=	1	1	1	0	1	0	0	1	1	0	0	0	0	0	0	0
C2	>=	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0
C3	>=	50	4	4	0,0001	6	2	2	4	2	1,1	8	15	7	9	0,001	0,0005
C4	>=	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
C5	>=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
C6	>=	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
C7	>=	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
C8	>=	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
C9	>=	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
C10	>=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
C11	>=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Where:

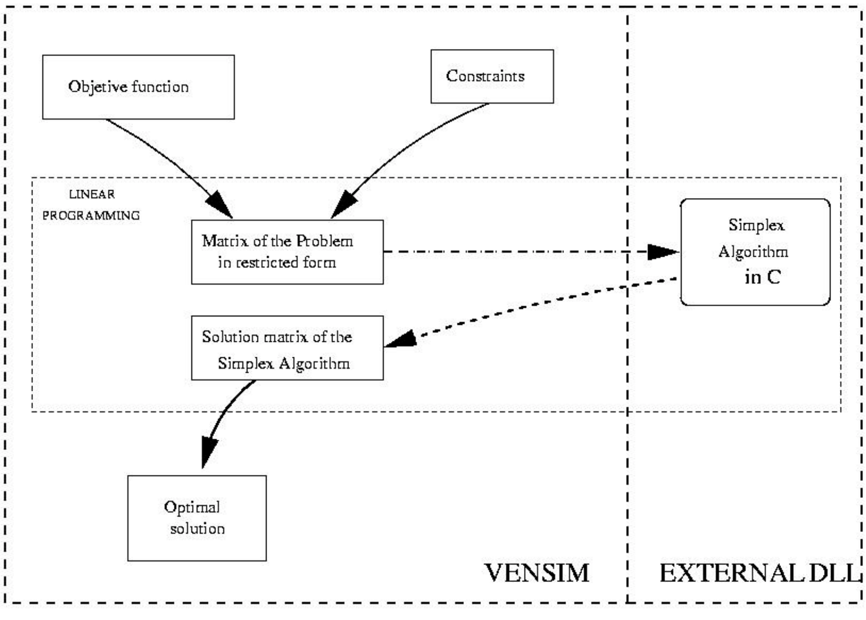
C1 = 1,5* extension of land for crop cultivation
C2 = Extensión of non-cultivated or idle land
C3 = Available labor force
C4 = 1800*(wheat+oats)
C5 = 150*Sheep+ 300*Cattle
C6 = 1.2*Sheep
C7 = 0,85*Cattle
C8 = 0,2*Sheep
C9 = 0,2*Cattle
C10 = 27*Cattle

Given the large number and complexity of the constraints considered, we do not believe that the optimization problem could be adequately written directly into the SD model. However, the linkage between the model and a linear programming library identifies the distribution of crops and livestock herds that provides the maximum return and introduces the optimum combination of economic activities into the model to provide information on the evolution of the other variables considered. Furthermore, this linkage guarantees automatic recalculation of the optimum distribution of economic activities when simulating changes in the restrictions, size of labor force, sets of prices, etc. The causal diagram below illustrates the steps to be followed in the construction of such a combined model.

Causal Diagram of the main relations of the Rapim Pirenaica Model



While the linkage between both instruments can be represented as:



For the purpose of this paper two different simulations have been run. The first one considers the set of constraints, the productivities, and the prices of each activity as illustrated above. The second simulation utilizes a different set of growth rates of some prices. More specifically, the base and alternative (base 1) scenarios are the following:

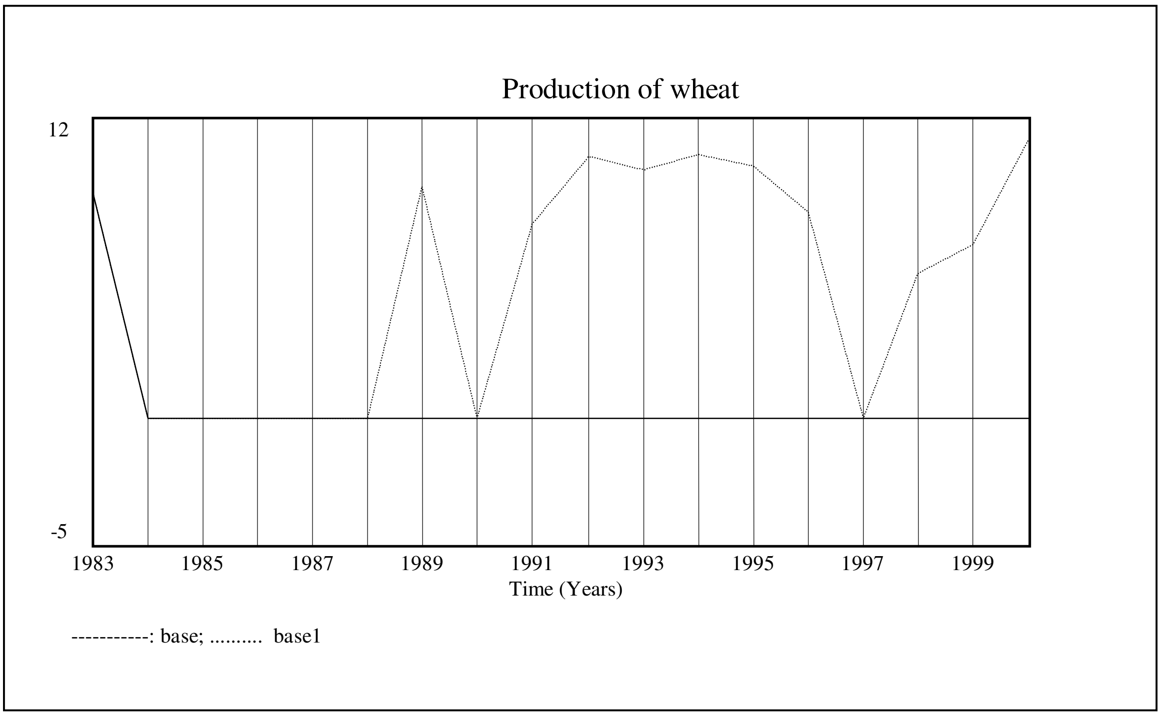
	Base Scenario	Alternative Scenario
X1	1,01	1
X2	1,025	1,16
X3	1,02	1,16
X4	1,03	1,16
X5	1,01	1,01
X6	1,01	1,01
X7	1,01	1,01
X8	1,01	1,01
X9	1,05	1,05
X10	1,01	1,01
X11	1,02	1,02
X12	1,02	1,02
X13	1,01	1,01
X14	1,01	1,01
X15	1,02	1,02
X16	1,01	1,01

Thanks to the linkages between the LP library and the SD model, a new set of results arises in just a few instances. The chart and diagrams that follow show the optimum combination of economic activities under both sets of assumptions. As can easily be seen, an increase in the rate of growth of the price of wheat, oats, and straw coupled with a reduction in the rate of growth of the price of potatoes, yields a rearrangement of the optimum production of the economic activities. The outcome in this case will be fewer potatoes and other crops, and more wheat, oats, and their subproduct, straw. Obviously, it is not the specificic results of this case study that matter, but the fact that the linkage proves to be the best way to integrate the results of a complex optimization problem into a System Dynamics model.

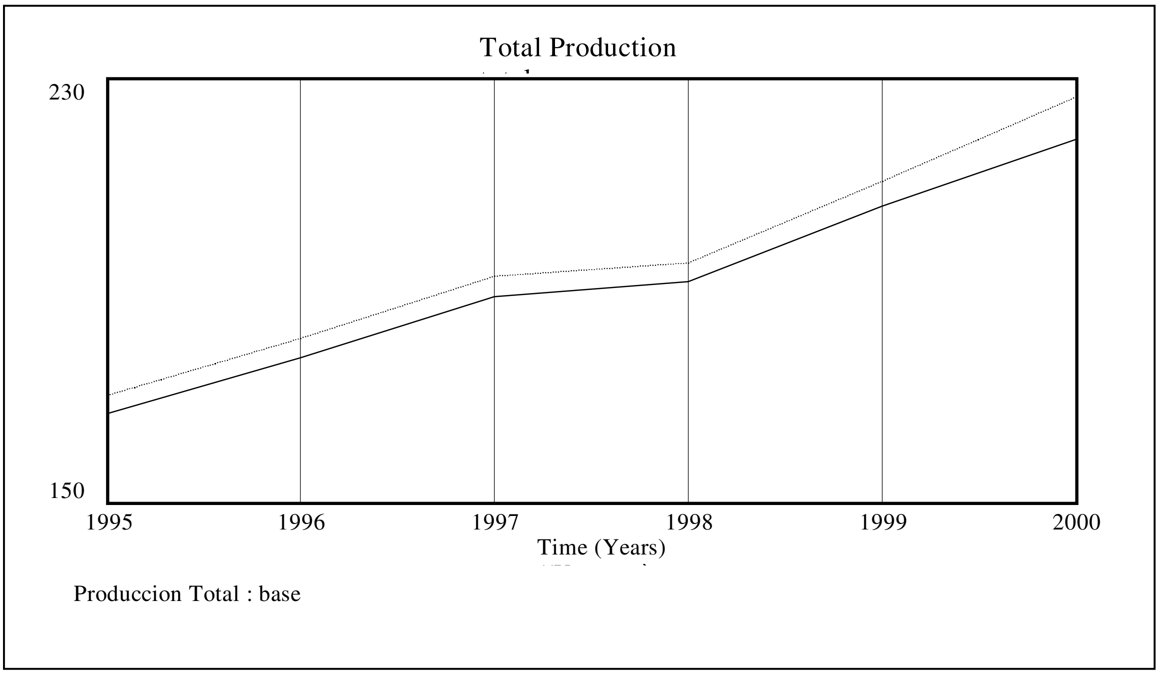
The table below summarizes the optimum output for each economic activity for both the base and alternative simulations. The diagrams that follow illustrate the evolution over time of the value of total output as well as that of the production of wheat, the crop more acutely affected by the price changes.

Year	Tot.Produc. (Mill ptas.)		Potatoes (Has.)		Wheat (Has.)		Oats (Has.)		Straw (Kg.)		Forrage (Has.)		Art. Meadow (Has.)		Nat. Meadow (Has.)		Other crops (Has.)	
1983	67.9	67.9	3.7	3.7	9.0	9.0	0.0	0.0	11.0	11.0	1.9	1.9	4.0	4.0	5.0	5.0	3.7	3.7
1984	38.8	38.9	3.7	3.7	0.0	0.0	0.0	0.0	16.7	16.7	1.9	1.9	12.7	12.7	0.0	0.0	21.4	21.4
1985	59.6	59.6	3.6	3.6	0.0	0.0	0.0	0.0	0.0	0.0	1.8	1.8	10.5	10.5	0.0	0.0	20.9	20.9
1986	63.8	63.7	4.9	4.9	0.0	0.0	0.0	0.0	0.0	0.0	2.4	2.4	11.6	11.6	0.0	0.0	28.8	28.8
1987	84.1	84.1	3.2	3.2	0.0	0.0	0.0	0.0	0.0	0.0	1.6	1.6	11.0	11.0	0.0	0.0	18.3	18.3
1988	116.4	116.3	3.4	3.4	0.0	0.0	0.0	0.0	0.0	0.0	1.7	1.7	10.3	10.3	0.0	0.0	19.4	19.4
1989	124.4	124.6	3.7	3.7	0.0	9.3	0.0	0.0	0.0	0.0	1.9	1.9	9.4	9.4	0.0	0.0	21.8	12.6
1990	141.7	142.7	4.7	4.7	0.0	0.0	0.0	11.6	0.0	17.5	2.3	2.3	12.8	12.8	0.0	0.0	27.2	15.6
1991	144.2	145.6	3.1	3.1	0.0	7.7	0.0	0.0	0.0	22.9	1.5	1.5	11.2	11.2	0.0	0.0	17.5	9.8
1992	162.2	163.9	4.2	4.2	0.0	10.5	0.0	0.0	0.0	14.7	2.1	2.1	9.5	9.5	0.0	0.0	24.9	14.4
1993	172.6	174.8	4.0	4.0	0.0	9.9	0.0	0.0	0.0	21.2	2.0	2.0	13.3	13.3	0.0	0.0	22.7	12.8
1994	156.8	159.5	4.2	4.2	0.0	10.5	0.0	0.0	0.0	17.8	2.1	2.1	11.1	11.1	0.0	0.0	24.6	14.1
1995	166.6	170.1	4.0	4.0	0.0	10.0	0.0	0.0	0.0	19.3	2.0	2.0	11.1	11.1	0.0	0.0	23.5	13.5
1996	177.2	180.8	3.3	3.3	0.0	8.2	0.0	0.0	0.0	19.4	1.6	1.6	11.5	11.5	0.0	0.0	18.8	10.5
1997	188.8	192.4	3.1	3.1	0.0	0.0	0.0	7.8	0.0	15.9	1.6	1.6	10.4	10.4	0.0	0.0	17.9	10.1
1998	191.5	195.2	2.3	2.3	0.0	5.8	0.0	0.0	0.0	15.0	1.2	1.2	13.0	13.0	0.0	0.0	12.2	6.4
1999	205.9	210.4	2.8	2.8	0.0	6.9	0.0	0.0	0.0	10.6	1.4	1.4	12.5	12.5	0.0	0.0	15.2	8.3
2000	218.5	226.4	4.5	4.5	0.0	11.2	0.0	0.0	0.0	12.6	2.2	2.2	12.1	12.1	0.0	0.0	26.2	15.0

Year	Fallow (Has.)		Forrest (Has.)		Lambs (Heads)		Calves (Heads)		Sheep (Heads)		Cattle (Heads)		Milk (Liters)		Manure (1000Kg)	
1983	4.7	4.7	3.0	3.0	0.0	0.0	6.0	6.0	0.0	0.0	1.0	1.0	23.0	23.0	6148	6148
1984	0.0	0.0	0.0	0.0	27.8	27.8	0.0	0.0	0.0	0.0	1.6	1.6	21.6	21.6	3150	3150
1985	0.0	0.0	0.0	0.0	10.2	10.2	0.0	0.0	0.0	0.0	3.3	3.3	43.9	43.9	4875	4875
1986	0.0	0.0	0.0	0.0	25.4	25.4	0.0	0.0	0.0	0.0	1.3	1.3	17.1	17.1	5398	5398
1987	0.0	0.0	0.0	0.0	24.8	24.8	0.0	0.0	0.0	0.0	2.5	2.5	34.2	34.2	7237	7237
1988	0.0	0.0	0.0	0.0	25.6	25.6	0.0	0.0	0.0	0.0	2.6	2.6	35.6	35.6	10237	10237
1989	0.0	0.0	0.0	0.0	13.3	13.3	0.0	0.0	0.0	0.0	1.6	1.6	21.5	21.5	10946	10946
1990	0.0	0.0	0.0	0.0	25.3	25.3	0.0	0.0	0.0	0.0	3.2	3.2	43.2	43.2	12063	12063
1991	0.0	0.0	0.0	0.0	20.0	20.0	0.0	0.0	0.0	0.0	0.7	0.7	9.4	9.4	12603	12603
1992	0.0	0.0	0.0	0.0	29.8	29.8	0.0	0.0	0.0	0.0	2.4	2.4	32.7	32.7	13707	13707
1993	0.0	0.0	0.0	0.0	7.4	7.4	0.0	0.0	0.0	0.0	2.0	2.0	26.8	26.8	14481	14481
1994	0.0	0.0	0.0	0.0	17.8	17.8	0.0	0.0	0.0	0.0	1.7	1.7	22.4	22.4	13091	13091
1995	0.0	0.0	0.0	0.0	27.9	27.9	0.0	0.0	0.0	0.0	2.3	2.3	31.2	31.2	13817	13817
1996	0.0	0.0	0.0	0.0	9.0	9.0	0.0	0.0	0.0	0.0	1.2	1.2	15.9	15.9	14625	14625
1997	0.0	0.0	0.0	0.0	20.3	20.3	0.0	0.0	0.0	0.0	2.8	2.8	38.0	38.0	15286	15286
1998	0.0	0.0	0.0	0.0	12.3	12.3	0.0	0.0	0.0	0.0	0.5	0.5	6.7	6.7	15941	15941
1999	0.0	0.0	0.0	0.0	31.2	31.2	0.0	0.0	0.0	0.0	2.2	2.2	30.2	30.2	16607	16607
2000	0.0	0.0	0.0	0.0	20.3	20.3	0.0	0.0	0.0	0.0	1.5	1.5	20.8	20.8	17435	17435



Production of wheat
12
-5
1983
1985
1987
1989
1991
1993
1995
1997
1999
Time (Years)
-----------: base; base1



Total Production
230
150
1995
1996
1997
1998
1999
2000
Time (Years)
Produccion Total : base

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