The Puzzle of Unemployment: Retrospecting Kaldor, Lipsey and Phillips on Wage, Employment and Profitability

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Abstract
As the number of jobless in the European Union (EU) has mounted, the classical controversy gains momentum again. Phillips (1958) argued, the higher the growth rate of employment, the higher is the rate of change of money wages, other things being equal. Challenging Phillips' view, Lipsey (1960) has claimed that times of falling unemployment were associated with lower growth rates of money wage than times of rising unemployment. Neither Phillips nor Lipsey included a profit rate as a factor of the wage rate in their models. Kaldor (1959) advised to relate wage increases to the increase in profitability. Computer simulations of the growth cycle and profit-wage spiral demonstrate that assertions of these renown scientists may be only partially true since they did not treat explicitly the wave-like pattern of economic dynamics. It is shown how highlighted relationships are contingent upon a particular phase of the long wave. This paper uncovers elements of proportional and derivative control inherent in the model economy. It stresses a probable positive impact of a big revival of investment on economic development and job creation.

The Basic Model of a Closed Capitalist Economy

The simplified Goodwill-like model of fluctuating economic growth has been extended to allow for the effects of composition of capital upon real wage, thus augmenting the key relationship – the real Phillips curve (see Ryzhenkov 1993, 1995). This modification also takes into consideration the historical and moral element in the value of labour power. It may be helpful for explaining the down-ward rigidity of the real wage. Typically, an increase in the labour income share enhances the capital-output ratio which, in turn, adversely affects the employment ratio. This model is abstracting from military expenditures, export and import.
The model is formulated in continuous time. Time derivatives are denoted by a dot, while growth rates are indicated by a hat. The simplified version of the model consists of the following equations:

$$P = K/s \tag{1}$$

$$a = P/L \tag{2}$$

$$u = w/a \tag{3}$$

$$\hat{a} = m_1 + m_2(\hat{K/L}),\ m_1 \geq 0,\ 1 > m_2 > 0 \tag{4}$$

$$(\hat{K/L}) = n_1 + n_2 u, \qquad n_1 \geq 0,\ n_2 \geq 0 \tag{5}$$

$$v = L/N \tag{6}$$

$$N = N_0 e^{nt}, \quad n = const \geq 0, \; N_0 > 0 \tag{7}$$

$$\hat{w} = -g_1 + rv, \quad g_1 \geq 0, \qquad r > 0 \tag{8}$$

$$M = (1 - w/a)P = (1-u)P \tag{9}$$

$$\dot{K} = (1 - u)P \; or \; P = wL + \dot{K}. \tag{10}$$

The equation (1) postulates a technical relation between the produced capital stock (K) and net output (P). The variable s is called capital-output ratio. The equation (2) relates labour productivity (a), net output (P) and labour input or employment (L). The equation (3) describes the share of labour in national income (u). The equations (4) and (5) represent a Kaldorian technical progress function and the Glombowski – Krüger mechanisation function, respectively. The equation (6) outlines the rate of employment (v) as a result of the buying and selling of labour-power. The equation (7) postulates an exponential growth of the labour force. Employees do not save at all. A private consumption of employers is not taken into account. The equations (9) and (10) show that profit (M), savings, investment and incremental capital ($\dot{K}$) are equal in this model for simplicity. The variable P designates a net domestic product.

A. Phillips put the relationship between the employment ratio and growth rate of real wage in a mathematical form. Equation (8) represents the linear approximation of the real Phillips curve. In this equation, g_1 and r are the intercept and slope, respectively. g_1 reflects the tendency of capitalist production to push the value of labour power more or less to its minimum level, r represents working men's bargaining power. A rising rate of employment is assumed to affect wage increases (in real terms). There is no money illusion.

The technical qualification of the workforce depends upon the availability of fixed assets (first of all equipment) on which to learn and gain experience. The higher the qualification, the higher is the capital intensity, and vice versa. In my opinion, the capital intensity may be used as the indicator of qualification in dynamics as well. The rate of change of the real wage rate (w) in the modified Phillips equation (8) depends only on the employment ratio (v) and on the rate of change of capital intensity (K/L).

The innovations should, of course, materialise in new investments. It first requires investment in capital goods industries, a process that can feed on itself for decades. "The process of rebuilding necessarily causes the economy to overshoot the long-run needs of replacement of depreciated assets and long-run growth," Sterman argues (Sterman: 7).

The long term dynamics of capital goods industries, excess capacities and self-ordering of durable assets are not treated explicitly in my model of the long wave. This limitation is done to emphasise endogenous technological progress.

Interconnections between the labour bill share in a national income (u), employment ratio (v) and capital-output ratio (s) are represented by a three-dimensional competitive–co-operative system of ordinary differential equations (11) – (13):

$$\dot{s} = -(m_1 + (m_2 - 1)(n_1 + n_2 u))s \quad (11)$$

$$\dot{v} = ((1 - u)/s - (n_1 + n_2 u) - n)v \quad (12)$$

$$\dot{u} = (-g + rv - m_1 + (b - m_2)(n_1 + n_2 u))u. \quad (13)$$

Different scenarios of evolution are possible depending on an initial state and values of the parameters. There is a good agreement between the properties of the steady-state growth in our model and the Kaldor five stylised facts: output per worker shows continuing growth with no tendency for a falling rate of growth of productivity; capital per worker is rising more or less in proportion to productivity; the rate of return on capital is steady; the capital-output ratio is steady; labour and capital receive constant shares of total income.

The necessary and sufficient conditions for a local stability in the neighbourhood of a nontrivial equilibrium (steady state) have been found. The formulas derived show the constancy of relative wage, employment ratio, profit rate and capital-output ratio at the steady state. The relative wage, employment ratio, profit rate are the higher and the capital-output ratio is the lower, the higher is the rate of technological progress $m_1/(1-m_2)$.

The interaction of an accelerator and multiplier generates typically cyclical growth in the model economy. In particular, there exist limit cycles in a vicinity of a locally unstable equilibrium that are generated via the Andronov – Hopf bifurcation, whereby b is the bifurcation parameter. The internal structure of capital accumulation drives long-wave rhythms under a definite constellation of the control parameters and initial values of the state variables (Table 1). The Lie derivative calculated for stable limit cycles for initial points sufficiently close to the fixed point equilibrium is negative thus indicating that the dissipative property of the initial system is retained. A further increase of b over b_0 brings about diverging fluctuations. Due to non-linearity in the system, a substantial increase of b over b_0 destabilises the system.

Table 1. An Example of the Long Wave

Steady state growth rate ($\hat{a}_2$)	Real eigen-value λ_3	Conjugate pair of eigen-values λ_1 and λ_2	$\frac{d(Re\lambda_{1,2}(b))}{db}$ for $b = b_0$	Period of fluctuations in the linerised model (years)
0.04	-0.004	0± 0.098i	0.075	64.127

Notes:

1. The length of the fluctuations is about 65 years.

2. $m_1 = 0.02$, $m_2 = 0.5$, $n_1 = 0.01$, $n_2 = 0.04$, $r = 0.062$, $b_0 = 0.5 - 1/7 \cong 0.357143$, $g = 0.02$, $n = 0.02$; the equilibrium vector $E_2 = (s_2, u_2, v_2) \cong (4.1667, 0.75, 0.7373)$, the initial vector $(s_0, u_0, v_0) = (4.1667, 0.75, 0.80)$.

3. At $b = b_0$, $a_1 a_2 = a_0$, $(\lambda^2 + a_1)(\lambda + a_2) = 0$, $\lambda_{1,2} = \pm (-a_1)^{1/2} =$
$\pm i[r u_2 v_2 (1/s_2 + n_2)]^{1/2}$, $\lambda_3 = (b_0 - m_2) n_2 u_2$.

It is worth to notice that the evolution of capital-output ratio is described by an equation (14)

$$\hat{s} = (\hat{K/P}) = (\hat{K/L}) - (\hat{P/L}). \qquad (14)$$

Using the equation (4), we obtain easily a linear relationship between the rate of change of capital-output ratio and the rate of change of labour productivity (15):

$$\hat{s} = (\hat{a}(1 - m_2) - m_1)/m_2, \; m_1 \geq 0, \; 1 > m_2 > 0. \qquad (15)$$

For $0 < \hat{a} < m_1/(1 - m_2)$, a rise in labour productivity is associated with an increase of capital productivity; for $\hat{a} > m_1/(1 - m_2)$, a rise in labour productivity is associated with a fall in capital productivity. Here $m_1/(1 - m_2)$ is the steady-state rate of growth of labour productivity ($\hat{a} = m_1/(1 - m_2) \Leftrightarrow \hat{s} = 0$).

The Profit-Wage Spiral

I have simulated limit cycles in the phase space using the soft-ware *Powersim.* In agreement with the Hopf theorem, numerical calculations do depict attraction of trajectories starting in the neighbourhood of $E_2 = (s_2, u_2, v_2)$ (at $b = b_0 \cong 0.357$) by different limit cycles. The magnitudes of the other parameters are given in Table 1. Numerical experiments will help to examine the views of renown economists in this section.

Let us tern our attention to the Phillips findings on the relationship between unemployment and the rate of change of money wage rates in the United Kingdom (1861-1957). He found first that there was a clear tendency for the rate of growth of the real wage to be high when unemployment was low and to be low when unemployment was high. If we look at the scatter graph of $\hat{w}$ and v on Figure 1**,** we will observe that model agrees with the first Phillips conjunction.

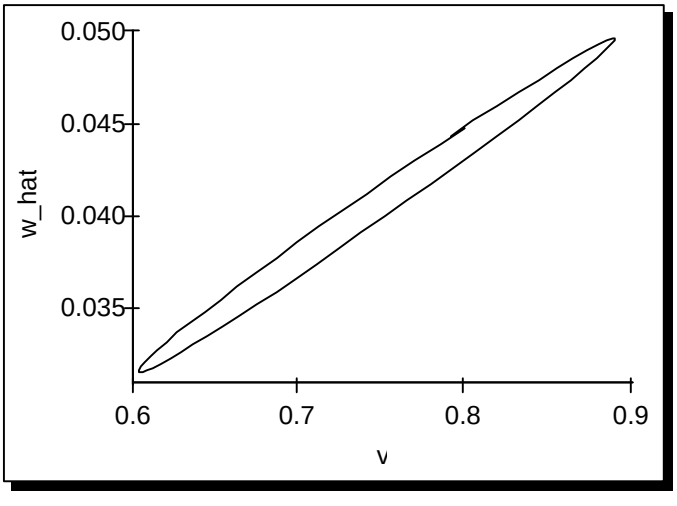

Figure 1. The Employment Ratio (v) as A Factor of the Rate

of Change of Real Wage (w_hat). Counter Clockwise Motion

The second Phillips conjunction was about the relationship between the rate of change of money wages and the rate of change of the unemployment ratio (see Phillips 1958: 290). He argued that the latter variable produces typically a negative influence on the former: the higher the growth rate of employment, the higher is the rate of change of money wages, other things being equal. He observed yet that this regularity was violated in 1953-1957.

Phillips put forward a hypothesis that this peculiarity of 1953-1957 resulted from a time lag in the real adjustment of wage rates to the unemployment ratio and to the rate of change of this ratio (see Phillips 1958: 297). Such a time lag appeared as a result of institutional changes on the labour market after the second world war. The importance of these changes has been demonstrated (Ryzhenkov 1994a, 1994b).

R. Lipsey has found out that, on the average experiences of the 1922-1957 period, times of falling unemployment were associated with lower rates of change of money wage rates than were times of rising unemployment. He challenged the Phillips point of view (see Lipsey).

Let us look at any given level of unemployment at Figure 2. There is a distinct tendency for the rate of change of real wage rate to be above the average for that level of unemployment when unemployment is growing during the downswings of the long wave and to be below the average for that level of unemployment when unemployment is decreasing during the upswings of the long wave. This result seemingly contradicts the Phillips second conjunction.

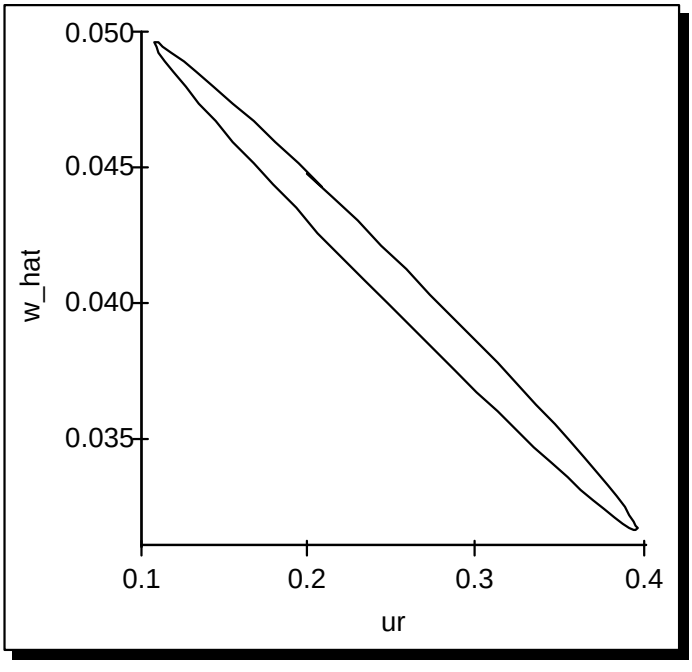

Figure 2. The Unemployment Ratio (ur) as a Factor of the Rate of Change of Real Wage (w_hat). Clockwise Motion

Figure 3 adds details to the co-movement of these variables over phases of the long wave. When the rate of change of real wage is average, the rate of change of em-

ployment is the highest or lowest. The rate of change of real wage, being greater than average, is declining along with a decrease of the employment ratio during the recession. This rate, being lower than average, is growing along with an increase of the employment ratio during the recovery of the long wave. The growth rates of real wage are generally higher during the recession when the employment ratio decreases than during the recovery when the employment ratio increases. The growth rates of real wage are generally lower during the depression when the employment ratio decreases than during the boom when the employment ratio increases.

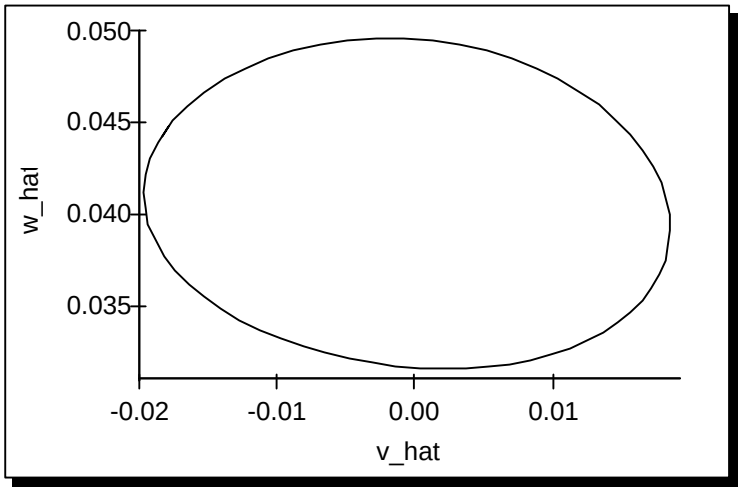

Figure 3. The Rate of Change of the Employment Ratio (v_hat) as a Factor of the Rate of Change of Real Wage (w_hat). Counter Clockwise Motion

At the limit cycles, the maximum of v corresponds to the minimum of s and vice versa, whilst $u = u_2$. A more detailed picture of the simulated long wave is as follows.

Phase 1 (recession): the labour bill share is growing from its average magnitude to its maximum value; the employment ratio is decreasing from its maximum to the average level; the capital-output ratio is moving from its minimum to the average magnitude; profitability is falling from its average level to the minimum. The greater relative over-population is the consequence of the lower rate of profit and the higher relative wage. Increasing unemployment is due to a growing mismatch between productivity growth and the reduced rate of growth in output (demand).

An industrial competition intensifies due to the fall in the rate of profit, therefore further temporary rise in the wage growth rate and a resultant temporary fall of the rate of profit. Chris Trinder, of the Public Finance Foundation in the UK, says: "To put it in a rather stylised way, there are companies where half the employees are losing their jobs and the other half are virtually doubling their pay. That is bad for employment and bad for the economy as a whole" (The Financial Times, April 13, 1993: 13). The growing capital intensity also fosters the increases in the wage rate that tend to slow capital accumulation.

The rate of accumulation lessens; but with its lessening, the primary cause of that lessening (the excess of u over u_2) vanishes during the next phase. This is the starting-point of large new investments.

Phase 2 (depression): the labour bill share is decreasing from its maximum magnitude to its average value; the employment ratio is decreasing from its average level to the minimum; the capital-output ratio is moving from its average magnitude to its maxi-

mum; profitability is rising from its minimum to the average level. The depression constitutes a structural crisis. Substituting labour by capital and destroying employment (and employees' bargaining power) help to restore profitability during the depression via the painful and conflict-ridden process that in practise leads to structural unemployment.

Phase 3 (recovery): the labour bill share is falling from its average magnitude to its minimum; the employment ratio is growing from its minimum to the average level; the capital-output ratio is falling from its maximum to the average magnitude; profitability is increasing from its average level to the maximum. The rise of real wage does not interfere with the progress of accumulation and new employment generation.

Phase 4 (boom): the labour bill share is increasing from its minimum to its average value; as output growth being greater than productivity growth, the employment ratio is growing from its average level to the maximum; the capital-output ratio is moving from its average magnitude to its minimum; profitability is falling from its maximum to the average level. Accumulation slackens in the consequence of the rise in price of labour power.

The growth rate of real wage ($\hat{w}$) is minimal at the beginning of the recovery phase, when the employment ratio is near its minimum and the rate of growth of capital intensity decreases below its average magnitude. The growth rate of real wage ($\hat{w}$) is maximal at the beginning of the recession phase, when the employment ratio is near its maximum and the rate of growth of capital intensity increases over its average magnitude. In view of technological progress, a growing employment ratio not necessarily requires a reduction in real wage in order to restore profitability.

The results of our simulations are not directly comparable with the Phillips or Lipsey findings since these prominent scientists have used data on the nominal wage. The difference between real and monetary wage does not allow a straightforward validation. Still both scientists did not take into account the possibility that the relationship between these variables is contingent on a particular phase of the long wave.

Notice that neither Phillips nor Lipsey included a profit rate as a factor of the wage rate in their models. Kaldor advised to relate wage increases to the increase in profitability. He wrote: "The *bargaining strength* of labour is...closely related to the prosperity of industry, which determines both the eagerness of labour unions to demand higher wages and the willingness and ability of employers to grant them" (Kaldor 1959: 293).

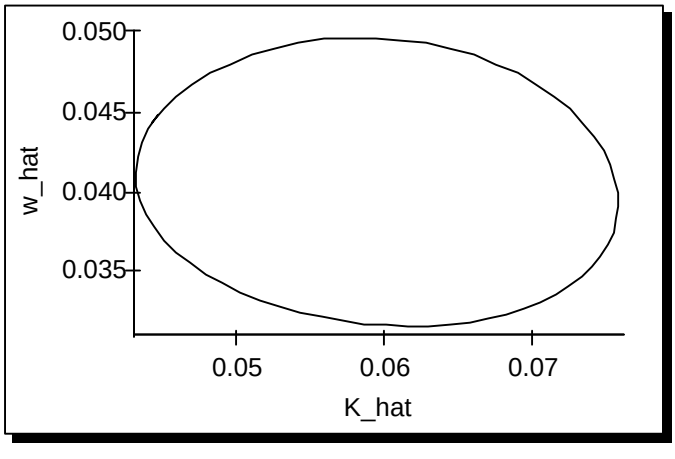

Figure 4. A Profit – Wage Spiral. Counter Clockwise Motion

Kaldor's advice in his paper of 1959 was right for the recovery of the fourth Kondratiev cycle. Presumably, Kaldor's assertion is only partially true: for a longer period the rate of change of real wage can decline simultaneously with an increase in the profit rate, and vice versa. Figure 4 displays the wave-like pattern of their change. So it is possible that Kaldor extrapolated the observed regularity beyond its proper limits. This conclusion may be tested empirically.

Types of Control

The initial Phillips equation corresponds to the simplest type of policy: $\hat{w}$ is made proportional to the excess of the employment ratio over its equilibrium, r measures the strength of this policy. I believe that the proposed modification in the modified Phillips equation (8) absorbs true aspects of the different views on factors of the rate of change of real wage. This equation can be transformed into

$$\hat{w} = -g_1 + rv + g_2 + b(\hat{K/L}) = -g + rv + b\hat{K} - b\hat{L} =$$

$$-g + rv + b(1 - u)/s - b\hat{v} - bn. \qquad (16)$$

Notice that the Phillips variables v and $\hat{v}$ are preserved. Lipsey would be satisfied with the sign with the sign of the term $b\hat{v}$ in this equation. Kaldor would appreciate the enforcing term $b(1 - u)/s$ on the right hand side of this equation.

The two enforcing terms rv and $b(1 - u)/s$ bring about the potent movements of the state variables near E_2. The system (11) - (13) loses its local stability at some critical b ($b_0 \cong 0.357$, in our example). The term ($- b\hat{v}$) is a drag on the amplitude of fluctuations.

Such a corrective action is a special case of a *derivative* control that has been added to the Phillips equation. As $\hat{v}$ is a leading indicator of the cyclical movement in relation to v, this derivative control has a somewhat similar effect to that which would be obtained by basing the corrective action on a forecast of a demand for labour power.

Let us examine what will happen after removing the term $b\hat{v}$ from the right hand side of the equation (16). Our modification affects the equation (13), whereas the equations (11) and (12) remain untouched. The modified system has the same unstable equilibrium E_2, but without a limit cycle in the vicinity. Numerical runs show that owing to very strong fluctuations the state variables leave the permissible region and the system terminates.

It is not clear whether the self-regulation in the extended model could be supported by another kind of proportional control, i.e., by policies designed gradually to raise the equilibrium level v_2 to any desired and feasible level of v, say 0.98. This question remains open especially in the context of globalisation. R. Goodwin wrote that these policies would require a tight discipline of employers and employees, their readiness to forgo raising real wages in consequences of tightness of the labour market, planning in advance of government expenditures and effective implementing of such a programme (see Goodwin 1990: 110).

In the first half of 1990s, trade unions in Germany, Great Britain and other OECD countries have generally preferred to forgo higher pay in order to retain jobs, indeed. Still a reduction of the share of GDP going to investment by one-third in the EU be-

tween mid-1970s and 1997 brought about the rise of unemployment ratio from 3 per cent in the 1960s and early 1970s to 11 per cent (see Modigliani and La Malfa).
The model explains why return to the EU level of unemployment of the 1960s is hardly possible without a recovery of investment to corresponding levels. To demonstrate this association, it is necessary to present $\hat{v}$ is the following way:

$$\hat{v} = (\hat{L/N}) = \hat{P} - \hat{a} - \hat{N} = (\hat{P/K}) + \hat{K} - \hat{a} - \hat{N} = -\hat{s} + (1 - u)/s - (\hat{a} + n) =$$

$$(1 - u)/s - \hat{s} - m_1 - m_2(\hat{K/L}) - n = (1 - u)/s - \hat{s} - m_1 - m_2(1 - u)/s + m_2(\hat{v} + n) - n =$$

$$(1 - u)/s - \hat{s}/(1- m_2) - n - m_1/(1 - m_2). \qquad (17)$$

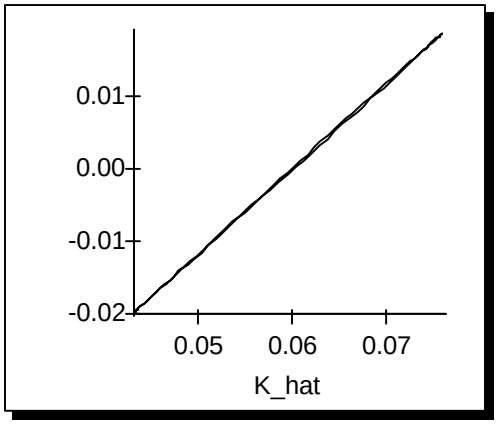

Figure 5. The Rate of Capital Accumulation (K_hat)
as a Factor of the Rate of Change of Employment Ratio (v_hat)

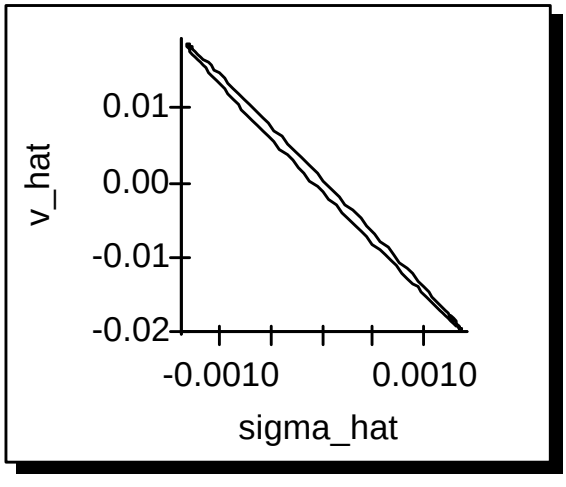

Figure 6. The Rate of Change of Capital-Output Ratio (sigma_hat)
as a Factor of the Rate of Change of Employment Ratio (v_hat)

The growth rate of the employment ratio, $\hat{v}$, depends on the growth rate of capital stock, $(1 - u)/s$, and the rate of change of capital-output ratio, $\hat{s}$. Thus, an economic policy for reducing unemployment would imply a revival of investment (Figure 5) and an improvement of capital productivity (Figure 6).

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Acknowledgement
This paper relates to a research project partially carried out at the *Industrieseminar of* Mannheim University. Professor Dr. Peter Milling and the other colleagues have facilitated this work. The A. von Humboldt Foundation has provided a financial support.