# Production Functions in THRESHOLD 21 and Their Preliminary Applications to Italy, Benin, and Cambodia

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The purpose of this paper is to summarize the work we have done on the production functions in our THRESHOLD 21 Model[*] , an integrated assessment tool for national development planning. Its current version, Version 1.0, includes an economic sector (foreign debt, national income, savings and investment, and balance of trade), production sectors (industry, services, and agriculture), a population sector, social sectors (primary education, health care, and food and nutrition), resource sectors (energy, water, forest, and agricultural land), environment sectors (air pollution and green house gas emission), the government sector, and a rest of the world sector.

This paper explains the general form of the production functions for the industry and service sectors in THRESHOLD 21. THRESHOLD 21 was applied to Italy with support from the Agenzia Nazionale per la Protezione dell'Ambiente (ANPA), of the Italian Government, and a quick exercise was done to apply THRESHOLD 21 to Benin and Cambodia. These three countries are representative of three different types: Italy is following a "normal" development path. Benin is not "normal" in the sense that during the 1970s and 1980s, its labor productivity (per worker real output) decreased. The reason, according to two Benin nationals, was the Russian-style economic system used in Benin during 1972 - 89. Cambodia's development is not "normal" due to decades of war.

As a means of model validation, THRESHOLD 21 starts from a date in history, such as 1965. The model first simulates from 1965 to the present, and the results are compared to existing and available historical data. If the simulation results fit reasonably well with historical data, then THRESHOLD 21 is ready to simulate alternative scenarios for the future, such as from the present to 2020 or 2050.

[*] The Millennium Institute's THRESHOLD 21 national sustainable development model integrate economic, social, resource, and environmental considerations. It answers questions about the future consequences of alternative policies guiding investments into alternative sectors of the economy. Versions of the model have been developed for Bangladesh, Benin, Cambodia, China, Italy, Malawi, Tunisia, and the United States. See: http://www.igc.apc.org/millennium.

## General production functions for industry and services

The general forms of production functions for industry and for services are similar, so only one of them is explained here: the production function for industry.

### *General form of the production function for industry*

Industrial production (or industrial value added GDP) measured in constant local currency is computed using the Cobb-Douglas Production Function:

$$\begin{aligned}&\text{industry produced} = A * \text{capital}^{\text{elasticity}} * \text{labor}^{(1-\text{elasticity})}\\&= A * (\text{capital}^{\text{elasticity}}/\text{labor}^{\text{elasticity}}) * \text{labor}\\&= \text{labor productivity} * \text{labor}\end{aligned} \qquad (1)$$

in which :

$$\begin{aligned}&\text{labor productivity} = A * (\text{capital}^{\text{elasticity}}/\text{labor}^{\text{elasticity}})\\&\quad = A * (\text{capital}/\text{labor})^{\text{elasticity}}\\&\quad = A * \text{capital intensity}^{\text{elasticity}}\end{aligned} \qquad (2)$$

and the variable *labor* is the number of people employed in the sector, or:

$$\text{labor} = \text{labor employed} \qquad (3)$$

*Capital intensity* is the capital per worker and is equal to total capital in industry divided by labor employed in industry, and A is the combined effect of education and training, and a constant:

$$A = \text{constant} * \text{industry education and training effect} \qquad (4)$$

The derivations of each variable is presented in the following sections.

### *Industry labor employed*

Historical data for *labor employed* in each sector can be estimated from the World Bank's World Data database. In the current version of THRESHOLD 21, labor employed is an exogenous. In future versions of THRESHOLD 21, sectoral employment should be related to total labor force (or urban labor force), per capita GDP (the higher the pc GDP, the higher the percentage employed in services, and the lower the percentage employed in agriculture), and the development strategy the country chooses (such as developing labor-intensive or capital-intensive industrial enterprises).

### *Industry capital elasticity*

The industry capital elasticity for a specific country can be estimated from the answers to three questions:

1) What is the overall capital rate of return in industry for the country?
2) What is the labor expense (salary plus fringe benefit) in relation to capital rate of return?
3) What is the marginal capital rate of return in industry for that country?

Assume the answer to question #1 is 0.1, which means for every dollar in industrial capital, the annual capital profit is $0.10

Assume the answer to question #2 is that labor expense is twice as much as the capital rate of return, which means for every dollar in capital, labor expense is $0.20. Further, assuming that the industry value added GDP is the sum of capital return plus labor expense, we will have $0.30 industry value added GDP for every $1 in capital.

Assume the answer to question #3 is 0.08, which means that for an additional $1 invested, another $0.08 will be generated as capital profit. Or in other words, an additional $1 capital to the existing $1 capital would increase value added from $0.3 to $0.38

From these answers we can estimate the elasticity of capital in industry:

ind capital elasticity = (% increase in value added)/(% increase in capital)
= ((0.38-0.30)/0.30)/((2-1)/1)
= 0.267 (5)

This elasticity value is assumed constant for the entire simulation.

### *Capital industry*

*Capital industry* is a stock variable which is accumulated through investment and reduced by depreciation, as the following equation shows:

$$\text{Capital industry}_t = \text{Capital industry}_{t-1} + (\text{investment rate industry} - \text{depreciation rate industry}) * \text{computation interval} \quad (6)$$

### *Ind education and training effect*

In the current version this variable depends only on the adult literacy rate. In different countries, education and training may have different effects on industrial production, and in future versions of THRESHOLD 21 additional inputs will be added.

To estimate this effect, we first establish a reference value of this effect, namely 1.0 at the beginning of the simulation (1965 by default). We then estimate the value of the effect for two other points. These two points are:

1) At the current adult literacy rate (known), what would be the value of the effect?
2) When adult literacy rate = 1 (i.e., all the population are literate), what would be the value of the effect?

Assume a country expert is available to answer questions. Assume further that the adult literacy rate for 1965 is 30%. Then the answer to question #1 is 1.5, and the current adult literacy rate is 60%. The answer to question #2 is 1.8. Then we will have the following table:

| | | | |
|---|---|---|---|
| X (adult literacy rate) | 0.3 | 0.6 | 1 |
| Y (edu and training effect) | 1 | 1.5 | 1.8 |

Based on this table, we can build a look-up function for the *Ind education and training* effect.

#### *Value of the constant in the production function*

From equations (2) and (4), we can obtain:

labor productivity =
constant * education and training effect *
capital intensity$^{elasticity}$ (8)

We will use the initial condition to estimate the value of the constant.

On the left-hand side of the equation, labor productivity at initial time is equal to initial industry GDP divided by initial industry employed, both of which are either available from existing data or can be estimated.

On the right-hand side, the *education and training effect* has a value of 1.0 at the beginning. Initial capital intensity is the initial capital (which is estimated in capital industry) over the initial industry employed, both of which are known.

Thus, we have:

initial labor productivity = constant * initial capital intensity$^{elasticity}$

or:

constant = initial labor productivity * initial capital intensity$^{-elasticity}$ (9)

## Application to Italy

#### *Parameters*

Labor employed is exogenous, as discussed in section II.2. Data shows that industry labor employed started at 8.7 million in 1965, then gradually fell to 8.5 million in 1975, 7.8 million in 1985, and 6.8 million in 1995. Assuming a

continuation of this trend, labor employed falls to 6.3 million in 2000, and 5.8 million in 2020.

Service labor employed showed a growing trend in the past: 7.1 million in 1965, 10.4 million in 1980, and 12 million in 1995. Assuming a slowed continuation of this trend, service labor employed will be 13.5 million in 2020.

Capital elasticity for industry and services are estimated to be 0.4 and 0.35 respectively.

Education and training effect, as discussed in Section II.5, depends on adult literacy rate, which has been high in Italy, and has not changed much since 1965, so this effect has very small influence on production.

***Historical comparison of industry GDP and service GDP***

The following two graphs show the historical comparisons between THRESHOLD 21 model output and data from the World Bank.

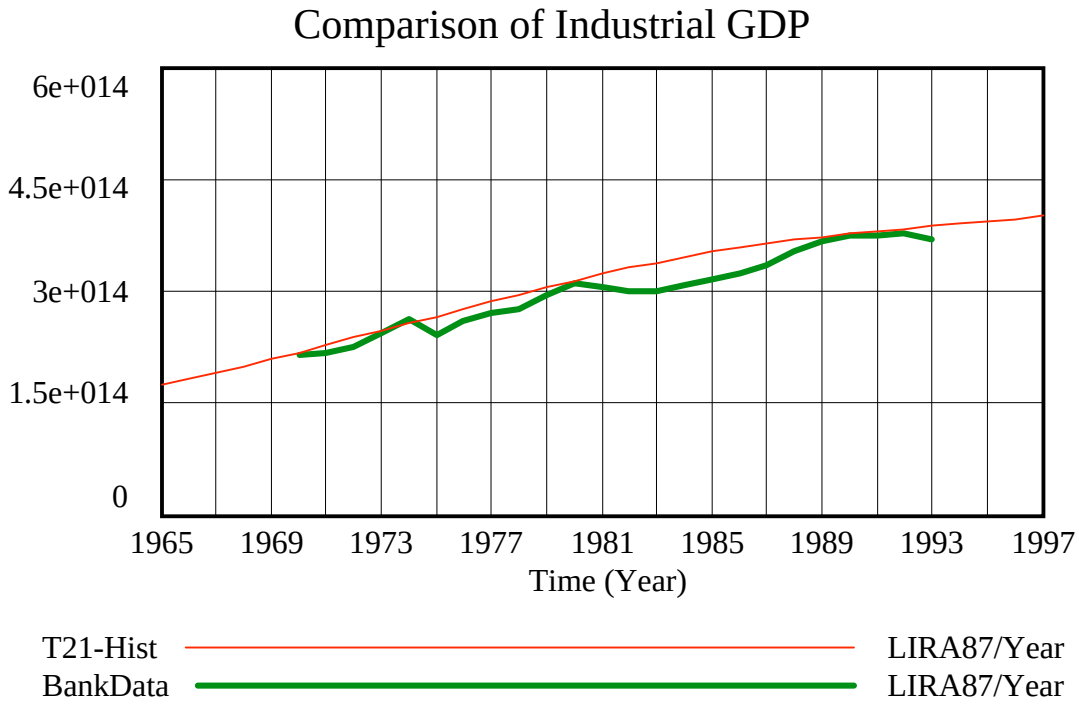



Figure 1: Historical comparison of industrial GDP for Italy

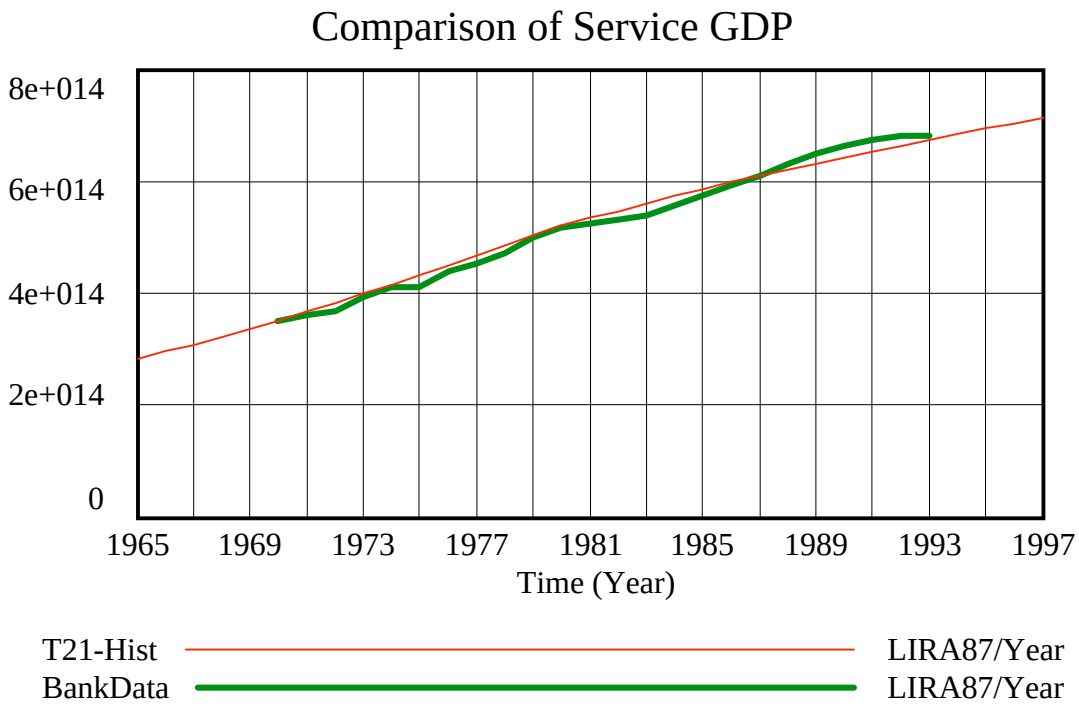



Figure 2: Historical comparison of service GDP for Italy

***Base scenario for the future***

After calibrating THRESHOLD 21 using past data, the model can be used to generate scenarios for the future. The base scenario, which is based on the assumption that past policies and performances will continue into the future, shows the industrial GDP and service GDP as follows:

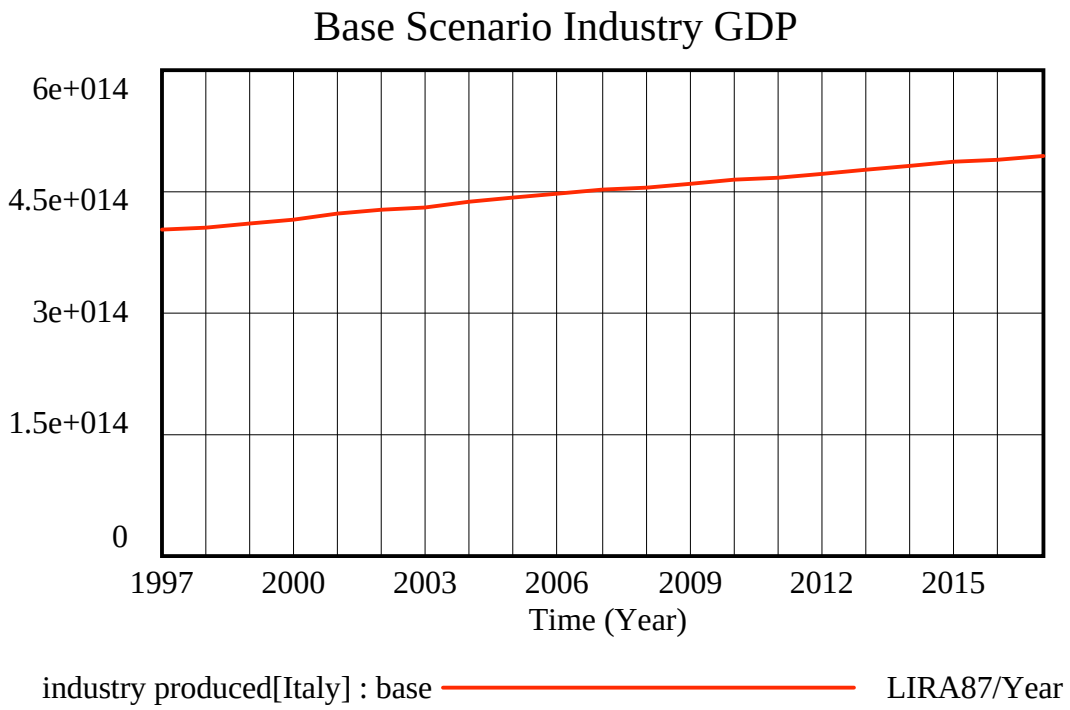



Figure 3: Projected industrial GDP for Italy

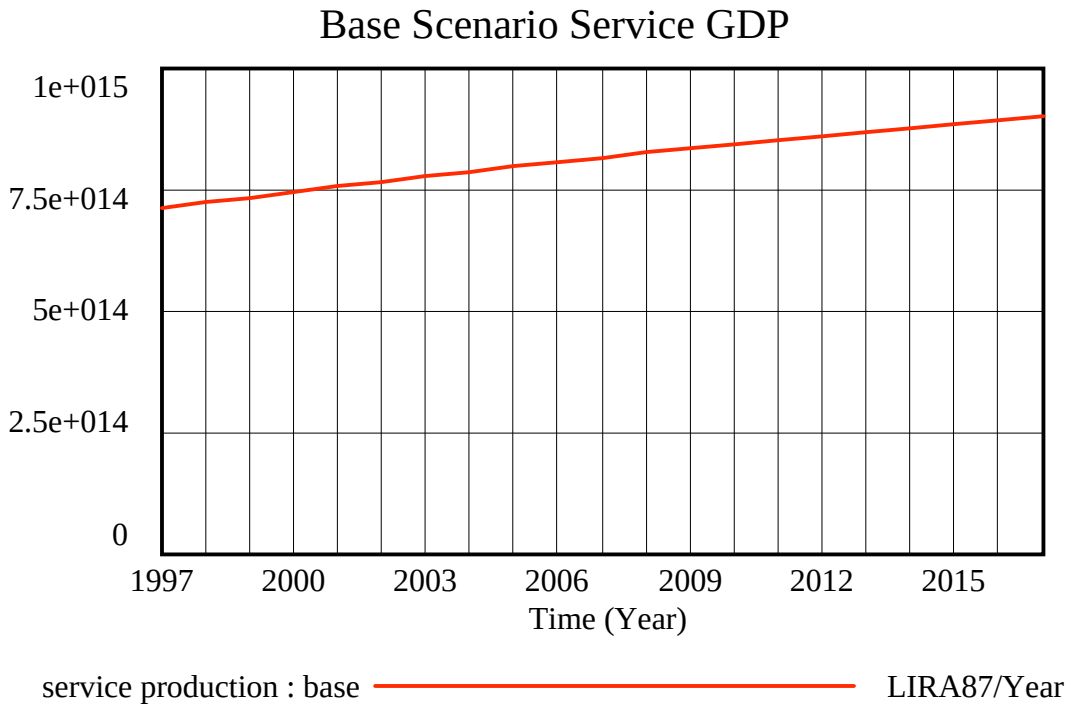



Figure 4: Projected service GDP for Italy

## Application of THRESHOLD 21 to Benin

### *Model modifications*

As explained earlier, Benin's labor productivity decreased during the 1970s and 1980s. To simulate that effect, a new variable, *productivity reduction effect*, was added to each of the industry and service sectors. The *productivity reduction effect* can be used to reflect the consequences of inefficient use of capital as a result of ineffective central planning (e.g., Benin).

The equation for labor productivity (Refer to Equation (2)) now is changed to:

$$\text{labor productivity} = A * \text{capital intensity}^{\text{elasticity}}/\text{production reduction effect} \qquad (10)$$

*Production reduction effect* has the value of 1.0 at the start of the simulation (1965) for both industry and services. From 1965 to 1989, it increases (meaning a reduction in labor productivity) with an annual increment, which is 0.01 for industry and 0.036 for services. After 1989, this variable remains constant at the 1989 level.

### *Parameters*

*Industry labor employed* was 62,400 for 1965, 118,000 for 1980, and 188,000 for 1995. It is assumed that it will grow to 255,000 by 2020.

*Service labor employed* was 165,400 for 1965, 410,400 for 1980, 662,000 for 1994. It is assumed it will continue to grow to 971,000 by 2020.

Capital elasticity for industry and services are estimated to be 0.2 and 0.1 respectively.

Education and training effect is estimated as: In 1965, adult literacy rate was about 10.5%, and the effect was 1. In 1997, adult literacy rate was about 28.1%, and the *ind education and training effect*, in the absence of a good country expert for Benin, was estimated to be 1.3. It is further assumed that increasing

literacy rate to 100% will contribute substantially to productivity in industry and services by a factor of 150%, as the following look-up function shows:

| | | | |
|---|---|---|---|
| Adult literacy rate | 0.105 | 0.281 | 1 |
| Ind edu and tr effect | 1 | 1.3 | 2.5 |

***Historical comparison of industry GDP and service GDP***

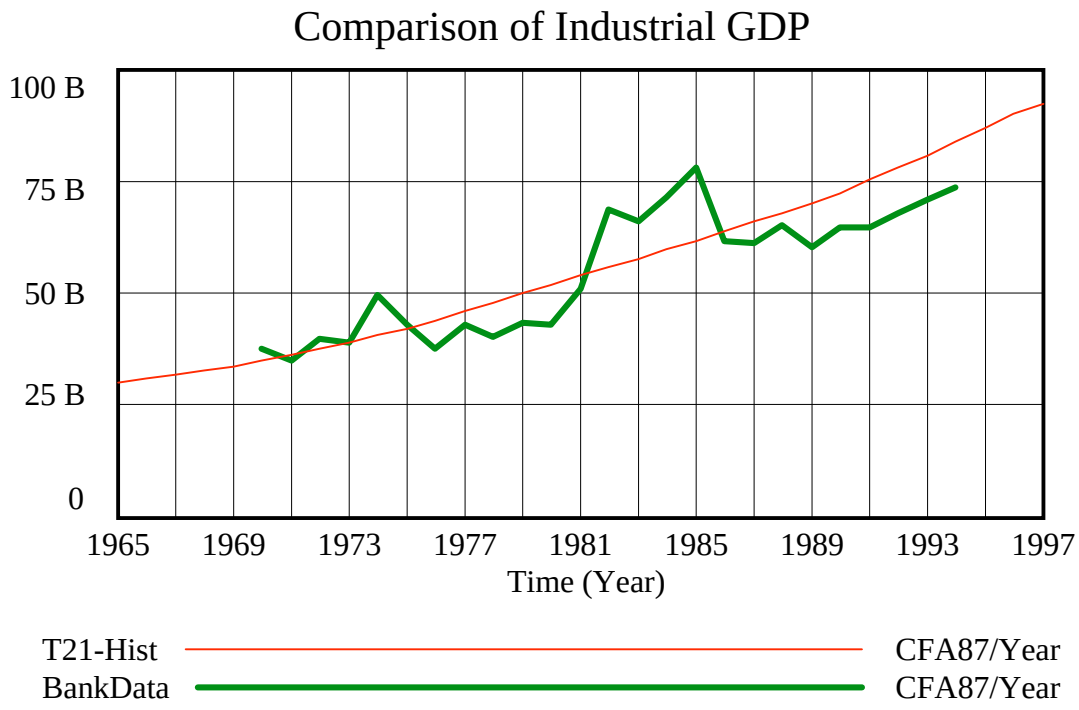



Figure 5: Historical comparison of industrial GDP for Benin

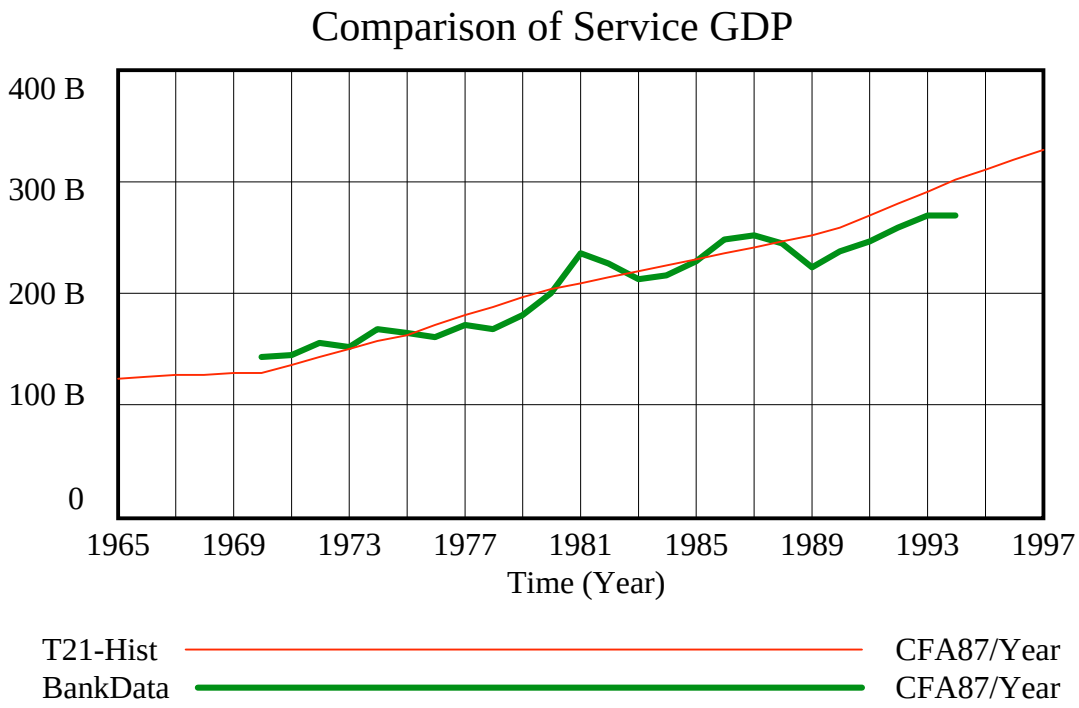



Figure 6: Historical comparison of service GDP for Benin

***Base scenario for the future***

Assuming the Benin economic development continues into the future with the new, more efficient policy implemented since 1989, its industrial and service GDP may grow as the following graphs.

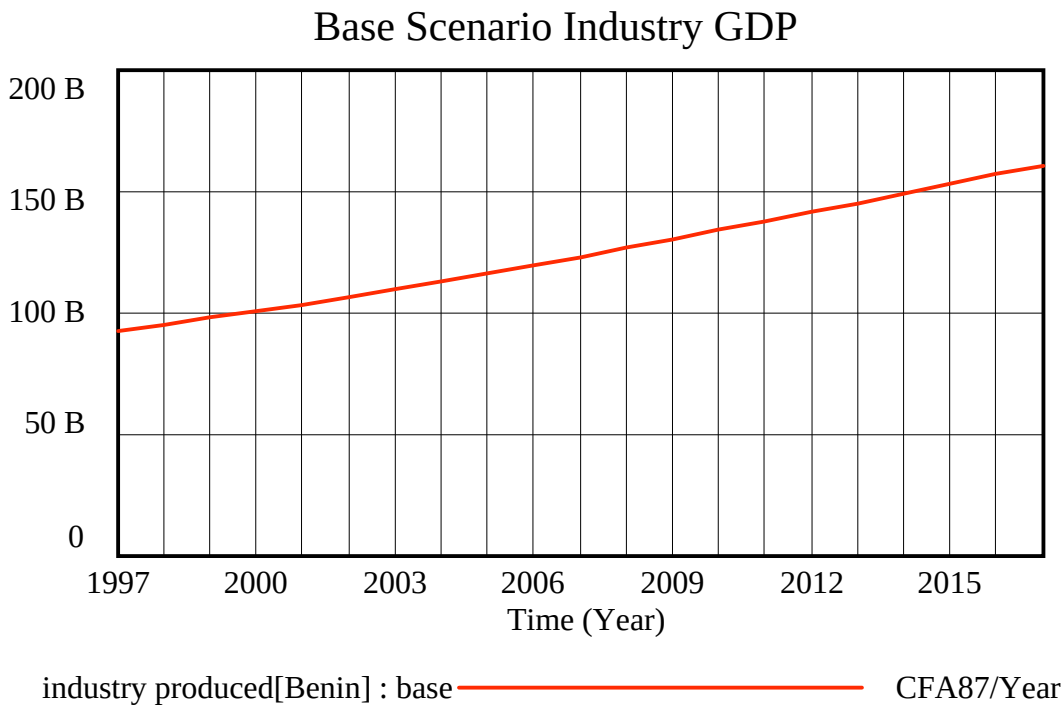
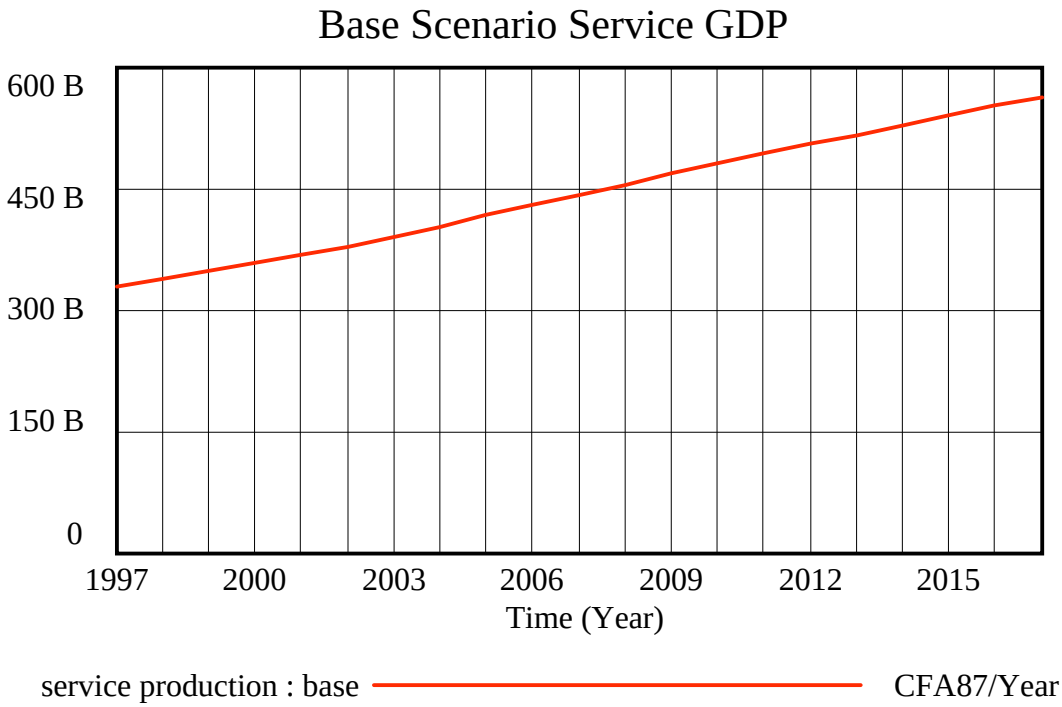



Figure 7: Projected industrial GDP for




Benin

Figure 8: Projected service GDP for Benin

## Application of THRESHOLD 21 to Cambodia

### *Model modifications*

Cambodia suffered much damage in a long war, and in THRESHOLD 21, the consequences of war are represented by *capital destruction*. Equation (6) is modified to:

$$\text{Capital industry}_t = \text{Capital industry}_{t-1} + (\text{investment rate industry}_t - \text{depreciation rate industry}_t - \text{industry capital destruction rate}_t) * \text{computation interval} \quad (11)$$

*Capital destruction* for each of the sectors is an exogenous time series variable which has non-zero values for years when war was going on, and zero values for peace time.

### *Parameters*

Industry labor employed was 106,000 for 1965, 220,000 for 1980, and 330,000 for 1995. It is assumed that it will grow to 430,000 by 2020.

Service labor employed was 444,000 for 1965, 625,000 for 1980, 750,000 for 1995. It is assumed it will continue to grow to 970,000 by 2020.

Capital elasticity for industry and services are each estimated to be 0.2.

*Education and training effect* is estimated as: In 1965, adult literacy rate was about 20%, and *education and training effect* was 1.0. In 1997, adult literacy rate was about 40%, and the effect was assumed to be 1.15. It is assumed that increasing literacy rate to 100% will contribute to productivity in industry and services by a factor of 60%, as the following look-up function shows:

| | | | |
|---|---|---|---|
| Adult literacy rate | 0.20 | 0.40 | 1 |
| Ind edu and tr effect | 1 | 1.15 | 1.6 |

***Historical comparison of industry GDP and service GDP***

Limited industry and service real GDP values for 1965 were estimated based on the GDP values measured in 1965 local currency from the World Bank's database.

GDP data for Cambodia from the Bank is not as complete as for other countries, as shown in the following graphs.

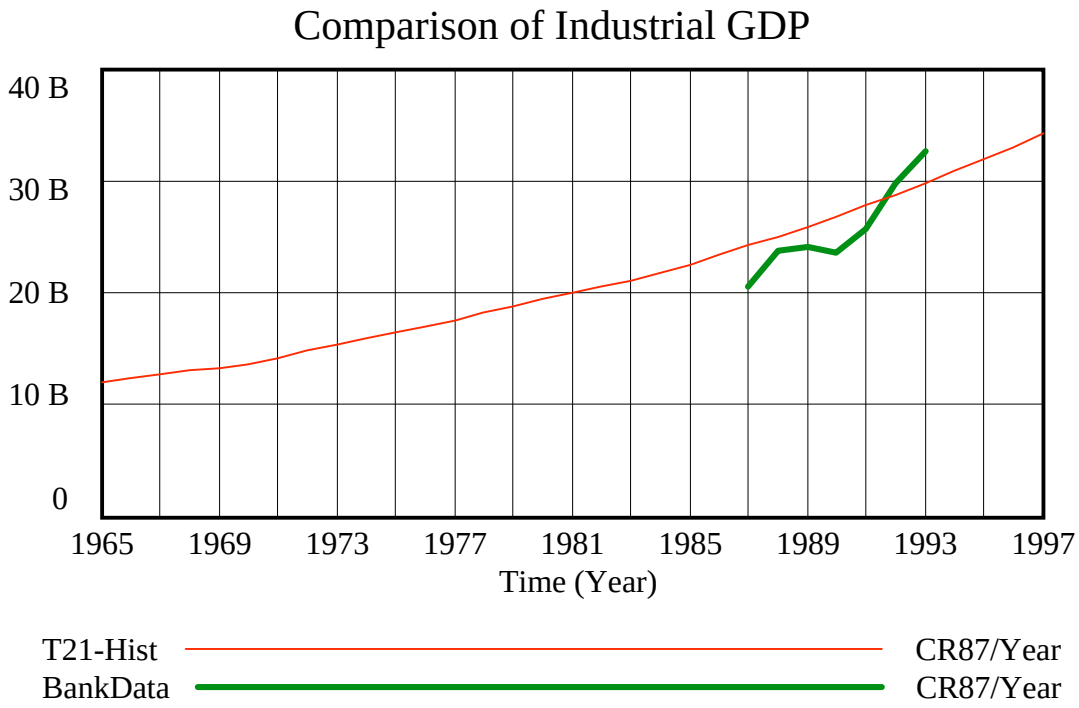



Figure 9: Historical comparison of industrial GDP for Cambodia

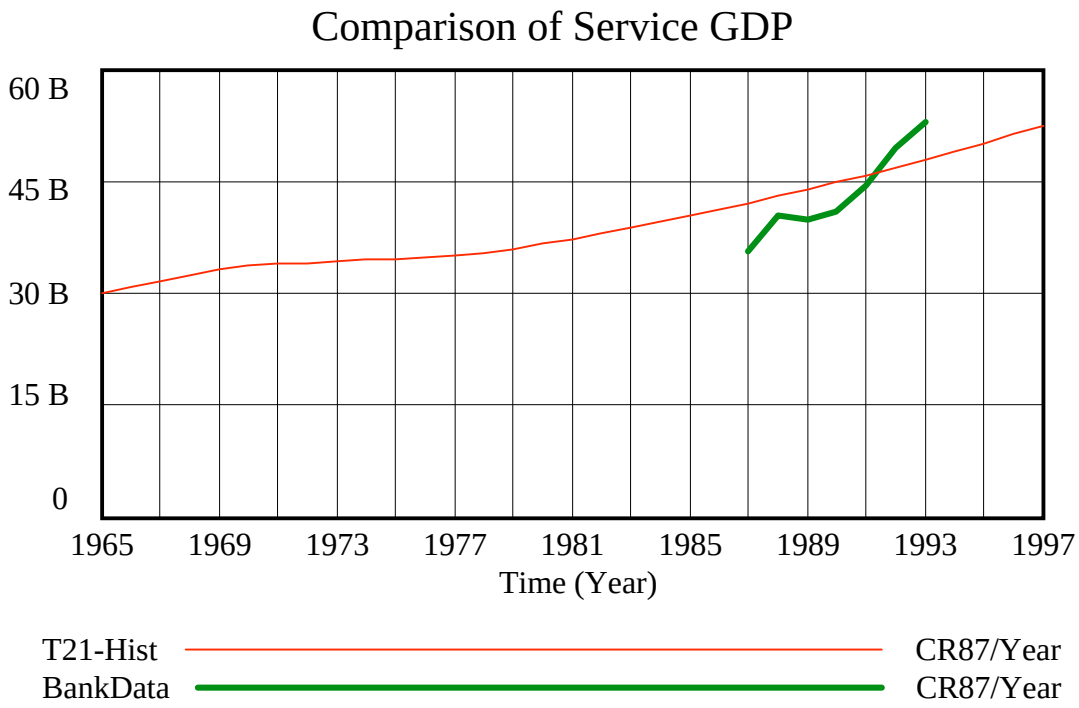



Figure 10: Historical comparison of service GDP for Cambodia

***Base scenario for the future***

Assuming no major wars in the future, industrial and service GDP in Cambodia may grow like the following graphs.

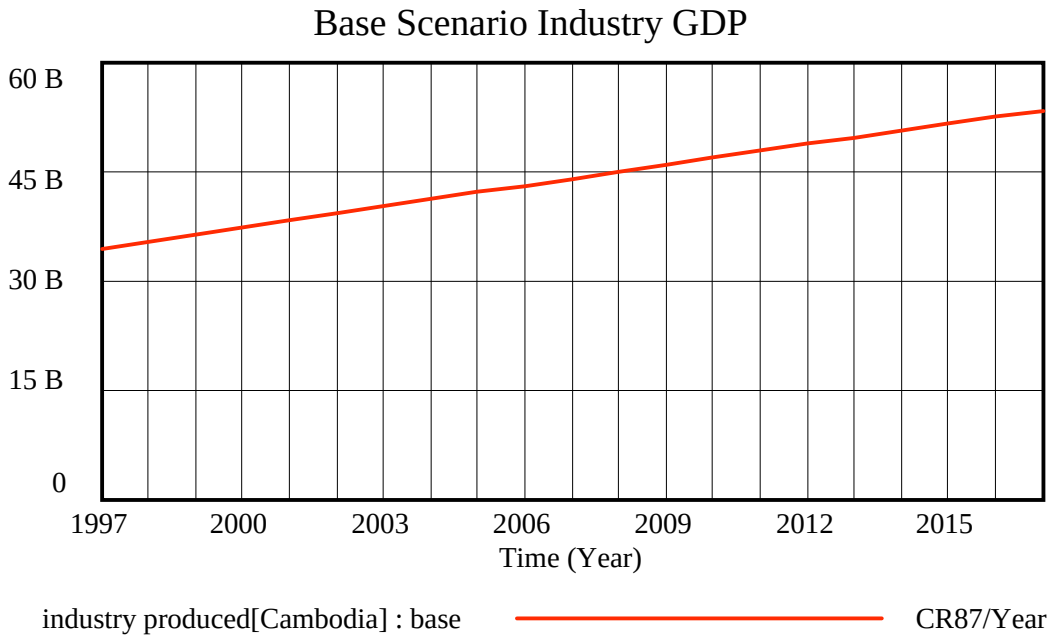


Figure 11: Projected industrial GDP for Cambodia

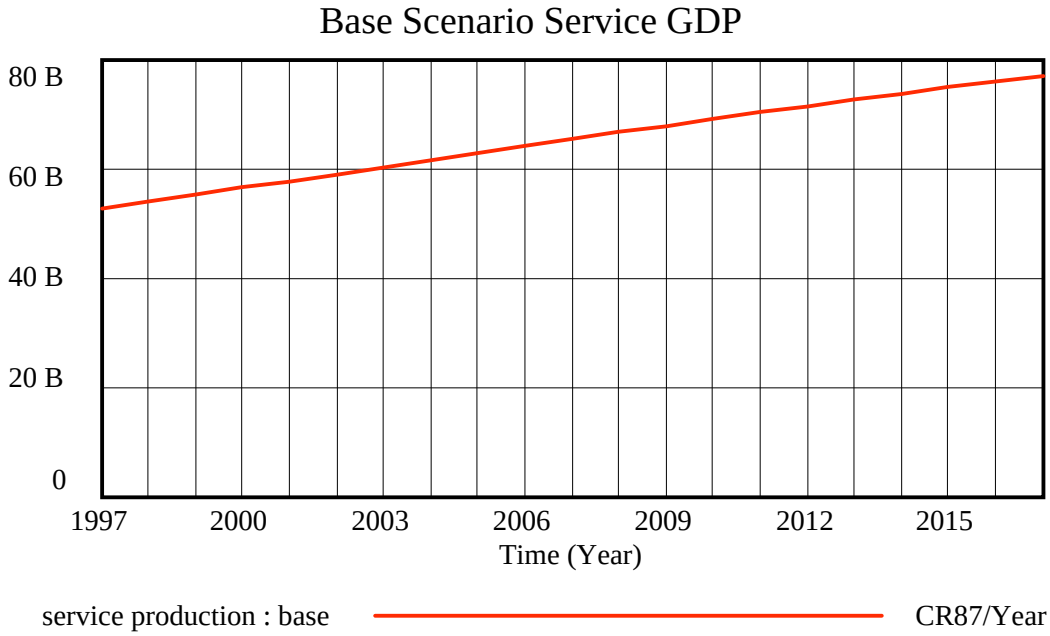



Figure 12: Projected service GDP for Cambodia

## Conclusions and further research

We have shown that by adding two factors to the Cobb-Douglas production function in the THRESHOLD model, it is possible to simulate the production of countries under both normal and unusual conditions, such as war or inefficient economic policies.

The modeling work done for Benin and Cambodia is preliminary, and more time and data are needed to bring this exercise to the level that the models can be used by national planners in these countries. In particular, more work is needed to improve on the exogenous treatment of employment and the subjective way of defining the *education and training effect.*

## References

1. Barney, G., Eberlein, R, Qu, W, and Sharma, P.D., *The Threshold 21 Sustainable Development Model*, Proceedings of 1995 International System Dynamics Conference, Tokyo.

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