A Behavioral Approach to Feedback Loop Dominance Analysis

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Abstract
Feedback loop dominance is a critical tool in explaining how structure drives behavior. Current analytic tools for loop dominance analysis are tacit, not codified, unable to accurately identify dominant loops or inapplicable to most models. Most loop dominance analysis tools focus on model structure to link structure and behavior. We use a behavioral perspective to define dominance, improve descriptions of behavior patterns and identify simultaneous dominance by multiple loops as an important and incompletely developed area of feedback analysis. A new analytic procedure is presented and illustrated. An evaluation of the behavioral approach is the basis for identifying new issues and future research opportunities.

Introduction

System dynamics explains how structure drives behavior. Linking dominant feedback loops and shifts in loop dominance to behavior patterns is critical in these explanations. Discovering these links requires an analysis which identifies the loops which dominate given time intervals of a simulation. How do system dynamicists analyze a model to identify the dominant loops? How much confidence should be placed in the conclusions of these analyses? The validity of most current analyses is implicitly based on the authority of the modeler as a qualified system dynamicist, the plausibility of the loop dominance explanation, the persuasiveness of the presenter or some combination thereof. These are adequate foundations for analysis validation in some contexts such as consulting and teaching but the rigorous research required to further solidify system dynamics as an independent scientific domain requires a feedback loop dominance analysis method which is independent of the analyst. System dynamics does not currently have such a method.

System dynamicists have traditionally used experimental model exploration, model reduction or both with their understanding of the behavior patterns typically generated by positive and negative feedback loops to identify dominant loops (Richardson, 1991, 1986). These approaches can lead to errors in identifying dominant loops (see Richardson, 1995 and Graham, 1977 for examples). System dynamics researchers have recognized the need for more rigor in feedback loop dominance analysis in general and the behavioral aspects of such analysis as a particularly important focus (Richardson, 1986). But no formal and unambiguous definition of behavior as it relates to dominance has been formulated. In addition the experimental method currently used for the majority of dominance analysis remains tacit and uncodified. This failure to map current practice prevents the evaluation and improvement of those practices, their comparison to recommended analysis procedures and their use in building improved tools and procedures. Finally, research has focused on the structural aspects of how feedback structures and behavior are linked far more than behavioral aspects (Richardson, 1995; Mojtahedzadeh, 1997; Kim, 1995; Kampmann, 1996a,b; Forrester, 1982; Davidsen, 1992). These structural approaches address only a portion of the possible feedback structures (e.g. linear or two-loop systems), are difficult to apply or are impractical for models of significant size. System dynamics needs an

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understanding of feedback loop dominance which balances structural and behavioral perspectives. A behavioral approach to feedback loop dominance analysis is an important step in developing such a balanced understanding. In this paper we seek to improve model analysis of dominance and provide a practical tool for dominant feedback loop identification by describing a behavior-based approach to feedback loop dominance, using that approach to formalize an analysis procedure, providing an example of its application and evaluating the approach to identify areas for improvement and further research.

A Behavioral Definition of Feedback Loop Dominance

Richardson and Pugh (1981, p. 285) provide a useful basis for developing a behavioral definition of loop dominance, "...a loop that is primarily responsible for model behavior over some time interval is known as a dominant loop." We expand on Richardson and Pugh's definition by adding specificity concerning three aspects: behavior patterns, location of dominance and gains. A rigorous approach must define the two system features it relates (feedback structures and variable behaviors) independently to prevent circular reasoning such as "The behavior is the kind generated by a positive loop so the dominant loop must be positive." Similarly, describing a behavior pattern in terms of its approach to a system feature such as a goal (as is commonly done with negative feedback loops) is less rigorous than a description based solely on the behavior of the variable. An improved definition of behavior patterns is required which relies only on the behavior of the variable of interest itself.

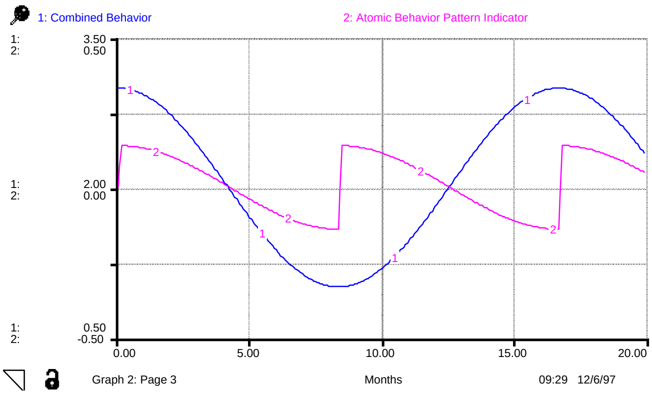
As the basis for an improved definition of behavior patterns we identify three unique behavior patterns based on the net rates of change of the variable of interest. The net rate of change of a variable is a characteristic solely of the variable and can be determined independently of how the structure and system conditions generate the change. The absolute values of these rates describe movement greater and less than their initial values. Trends in the absolute values of net rates of change can be used to uniquely identify the three atomic behavior patterns. The first atomic behavior pattern is linear behavior. When the absolute value of the net rate of change of a system variable is constant the variable grows or declines steadily and the behavior is linear. Equilibrium conditions are a special case of this pattern in which the net rate of change is zero. The second atomic behavior pattern is exponential growth or decay. When the absolute value of the net rate of change of a system variable increases over time the variable moves away from its initial value faster over time. The typical behavior generated by positive feedback loops is exponential. The third atomic behavior pattern is logarithmic growth or decay. When the absolute value of the net rate of change decreases over time the variable moves away from its initial conditions at a slower rate over time.

The three atomic behavior patterns can be described mathematically with the derivatives of the value of the variable of interest (denoted x). The net rate of change is the first derivative of the variable's value ($\partial x/\partial t$). The variable's second derivative describes the movement of the net rate of change, A positive second derivative indicates an increasing absolute size of the net rate of change, a negative second derivative indicates a decreasing absolute size of the net rate of change and a second derivative equal to zero indicates a constant rate of change.

Including absolute values to describe behavior greater and less than initial values produces the following definitions:

Linear atomic behavior pattern	$\partial(\lvert(\partial x/\partial t)\rvert) / \partial t = 0$	(1)
Exponential atomic behavior pattern	$\partial(\lvert(\partial x/\partial t)\rvert) / \partial t > 0$	(2)
Logarithmic atomic behavior pattern	$\partial(\lvert(\partial x/\partial t)\rvert) / \partial t < 0$	(3)

Combinations of the three atomic behavior patterns can describe most behavior simulated by system dynamics models[2]. As an example pattern 1 in Figure 1 illustrates a combination of two of the three behavior patterns as generated by a single negative feedback loop. In pattern 2 of Figure 1 our mathematical definition of the atomic behavior patterns identifies the exponential behavior patterns as those occurring when the atomic behavior pattern indicator value is positive and logarithmic patterns as those occurring when the atomic behavior pattern indicator value is negative.



Time Period	0 - 4.25	4.25 - 8.50	8.50 - 12.75	12.75 - 17.00	17.00 - 20.00
Test polarity	positive	negative	positive	negative	positive
Behavior Pattern	Exponential	Logarithmic	Exponential	Logarithmic	Exponential

Figure 1: Combinations of Atomic Behavior Patterns

A traditional description of the behavior patterns in Figure 1, pattern 1 would describe them as goal seeking. Our description of behavior patterns differs from traditional descriptions in its use of only the time varying value of the variable of interest and not other system features such as structure or system conditions to describe behavior. Conspicuously absent is any reference in our definition to goals to describe behavior patterns. The three atomic behavior patterns are also better discriminators of behavior patterns than traditional means. For example a traditional description of the behavior in Figure 1, pattern 1 as goal seeking does not distinguish between exponential and logarithmic behaviors in the pattern although they are fundamentally different shapes. Scaling and timing changes can make this distinction very important by causing the growth portion of a negative loop such as begins at the minimum value in a limit cycle (e.g. time 8.50-12.75 in Figure 1) to appear to be exponential growth due to a positive loop, thereby leading to errors in identifying dominant loops.

In addition to an improved definition of behavior patterns a rigorous behavioral definition of dominance requires specificity concerning the location of dominance. The location of

[2] One exception is discrete flows such as those caused by pulses or the instantaneous release of work described in Ford and Sterman (1998).

dominance must be identified more specifically than at the level of a model because different variables in a model can have very different behavior patterns in the same time interval. Therefore the identification of feedback loop dominance requires the specification of a single system variable for which dominance is considered important. We refer to this variable as the "variable of interest." Specificity is also needed concerning the gains which determine feedback loop strength and therefore determine dominance. The sizes of these gains can change over the simulation period, potentially changing loop dominance (Mojtahedzadeh, 1997; Kim, 1995). Therefore loop dominance depends upon the conditions of the rest of the system during the time interval. The system structure and combinations of parameter values which determine loop gains in a time interval define the conditions in which the results of dominance analysis are valid. Completely specifying loop dominance requires specifying the system structure and conditions under which a given loop dominates.

Based on the above we define feedback loop dominance as follows: *A feedback loop dominates the behavior of a variable during a time interval in a given structure and set of system conditions when the loop determines the atomic pattern of that variable's behavior.* This definition provides the basis for an unambiguous and objective test of dominance. As will be illustrated our definition allows multiple loops to simultaneously dominant a single variable. This is consistent with an intuitive meaning of dominance (Richardson, 1986) because more than one loop may be required to produce a given behavior pattern. For example the success-to-successful system archetype (Senge, 1990) consists of two loops which are both required to maintain linear behavior when the system is in equilibrium. While simultaneous multiple dominance is not precluded in some structural approaches to loop analysis a behavioral approach expands an analyst's perspective from a search for a single dominant loop with non-dominating influences toward a richer view of how loops drive behavior which includes multiple dominant loops.

The Feedback Loop Dominance Analysis Procedure

Our analysis procedure, portions of which are currently performed by many system dynamicists informally, is purposefully behavioral in nature and structurally simple. We use changes in atomic behavior patterns in the presence and absence of a feedback loop to signal loop dominance in a chosen time interval. Shifts in loop dominance across adjacent time intervals are identified by identifying the dominant loops in each time interval. The procedure structures the following eight steps into an iterative process for identifying dominant loops in selected time intervals.

1. Identify the variable of interest which will determine feedback loop dominance and simulate the behavior of the variable of interest over time.
2. Identify a time interval during which the variable of interest displays only one atomic behavior pattern[3]. This is the reference atomic behavior pattern and time interval. The system structure and parameter values during this time interval define the conditions in which dominance is specified.

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[3] Due to the use of difference equations by simulation software to perform calculations two timesteps are required to generate an acurate second derivative. Therefore the first two timesteps may gernerate false positive values for the atomic behavior pattern indicator. These should be ignored.

3. Use the feedback structure of the model to identify the feedback loops which influence the variable of interest. Select one of those feedback loops as the candidate feedback loop, beginning with a loop which contains the variable of interest if possible.
4. Identify or create a control variable in the candidate feedback loop which is not a variable in other feedback loops and can vary the gain of the candidate loop. Use the control variable to deactivate the candidate loop.
5. Simulate the behavior of the variable of interest over the reference time interval with the candidate feedback loop deactivated and identify the atomic behavior pattern or patterns of the variable of interest during the time interval.
6. Identify time intervals, each which contains a single atomic behavior pattern[4]. If the atomic behavior pattern in a time interval generated in step 5 is different than the reference pattern identified in step 2 the candidate feedback loop dominates the behavior of the variable of interest under the system conditions during that time interval, otherwise the candidate feedback loop does not dominate[5].
7. Repeat steps 3 through 6 with the candidate loop active to test for multiple dominant feedback loops during the time interval.
8. Repeat steps 1 through 7 for different time periods to identify shifts in feedback loop dominance and feedback loop dominance over other variables of interest.

Application of the Behavioral Loop Dominance Analysis Approach

An example using a simple structure with simultaneous multiple loop dominance based on a project model (Ford and Sterman, 1998) will be used to illustrate the application of the behavioral approach to analyzing feedback loop dominance. The equations which describe the system used in the example are available from the author. The structure used in this example (Figure 2) simulates the availability and completion of work based on the amount of work which has already been completed. A simple but realistic relationship is assumed in which 10% of the work is available initially and the completion of each task releases another task for completion until all work is available. Such a constraint could describe the availability of floors in the construction of a ten story building as lower floors are completed.

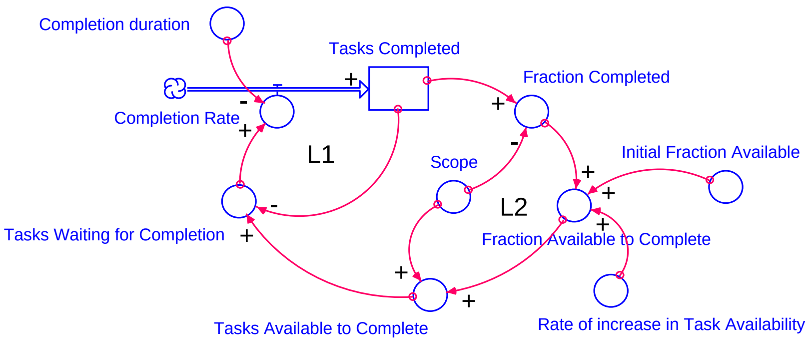

Figure 2: Feedback Loop Example Structure Diagram

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[4] Deactivating some candidate loops which do not include the variable of interest alters only dominance shift timing. Deactivating these timing loops can generate two behavior patterns in a single reference time interval as well as candidate loops which change the loops which dominate.

[5] A third possibility is the presence of one or more shawdow Feedback loops, which are beyond the scope of this paper. See Ford (1997).

Step 1: The variable Tasks Completed is selected as the variable of interest. The behavior of the variable of interest and atomic behavior pattern indicator over 40 days are shown in Figure 3.

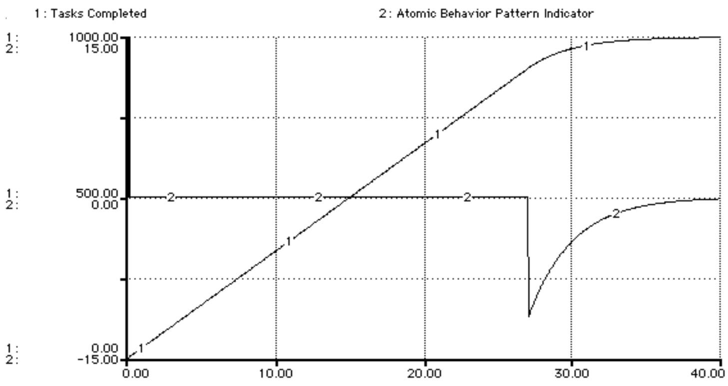

Figure 3: Feedback Loop Example Reference Behavior

Step 2: The atomic behavior pattern indicator identifies the behavior is linear from day 0 through 27 and logarithmic from day 27 through 40. Days 0 through 27 are selected as the time interval for analysis. The system conditions under which analysis is valid are described with the structure equations and the parameter values sets as they vary from day 0 to day 27.

Step 3: Feedback loop L2 is selected as the candidate loop.

Step 4: Feedback loop L2 is deactivated by severing the causal link between the Fraction Available to Complete and the Tasks Available to Complete variables by changing the equation for the Tasks Available to Complete from:

Tasks Available to Complete = Scope * Fraction Available to Complete
to
Tasks Available to Complete = Scope

Step 5: The behavior of Tasks Completed with loop L2 deactivated and atomic behavior pattern indicator are shown in Figure 4. The atomic behavior pattern during the time frame day 0 - 27 is logarithmic.

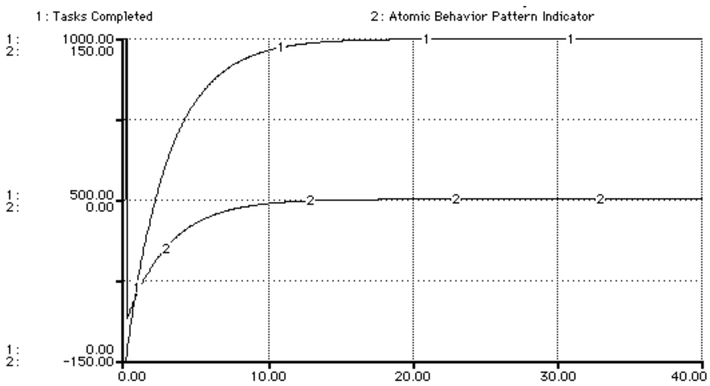

Figure 4: Feedback Loop Example Feedback Loop L2 Inactive

Step 6: The atomic behavior pattern changed from linear to logarithmic over the entire time interval, indicating that feedback loop L2 dominates the behavior of Tasks Completed during the time interval 0 - 27 under the conditions of the system identified in step 2. This helps analysts understand that loop L2 constrains the system into a linear pattern of completing tasks less quickly than it would otherwise.

Step 7: To test for multiple dominant loops feedback loop L2 is reactivated and the test is repeated with feedback loop L1 (step 3) as the candidate loop. Loop L1 is deactivated (step 4) by redefining the equation for the Tasks Waiting for Completion from:

Tasks Waiting for Completion = Tasks Available to Complete - Tasks Completed
to
Tasks Waiting for Completion = Tasks Available to Complete

The behavior of the variable of interest over the time interval with feedback loop L1 inactive and atomic behavior pattern indicator are shown in Figure 5 (step 5).

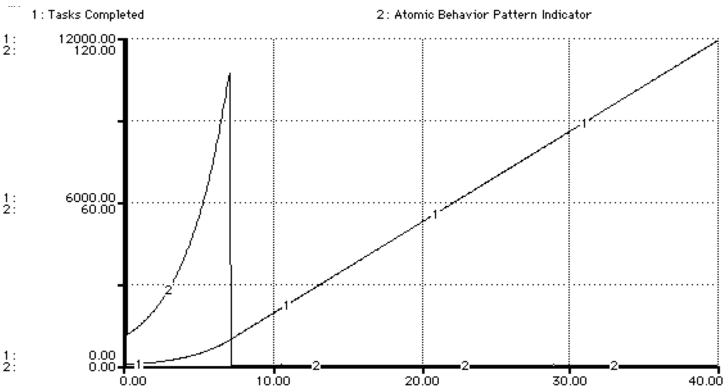

Figure 5: Feedback Loop Example Feedback Loop L1 Inactive

The atomic behavior pattern changes from linear (Figure 3) to exponential (Figure 5) during days 0 to 7 when loop L1 is deactivated (step 6). This indicates that feedback loop L1 dominates during that time interval. From day 0 through 7 the system behaves logarithmiclly under the control of L1 alone (Figure 4) and exponentially under the control of loop L2 alone (Figure 5) but linearly when both loops L2 and L1 are active (Figure 3). This indicates that the interaction of the two loops generate the linear behavior and not either loop alone and therefore they both dominate during days 0 - 7. The procedure and results help the analyst understand that loop L1 restrains the exponential completion of tasks by loop L2 into a linear pattern and that Loop L2 restrains the logarithmic completion of tasks by loop L1 into a linear pattern.

The atomic behavior pattern with loop L1 inactive remains unchanged (linear) over the time interval 7 - 27, indicating that loop L1 does not dominate this time interval. Since there are no other candidate feedback loops analysts can use this to understand that during days 7 - 27 loop L2 dominates the system behavior alone (step 6). Applying the analysis to the time interval 27 - 40 identifies feedback loop L1 as the only dominant loop. The results of the analysis can be concisely shown by identifying the time intervals during which specific loops dominate on a behavior graph of the variable of interest with all loops active (Figure 6).

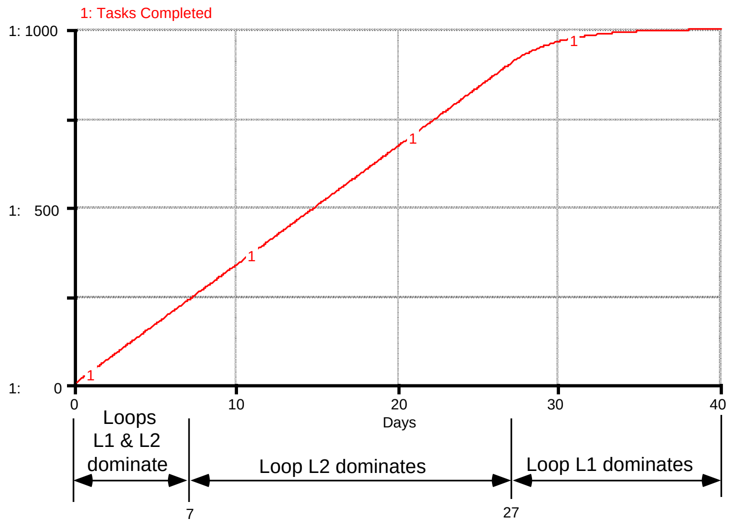

Figure 6: Feedback Loop Example Results of Feedback Loop Analysis

Since there are no other variables of interest (step 8) the analysis is concluded.

The simultaneous multiple dominant loop example demonstrates how the test identifies time intervals in which different loops dominate the behavior of a variable of interest, simultaneous multiple loop dominance and the iterative application of the procedure.

Evaluation of the Behavioral Loop Dominance Analysis Approach

Based on the preceding and other applications of the behavioral approach to feedback loop dominance analysis we make the following assessments.

1. The method generates explicit and precise results and the conditions in which those results are valid.
2. The procedure is explicit and applicable to any system dynamics model for the analysis of any variable with simple modeling tools and methods.
3. The method can isolate specific model variables and feedback loops for analysis. This allows the analyst to perform partial model analysis and investigate feedback loops of particular interest (e.g. loops believed to be high-leverage or loops which can be influenced).
4. The approach rigorously separates structure and behavior. Behavior patterns and analysis results do not depend on the polarity of feedback loops or behavior pattern estimates based on loop polarity. The procedure does not require distinctions among structural components such as stocks, flows and auxiliaries.
5. The procedure identifies simultaneous domination by multiple loops. This provides a more accurate description of how structure drives behavior than procedures which identify only one dominant loop. However this ability may increase the challenges of communicating analysis results since simultaneous multiple loop dominance may be

more difficult to understand and describe than the results of tests which identify only one dominant loop.

6. The procedure fully validates its results only through the testing of many loop combinations. Its rigor is therefore limited by the difficulty of identifying and isolating feedback loops. This can be partially addressed through a partial-model testing approach (Homer, 1983) to feedback loop analysis but does not resolve the challenges of analyzing large complex models and loops which integrate model subsystems.
7. The difficulty of specifying the system conditions under which loop analysis results are valid increases with model size.
8. The procedure allows non unique methods of deactivating feedback loops. Different deactivation methods may generate different behavior patterns.

Our assessment indicates that the behavioral approach to feedback loop dominance analysis provides clear advantages over informal and undocumented analysis procedures but has weaknesses which can act as warnings for its application and guidance in the development of automated feedback loop analysis tools.

Conclusions

The rigorous identification of dominant feedback loops in models is required to explain how structure drives behavior. Our behavioral approach defines dominance with atomic behavior patterns and identifies simultaneous dominance by multiple loops. Our analysis procedure uses changes in atomic behavior patterns to signal dominance. An illustration of the application of the procedure shows its ability to identify and distinguish among dominant structures better than informal analysis methods. Our approach contributes to the analysis of feedback loop dominance by improving the definition of loop dominance and defining multiple dominance as an important type of feedback structure. We have also codified an applicable analysis procedure. This will allow analysis methods to be implemented, evaluated and improved by many dynamists simultaneously and, as suggested by Richardson (1995), bridge between intuition-based analysis and more formal solutions .

Our behavioral approach to feedback loop dominance analysis shows promise as a tool for the analysis of small-to-medium size and large simple models and for the development of improved understanding and explanation of how feedback drives behavior. However the limitations of its manual application and the complexity of dominance emphasize the need for automated model analysis tools based on rigorous mathematical definitions of both feedback loop dominance and behavior patterns. Future research can further validate our procedure, expand our initial investigations and integrate our behavioral perspective with structural approaches to feedback loop dominance analysis.

By proposing and testing a more specific and explicit defintiion of feedback loop dominance and analysis method our work raises new questions which must be resolved to fully address the feedback loop analysis of system dynamics models. Is the purpose of feedback loop analysis to diagnose nominal model behavior, explain the shapes of behavior plots, educate about how structure drives behavior or some combination thereof? Are different types of dominant loops such as those which determine patterns versus those which determine the timing of dominance shifts more or less important for different purposes? Are different definitions, tools and methods needed for different loop analysis purposes? How can feedback loop analysis bridge the gap between the complexity of most feedback models and the limited capacity of humans to comprehend dynamic complexity? The continued development of analysis tools and methods can provide the basis for improving our understanding and use of feedback loop dominance to improve system performance.

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