Stochastic optimization in policy space using simulation models.

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Optimization has not been an important part of system dynamics thus far, and for several good reasons: Simulation models have been sufficient to point out greater potentials for improvement than what policy makers have been willing to embrace. Hence presumably minor improvements of policies might have seemed of little value. Even simple simulation results have been difficult to communicate, indicating that the results of complicated optimization efforts would be even harder to convey. Optimization methods have not been able to capture the richness of real decision problems characterized by simultaneous appearance of dynamics, nonlinearities and uncertainties. Hence there has been uncertainty about the transferability of solutions from simple to complex environments.

While many of these reasons will remain valid for policy analysis, there is an increasing need for optimization at the more scientific end of the spectre of system dynamics analysis: When testing behavioural assumptions with experiments, Sterman (1987), optimal policies are required to characterize the results and to establish proper benchmarks. Optimization is needed to get precise estimates of how rational policies change as complexity is added. This is an approach to behavioural analysis which might have started with Simon (1955). Furthermore, a simulation study which produces optimal solutions is likely to gain increased validity and acceptance among decision makers and in journals.

Analytical solutions to dynamic, nonlinear, stochastic models are hard to come by. Numerical solutions by the use of stochastic dynamic programming require skill and time, and is hampered by the so-called 'curse of dimensionality'. Here a method called 'stochastic optimization in policy space' is presented and tested. Basically the idea is to refine the trial and error search for appropriate policies often used in simulation studies. We start out with a policy function with unknown parameters. Then we search for the parameter values of the policy function that maximize the criterion. In the case of uncertainty, repeated (Monte Carlo) simulations are made to calculate the expected (average) criterion value. A nonlinear optimization technique is used to search for parameter values. The idea is discussed in Polyak (1987), Ermoliev and Wets (1988) and Walters (1986). Details and caveats are presented in Moxnes (1996).

We apply the method to seek optimal quota policies for the management of a stochastic predator-prey system. The example is cod and capelin in the Barents Sea, two species which can be harvested separately. There is a growing awareness of the importance of the interconnectedness of marine species as well as the importance of the uncertainty in our knowledge about ocean ecosystems. Thus, an effort to improve our understanding of the management of multispecies fish stocks under uncertainty seems pertinent. Previously Mendelssohn (1980) has characterized the solution to a much simplified multispecies model.

We want to maximize the expected net present value *ENPV* of the two fisheries i=1,2. Profits (price p_i times harvest H_t^i minus costs c_i times effort $F_t^i = H_t^i\,(x_t^i)^{-\beta_i}$) are discounted with the rate δ and are summed over the time period T. To get the expected value, the average net present value over M Monte Carlo simulations are calculated.

$$ENPV = \frac{1}{M}\sum_{m=1}^{M}\{\sum_{t=0}^{T} e^{-\delta t}\sum_{i=1}^{2}(p_i H_t^i - c_i F_t^i\, dt)\} \qquad (1)$$

The resource dynamics are given by a frequently used modification of the Lotka-Volterra equations:

$$X_{t+1}^i - S_t^i = S_t^i\,(\,r_i + \sum_{j=1}^{2}\alpha_{ij} S_t^j + \varepsilon_t^i\,) \qquad (2)$$

where X_t^i is the biomass of species i at time t, S_t^i is the survival after harvest, $S_t^i = X_t^i - H_t^i$. The challenge is to find the harvesting policies as functions of the biomass vector X_t and a policy parameter vector θ (a vector which contains only the parameters of the policy function).

$$H_t^i = \max[0, f_i(X_t, \theta)] \qquad (3)$$

The functions f_i are both two dimensional interpolations between 9 grid points. Hence the functional form is very flexible and do not impose strong restrictions on the solution. The max-function ensures that the quotas are non-negative. Nonlinear optimization is used to search for the parameter vector θ^* which maximizes the criterion *ENPV*.

Finally note that the method is not guaranteed to yield a global solution for all problems. Hence we repeat the search for θ^* from different starting points for θ. This resulting solutions also enable us to estimate the accuracy of our findings.

Figure 1 shows the optimal solution. Both diagrams show quotas on the y-axis, and biomass of own species on the x-axis. The different types of lines (dashed, thin, and thick) denote the biomass of the other species, see the figure text for values. The bands between lines of the same type denote expected quota policies plus minus two standard deviations.

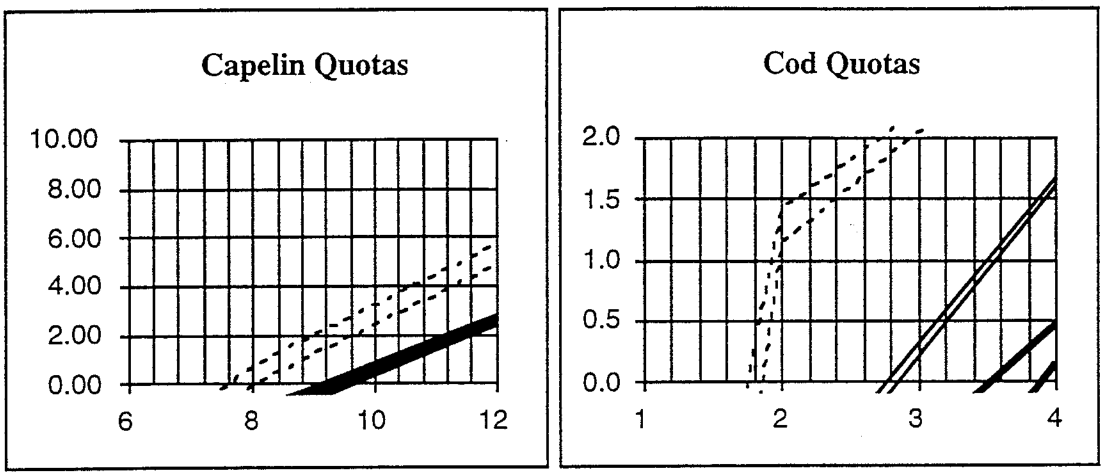

Figure 1: Expected quota policies for capelin, plus-minus two standard deviations, as a function of the amount of capelin. Dashed lines denote 0.5 mill. tonnes of cod, thin line 2 mill. tonnes and thick line 4 mill. tonnes.
Expected quota policies for cod, plus-minus two standard deviations, as a function of the amount of cod. Dashed lines denote 2 mill. tonnes of capelin, thin line 8 mill. tonnes and thick line 14 mill. tonnes.

Comparing to a solution found by stochastic dynamic programming, Brekke (1994), our solution turns out to give a slightly higher *ENPV*. This indicates that the restrictions imposed by the chosen policy function are less important than the discretizations of the state and policy spaces required for dynamic programming.

Sensitivity tests show that the solution does not change much when the "infinite" time horizon is reduced from 50 to 20 years or when the number of Monte Carlo runs is reduced from 100 to 10. Due to compensating mechanisms the criterion is not very sensitive to the exact policy values. Starting out with the optimal solution for the *deterministic* version of the model leads to a criterion value 6 percent below the optimal one. This indicates a considerable potential for trial and error when searching for satisficing policies in a deterministic model.

The effect of stochastic variation is not easily determined by trial and error. By varying the standard deviations for the stochastic terms ε_t^i we find that the two-species policies are more sensitive to stochasticity than what has been reported for single-species models, see Clark (1985). Still the effects are modest. If the standard deviations were to be underestimated by 50 percent, the expected loss would be about 4 percent.

By adding nonlinearities to the original model, making it more realistic, we find strong effects on the optimal policies. Using the solution for the original model in the complex one, leads to a loss of 90 percent! The importance of uncertainty tends to increase with complexity. We also find that the solution to the complex model is more in line with historical policies than the solution to the simple model.

In ongoing research, we use 'stochastic optimization in policy space' to estimate the value of accuracy in resource measurements and in model parameters. Such estimates are difficult to make without a technique of the sort we use since optimal polices change with the amount of measurement or model error.

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