

## A DYNAMIC MODEL OF LABOR MARKET

M. D. Soto Torres  
R. Fernández Lechón

Facultad de Ciencias Económicas y Empresariales  
Departamento de Economía Aplicada  
Universidad de Valladolid  
Avda. Valle Esgueva 6  
47011 VALLADOLID, SPAIN

### Abstract

*We present a system dynamic model about labor market. The model is based on four equations mainly: demand and supply for labor, and evolution of prices and wages. To find the supply labor we consider characteristics of the labor market in Spain. The simulation of the model allow us to determine the evolution of variables as demand employment, male and female supply labor, wages, prices, inflation and unemployment rate.*

### INTRODUCTION

Many economic models involve feedback loops, thus, it is possible to use the system dynamics methodology to study their behavior. The papers of Radzicki and Sterman (1993), Saeed and Radzicki (1993) or Rasmussen, Mosekilde and Sterman (1985), are an example of the use of this methodology on different economic models.

In this paper, we try to build a system dynamic about labor market. The system attempts to take into account some characteristics on the labor market in Spain. The building of the system is based on a set of equations which are considered from different authors. We will take functions of supply and demand of employment over empirical evidence and following some considerations from the works of Beenstock and Warburton (1988) and Maté (1994). The former experiment with a model of three equations to analyze the U.K. labor market. The last, considers analogous equations, but in the Spanish case.

The third equation of our model concerns to the variation of real wages, that is fitted by means of a Phillips curve following the above authors. This equation closes the feedback loop. But, like the supply for labor depends on the consumer prices, we will consider a fourth equation on prices.

These equations allow us to build a flow diagram based on them. The diagram will

have four parts interconnected: male and female supply for labor, demand of employment and formation of prices and wages.

The distinctness between male and female supply is justified assuming that men and women have different reasons to participate in the labor market which are considered in the flow diagram. The demand of employment is obtained for a economic sector. From that value we estimate the demand of employment for the economy.

The comparison between demand and supply allow us to find the unemployment rate, that affects to the evolution of prices and wages.

With the simulation of model we can find the evolution of variables as demand for labor, male and female supply, wages, prices, inflation and unemployment rate.

## THE MODEL

We consider a closed economy with two main elements, which determine the labor market, the population and the business sector. The public sector is not considered in this paper. From the population, we estimate the labor supply and the firms determine the demand of employment.

Beginning with the demand for labor, we assume that the business sector consists of a number of identical firms. All the firms produce a good with two factors, labor and capital, and moreover, want to maximize profits. Then, if the production function is denoting by the expression:

$$Y = F(K, L) ,$$

where  $K$  is capital stock,  $L$  is employment, both measured in homogeneous units, the profit function will be:

$$\Pi = pF(K, L) - cK - wL ,$$

where  $p$  is output price,  $c$  is cost of capital and  $w$  is wage.

All these variables and the followings are fixed in each moment, but, they can change over the time.

Assuming perfect competition and maximizing  $\Pi$  with respect to  $L$ , we have the demand for labor of the business sector. Particularly, if the production function is Coob-Douglas, the expression for labor is:

$$L^D = \left( \frac{pA(1 - \alpha)}{w} \right)^{\frac{1}{\alpha}} K ,$$

where  $A$  is technological coefficient,  $\alpha$  is elasticity of capital and  $(1 - \alpha)$  is elasticity of employment.

To find the labor supply, we consider the population divided between males and females. The reason of this fact, is explained from different attitudes that men and women have

with respect to labor market.

So, we assume that the men, from 16 years to 65 years, want to participate in labor market. Only two causes limit the participation. On the one hand, the young men, less than 25 years, generally prefer to have more qualification, thus only a percentage of them will belong to the labor supply. (Spain is the country that has more University students of the E.U.). On the other hand, the men aged 50 years and over, can decide to leave of the labor market. This decision can be explained from the substitution effect between the real wage and real value of pensions or social benefits. In Spain the male participation rate was, in 1994, of 70 percent. This same rate calculated for men 50 years and over, was 55 percent.

Then if  $L^m$  denotes the male labor supply, we can express it as:

$$L^m = L^m(\text{age, qualification, } \frac{w(1-b)}{p_c}),$$

where  $(1-b)$  is a rate of benefit socials and  $p_c$  is the consumer price.

In the case of females, the same considerations we realize about of the young men can be made with the young women. But their participation in the labor market is less high. The female participation in Spain was a 30 percent in 1994. Then, we assume that the female participation is determined, moreover, for other factors: familiar situation, real wage and unemployment rate, though, these last factors, are of difficult empirical evidence.

The familiar situation will affect to young women with children, especially if the children are under 5 years. This perspective can require that the woman remains at home. The other factors: real wage and unemployment rate, wich are not considered in the male participation, affect to all women. Generally, a woman earns less than a man in the same position. In Spain, the porcentage can be close to 20 percent. Moreover, the women are more favorables to participe in the labor market when job prospects are better than when are worse. There are some women that prefer to stay at home if the unemployment is high.

Then, if  $L^f$  is the female labor supply we have, following these observations:

$$L^f = L^f(\text{children, qualification, } \frac{w^f}{p_c}, U),$$

where  $U$  is the unemployment rate, and  $w^f$  is the female wage.

Thus, the total labor supply will be:

$$L = L^m + L^f,$$

and  $U = \frac{L^D - L}{L}$ .

Note that a share of  $L$  will work or works in other sectors no business. Moreover, in  $L$  is contained the natural rate of unemployment  $U^*$ .

We assume that the behavior of the nominal wage follows a Phillips curve:

$$\frac{\dot{w}}{w} = \Pi^e + f(U - U^*) + \Phi ,$$

donde  $\Pi^e$  is expected inflation, the function  $f$  explains the influence of unemployment rate on real wage variation rate. So, if  $U = U^*$ , the function  $f$  is null, is positive if  $U > U^*$ . The opposite occurs if  $U < U^*$ . The parameter  $\Phi$  measures the annual average of marginal productivity of the labor. ( Maté 1994, p. 84, estimates that in Spain its value is next to 0.0424).

The output price equation, is obtained, from Rojo (1978, p. 398) assuming that the firms fit the output price considering mark-up and the production function is Coob-Douglas. So, its variation rate will satisfy the equation:

$$\frac{\dot{p}}{p} = (1 - \alpha)\left(\frac{\dot{w}}{w} - \Phi\right) + \alpha\frac{\dot{c}}{c} ,$$

where, all the variables and parameters have been defined before.

The output price variation rate is the inflation. If, we assume that the expected inflation follows a process of adaptatives expectatives, we have from Chiang (1987, p. 638) that its behavior will be:

$$\dot{\Pi}^e = \lambda(\Pi - \Pi^e) ,$$

where  $\Pi = \frac{\dot{p}}{p}$ , is the inflation and the parameter  $\lambda$  satisfies  $0 < \lambda \leq 1$ .

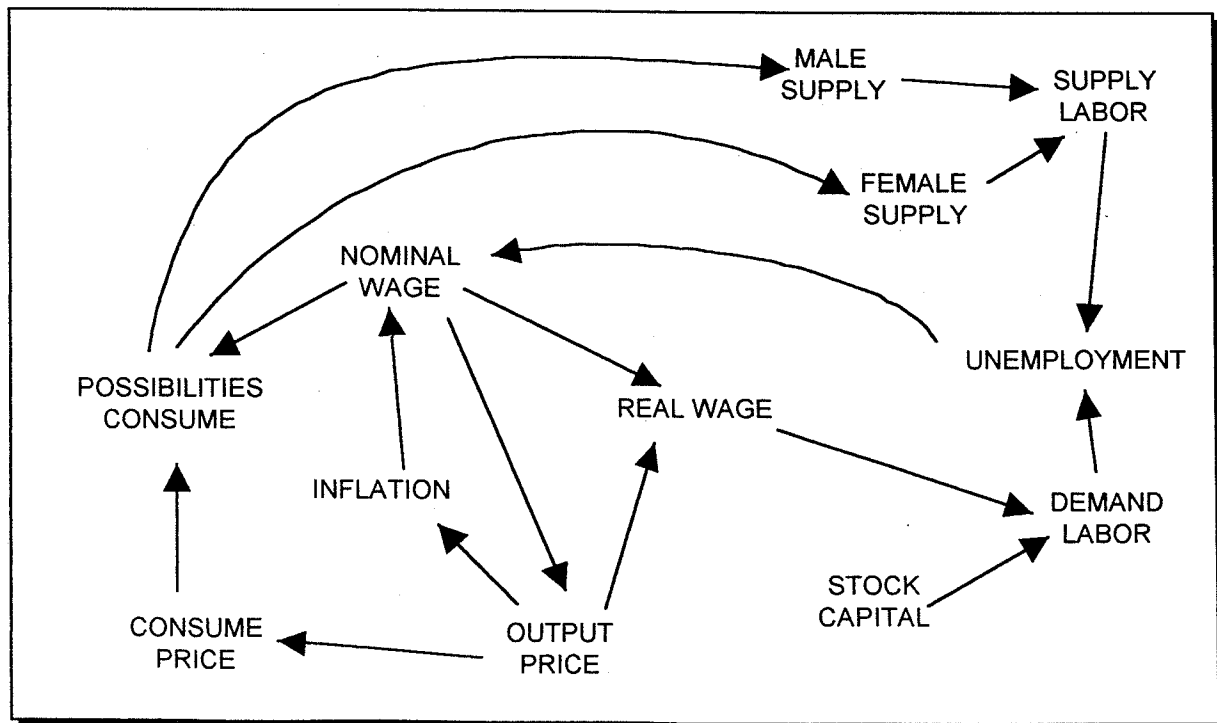


Figure 1. Causal Diagram

In all cases, the dot means the derivate of a variable over the time.

This last equation closes the model. A causal diagram between the variables more representatives is showed in figure 1.

## THE FLOW DIAGRAM

The flow diagram is based on the equations established in the above section, but it contains, moreover, aspects necessities to simulation. To obtain supply of labor, we classify the population aged 16 years to 65 years, by age. In this manner, we can determine the women and men that there are in each group of age annually. After, it is possible to determine the women and men that want to participate in labor market. We assume that the population grows a 3 percent per year and there is in it a 51 percent of women.

For the women, will be necessary detract the students (20 percent) and those that take care of children (20 percent). The rest of women, only a percentage of them will belong to female supply, because this group is affected by the rate of unemployment and by their wages. See figure 2. The influence of the unemployment is measured considering an average annual of unemployment rates. So, if  $U^A$  is this average, we assume that  $1 - \frac{U^A}{2}$  is the rate that affects to the women. In this manner, if  $U^A$  increases, the female participation decreases. The effect that on the participation have their wages, is obtained considering their possibilities of consume. If this last grows, the participation too.

For the men, we have two groups affected by percentages: the young and old men. For the young men, we also detract, as for the young women, the students. For the old men, we consider that a percentage leaves of the labor market by motives of voluntary retirement. This decision depends on the wage after retirement-consumer price ratio. Again, the possibilities of consume are fundamentals. A percentage analogous, as the women, is taked. We have assumed that the wage after retirement is 75 percent of the real wage in each moment and that the consumer price is 10 percent higher than output price. Figure 3 shows the flow diagram of these variables.

The demand for labor in business sector follows the equation of the above section. We assume that the capital is modified by depreciation (5 percent per year) and inversion (10 percent per year). The technological coefficient and the elasticities for labor and capital are considered constants and dimensionales. The value of the real wages is obtained from the evolution of the output price and nominal wage.

The demand for labor of the economy, we assume that is 4.2 times the demand for labor in bussines sector. This value is calculated like average for the last five years in the spanish economy. Comparing the supply and demand we have the unemployment rate of the economy. Figure 4 takes into account these considerations.

The last part of the flow diagram attends to determine the behavior of prices and wages. The evolution of output prices and nominal wages is conditioned by the unemployment rate

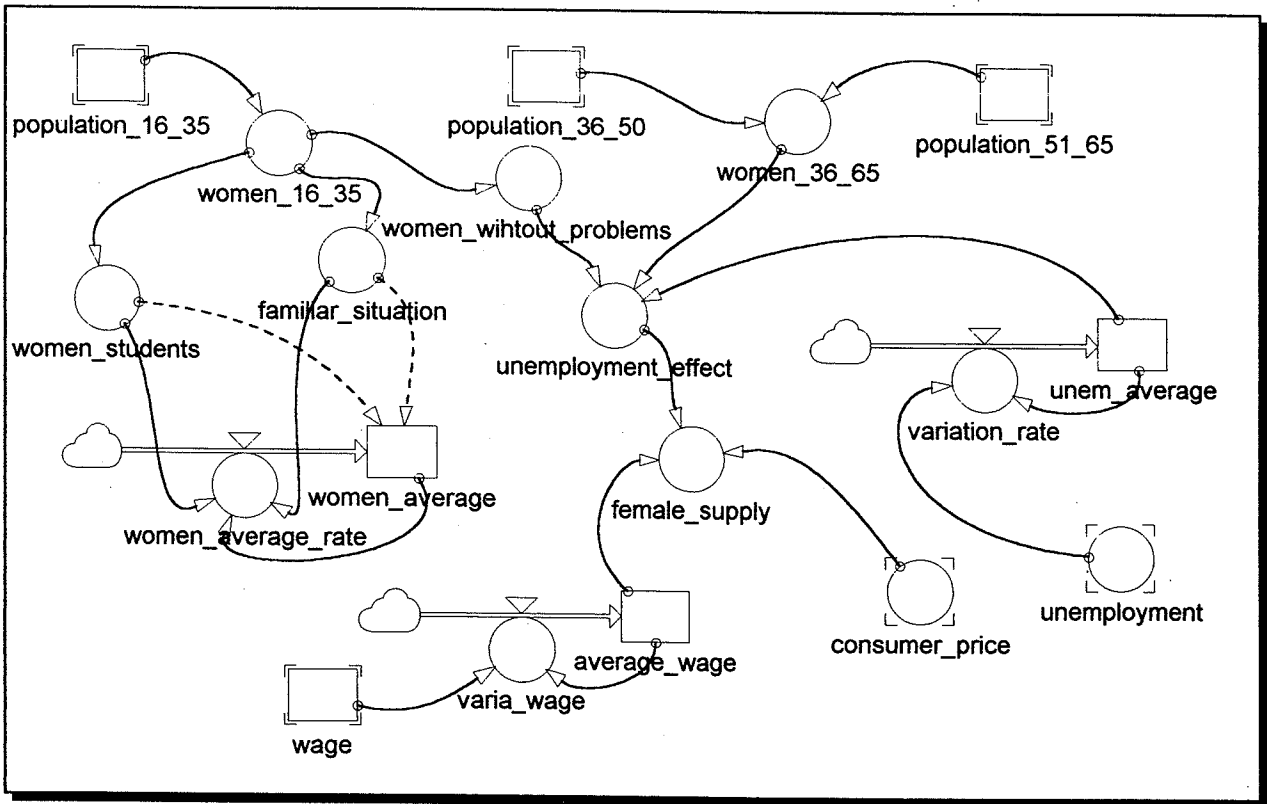


Figure 2. Female Supply

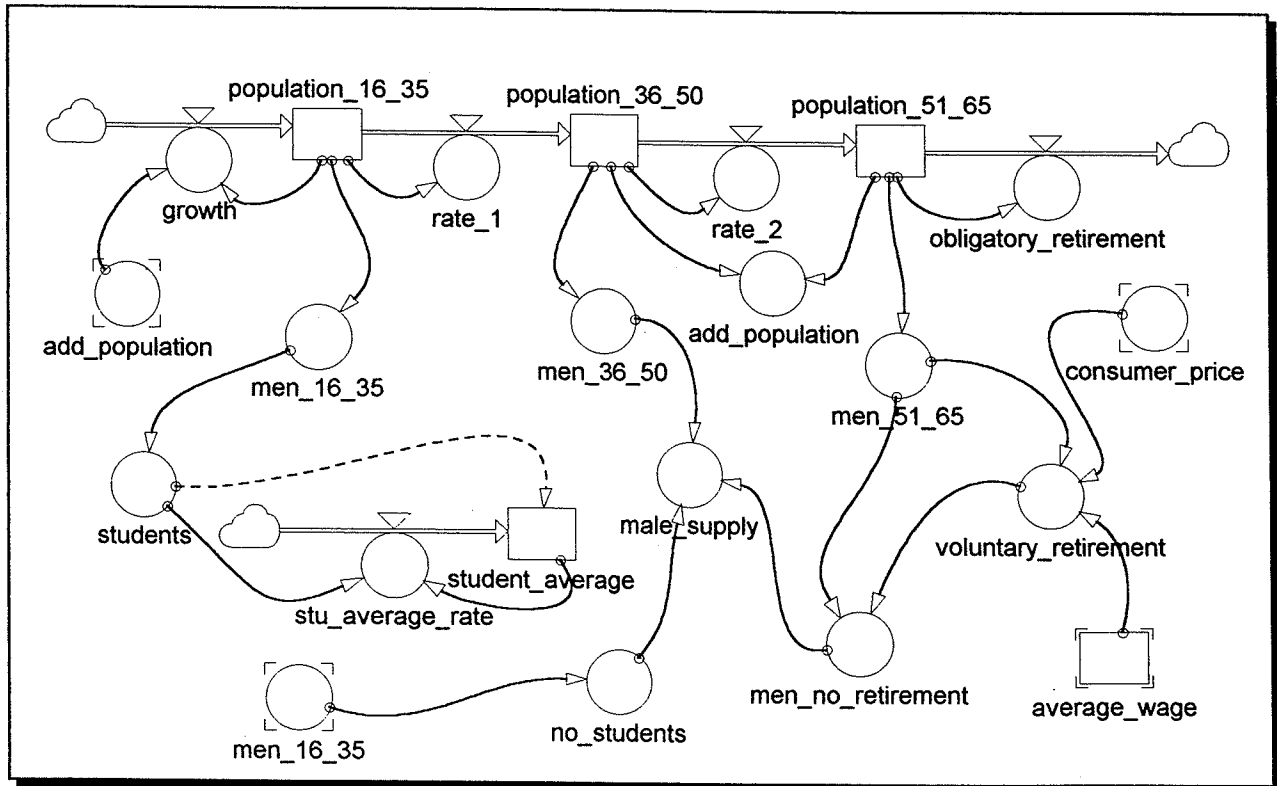


Figure 3. Male Supply

Parallel Program

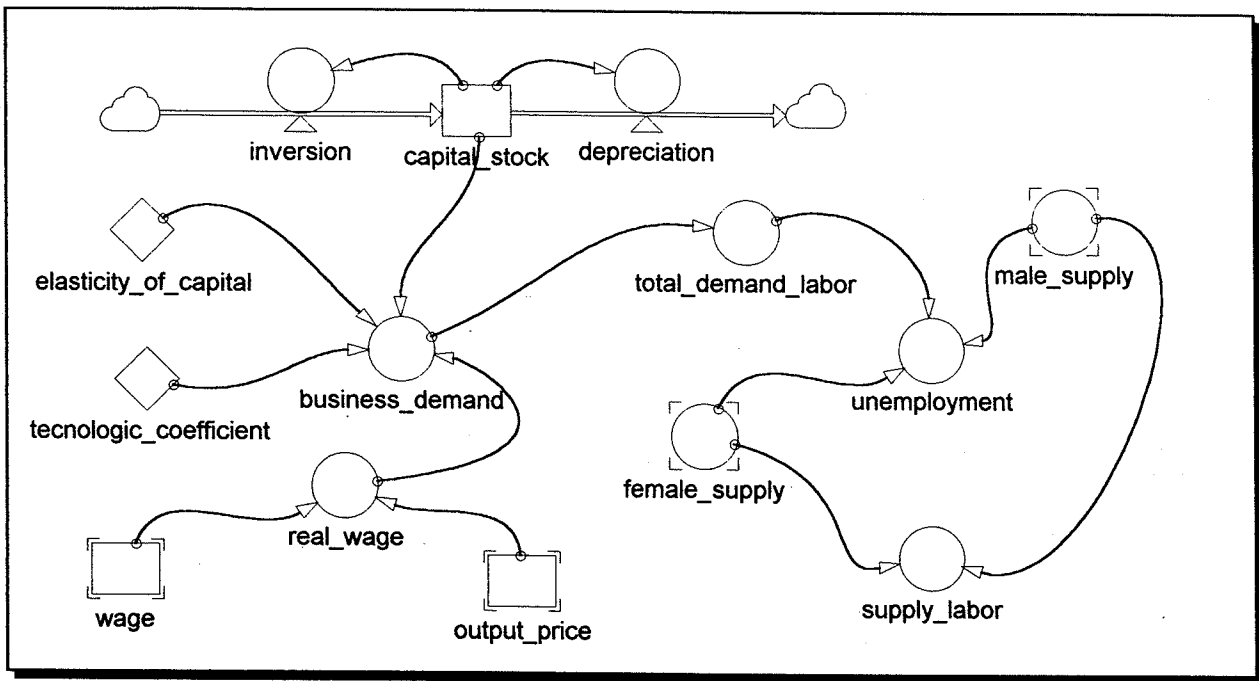


Figure 4. Demand of Employment

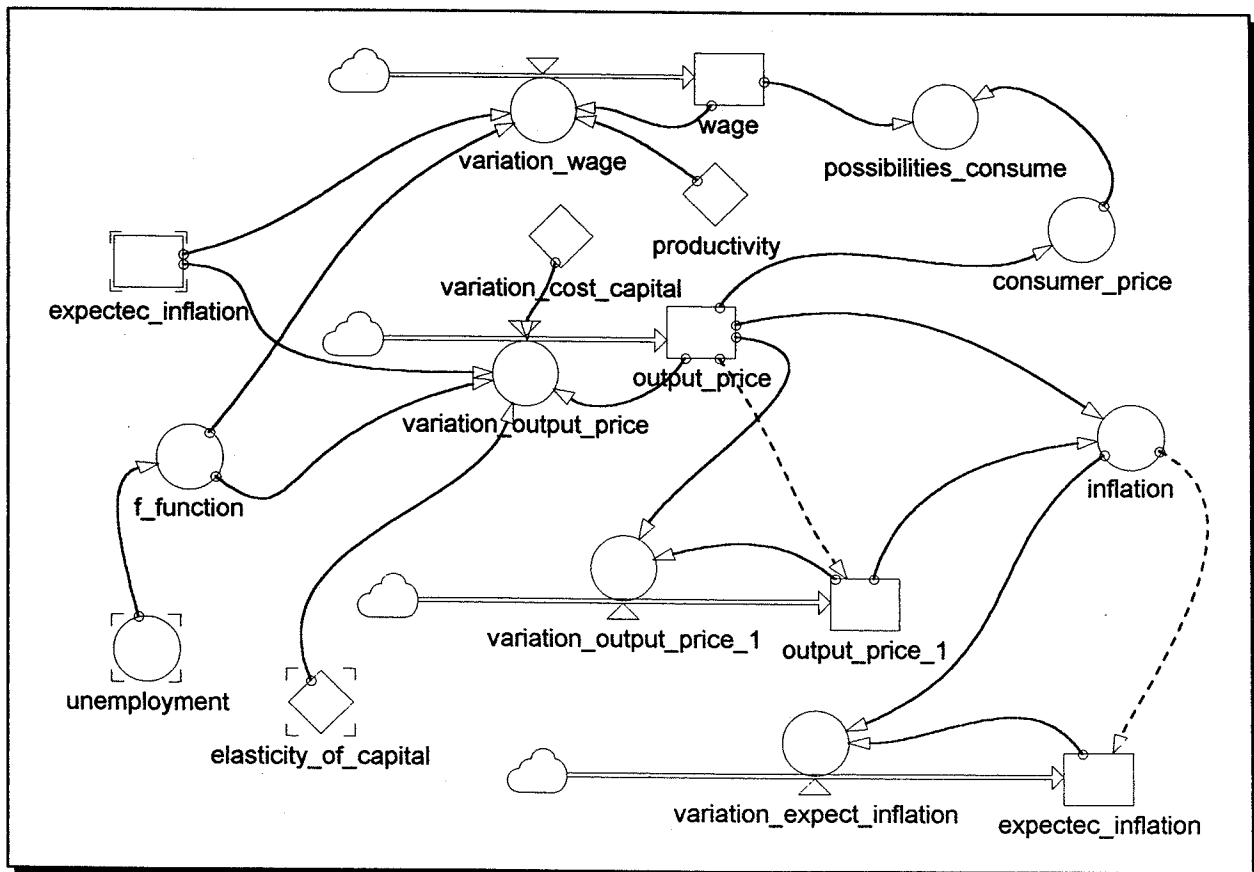


Figure 5. Formation of Prices and Wages

and expected inflation. That's why, in this part of the flow diagram, we take four levels. Wages and output prices that are affected by expected inflation and  $f$  function according of the Phillips curve. A delay about output price to obtain inflation and expected inflation, average of annual inflation. The definition of  $f$  function is obtained from Rojo (1978, p. 404). In this definition influences the rate natural of unemployment that we have considered between a 6 percent to 10 percent following to Maté (1994, p. 84). From this last author, we also have taked the value of productivity of labor, wich conditioned the evolution of wages directly. Figure 5 shows this development.

## RESULTS SIMULATION

To simulate the model, we have considered the next specifications: initial values for nivels

population 16-35	1250	people
population 36-50	1250	people
population 51-65	1250	people
capital stock	20000	ptas
wage	30	ptas/people
output price	3	ptas/unit

The rest of nivels are averages. Their initial values are chosen from the variables associated. We also have chosen as unit of time the year, Euler (fixed step) method to simulation and time step equal to 0.125 years.

The selected variables for simulation have been: female supply, male supply, supply labor, total demand for labor, inflation, unemployment, consumer price that is proportional in our model to output price and nominal wage-consumer price ratio that measures the possibilities of consume of the workers.

The building of the model is based on differents hypothesis. This suggests the possibility of analyce differents scenarios if we change some hypothesis. In this paper, we have studied the differents behaviors that the selected variables have when we modify the conditions of inversion. So, we porpose a change from 10 percent of the capital to 15 percent. Figures 6, 7 and 8 show the results, where the variables with an one additional correspond to 15 percent inversion.

In these results we can observe some differences on the behavior of the variables. As the female syppy as male supply grow. But, the former grows quicker if the inversion is higher, seeming that it shows convergence to male supply. The demand for labor grows in both cases. This fact, seems logical, because the demand of employment is proportional to the capital though it also is inversely proportional to real wage.

The inflation increases from a value next to 9 percent to 11 percent. However, the



## Parallel Program

unemployment rate decreases from 11 percent to 10 percent. The output price grows with both assumptions of inversion, but their rise rather faster when inversion is high. This fact, can be justified, by the increment of the use of capital. Moreover, the consumer prices and the nominal wages increase, but the formers grow higher than the last, then, the possibilities of consume grow, but slowly.

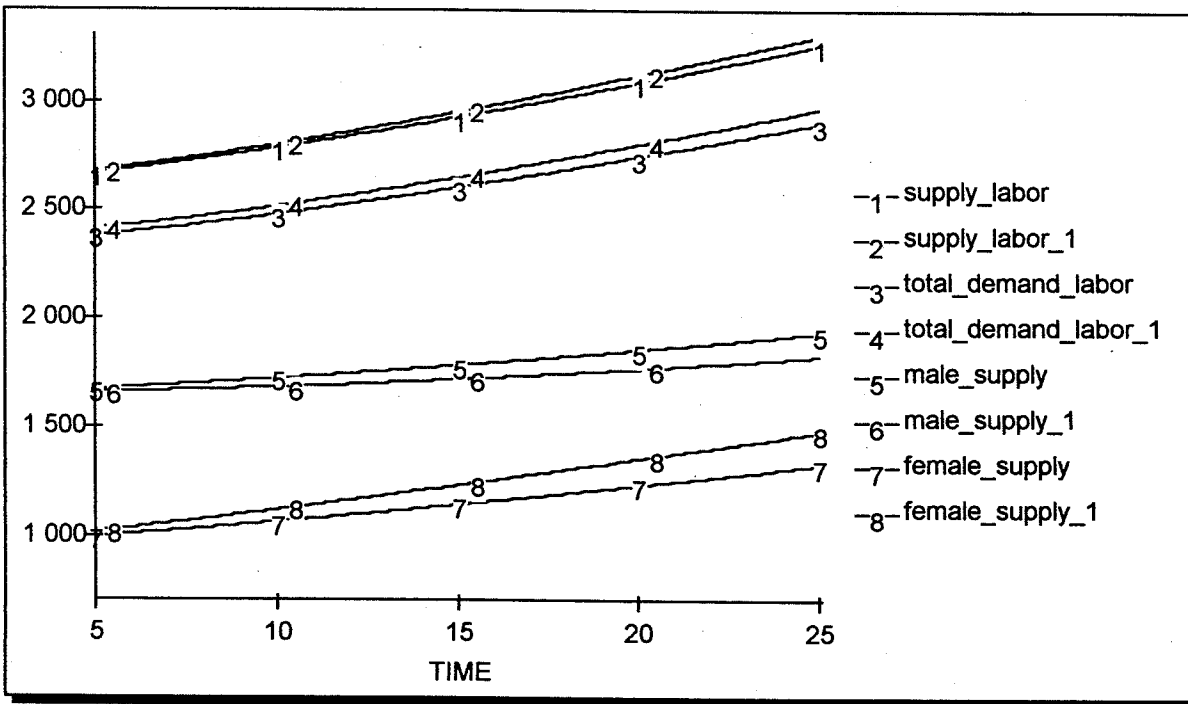


Figure 6. Demand and Supply of Employment

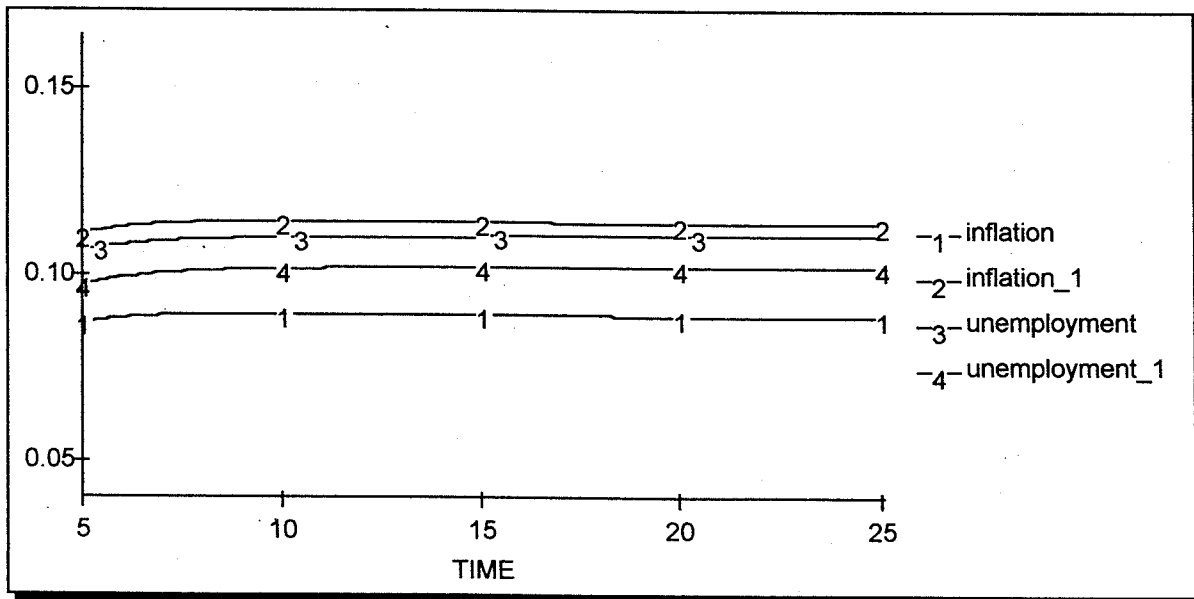


Figure 7. Inflation and Unemployment

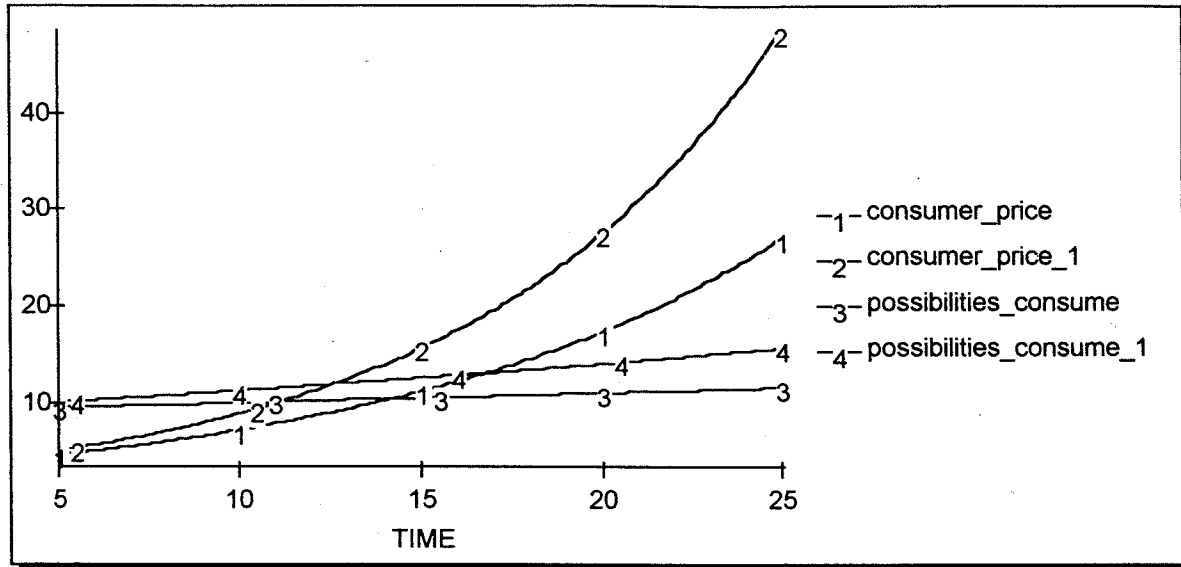


Figure 8. Prices and Possibilities of Consume

## REFERENCES

- Beenstock, M. and Warburton, P. 1988 "A neoclassical model of the U.K. labour market", in M. Beenstock, ed., *Modelling the Labour Market*. Chapman and Hall, 71- 104.
- Chiang, C.A. 1987 *Métodos Fundamentales de Economía Matemática*. Mexico: McGraw Hill.
- Maté, J.J. 1994 *Demanda, oferta y ajustes salariales en el mercado de trabajo español*. Valladolid: Universidad de Valladolid.
- Radzicki, M.J. and Sterman J.D. 1993 "Evolutionary Economics and System Dynamics", in E. Zepeda and J.A.D. Machuca, eds., *System Dynamics'93*. Cancun, 388 I - 388 XXVI.
- Rasmussen, S., Mosekilde, E. and Sterman J.D. 1985 "Bifurcations and Chaotic Behavior in a Simple Model of the Economic Long Wage". *System Dynamics Review*. 1 (1): 92 - 110.
- Rojó, L.A. 1978 *Renta, precios y balanza de pagos*. Madrid: Alianza Universidad.
- Saeed, K. and Radzicki, M.J. 1993 "A Post Keynesian Model of Macroeconomic Growth, Instability and Income Distribution", in E. Zepeda and J.A.D. Machuca, eds., *System Dynamics'93*. Cancun, 435 - 443.