USE OF SYSTEM DYNAMICS DIAGRAMMATIC TOOLS AS REPRESENTATION SCHEMES FOR LINEAR PROGRAMMING MODELS

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ABSTRACT

The paper suggests that the diagrammatic tools available in the system dynamics methodology are very useful to build external representation schemes for linear programming models. The paper further suggests that these tools can be used even for problem conceptualization and model building. An example is cited to demonstrate the power of the system dynamics diagrammatic tools to conceptualize the problem, fix the model boundary, and define the decision variables and the constraints.

1. Introduction

A recent paper by Murphy et al. (1992) highlights the difficulties that are generally faced during the phase of formulation of linear programming models. The paper presents eight different approaches that are generally followed to represent these models and compares these approaches using a common example taken from Schrage (1987). The paper emphasizes graphic representation schemes that take into account limited cognitive capability of human mind.

In their paper Murphy et al. are concerned with rigorous conceptual framework for problem formulation and the mapping between real world objects and relationships (p. 966). This process, they say, is painful even for experts. They refer Shneiderman (1987) to emphasize the help the external representation schemes provide in visualizing the real world in concrete terms and thereby facilitating the generation of correct models.

Murphy et al. undoubtedly address a very important problem. But this problem is only one aspect of the real mega-problem that the novice and even the expert management scientists face. This is the problem of conceptualizing the real-life problem in all its dimensions. The issues in problem conceptualization are much deeper than that visualized by the authors. The issues are with regard to capturing the values of the stakeholder, visualizing the nature of the problem, identifying the boundary within which the problem can be defined, recognizing the major sectors and their interactions that are relevant in the context of the problem, delineating the cause-effect relationships, defining the decision variables. and recognizing the constraints that are imposed both from within and outside the boundary of the system. The issues also include the choice of the plan horizon, the choice with regard to a static or a dynamic model, and with regard to the linearity or the nonlinearity of the constraints and the objectives.

A problem associated with any management science model is its validity. While the solution procedure of a management science model more or less guarantees its internal consistency (validity), the problems associated with the identification of the real-life problem, the choice of the type of the model, the value system as represented by the objective function(s), the selection of the decision variables and the constraints have not received their due share of attention. Therefore, though the management science models follow precise, unambiguous, objective and well-documented solution procedures, conceptualization of real life problems, the appropriateness of a particular model, and the actual construction of a model are highly subjective.

In contrast, the simulation approach of management science lays heavy emphasis on the issues related to problem conceptualization and model validity (for example, see Naylor et al. 1966). During the last three decades, system dynamics has shown great promise of a methodology whereby real-life problems can be easily conceptualized. The various diagrammatic tools that are popularly used in system dynamics, are excellent problem conceptualization tools. This paper demonstrates the power of these system dynamics diagrammatic tools as external representation schemes for linear programming problems for both problem conceptualization and model representation. Schrage's (1987) problem of growing wheat and/or corn and raising pig and/or hen, that was used by Murphy et al., is used here for this purpose.

2. System Dynamics Framework for LP Problems

A question that comes naturally to mind is: how can a dynamic model represent a static problem? To answer this question, we first note that a static problem is a single period case that tacitly assumes that the decision variables are fully realized in a single time period. These decision variables usually reflect physical flows. Taking examples from the problem used by Murphy et al., hiring of labour can be seen as a flow of man, buying and selling of wheat or corn as a flow of material, buying and selling of pig or hen as a flow of livestock, using existing land for cultivation of corn as a flow of production capacity, and so on. Thus the decision variables in a static problem can be seen as flow variables in physical flows of system dynamics models.

No realistic decision variable in a linear programming problem is allowed to take negative values. This property holds also for flow variables in system dynamics models.

There is another similarity between the managerial decisions and the system dynamics flow variables. Both of them attempt to achieve certain explicitly or implicitly defined goals.

Linear programming models consider both equality and inequality constraints. It is also possible to find their equivalence in the system dynamics models. Equality constraints arise due to two reasons. First, certain decision variables are inadvertently related to other decision variables and/or to state variables. Second, the decision variable values are to be so chosen as to achieve the intended goals within the stipulated single time period.

The inequality constraints arise from the consideration that system dynamics models do not allow the level variables to be negative. However, outflow variables can, theoretically, cause negative values of the level variables from which they emerge. Since a negative level value is not permitted, the total outflow from such a level variable has to be constrained by its initial value.

It is to be noted that in a static model, all decision variables are realized in a single period and that the model does not distinguish between the start and the end of a period. In the context of a static model considered in a dynamic setting, therefore, one has to consider the total accumulation of the inflow and the outflow variables to take place within a single time period while writing down the constraints.

3. The Sample Problem

We now attempt to build system dynamics representation schemes for the farmer's sample problem, that was taken from Schrage (1987) and used by Murphy et al. (1992).

The Overview Diagram

To begin with, we try to define the boundary of the system and the major inflows and outflows with regard to this boundary. We understand that the farmer grows crops and livestock. Therefore, the model must have two sectors, one each for the crops and the livestock. The farmer acquires five types of input resources from the input resource market sector. They are the labour, the wheat seed, the corn seed, the wheat and the corn. Therefore these resources constitute the inflows into the model system. The farmer sells wheat, corn, pig, and poultry to the product market. Therefore, they constitute the outflows from the model system.

Having identified the inflows and outflows that take place across the model boundary, we now look inwards into the interior of the model boundary. As discussed earlier, the model has two sectors: crops and livestock. We note that crops are required as feed for livestock. Therefore wheat and corn must flow from the crops sector to the livestock sector. Going deeper into each sector, we note that land and labour are the most important resources in the crops sector while floor space is the most important resource in the livestock sector. These sectors of course use resources which flow into them from either the environment or the other sector. Figure 1 is an overview diagram of the problem under consideration. It depicts the model boundary, the major flows that take place across this boundary, and the intramodel details.

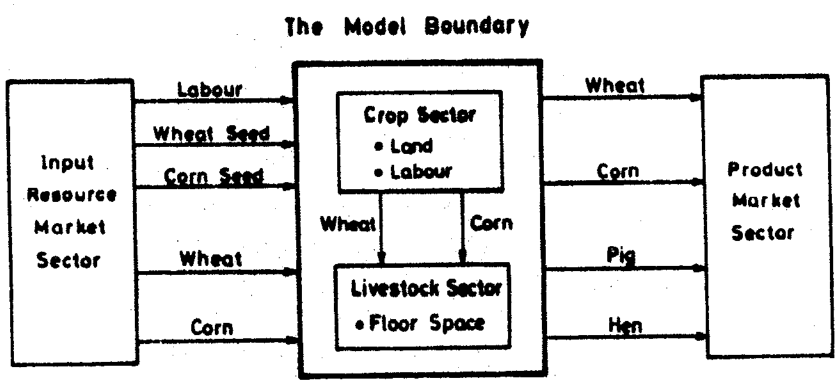


Figure 1 : The Overview Diagram for the Farmer's Problem

The Decision Structure Diagrams

We now concentrate on the individual physical flows. The overview diagram has already indicated that the major physical flows occur for labour, wheat, corn, pig, and hen. There are two additional flows: flows for the land and for the floor space. Although no inflow from environment takes place for either of them, these resources get diminished as they are utilized for cultivation and livestock rearing respectively. We now construct the 'policy structure diagram' associated with each of these flows. We may rename them as 'decision structure diagrams' since they relate to single shot decisions rather than to streams of decisions.

Figure 2 is the decision structure diagram for hiring labour. The valve depicts the flow (decision) variable 'Hire Labour (HL)'. It shows that labour hiring decision depends primarily on the discrepancy between the required labour and the available labour with the farmer. However, since extra time of the farmer is expended on supervising the hired labour, there is an information flow also from 'Labour Hired'. Figure 2 also shows that computation of required labour depends on land for corn and wheat, number of pig and hen raised, and harvested amount of corn and wheat.

A detailed flow diagrammatic representation of the labour hiring decision is presented in Figure 3. This diagram shows, in addition to the variables shown in Figure 2, constants (or conversion coefficients) that define 'Required Labour'.

We note here that 'Required Labour ' and 'Labour Hired' have the dimension (Hours of Labour). But the decision variable 'Hire Labour' has the dimension (Hours of Labour/unit time). Since the time period under consideration is one year, the dimension of 'Hire Labour' is (Hours of labour /year) and the time constant associated with this hiring decision has the value 1 and the dimension (Year). Thus 'Hire Labour' is numerically equal to the discrepancy between 'Required Labour' and 'Available Labour'. Again, since the single period assumption makes the decisions

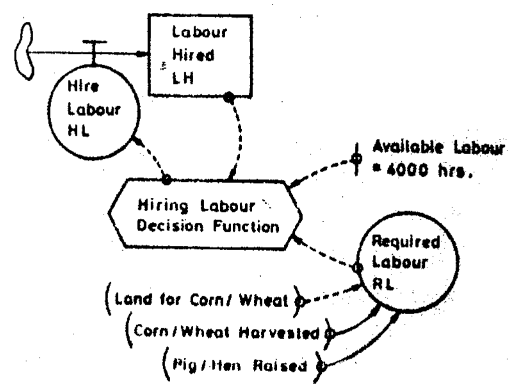


Figure 2 : Decision Structure Diagram for Hiring Labour

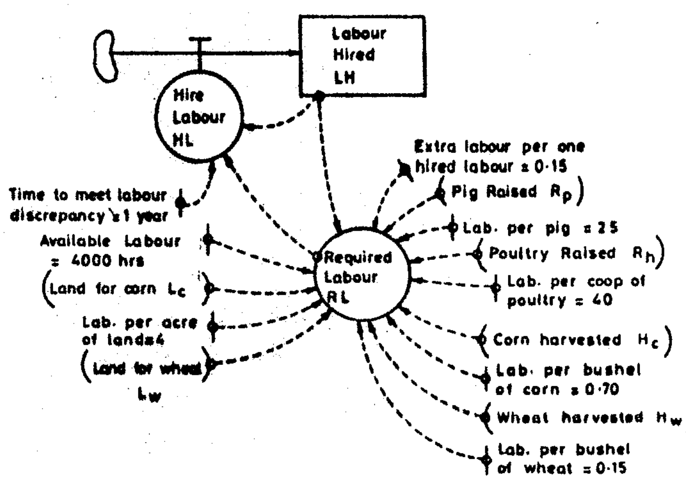


Figure 3 : Flow Diagram for Labour Hiring

We note here that 'Required Labour ' and 'Labour Hired' have the dimension (Hours of Labour). But the decision variable 'Hire Labour' has the dimension (Hours of Labour/unit time). Since the time period under consideration is one year, the dimension of 'Hire Labour' is (Hours of labour /year) and the time constant associated with this hiring decision has the value 1 and the dimension (Year). Thus 'Hire Labour' is numerically equal to the discrepancy between 'Required Labour' and 'Available Labour'. Again, since the single period assumption makes the decisions realizable in the same period, 'Hire Labour' is numerically equal to 'Labour Hired', although they are different dimensionally.

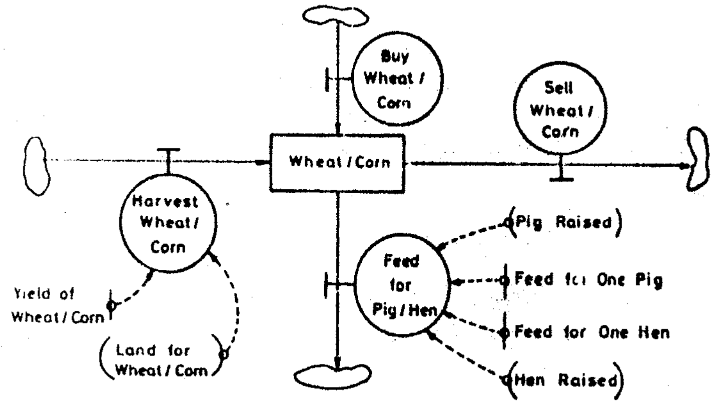


Figure 4 : Decision. Structure Diagram for Wheat / Corn

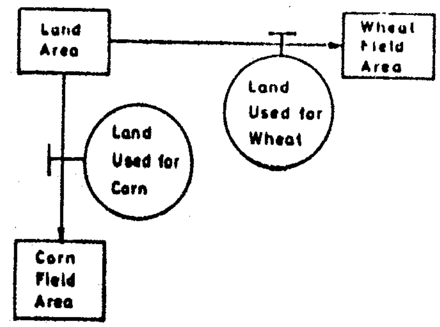
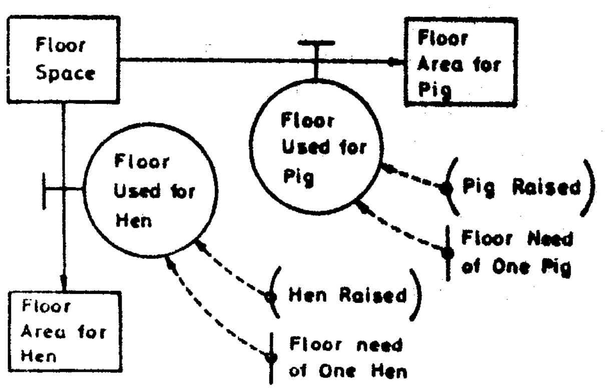


Figure 5 : Decision Structure Diagram for Land Use

Figure 4 is the decision structure diagram involving wheat/corn. Here two inflows to the level of wheat are 'Buy Wheat/Corn' and 'Harvest Wheat/Corn'. The two outflows are 'Sell Wheat/Corn' and 'Feed for Pig/Hen'. Harvest depends on land and the yield, while feed depends on pig and hen raised and the feed for unit livestock.

Figure 5 is a decision structure diagram involving land use. The two decisions are land to be used for cultivation of wheat and that for cultivation of corn. They are outflows form Land Area and they accumulate into 'Wheat Field Area' and 'Corn Field Area' respectively.

Figure 6 is the decision structure diagram involving floor space use. The two decisions are with regard to the use of floor space for pig and for hen. These decisions depend on the number of pigs and hens raised respectively. These decisions are represented by outflows from 'Floor Space' and accumulate into levels 'Floor Area for Pig' and 'Floor Area for Hen' respectively.



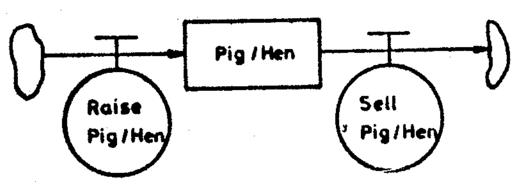


Figure 7 : Decision Structure Diagram for Raising and Selling of Pig / Hen

Figure 6 : Decision Structure Diagram for Floor Space Use

Figure 7 is the decision structure diagram for raising and selling of pig/hen. The level indicates the net pig/hen population. As the pigs/hens are raised, their populations rise. As they are sold, their populations fall.

The Influence Diagram

Figure 8 portrays the influences that exist among the variables in the sample problem.

When land for wheat increases, the harvest of wheat is also increased. When wheat is sold, stock of wheat is reduced.

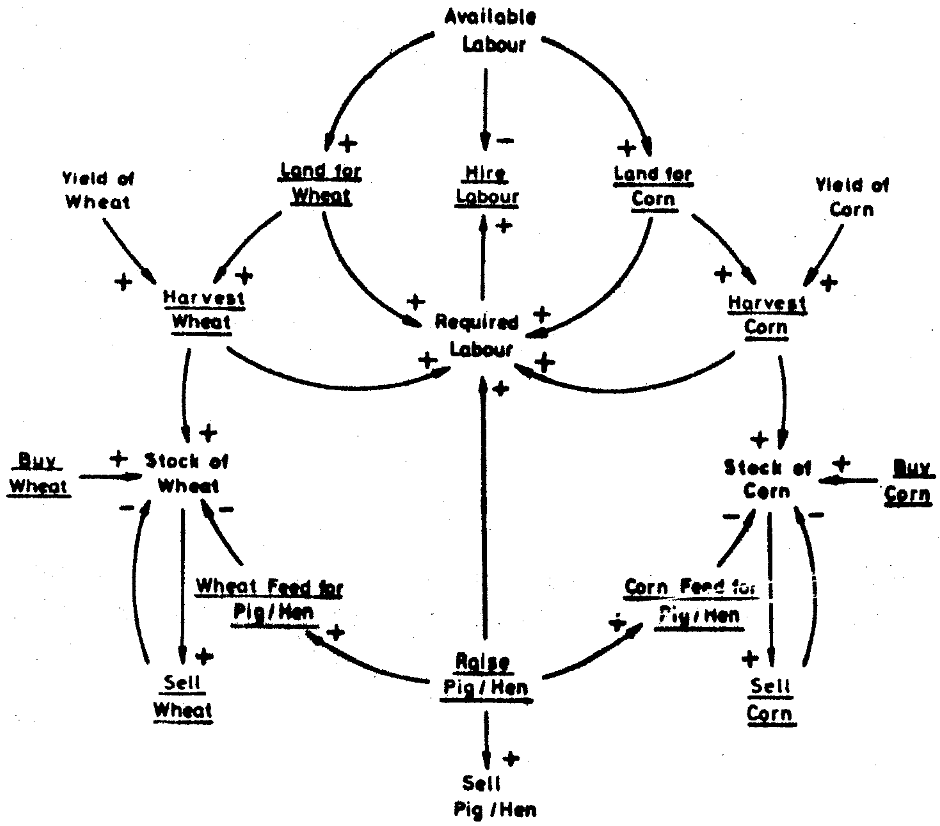


Figure 8 : The Influence Diagram for the Farmer's Problem

Figure 8 depicts cause-effect relationships among pertinent variables in the farmer's problem. This influence diagram brings together the various component cause-effect relationships that were depicted in different forms in the decision structure diagrams (Figures 2 through 7)

4. Formulating the Linear Programming Model for the Farmer's Problem

The Decision Variables

The flow variables appearing in the decision structure diagrams are the decision variables. They are the following:

HL (Hire Labour, Hours per year), BW (Buy Wheat, Bushels per year), BC (Buy Corn, Bushels per year), HW (Harvest Wheat, Bushels per year), HC (Harvest Corn, Bushels per year), FHW (Feed used for Hen as Wheat, Bushels per hen per year), FHC (Feed used for Hen as Corn, Bushels per hen per year), FPW (Feed used for Pig as Wheat, Bushels per pig per year), FPC (Feed used for Pig as Corn, Bushels per pig per year), SW (Sell Wheat, Bushels per year), SC (Sell Corn, Bushels per year), LW (Land for Wheat, Acre per bushel per year), LC (Land for Corn, Acre per bushel per year), FH (Floor space for Hen, Acre per hen per year), FP (Floor space for Pig, Acre per pig per year), RH (Raise Hen, Hens per year), RP (Raise Pig, Pigs per year), SH (Sell Hen, Hens per year), SP (Sell Pig, pigs per year).

Thus we see that there are nineteen decision variables. But we observe that certain decision variables are linearly related to each other. Therefore one can reduce the number of decision variables. For example, it is implicitly assumed in the problem statement that all the pigs or hens that are raised are sold. So RH = SH, and SH = SP (see Figure 7). In figure 6 we notice that the floor spaces used for hen (FH) and for pig (FP) are directly proportional to the hens and pigs raised (RH and RP). In figure 4 we notice that harvest of wheat/corn (HW/HC) is dependent linearly on land for wheat/corn (LW/LC). Thus, it is possible to exclude SH, SP, FH, FP, HW, and HC from the list of decision variables. We have however retained variables HW and HC in our list of decision variables just to indicate that any redundancy in terms of the definition of the decision variables will not affect the results.

We may comment here that all the decision variables are also present in the influence diagram (Figure 8). They are underlined to distinguish them from other variables (levels and auxiliaries) and parameters of the system.

The Constraints

Land

From the decision structure diagram for land use (Figure 5) it is obvious that the total land to be used for wheat and for corn during one year must not exceed the available land area of 120 acres:

$$(LW + LC) * 1 \quad <= \quad 120$$
$$\text{Or, } LW + LC \quad <= \quad 120 \qquad \ldots (1)$$

Labour

Figure 2 gives the decision structure diagram for the hiring of labour. During the time period of one year under consideration, the farmer must hire at least the labour required in excess of the available labour of 4000 hours:

$$(HL) * 1 \quad >= \quad (\text{Required Labour - Available Labour})$$
$$\text{or, } HL \quad >= \quad \text{Required Labour - 4000.}$$

From the detailed flow diagram (Figure 3), Required Labour is defined as

$$\text{Required Labour} = 4 * (LC+LW) + 0.15 * HW + 0.7 * HC + 40 * RH + 25*RP + 0.15 * HL$$

Using this expression for required labour in the inequality (1) for hiring labour, one obtains the labour constraints as:

$$- 0.85 * HL + 4*LC + 4*LW + 0.15*HW + 0.7*HC + 40 * RH + 25 * RP <= 4000 \qquad \ldots (2)$$

Wheat

It is implicitly assumed that all the wheat that is harvested and bought is used as feed for pig or hen and the rest is sold (Figure 4) leaving no stock at the end of the year. Thus the inflows to the level in Figure 4 must equal the outflows:

$$HW + BW = SW + FHW + FPW$$

Or, $HW + BW - SW - FHW - FPW = 0$ ... (3)

Corn

Making considerations similar to those for wheat, Figure 4 also helps in writing the following balance equation for corn:

$HC + BC - SC - FHC - FPC = 0$ ... (4)

Floor space

From Figure 6 it is obvious that floor space used for hen and pig in a period of one year must not exceed the available floor space of 10,000 sq. ft.:

$$(25 * RP + 15 * RH) * 1 <= 10,000$$

or, $25 * RP + 15 * RH <= 10,000$ ... (5)

Equality Constraints due to Direct Relationships between Decision Variables:

We recall that we have included LW and HW, and LC and HC. Their linear relationships are therefore to be expressed as equality constraints (Figure 4):

$$HW = 55 * LW$$
$$HC = 95 * LC$$

or, $HW - 55* LW = 0$ ... (6)

$HC - 95 * LC = 0$ ... (7)

Constraints due to 'OR' Type Relationships

There are also relationships between the animal feeds and the animals raised. The relationships are however not direct, but simultaneous, due to the presence of logical 'OR' type relationship. This requires a special type of treatment to unearth the constraints.

Hen Feed

A hen takes either 10 bushels of wheat or 25 bushels of corn. If FHW bushels of wheat and FHC bushels of corn are fed to hens, then the number of hens who feed on wheat is (FHW/10) and that feeding on corn is (FHC/25). Since all the hens are to be fed, these two numbers must add up to the number of hens raised:

$$0.1 * FHW + 0.04 * FHC = RH$$

So, $0.1 * FHW + 0.04 * FHC - RH = 0$ ... (8)

Pig Feed

Making considerations similar to those for hen-feed, one obtains the following equality constraint for the pig-feed:

0.04 * FPW + 0.05 * FPC - RP = 0 ... (9)

Objective Function

Figure 9 depicts a flow diagram for computation of profit. Profit is taken as a level variable. Inflows to profit occur due to sale of wheat and corn and of pig and hen. Outflows from profit occur as payments are made against purchase of seed for wheat and corn, purchase of wheat and corn, and against hiring of labour. Dependence of flow variables on various other variables and the associated constants is also shown in Figure 9.

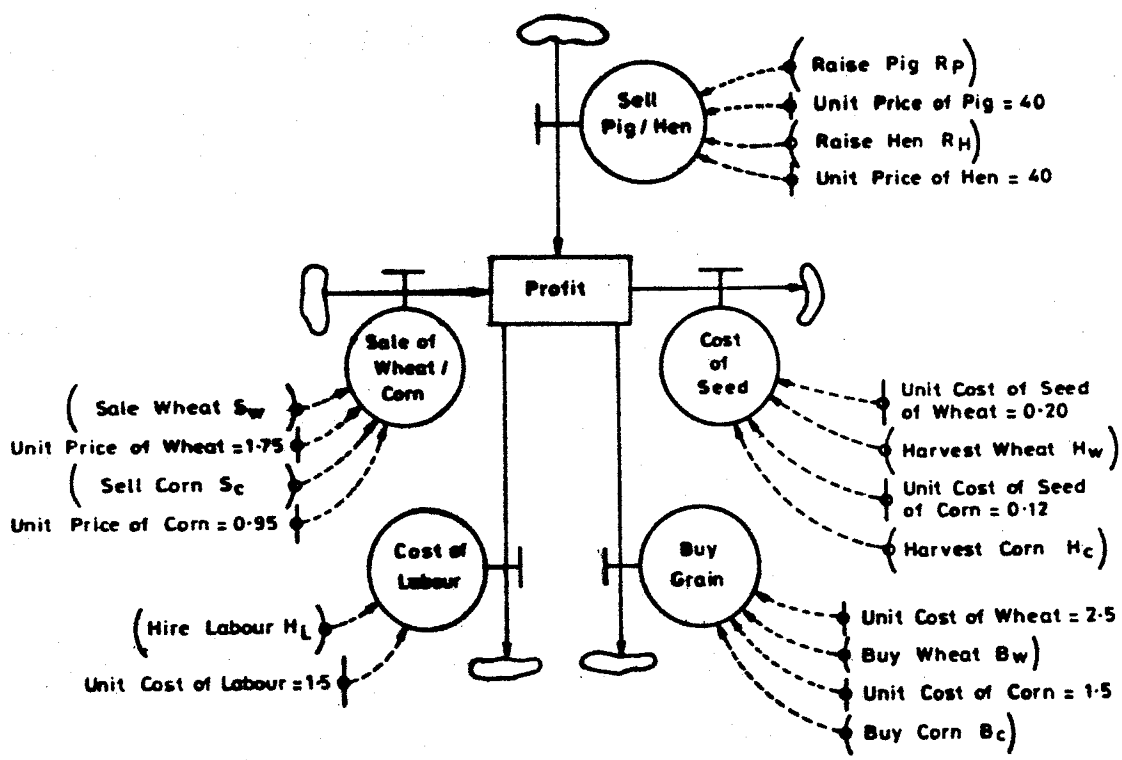


Figure 9 : Flow Diagram for Profit

As before, the computation of the level variable 'Profit' has to be done by considering the accumulations due to inflows and outflows during a period of one year. So the objective function is given by

$$\text{Profit} = 1.75 * SW + 0.95 * SC + 40 * RP + 40 * RH - (0.20 * HW + 0.12 * HC) - (2.5 * BW + 1.5 * BC) - (1.5 * HL) \quad \dots (10)$$

The LP problem for the farmer's problem is then given by the objective function (10) which is to be maximized subject to the constraints given by (1) through (9). The decision variables are also to be nonnegative.

The initial LP tableau for the Farmer's problem can now be constructed. On comparing our LP tableau with that given by Murphy et al., we notice, as expected, that we have two more decision variables (corresponding to land used for each of the two grains) and two more constraints (depicting the two land-grain relationships). The advantage of choosing a higher dimension of the problem is that it has eased the problem of computing the technological coefficients for the land and the labour constraints.

5. Conclusions

This paper is a sequel to the paper by Murphy et al. (1992) that discusses various representation schemes for linear programming models. The paper advances graphic aids available in system dynamics methodology to both conceptualize a problem and formulate it in a linear programming format. The sample problem that was used by Murphy et al. is used here to demonstrate the utility of these graphic aids. We feel that these aids are extremely effective in understanding the problem context, recognizing major sectors and their interrelationships, delineating cause-effect relationships, and, most importantly, guiding the fixing of the model boundary, the choice of decision variables, and the formulation of the constraints and the objective function.

We would like to stress here that external representation schemes based on system dynamics graphic aids can help in conceptualizing the real problem in all its entirety, fixing the model boundary, deciding on the decision variables, and even in deciding on the modeling methodology to be adopted for the problem. We therefore suggest that representation schemes should also be judged in terms of additional dimensions such as problem conceptualizing power and modeling guidance, in both of which system dynamics graphic aids undoubtedly score very high.

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