

Optimal Control Modeling with Vensim: Applications to Public Finance.

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Abstract

The purpose of this paper is to improve the results obtained by Fernández and Mantel (1989), referred to the price control inconvenience in the application of a stabilization plan for Argentina's hyperinflation in the Eighties. First, the model's correct translation is checked, by replication of the original experiments. Secondly, an intuitive policy suggested by the shape of the original experiments, that proposes a timely starting of the original price control policy is tested, achieving better results. Finally, an optimization process, currently available in Vensim programme, penalizes, on the one hand, the oscillations which are shown by the path that the instantaneous inflation follows to reach the equilibrium inflation rate; on the other hand, the slowness to reach that equilibrium point. Vensim's advise nearly matches previous intuitive timing for starting to control prices.

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1. Introduction.

Nowadays the available software can force a simulated system to follow trajectories that optimize objective functions. Vensim's technical abilities to optimize allow the examination of the effects of imposing optimal policies on endogenous processes. It is pertinent to experiment with the tool, while analyzing the consequences for the traditional style of simulation experiments in System Dynamics.

2. The model.

For the purpose announced in the introduction a small model is used, referred to the ineffectiveness of price controls when it is required to stabilize inflationary processes (Fernández and Mantel, 1989). The model, built upon two state variables, the rate of inflation " π " and the market interest rate " i ", is highly non-linear. The present section presents a rigorous translation of the original model into System Dynamics terms.

2.a. Government budget restriction.

The sources for financing the government deficit are real monetization, " dm/dt ", that is to say the rate of change of real money balances, " m ", over time, " t ", and the inflation tax. The deficit includes besides the difference between expenditures and taxes the expenses incurred in servicing the external debt, assumed to be constant. The international inflation rate is assumed to be zero, whereas the rate of devaluation of the exchange rate equals the domestic inflation rate. The service of the internal debt, " bxr ", is included as a separate term to take into account the effect of the real interest rate, " r ". Also the action of the "Olivera-Tanzi" effect, due to tax collection lags, is explicitly considered by the product " $\lambda\pi$ " of the impact intensity " λ " times the rate of inflation " π ". Note that in effect this reduces the revenue from the inflation tax to " $(m-\lambda)\pi$ " due to the fiscal lags in tax collection. Equating the sources of financing to these concepts of the deficit, one has the government's budget restriction

$$\frac{dm}{dt} + m\pi = d + br + \lambda\pi \quad (1)$$

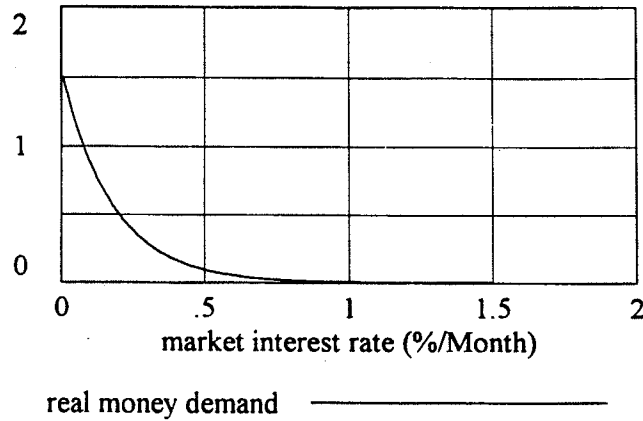
2.b. The money market.

The model incorporates Cagan's hypothesis on the behavior of cash balances, according to which during a hyperinflation, the change in the yield of money is the most important factor that explains the amount of cash that people desire to hold (Cagan, 1956). The following equation 2 introduces the demand of real money balances as a function of the market interest rate, " i ". It is a behavioral equation, where " β " means the semi elasticity of demand of money with respect to changes in nominal interest rate:

$$m = qe^{(-\beta i)} \quad (2)$$

In figure 1 the proposed static relationship between the market interest rate and real money balances can be seen. The holdings of real money " m ", meaning the currency plus current account deposits "M1" as a fraction of GDP, falls exponentially as the nominal interest rate decreases.

Fig. 1 Cagan's hypothesis



Tue Mar 15, 1994 8:56AM

It would be ordinary System Dynamics' practice to model the real money supply "m" as a state variable regulated by a controlling flow, as suggested by equation 1. The latter would be ruled by the discrepancy between money demand and supply. However, as can be expected in hyperinflationary circumstances, the money market clears almost instantaneously. Therefore, instead of following that line of thought, the model merges both variables into a single variable "m", omitting the corresponding loop. This variable represents not only the demand for real money, "m^d", but also the stock of money holdings, "m^s", to which the inflation tax rate "π" is to be applied to compute the tax, "mπ". The model assumes that the actual inflation rate is also the expected rate. This reduces Fisher's equation for the arbitrage between money and bonds to

$$i = r + \pi \tag{3}$$

Inserting these static equations 2 and 3 into the government's budget equation, equation 1, results one of the reduced form dynamic equation, controlling the accumulation of real money balances "m",

$$\frac{dm}{dt} = d + b \frac{\log(\frac{q}{m})}{\beta} - (m + b - \lambda) \tag{4}$$

The money differential equation 4 organizes, in control terms, a loop presented in the upper sector of figure 2. It regulates the state stock of real money holdings. The monetization real rate "dm/dt" secures that the monetization level of the economy will be sufficient to cover that portion of the expenditures not satisfied with the normal tax collection.

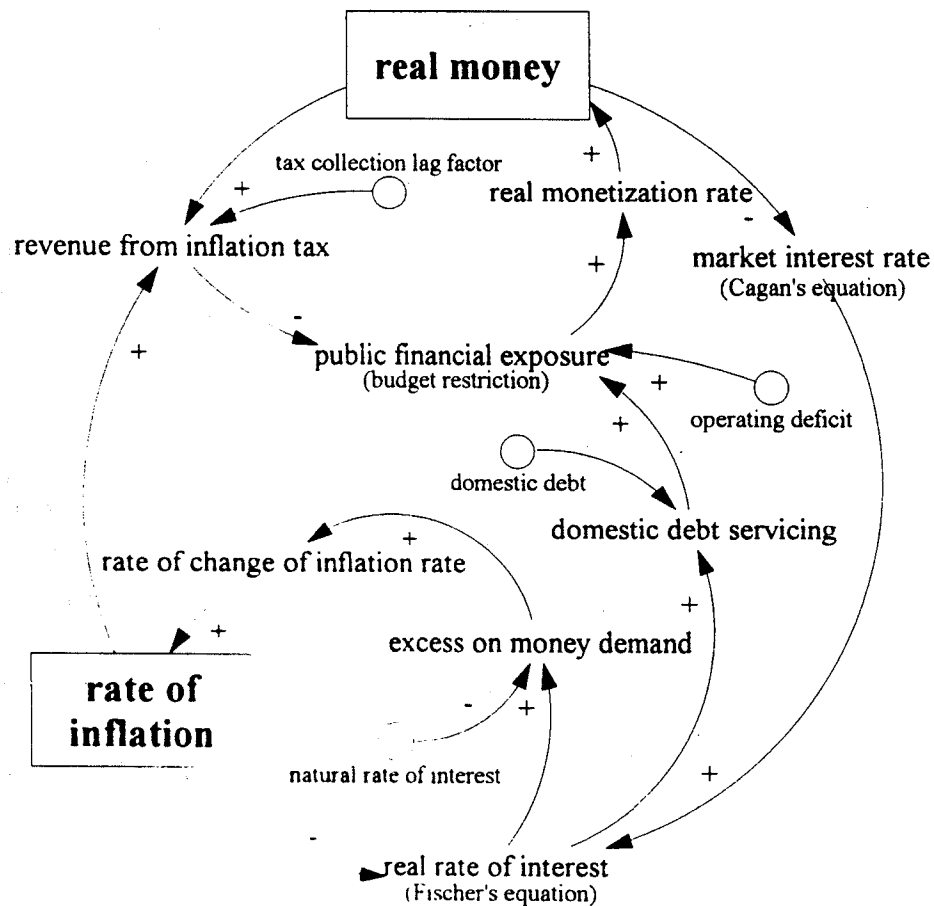
The other nucleus of the model is the inflation rate "π". Modeling the inflation rate as a state variable is inspired in Wicksell (1907). It is assumed that there is a "natural" real rate of interest, "n". It characterizes the national economy as such. This rate can be estimated by adding the country risk premium to the long run growth rate of the economy. The discrepancy between the real interest rate and the natural interest rate, "r-n", is an indicator of the excess of money demand. That excess relates to a "natural" level of demand for money, harmonious with the idea

of a natural rate of interest. The loop presented in the lower region of figure 2 dissipates such excess of money demand, letting the inflation rate to fall. The change in prices is adjusted in an amount that it is proportional to that excess money demand, though in the opposite direction. The parameter "a" shows the speed of the adjustment

$$\frac{d\pi}{dt} = \alpha(r - n) \tag{5}$$

The loop controls inflation to put it in line with the natural interest rate. This was something to be expected, given the pivotal role assigned to the natural interest rate. The above described differential equations 4 and 5, in addition to the function presented in the figure 1, complete the model of the system exhibited in figure 2. The strong endogenous causal structure that ties up both state variables is marked with arrows.

Fig. 2 Causal structure of the model

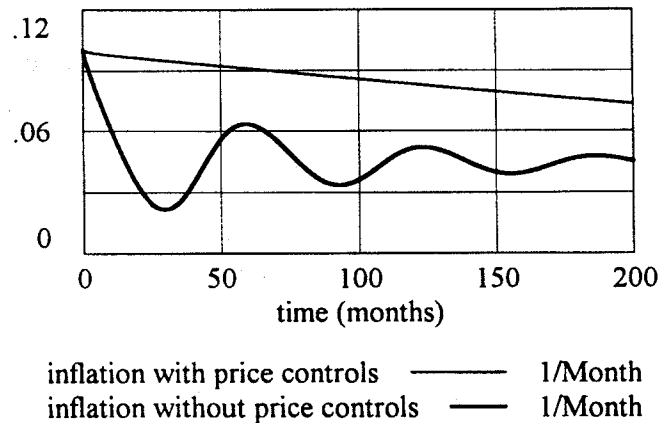


3. Model validation

This section presents the reproduction of main experiments with the original model, by comparing the results of the corresponding simulations with Fernández and Mantel's (1989) results. The announcement of the stabilization plan in 1985 surprised Argentine economy operating at a 30 percent monthly inflation rate. Immediately after the announcement, the plan accomplished a reduction of the inflation to 10 percent monthly. It was accompanied by a 15 percent interest market rate. Also there was a reduction of the operating deficit "d", to 2 percent

monthly. These are the initial conditions of the simulated system. Figure 3 exhibits the consequences of the application of the plan with and without controls on the prices. Such controls are introduced in the model manipulating the speed "a" of adjustment of the inflation rate. It was, before the application of the program, about 0.15. The experiments assume that the efficiency of control price policy is 90 per cent. This equals to say that adjustment speed of the inflation decreased to the tenth part: 0.015. As it is observed in the figure, the program without controls reaches, with oscillations, an equilibrium 5 percent inflation rate. The application of price control also diminishes inflation, although does not check it that much. It does, though, without oscillations. These experiments ratify the results that Fernandez and Mantel (1985) reported.

Fig. 3 Dynamics of inflation with and without price controls



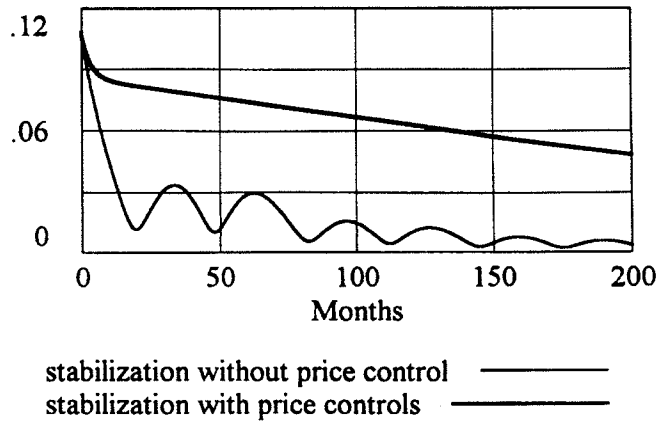
Optimal control policies

The purpose of the present investigation is to try to improve upon the two trajectories analyzed by Fernández and Mantel (1989). As mentioned above, these correspond to two situations, with or without controls, reflected in the size of the inflation rate adjustment velocity "a". With an effectiveness of controls of an assumed 90 % this coefficient is ten times as large in the second situation as compared to the first. In an implementation of economic policy it does not seem to be reasonable to decide to apply or not a given policy instrument from the beginning and then keeping it at the same level forever. Consider the more flexible case in which controls can be applied strictly or not at any time. Thus "a" can now attain its two extreme values in an alternating pattern. If the objective of the Minister of Economics were to stabilize the economy as much as possible within his tenure (some 28 months at that time in Argentina) a reasonable measure to minimize is the distance "δ" from the equilibrium attained, where the function "δ(t)" is some measure of disequilibrium, say

$$\delta(t) = \sqrt{[i(t) - i_e]^2 + [\pi(t) - \pi_e]^2} \quad (6)$$

No rescaling of the variables involved in equation 6 needs to be done, since "i" and "π" are of comparable magnitude. The dynamics of distance to equilibrium is shown in figure 4 for the two constant policies analyzed by Fernández and Mantel (1989).

Fig. 4 Distance to equilibrium under constant policies



A first intuitively good policy strategy seems to be to choose the high adjustment velocity when moving in the "right" direction - for example when inflation exceeds its equilibrium value and is falling -and the low velocity when moving in the "wrong" direction -inflation falls and is below its equilibrium value -. Equation 7 expresses the corresponding policy function, at each instant, and the resulting trajectories are shown in figure 5.

$$\alpha = 0.0792 + 0.0648 * \text{IF THEN ELSE} ((r - n) \leq 0, -1, 1) * \text{IF THEN ELSE} ((\pi - \pi_e) \leq 0, -1, 1) \quad (7)$$

Fig. 5 Distance to equilibrium

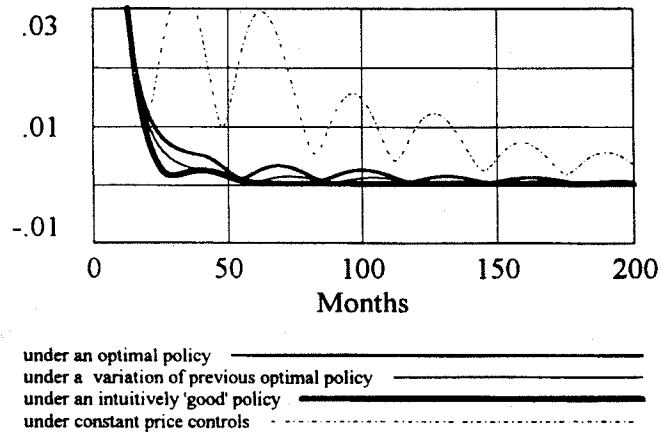
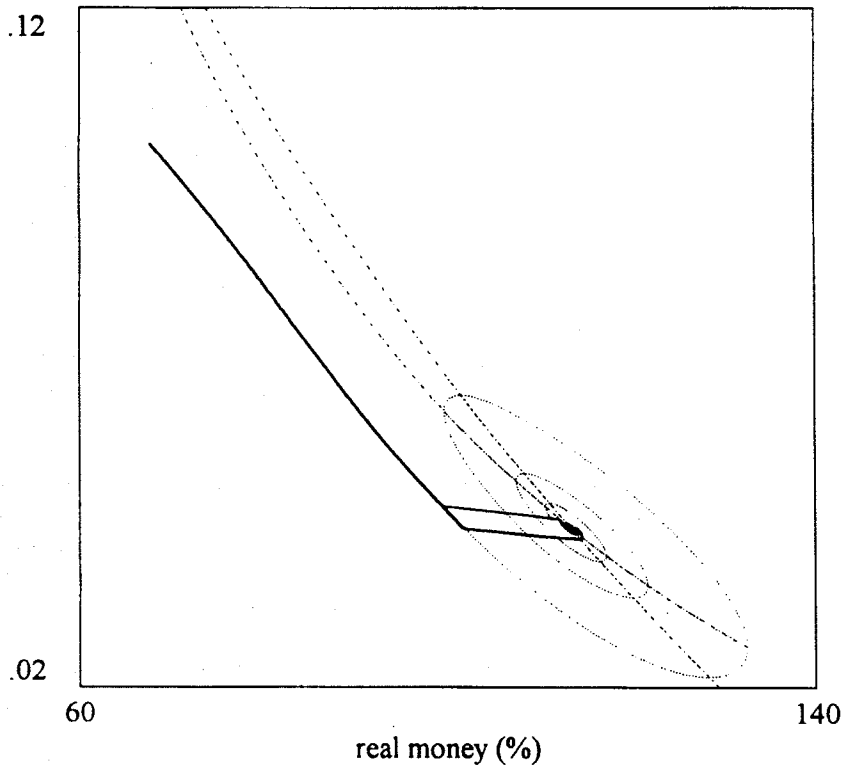


Fig. 6 Phase's Diagram: rate of inflation vs. real money



intuitively 'good policy'	—————	1/M
optimal policy	—————	1/M
policy without price controls	1/M
locus dpi/dt=0	1/M
locus dmi/dt=0	1/M

The final exercise consists in allowing Vensim (Ventana Systems 1993) to choose the timing of the policy switches related to the application of price controls, so as to optimize the performance criterion that has been mentioned. Parameters "p₀" and "p₁", define a two-step function, equation 8, for the behavior of the inflation rate adjustment speed, "a". The first step switches the price control on and the second one switches it off.

$$\alpha = 0.144 + \text{IF THEN ELSE} (\text{Time} < p_0, 0, -0.1296) + \text{IF THEN ELSE} (\text{Time} < (p_0 + p_1), 0, 0.1296) \quad (8)$$

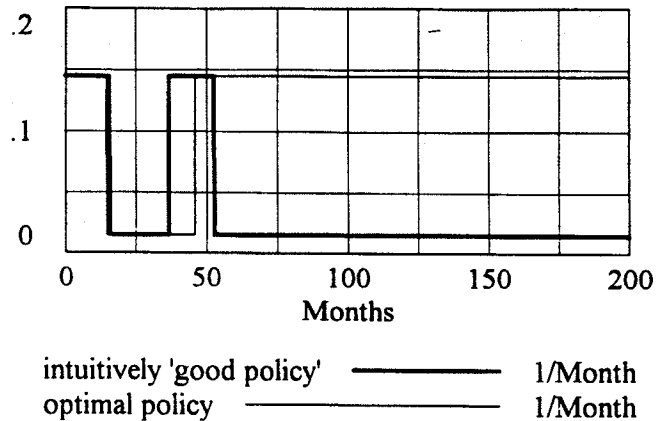
The objective function J is expressed in the form of an integral over a time period. In System Dynamics terms it is a level having nothing inside at the beginning of the simulation. The function under the integral sign reveals the goals of the policy maker.

$$J(t) = \int_{t=0}^{t=200} [100(\delta_t - \delta_{t-1}) + 3(\delta_t)^2] dt \quad (9)$$

Such a measure J of the performance of the system seems reasonable since it avoids simultaneously costly cyclical behavior and deviations from the stationary solution. The effort to

reduce the addition of the squared difference of the instantaneous value of distance to equilibrium to its lagged version, $_{-}(\partial_t - \partial_{t-1})^2$, takes care of the oscillations. Minimizing the addition of the squared distance, $_{-}(\partial_t)^2$, leads to the equilibrium point (i_e, π_e) ; the quicker the better.

Fig. 7 Timing for starting and ending price controls



Trying to achieve different goals synchronously brings forward the issue of scaling the terms inside the objective function. In this case this was done in two shoes. First, they were made comparable. One hundred times the squared difference of distances and the squared distance to equilibrium is a well matched pair. Vensim graphical capability is highly convenient to this purpose. Once they have similar magnitudes, fine tuning lets accentuate the goal one desires to, modifying the weights of the variables that appear in the integrand. Therefore factor 3 emphasizes the desire to achieve equilibrium as soon as possible.

The selected option for the procedure, MULTIPLE START=VECTOR, restarted optimization multiple times from different starting points, changing only one parameter at a time. It searched the first parameter from its minimum to maximum values and then the second. After 245 simulations, Vensim selected PARAM0 = 14.14 and PARAM1 = 27.281. That means that price control starts at month 14 and ends at month 41. Enlarging the weight of term $_{-}(\partial_t)^2$ to 5, provokes a late end of price controls, flattening even more the oscillations, as can be seen in figure 5.

Figure 6 exhibits the phase's diagram with the corresponding isoclines $dm/dt=0$ and $d\pi/dt=0$. There the approach of inflation rate to equilibrium inflation rate under the above mentioned policies can be seen, and figure 7 compares the timing of starting and ending price control under those policies.

Conclusions.

A fitting SD version of Fernández and Mantel's model of public finance (1989) exposes the causal structure implied in the original reduced form. As the figure 2 tells, not only two loops are involved, controlling both the real money stock and the inflation rate, formulated as system's states, but other longer loops also, forming a highly non linear system. This model reproduces the response of the system to the stabilization policies tested by Fernández and Mantel. New experiments with the SD version of the model show the inconvenience of trying to maintain indefinitely both original proposals. To start the price control the moment the inflation rate is below the equilibrium inflation rate, turns out to be very efficient, as can be seen in figures 5 and 6. This policy, suggested by the visual examination of the original experiments, in the best style

of Systems Dynamics, could be confirmed by letting Vensim's optimizer to choose the most convenient moment to start with the price control policy. Even though optimum control systems are often of the open loop type, the results shown here prove the convergence of the two criteria, optimization and intuition, when applied to closed loop, feedback systems.

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Appendix: Vensim's equations

absolute tolerance = 0.001
 ~ ~ maximum acceptable error for Runge-Kutta 4

beta = 5.645
 ~ ~ semielasticity of demand of money to changes in nominal interest rate

content of delay = INTEG((distance from equilibrium-(lagged distance from equilibrium)),(0.1076*time delay))
 ~ ~

distance from equilibrium =SQRT((rate of inflation-stationary rate of inflation)^2+(market interest rate-stationary market interest rate)^2)~ ~

domestic debt = 0.8
 ~ ~ Internal debt as proportion of the gross domestic product

domestic debt servicing = domestic debt*real rate of interest
 ~ ~ monthly servicing of domestic debt

excess on money demand = natural rate of interest-real rate of interest
 ~ ~ discrepancy between the natural interest rate and the real rate of interest, as indicator of excess of money demand with respect to the "natural" demand of money

FINAL TIME = 200
 ~ ~ The final time for the simulation.

inflation rate adjustment speed =switch0*0.0144+(1-switch0)*((switch1)*0.144+(1-switch1)*(switch2)*(0.0792+0.0648*IF THEN ELSE((real rate of interest-natural rate of interest)<=0,-1,1)*IF THEN ELSE((rate of inflation-stationary rate of inflation)<=0,-1,1)))+(1-switch2)*(0.144+IF THEN ELSE(Time<param0,0,-0.1296)+

of Systems Dynamics, could be confirmed by letting Vensim's optimizer to choose the most convenient moment to start with the price control policy. Even though optimum control systems are often of the open loop type, the results shown here prove the convergence of the two criteria, optimization and intuition, when applied to closed loop, feedback systems.

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beta = 5.645
  ~ ~ semielasticity of demand of money to changes in nominal interest rate
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content of delay = INTEG((distance from equilibrium-(lagged distance from
  equilibrium)),(0.1076*time delay))
  ~ ~
|
distance from equilibrium =SQRT((rate of inflation-stationary rate of inflation)^2+
  (market interest rate-stationary market interest rate)^2)~.~
|
domestic debt = 0.8
  ~ ~ Internal debt as proportion of the gross domestic product
|
domestic debt servicing = domestic debt*real rate of interest
  ~ ~ monthly servicing of domestic debt
|
excess on money demand = natural rate of interest-real rate of interest
  ~ ~ discrepancy between the natural interest rate and the real rate of interest, as indicator
  of excess of money demand with respect to the "natural" demand of money
|
FINAL TIME = 200
  ~ ~ The final time for the simulation.
|
inflation rate adjustment speed =switch0*0.0144+
  (1-switch0)*((switch1)*0.144+
  (1-switch1)*(switch2)*(0.0792+0.0648*
  IF THEN ELSE((real rate of interest-natural rate of interest)<=0,-1,1)*
  IF THEN ELSE((rate of inflation-stationary rate of
  inflation)<=0,-1,1)))+(1-switch2)*(0.144+
  IF THEN ELSE(Time<param0,0,-0.1296)+

```

```

IF THEN ELSE(Time<param0+param1,0,0.1296))
  ~ ~ velocity of adjustment of inflation rate to changes in real rate of interest
  |
initial rate of inflation = 0.1
  ~ ~ initial value for inflation state variable
  |
initial real money =q*EXP(-beta*0.15)
  ~ ~
INITIAL TIME = 0
  ~ ~ The initial time for the simulation.
  |
isocline m = (operating deficit+domestic debt*market interest rate)/
(real money+domestic debt-tax collection lag factor)
  ~ ~
  |
isocline pi=(LN(q/real money)/beta)-natural rate of interest
  ~ ~
  |
lagged distance from equilibrium=content of delay/time delay
  ~ ~
  |
market interest rate = (LN(q)-LN(real money))/beta
  ~ ~ nominal rate of interest
  |
natural rate of interest = 0.015
  ~ ~ economy 's natural rate of growth plus the country risk
  |
objective function =INTEG(100*((distance from equilibrium-lagged distance from
equilibrium)^2)+5*((distance from equilibrium)^2),0)
  ~ ~
  |
operating deficit = 0.02
  ~ ~ deficit that includes, besides the government primary deficit, the service of foreign debt
and excludes service of national debt
  |
public financial exposure = domestic debt servicing+operating deficit
-revenue from inflation tax
  ~ ~ government financial balance
  |
q = 1.58
  ~ ~
  |
param0 = 0
  ~ ~ the time at which decision to control prices is made
  |
param1 = 0
  ~ ~ the time at which decision to end control prices is made is: time=param0+param1
  |
real monetization rate = public financial exposure
  ~ ~ real money state derivative
  |
real money =INTEG(real monetization rate,initial real money)
  ~ ~
  |
real rate of interest =market interest rate- of inflation
  ~ ~ Remaining of nominal rate of interest after discounting inflation rate
  |

```

```

revenue from inflation tax =(real money-tax collection lag factor)*rate of inflation
  ~ ~ government financing through inflation
|
real money 2 = real money*100
  ~ ~
|
rate of change of inflation rate = excess on money demand*inflation rate adjustment speed
  ~ ~ inflation rate state derivative
|
rate of inflation = INTEG(rate of change of inflation rate,initial rate of inflation)
  ~ ~ monthly rate of inflation state variable
|
relative tolerance = 0.001
  ~ ~ maximum acceptable relative error for Runge-Kutta 4
|
SAVEPER = 0.1
  ~ ~ The frequency with which output is stored.
|
stationary market interest rate = 0.0584
  ~ ~
|
stationary rate of inflation = 0.0434
  ~ ~
|
switch0 =0
  ~ ~ switch0=1 means constant price control and switch0=0 allows other policies.
|
switch1 = 0
  ~ ~ combination switch0=0, switch1=1 means absence of price control and switch1=0
indicates the application of stabilization policies with price control, either alternating or
optimal, manipulating the velocity of adjustment of inflation rate.
|
switch2 = 0
  ~ ~ combination switch0=0, switch1=0, switch2=1 means the application of alternating
pattern of price controls, and combination switch0=0, switch1=0, switch2=0 indicates the
application of stabilization policies with optimal price controls.
|
tax collection lag factor = 0.4
  ~ ~ magnitude of the reduction of taxable stock of real money due to tax collection delays
in inflationary conditions (Olivera-Tanzi)
|
time delay = 2
  ~ ~
|
TIME STEP = 0.1
  ~ ~ The time step for the simulation.
|

```