EVOLUTIONARY ECONOMICS AND SYSTEM DYNAMICS

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ABSTRACT

The purpose of this paper is fourfold: 1) to survey the literature on evolutionary economics in general; 2) to survey the literature on evolutionary economic modeling in particular; 3) to outline the contribution that system dynamics can make to evolutionary economic modeling; and 4) to present two original, evolutionary, system dynamics models.

The paper begins by noting that the evolutionary perspective has a long and distinguished history in the field of economics. Well-known economists such Karl Marx, Richard Eli (founder of the American Economic Association), Thorstein Veblen, Joseph Schumpeter, Gunnar Myrdal ("circular and cumulative causation"), Kenneth Boulding (general systems theory), and Nicholas Kaldor ("increasing returns"), for example, have utilized the evolutionary perspective. Despite this rich history, however, the paper notes that the evolutionary perspective does not dominate economic theory. Two explanations for this are offered: 1) it is not in harmony with neoclassical theory; and 2) it has historically been seen as not amenable to formal modeling.

The paper then presents a survey of the literature on evolutionary economics. The survey indicates that the writing on evolutionary economics usually involves one or more of the following ideas: 1) structural change versus change within a given structure; 2) time irreversibility; 3) the second law of thermodynamics; 4) hysteresis; 5) co-evolutionary processes; and 6) the behavior of thermodynamically open, nonlinear, systems in a far-from-equilibrium state.

The paper next proceeds to survey the literature on evolutionary economic modeling. This survey indicates that economic models classified as evolutionary usually exhibit one or more of the following characteristics: 1) path dependency; 2) multiple equilibria; 3) the ability to self-organize; 4) the ability to behave chaotically.

Next, the paper provides an overview of the field of system dynamics and notes that, among other things, it can bring an evolutionary economic modeling process to the field of evolutionary economics. Further, it can be used to create individual models that can be classified as evolutionary, given the criteria mentioned above. Care is also taken to discuss the fundamentals of system dynamics modeling, including the systematic and formal treatment of dynamics and feedback and the creation of models that portray realistic decision making structures.

The paper concludes with a detailed presentation of two evolutionary system dynamics duopoly models that generate path dependency, multiple equilibria, and the ability to self-organize.

ADDITIONAL COMMENTS

This paper is forthcoming in a book titled: Evolutionary Concepts in Contemporary Economics. Ann Arbor, MI: University of Michigan Press. Richard W. England, ed. The authors found it impossible to condense the paper for these proceedings. Anyone interested in obtaining a copy should contact Professor Sterman at the above address.

Evolutionary Economics and System Dynamics

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Time is a device to prevent everything from happening at once.[1]

Introduction

The evolutionary perspective has a long and distinguished history in the field of economics. Indeed, it was adopted by economists such as Karl Marx (1867) and Thorstein Veblen (1898) as early as the nineteenth century, and Joseph Schumpeter (1934, 1939), Gunnar Myrdal (1944), and Kenneth Boulding (1981, 1991) during the twentieth. Unfortunately, although provocative and insightful, the writings of the early evolutionary economists were unable to catapult the evolutionary perspective to the forefront of the economics profession. Two common explanations for this failure are that: (1) the evolutionary approach is at odds with the corpus of nonevolutionary theory which dominates economic thinking, and (2) evolutionary economics has traditionally been seen as not amenable to mathematical formalization.

With regard to its incompatibility with mainstream economic theory, there is a great deal of evidence (e.g., Mirowski 1988; England 1993) indicating that the economics profession grew up trying to imitate classical mechanics. As a result, the body of theory that emerged and still largely dominates economic analysis (i.e., neoclassical economics) is based upon the notion of conserved or Hamiltonian systems and hence on a Newtonian or time reversible view of the world (Hamilton 1953). Theories that are out of harmony with this view are, at best, treated with suspicion and, at worst, rejected or relegated to less-visible scholarly outlets by the invisible college of economists.

In terms of the historical lack of mathematical formalization in evolutionary economics, it is clear that most of the classic evolutionary theories were created by economists who either wrote at a time when formal modeling was not practiced, lacked the necessary training in mathematics, or felt that the mathematical tools of the day were insufficient for representing evolutionary change. Richard Goodwin (1991: 30), for example, remembers Schumpeter's "sadly deficient mathematical capability" and both Myrdal (1944: 1069) and Boulding (1962) expressed their pessimism regarding the possibility of mathematically representing evolutionary change.[2]

Of the two explanations for the failure of the evolutionary perspective to become the normal science of the economics profession the first -- its incompatibility with neoclassical theory -- is of primary importance. The second -- its presumed inability to be mathematically formalized -- is really something of an historical stereotype and clearly not correct. Nonlinear dynamic computer simulation modeling has made the building of mathematical evolutionary economic models possible since the 1950s.

The purpose of this paper is to discuss the types of structure and behavior associated with

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[1] Joan Robinson (1962: Epigraph) attributes this quote to Henri Bergson (1911/1944).

[2] More precisely, Boulding has argued that dynamic economic models created with ordinary differential equations are deterministic and hence nonevolutionary, while Myrdal expressed doubt that the process of "circular and cumulative causation" -- his engine of evolutionary economic and social change -- could be represented mathematically. Similar positions have been taken by K. William Kapp (1968: 13) and Allan Gruchy (1972: 305). As is shown below, however, Boulding is incorrect, unless he takes a very narrow view of differential equations. Recent developments in nonlinear dynamics, moreover, happily reveal that Myrdal was unduly pessimistic.

mathematical models that are typically categorized as evolutionary, and show that a particular type of computer simulation modeling -- system dynamics -- can be used to create models that possess these characteristics. To support this claim, a number of evolutionary system dynamics models will be discussed and an original evolutionary system dynamics model will be presented. Software and other resources available for the creation and analysis of evolutionary system dynamics models will also be discussed.

But What Exactly is Evolutionary Economics?

In order to review the fundamentals of evolutionary economic modeling, the characteristics of evolutionary economic change must, arguably, first be identified and understood. Although a survey of the literature would seem to indicate that no single, comprehensive definition of the phenomenon exists, it is possible to identify a number of recurring themes.

According to David Hamilton (1953), evolutionary or "Darwinian" change is caused by changes in system structure, while nonevolutionary or "Newtonian" change represents change within a given structure. He used this distinction, as did Veblen (1898), to show the nonevolutionary nature of neoclassical microeconomic theory. On the macroeconomic side, the distinction between structural and nonstructural change has been used by Johansson et al. (1987: 4) and Boulding (1981) to draw a distinction between economic growth and economic development. In their view, the former implies "more of the same" while the latter implies structural change.

Louis Perelman's (1980) view of evolutionary change emphasizes the idea of time irreversibility -- i.e., the notion that it is impossible to reverse time and make events undo themselves. England (1993) points out that most modern growth theoretic models violate this canon because their time paths can be reversed by switching the signs of their parameters.

The concepts of time irreversibility and structural change are closely related to the second law of thermodynamics which shows that dissipative dynamical systems generate increased entropy or disorder over time, preventing them from returning to their previous states. Nicholas Georgescu-Roegen (1971, 1980) and Boulding (1981, 1991) have applied the second law to the analysis of economic systems.[3] Time irreversibility, structural change, and the second law of thermodynamics are themselves closely related to the idea of hysteresis, or the inability of a system that has been changed by an external force to return to its original state after the external force is removed. Olivier Blanchard and Lawrence Summers (1986) have used this concept to explain European unemployment, Dixit (1992) has used it to explain the failure of firms to withdraw from investment projects after the conditions that initially made them appear profitable disappear, and Evans and Ramey (1992) have used it to create a Phillips curve that embodies rational expectations with explicit calculation costs.[4]

The view that economic systems evolve toward increased levels of disorder and entropy has sometimes been referred to as the "engineering view" of evolution. Of note is that this view conflicts with the view of evolution originating in biology, which posits that systems evolve toward greater levels of order and complexity. "Co-evolutionary economists" such as Richard Chase (1985) and James Swaney (1985) have developed theories that enable this conflict to be reconciled. In these theories, dissipative economic systems generate increased levels of entropy and disorder that motivate humans to develop increasingly complex entropy-skirting technical innovations and social institutions.[5]

Ilya Prigogine's original work on far-from-equilibrium thermodynamic systems is similar to the theories of the co-evolutionary economists.[6] Prigogine and theorists in physics, chemistry, and biology have shown how thermodynamically open, dissipative, entropy generating systems, operating in a far-from-equilibrium state, can reorganize themselves into more complex temporal and/or spatial structures when they are pushed against their nonlinear constraints. This view is thus also able to reconcile the engineering and biological views of evolution.

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[3] See the discussion in Radzicki (1988a).

[4] An overview of hysteresis effects in economics is contained in Cross and Allan (1988).

[5] For a further discussion of these ideas see Radzicki (1990b).

[6] See Nicolis and Prigogine (1977), Jantsch (1980), Prigogine and Stengers (1984), Laszlo (1987), Allen and McGlade (1987), and Allen (1988).

But What Exactly Is Evolutionary Economic Modeling?

An examination of the types of mathematical models that are commonly classified as "evolutionary" indicates that they are constructed in both discrete and continuous time, utilize a variety of mathematical techniques, exhibit different types of dynamical behaviors and, in some cases, can be solved without the aid of a computer. This lack of uniformity, however, does not preclude the identification of some common characteristics. In addition to being dynamic and able to exhibit some form of disequilibrium behavior, evolutionary economic models tend to possess one or more of the following traits: 1) path dependency; 2) the ability to self-organize; 3) multiple equilibria; or 4) chaotic behavior.

Path dependency is a characteristic of models that can get locked into the particular dynamical path they initially "choose" (usually by chance). Paul David (1985) and Brian Arthur (1988, 1989, 1990) have described numerous real-life instances of this behavior involving the adoption of new technologies and the location decisions of firms, while Arthur (1989, 1990), Arthur et al. (1987), and Krugman (1991) have developed formal models of the phenomenon. Economic models exhibiting hysteresis (e.g., Blanchard and Summers 1986, Dixit 1992, Evans and Ramey 1992) can also be considered path dependent, as can system dynamics models possessing "floating goal" structures.

Floating goal structures are aspiration levels used by agents in decision making, which themselves adapt to past experience and hence cause present goals and activities to be influenced by past results (see Forrester 1968, Meadows 1982). In a floating goal structure system, the direction taken in the future depends upon the cumulative impact of the potholes, actions, and obstacles it meets along the way, and not solely on its current physical state. Thus, random events become critical determinants of the system's path and even its qualitative character; as when the chance formation of a few businesses in a region causes the growth of a cluster of related industries through the cumulative advantage of co-location and access to developing knowledge, infrastructure, and other resources (e.g., the Silicon Valley, the New York Diamond District).

Self-organization is exhibited by models that undergo abrupt changes in their temporal or spatial structures through changes in their parameters or via the amplification of random, microscopic, fluctuations. Self-organizations of the former type include models that can exhibit bifurcations and catastrophes, such as those developed by May (1976), Varian (1979), Stutzer (1980), Mosekilde et al. (1988), Andersen and Sturis (1988), Sterman (1988b), Sterman (1989b), and Lorenz (1989). Richard Day (1983) has described bifurcations and catastrophes as being akin to a marching band suddenly breaking formation, scrambling around, and regrouping in another formation.

Examples of self-organizations that occur due to the amplification of microscopic fluctuations can be found in the behavior of many nonlinear dynamic models residing within, and outside of, the fields of economics and system dynamics. Of particular note is the work of Forrester (1961: Appendix N), Nicolis and Prigogine (1977), Jantsch (1980), Montano and Ebeling (1980), Mosekilde et al. (1983), Prigogine and Stengers (1984), Mosekilde and Rasmussen (1986), Laszlo (1987), Arthur et al. (1987), Allen and McGlade (1987), Allen (1988), Dosi (1988), Silverberg (1988), Silverberg et al. (1988), Arthur (1989), Radzicki (1990b), Moxnes (1992), and Wittenberg and Sterman (1992). In these models, random fluctuations, often representing the idiosyncratic actions of individual economic agents, become amplified by positive feedback processes and grow to dominate the macroscopic behavior of the systems.

Yet another way to identify models that are typically classified as evolutionary is via the presence of multiple equilibria. The particular equilibrium "chosen" by these models usually reflects the effects of random shocks that direct it down a particular path. Models with multiple equilibria can also be path dependent and exhibit time irreversibility and the ability to self-organize. Peter Diamond (1987) has shown that multiple equilibria can arise in economic models that explicitly represent market imperfections.

Deterministic chaos is an irregular oscillatory behavior that arises in nonstochastic, nonlinear, feedback systems. Although it is generated by models that are completely devoid of exogenous randomness, its period and amplitude never repeat and it functions much like the idealized random variates of probability theory, generating variety and causing deviations from "average" behavior. A small sample of economic models that can exhibit chaos includes those created by Stutzer (1980), Day and Shafer (1986), and Goodwin (1991).[7] A small sample of system dynamics models that can generate chaos includes those developed by Andersen and Sturis (1988), Sterman (1988b), Sterman (1989b), and Mosekilde et al. (1992). An excellent overview of the issues associated with chaotic dynamics is presented by Mosekilde et al. (1988).

[7] See also the collection of economic models contained in Lorenz (1989).

The tie between evolutionary behavior and models that can produce chaos involves the notion of an attractor. An attractor is the set of points that defines the steady state behavior or "temporal structure" of a dynamical system. A fixed point (defining an equilibrium steady state) is the only type of attractor possible in linear systems, while fixed points, saddle loops, limit cycles, tori, higher dimensional orbits of some complexity, and chaotic attractors are possible in nonlinear systems. Of note is that many nonlinear systems exhibit bifurcations by which they switch their trajectories from one attractor to another via a small change in one of their parameters. Such switches are examples of system self-organizations and hence of model-based evolutionary change.

An important characteristic of a model whose motion is defined by a chaotic attractor is that its behavior is sensitive to its initial conditions. This means that a minute change, ε, in its vector of state variables will cause it to travel down a time path that is significantly different (i.e., much greater than ε) from its previous trajectory. In fact, the chaotic attractor will stretch and fold the motion of the system so severely that it will cause an exponential divergence of the two time paths. As a result, models that produce chaos can also be said to produce path dependent behavior.

One last point concerning dynamical models whose steady state behaviors are defined by attractors and whose time paths have transient components, is that it is not possible to reverse the signs of their parameters and "backward predict" their trajectories, unless their initial values are known with exact certainty (Lorenz 1989: 61-63). In this sense then, they are time irreversible and hence evolutionary.

Characteristics of System Dynamics Models

System dynamics was originally created in 1950s to address problems encountered by managers in corporate systems (Forrester 1961). Its use was extended during the 1960s, 70s, and 80s to include economic, social, biological, and physical systems (Forrester 1969, 1972; Roberts 1978; Sturis et al. 1991). Today system dynamics is applied to diverse problems in the behavioral, economic, and natural sciences. It is used as a modeling methodology in academic research (e.g. Sterman 1989a, 1989c), as a method to stimulate learning among corporate executives (e.g. Senge 1990, Morecroft and Sterman 1992) and as a tool for teaching at the pre-college level (e.g. Hopkins 1992; Gould 1993).

The intellectual roots of system dynamics lie in control engineering and the theory of servomechanisms developed in the early part of the twentieth century. Richardson (1991) has traced the history of system dynamics and the concept of feedback in the social sciences from the use of feedback in ancient mechanical devices, through the theory of feedback control systems in steam engine governors and servomechanisms, to its diffusion into the social and behavioral sciences beginning in the 1940s. Over the years, system dynamicists have developed a distinct set of guidelines for helping them build dynamic models.[8] Among the most important are that: 1) the dynamic behavior of any system emerges from its structure; 2) the modeling, and subsequent understanding, of any system requires the identification and representation of that structure; 3) decision making in human systems is boundedly rational; and 4) discovery of the decision rules people actually use requires empirical work, including field observation of decision making behavior.

System dynamics models, from a mathematical point of view, consist of systems of ordinary nonlinear differential equations. Typically, system dynamics models are formulated in continuous time and assume continuous variables, though the use of simulation to solve the models means continuity is not essential to the method. Indeed, where necessary for fidelity to the problem being modeled, a good system dynamics model will contain discrete elements such as queues, quantized flows (e.g. integer flows of people), probabilistic decision rules, and other departures from deterministic lumped models.[9]

System dynamics models can be characterized as structural, disequilibrium, behavioral models. They differ, therefore, from the familiar econometric models, general equilibrium models, and rational expectations models in a variety of ways:

Macrobehavior from Microstructure: The concept of feedback is central to system dynamics. Feedback exists whenever decisions made by agents in a system alter the state of the system, thus giving

[8] Day (1987) has developed a similar set of guidelines for economic modeling. See also Radzicki (1988b, 1990a).

[9] Software tools such as STELLA (Richmond and Peterson 1992) support both continuous and discrete elements, so it is a simple matter to simulate any system of mixed continuous-discrete elements, systems of difference equations, delay-differential models, markov models, and so on.

rise to new information that conditions future decisions. The dynamics of a system emerge out of the interaction of the multiple feedback loops in its structure. Feedback loops may be self-reinforcing (positive feedback) or self-correcting (negative feedback). Positive loops are self-reinforcing processes such as the compounding of interest or the growth of a population. Negative loops define goal-seeking processes such as the regulation of inventory by adjustments of production, the equilibration of demand and supply via changes in price, or the adjustment of a firm's capital stock to appropriate levels via changes in investment. A system dynamics model is an explicit mapping of a system's positive and negative feedback loops.

System dynamics models seek to portray the microstructure of a system at an operational level. The feedback loop structure of any dynamic system consists of the physical structure of the system, the flows of information characterizing the state of the system, and the decision rules of the agents in the system, including the behavioral decision rules people use to manage their affairs.

The physical structure of any system is represented by networks of stocks and flows. Stocks characterize the states of a system while flows represent the rates of change of the stocks. A model of a firm, industry, or national economy, for example, would explicitly portray the stocks and flows of people, resources, money, goods, capital, information, and so on. The stock-flow representation is a very general idea that can be applied to the dynamics of any system. Sturis et al. (1991), for example, have created a system dynamics model of human glucose-insulin interaction that includes stocks of glucose, insulin, glucagon, and flows representing the synthesis, transport, and metabolism of these compounds. A system's stocks accumulate or integrate its rates of flow and determine its state at any point in time. As a result, each stock represents the accumulated history of its flows and serves as a source of system inertia and as part of its memory.

A second characteristic of stocks is that they decouple a system's inflows from its outflows. In equilibrium, the net inflows to all stocks are zero, and the stocks are thus unchanging. For example, in equilibrium orders for products must equal shipments which must equal production (ignoring cancellations and scrappage). Since the stocks in traditional equilibrium models are unchanging they are often omitted. To capture disequilibria in a system, however, stocks must be explicitly represented since they accumulate the imbalances between inflows and outflows. In reality, orders for products need not, and usually do not, equal shipments; the difference between these flows accumulates in order backlogs. Likewise, differences between production and shipments accumulate in inventories. Explicit representation of stocks also enables their inflows and outflows to respond to the decisions of the distinct economic agents who, in the real system, control these separate flows (e.g., buyers and sellers may place orders and produce goods at different rates, according to the separate decision rules and constraints they each face).

As a system's stocks rise and fall, agents take various actions to alter the rates of flow, thus closing the feedback loops that may bring the system into equilibrium or reinforce current trends. For example, excessive inventories may cause a firm to lay off some workers to reduce production or cut price to stimulate orders, thus reducing inventories to desired levels. Whether such corrective actions in fact bring the system into equilibrium is determined by the interaction of all the feedback processes in the system, as are the characteristics of the adjustment path itself. However, often the interaction of multiple feedback processes in complex nonlinear systems cause disequilibria to persist. For example, in the case of a speculative bubble, it has been repeatedly demonstrated empirically (e.g. Andreassen 1990, Sterman 1987) that people tend to form expectations of future asset prices (e.g., real estate prices, the price of gold, the price of tulips) by extrapolating recent price trends. An exogenous price rise may thus cause new buyers to enter the market and reduce offerings by current holders, so that the price in fact rises in a self-fulfilling prophecy, as described by John Stuart Mill (1848, Volume II: 45ff), Robert K. Merton (1936) and Charles Kindleberger (1978).[10] Here the intendedly rational decisions of individuals create and reinforce disequilibrium.

Another important component of any system's structure is its nonlinear relationships. Every significant economic process and institution involves nonlinearities (Forrester 1987), though much of the history of economic theory in general, and business cycle theory in particular, has been an attempt to work around nonlinearity for reasons of analytic tractability (Richardson 1991, Zarnowitz 1985: 540). Nonlinearities are responsible for a system's robustness or ability to stay within certain boundaries. For example, output suffers diminishing returns as individual factors of production are increased relative to others, gross investment remains nonnegative no matter how much a firm's capacity exceeds its orders, shipments are

[10] See the discussion in Richardson (1991: 77ff).

determined primarily by orders when warehouses are full but must drop to zero as inventories are depleted, the cash position of a firm has little influence on its capital investment or employment decisions unless a severe liquidity crisis appears and dominates all other considerations, nominal interest rates do not become negative no matter how rapid deflation may be, and so on.

In addition, and perhaps most importantly, nonlinearities contribute significantly to a system's evolutionary behavior because they cause the strength of its feedback loops, and hence its "active structure," to change over time (Richardson 1991). Returning to the example of the speculative bubble, it is clear that the positive feedbacks of extrapolative expectations are opposed by the negative feedbacks created by substitution to other products, increases in production of the commodity, declining real incomes as prices rise, and arbitrage opportunities. However, if the lure of speculative profit is strong enough, the positive feedback loops created by extrapolative expectations can overwhelm the negative feedbacks that might restore equilibrium – at lease for a time. As prices are bid higher and higher relative to fundamental value, however, the credibility of projections of further increases falls, weakening the positive loops. At the same time, the negative loops gain in strength. That is, the relative strength of the different loops is nonlinearly dependent on the balance between current prices and fundamental value. Eventually, the negative loops become dominant and price increases slow. As soon as this occurs, of course, some seek to liquidate their holdings, and prices begin to fall. Now the same positive feedback loops dominate again as falling prices lead to panic selling. Eventually, the negative feedback loops reassert themselves once prices are low relative to fundamental value, halting price declines. Of note in this account is the shifting dominance of the positive and negative feedbacks due to nonlinearities. The nonlinearities cause the active feedback loops, and hence the dynamic behavior of the system, to change endogenously through time, and ensure the global robustness of the system. No linear model can capture such shifts.

Together, these elements of structure (stocks and flows, information feedbacks, decision rules, and nonlinearities) define the feedback loops in any system. By modeling decision making behavior and the physical structure of the system at the micro-level, the macro-level dynamics emerge naturally out of the interactions of the system components. Because such models provide a behavioral description firmly rooted in managerial practice they are well suited to an examination of the dynamic effects of policy initiatives.

Disequilibrium Dynamics: System dynamics models are disequilibrium models. It is not assumed that economic systems are always (or ever) in equilibrium, nor that they move smoothly from one equilibrium to the next. To model dynamics, including evolution, properly, the stability of the system must not be assumed. Rather, the decision processes of the agents in the system must be modeled, including the way people perceive and react to imbalances, as well as the delays, constraints, inadequate information, and side-effects that often confound them. Stability, adjustment paths, the response to shocks, and the nature of equilibria are viewed as behavioral outcomes of a model. They are properties that emerge from the underlying assumptions about system structure and the interaction of the feedback loops created by the stock and flow networks, information flows, and decision rules of the actors in the system. Thus system dynamics models are well suited to modeling evolutionary environments where path-dependent behavior and multiple and changing equilibria often arise.

Bounded Rationality: The decision rules in system dynamics models govern the rates of flow that alter system stocks. The decision rules in models of human behavior rest on the theory of bounded rationality (Cyert and March 1963, Merton 1936, Nelson and Winter 1982, Simon 1947, 1957, 1979). The essence of the theory is summarized in Herbert Simon's principle of bounded rationality (1957: 198):

> The capacity of the human mind for formulating and solving complex problems is very small compared to the size of the problem whose solution is required for objectively rational behavior in the real world or even for a reasonable approximation to such objective rationality.

Boundedly rational decision making means agents at each decision point in a system use heuristics to select from among the available information cues, process and combine those cues, and make a decision. These decisions then alter the rates of flow in the system, altering its stocks, and giving rise to new information, thus closing various feedback loops as the decision makers perceive and react to the new information. Though there is often a rationale, or intended rationality, to the decision making heuristics of the agents, there is no presumption in system dynamics that these heuristics are optimal, or even consistent; nor that decision making is based only on rational cognitive factors.[11] The theory of bounded

[11] For example, emotions, habit, rules of thumb, and culture often play roles in decision making.

rationality provides both theoretical underpinnings and a rich data base for the development and testing of behavioral models of decision making in economics. Psychological, contextual, cultural, and other social and economic forces may all influence the heuristics people use. For example, cognitive and social psychology provides a rich database of theory and experimental results documenting numerous cognitive limitations on human information perception and processing, errors and biases in common heuristics used in judgment and decision making, and other deviations from the axioms of rationality (Tversky and Kahneman 1974, Hogarth 1987, Kahneman, Slovic, and Tversky 1982).

Empirical methods in system dynamics: A good model of economic dynamics must be descriptive. To simulate, in the root sense of "mimic," the behavior of a system accurately, decision making must be portrayed as it is, and not as it might be if people conformed to the axioms of economic rationality. Discovering, representing, and testing models of decision making heuristics is intrinsically an empirical task. Because the focus is on the process by which people make decisions, good system dynamics modeling involves field work and direct observation of the system under study, as well as the traditional tools of statistical estimation.[12] The modeler must often use ethnographic and anthropological methods to elicit the decision rules of the actors (Forrester 1961, Morecroft and Sterman 1992). Additional techniques to elicit decision making behavior include laboratory experiments (Sterman 1989a, 1989b, 1989c, 1988b) and cognitive mapping (Axelrod 1976, Checkland 1981, Vennix and Gubbels 1992, Richardson et al. 1992). When well done, complementary field-based, laboratory, and statistical methods yield a rich representation, grounded in multiple data sources, of the decision making heuristics of agents and how these rules might change over time. Evolutionary models need to be grounded in such direct observation of decision making, lest the axioms of individual profit and utility maximization be replaced by equally whimsical and arbitrary assumptions about decision making behavior.

The attributes described above make system dynamics modeling well suited to the study of evolutionary dynamics in human systems. The flexibility of the modeling method and emphasis on empirical assessment of the decision rules of the actors means the microstructure of a system can be represented with great fidelity. The resulting high-order, nonlinear systems typically contain dozens or even more interacting positive and negative feedback loops. The nonlinearities in dynamic systems mean the active structure or dominant feedback loops can change endogenously. As a result, system dynamics models may possess multiple equilibria. The equilibria in a system dynamics model may or may not be stable. They can (and do) exhibit path-dependent, irreversible dynamics. They can learn and evolve. For example, one of the earliest system dynamics models (Forrester 1961: Appendix N) represents a manufacturing firm that "learns" to detect seasonal cycles in incoming orders, then adjusts production accordingly. The customer order rate has no exogenous seasonality but does contain random disturbances. As the firm responds to these random fluctuations, the resulting changes in price and product availability, in turn, induce the simulated customers to alter their ordering patterns until the system generates strong seasonal patterns, when none existed before. Other examples of evolution and learning in system dynamics models are provided by Merten, Löffler, and Wiedmann (1987), whose model of a multinational firm learns to reorganize itself as it grows; Nancy Roberts' (1974) model of elementary schools, in which each student's achievement is dependent on teacher, student, and parent expectations, which in turn are dependent on student achievement; and Levin et al's. (1976) models of human service organizations, in which service provider standards and client expectations are conditioned by the quality of services received, thus creating path dependent dynamics.

Two ways in which System Dynamics Modeling is Evolutionary

There are really two ways in which system dynamics modeling can be considered evolutionary. The first, as discussed above, is in terms of the behavior of a particular system dynamics model. System dynamics models can possess multiple equilibria and exhibit path dependency, self-organization, chaos, time irreversibility, and evolution to increased levels of complexity and entropy. Moreover, their nonlinear

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Homer (1985), Shantzis and Behrens (1973), Levin, Roberts, and Hirsch (1975), and Homer (1992) provide examples including worker burnout, tribal rituals, and drug use.

[12] See Senge (1980) for an example of econometric tools applied to system dynamics models. Validation is discussed in Forrester and Senge (1980), Sterman (1984), Radzicki (1988b), and Radzicki (1990a).

relationships can cause their "active structures" to change as a simulation unfolds. In terms of a number of criteria therefore, individual system dynamics models can be classified as evolutionary.

A second way in which system dynamics models can be considered evolutionary comes from the notion that the true value of modeling arises from the modeling process, rather than from any particular model (Forrester 1985). In other words, system dynamicists believe that it is the iterative process of making one's perceptions explicit and then testing their adequacy via simulation, that generates insight and hence the value from modeling, and not any one run or version of a model. As a result, system dynamicists never consider a model as being complete, but only in its latest stage of development. Moreover, they note that as new insights and ideas are generated from the modeler's participation in the process, the structure of the model will change to accommodate them. Given this perspective then, the system dynamics modeling process can clearly be classified as evolutionary. Of note is that evolutionary economists have put forth essentially the same argument vis-á-vis their pattern modeling process since the time of John Dewey (1910, 1938).[13]

An Illustration

To illustrate some of the ideas put forth in this paper, a simple evolutionary system dynamics model will now be presented. The model depicts the competition for market share between firms where each benefits from a significant learning curve. For clarity of exposition and considerations of space, the model is highly simplified compared to typical theories of industry and firm structure in the system dynamics literature (e.g. Forrester 1961, Mass 1975, Lyneis 1980, Beinhocker et al. 1993), yet it illustrates the path-dependent, self-organizing dynamics typical of evolutionary models. Further, for brevity, empirical tests of the model are not described. The reader interested in empirical testing is referred to Paich and Sterman (1992) for an experimental study of decision making behavior in a setting similar to the one assumed below.

Figure 1 shows the system dynamics stock-flow diagram for the learning curve model. The model's stocks are represented by the rectangles (e.g., Firm 1 Cumulative Experience), and its flows are represented by the pipe and valve-like icons that appear to be filling and draining the tubs (e.g., Firm 1 Production). The solid arrows in Figure 1 represent flows of information while the circular icons depict constants, behavioral relationships, or decision points where the simulated agents transform flows of information into decisions (e.g. Firm 1 Price is determined by Firm 1 Unit Costs and Firm 1 Margin).

There is a one-to-one correspondence between the structural diagram and the equations. The diagram and equations are the actual output from STELLA, the software program used to develop the model (Richmond and Peterson 1992). The model was created by drawing the structural diagram on the screen of the computer, then specifying the form of the equations. The software enforces consistency between the diagram and the equations, and provides numerous built-in functions to assist the model builder. Experience has shown that business people and students, from grade school to CEOs, can learn the mechanics of the software in a few hours. A caveat, however: learning the software mechanics is easy, learning how to build good models is difficult. The ease of use of the software tools means complex nonlinear dynamic modeling is now accessible to anyone, regardless of computer skills or mathematical background. Obviously, some training in mathematics and an understanding of decision making behavior and complex dynamics are important for developing insightful, robust models. The software allows a modeler to spend his or her time thinking about system structure and behavior, rather than programming. Researchers interested in evolutionary dynamics will find that such software can be used for "rapid prototyping" and testing of models with considerable complexity.

The model represents the competition among firms in the presence of a learning curve. The simplest version of the model, presented first, is one in which the only feedback loops are those created by the learning curve. This version shows how a learning curve can create path-dependent dynamics. The model is then extended to consider imperfect private appropriability of experience, introducing additional feedback complexity and yielding much richer dynamics.

The model assumes that all firms are identical in structure, parameters, and initial conditions. Two firms are assumed for simplicity, although the model readily generalizes to a population of N firms, which may be heterogeneous. The equations[14] are:

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[13] See also Wilber and Harrison (1978) and Gruchy (1972).

[14] For brevity of exposition only the equations for firm 1 are shown. The equations for firm 2 are identical.

(1) Firm_1_Demand = Firm_1_Market_Share*Industry_Demand

Each firm's demand is the industry demand multiplied by the firm's share of that demand.

(2) Firm_1_Market_Share = Firm_1_Attractiveness/Aggregate_Attractiveness

Each firm receives a share of the industry demand proportional to the "attractiveness" of that firm's product compared to that of other firms (see equation 12).

(3) Firm_1_Attractiveness = Firm_1_Random_Disturbance*(Firm_1_Price ^ (Consumer_Sensitivity_to_Price))

(4) Firm_1_Random_Disturbance = 1+STEP(1,1)*NORMAL(0,.1)

The attractiveness of each firm's product is determined by price and a random disturbance. The elasticity of attractiveness with respect to price is high but finite: the products are not perfect substitutes but somewhat differentiated. In addition, each firm's attractiveness is influenced by an independent random variable representing the stochastic influence of factors of attractiveness not captured in price and variations in consumer preferences. The disturbances are specified as normal random variables with standard deviations of 10% (the STEP function prevents the random disturbances from having any impact until time 1, so that the model begins in an initial equilibrium where the two firms are identical). Models with more sophisticated determinants of product attractiveness, including product attributes such as delivery delay and reliability, product quality and functionality, service, network externalities, and so on are described in Paich and Sterman (1992) and Sterman (1988a).

(5) Firm_1_Price = Firm_1_Unit_Costs*(1+Firm_1_Margin)

(6) Firm_1_Margin = 0

Price is determined by unit costs and a target margin, assumed to be constant and set to zero for simplicity. In more complex models the margin is a strategic variable which can be used to capture firm strategy such as an attempt to gain initial market share advantage to profit from the learning curve (Beinhocker et al. 1993).

(7) Firm_1_Unit_Costs = (Firm_1_Cumulative_Production)^(Firm_1_Learning_Rate)

(8) Firm_1_Learning_Rate = LOGN(.80)/LOGN(2)

In the spirit of Arrow's (1962) original work, equations 7 and 8 portray the learning curve. Following standard learning curve theory and empirical research, the unit production costs of each firm fall by a fixed percentage with each doubling of cumulative production experience.[15] An 80% learning curve is assumed; that is, unit costs fall 20% with each doubling of cumulative experience. The model also assumes, for now, that learning is privately appropriable – each firm can prevent rivals from benefitting from its own experience.

(9) Firm_1_Cumulative_Production(t) = Firm_1_Cumulative_Production(t - dt) + (Firm_1_Production) * dt
INIT Firm_1_Cumulative_Production = 1

(10) Firm_1_Production = Firm_1_Demand

Cumulative production is simply the integral of production. The initial cumulative production levels are set to unity (as specified by the INIT statement). Production is assumed to equal demand. For simplicity, capacity constraints, production lags, inventories, and backlogs that can cause disequilibria in the goods markets are ignored. Models treating disequilibrium dynamics caused by inventories and capacity are plentiful in the system dynamics literature (e.g. Forrester 1961, Mass 1975, Lyneis 1980, Sterman 1989a,

[15] Arrow (1962), however, originally assumed that learning was a function of cumulative investment.

Sterman 1989b, and Sterman 1989c). Models of learning curve environments that treat these sources of disequilibrium include Beinhocker et al. (1993), and Paich and Sterman (1992).

(11) Industry_Demand = STEP(4,1)

The evolution of the industry commences when industry demand, initially zero, increases suddenly to four units per year in year one. For simplicity industry demand price elasticity and other factors that may affect industry demand such as word of mouth, marketing, demographic changes, etc. are ignored. See Paich and Sterman (1992) and Beinhocker et al. (1993) for models with dynamic, endogenous industry demand.

(12) Aggregate_Attractiveness = Firm_1_Attractiveness+Firm_2_Attractiveness

The aggregate attractiveness of all firms is the sum of the individual attractiveness levels, ensuring that the sum of the market shares is unity for any number of firms.

(13) Consumer_Sensitivity_to_Price = 10

Each firm is assumed to operate in an imperfectly competitive environment. Each firm's demand curve is highly, but not infinitely, elastic (assuming no reaction by the other firm).

Obviously the model is highly simplified. Yet it contains sufficient feedback complexity to show interesting path-dependent behavior. The feedback structure of the model is shown in Figure 2. The learning curve creates a positive or self-reinforcing feedback process within each firm (loops 1 and 2 in the figure). These loops act to differentiate the two firms from one another by progressively reinforcing and amplifying any initial difference in prices and market shares. In addition, the coupling of the two firms through competition creates a third positive loop (the "Figure 8" loop denoted as loop 3 in Figure 2) whereby greater market share of, say, firm 1 boosts its cumulative output, lowering its price, and reducing firm 2's market share, thus slowing the rate at which firm 2 gains experience and can lower its price, further boosting firm 1's market share. Though both firms are identical at the start of the simulation, the random disturbances in product attractiveness will give one firm a small initial advantage in market share. In the simulation shown in Figure 3, the initial edge goes to firm 1. Firm 1 develops a slight lead in the accumulation of production experience, and moves down the learning curve faster than its competitor, yielding a slight price advantage. Lower price then yields additional market share and still faster accumulation of production experience, while the competitor's rate of experience accumulation slows. The process continues until the leading firm captures essentially the entire market, driving the competitor out of business. The competitor's costs stabilize well above those of the dominant firm.

Figure 4 shows the result of fifty simulations, differing only in the particular sequence of random disturbances realized in each case.[16] As expected, each firm dominates about half the time, and the envelope of market share paths traces out a "lobster claw" shape. Because costs fall most rapidly in the early years when cumulative production is doubling rapidly, small initial advantages rapidly differentiate the two firms. Later, the cumulative cost advantage of the dominant firm is simply too great to overcome and the system locks in to the particular equilibrium chosen. Indeed, in most cases the loser has been driven out by year 10. Occasionally, however, the random disturbances roughly balance during the period in which the learning curve is strongest, leading to slower differentiation. However, the positive feedback loops through which success begets success always lead eventually to the dominance of one of the firms – that is, the model has only two equilibrium states: Firm 1 market share must tend towards 100% or 0%. Further, the particular equilibrium realized depends on the particular sequence of events in the early history of the industry. Here these events are modeled as random, though in reality they also depend on the strategic moves of the contending firms as well as the parameters governing the learning curve and other aspects of the firms' structure and decision making behavior (which need not be the same).

It is worthwhile to consider more subtle dynamics which can arise when the feedback environment is richer, containing multiple positive and negative feedbacks, some of which are nonlinearly coupled, so that the dominant loops or active structure can shift endogenously as the system evolves. To illustrate, the model is now generalized to include imperfect appropriability of learning. In reality, a firm may often benefit from the production experience of its rivals by imitating their practices and techniques, learning

[16] The simulations were run under the Euler integration method with a time step DT = .25 years.

from suppliers or customers they have in common, sending their employees to trade shows and professional conferences, hiring competitor employees, and reverse-engineering rival's products (von Hippel 1988). The equations of the model are modified as follows:

(7') Firm_1_Unit_Costs = (Firm_1_Cumulative_Production + Firm_1_Cumulative_Learning_from_Competitors)^(Firm_1_Learning_Rate)

Unit costs are now determined by the sum of the firm's own cumulative production experience and the stock of cumulative experience the firm has been able to glean from its competitor.

(14) Firm_1_Cumulative_Learning_from_Competitors(t) = Firm_1_Cumulative_Learning_from_Competitors(t - dt) + (Firm_1_Learning_from_Competitors) * dt
INIT Firm_1_Cumulative_Learning_from_Competitors = 0

The stock "Cumulative Learning from Competitors" reflects the amount of the competitor's relevant production experience the firm has been able to acquire. Thus to the extent a firm can learn from its competitor, it will move down the learning curve faster than when learning is privately appropriable. Initially, none of the competitor's experience is known to the firm.

(15) Firm_1_Learning_from_Competitors = (1-Appropriability_of_Firm_2_Experience)*MAX(0,(Firm_2_Cumulative_Production-Firm_1_Cumulative_Production)*NORMAL(1,.1)/Firm_1_Experience_Diffusion_Delay)

(16) Firm_1_Experience_Diffusion_Delay = 1

The rate at which each firm accumulates knowledge about the production experience of its competitor depends on several factors. First, each firm may benefit from the competitor's experience only to the extent the competitor's production experience is not privately appropriable (hence the {1-appropriability} term). Second, the model assumes that learning is only beneficial to the firm (hence the MAX function to ensure nonnegativity of the learning rate). Third, the model assumes that the firm can only learn what it does not yet know. Thus the rate of learning is proportional to the difference between the competitor's knowledge and the firm's: the greater the lead of the competitor, the more the firm might benefit. The time constant over which the gap in knowledge is closed is determined by the Experience Diffusion Delay. The diffusion delay represents the time required for one firm to learn about and implement the knowledge of its competitor. A one year average delay is assumed in the simulations below. Finally, it is assumed that a firm's learning from its competitor is stochastic, with multiplicative disturbances in the learning rate of each firm determined by an independent normal random variable, with a standard deviation equal to 10% of the expected learning rate.

(17) Appropriability_of_Firm_1_Experience = GRAPH(Firm_1_Market_Share)
(0.00, 0.00), (0.1, 0.00), (0.2, 0.00), (0.3, 0.05), (0.4, 0.15), (0.5, 0.3), (0.6, 0.7), (0.7, 0.95), (0.8, 1.00), (0.9, 1.00), (1.00, 1.00)

There are many possible hypotheses regarding the appropriability of learning. To illustrate the concept of shifting feedback loop dominance, the appropriability of each firm's experience is assumed to vary nonlinearly with market share, where market share is used here as a proxy for market power (e.g., control of suppliers from whom competitors might glean knowledge of the firm's practices and techniques). When the competitor's market share is low, their production experience is assumed to be nonappropriable -- i.e., the firm cannot protect its knowledge from larger and more powerful rivals. As a firm's market share rises, however, the degree of appropriability rises until, for high market shares, its knowledge is assumed to become fully appropriable (Figure 5). The software program STELLA allows this relationship to be captured through a GRAPH function. The GRAPH function allows the model-builder to specify arbitrary nonlinear relationships as a series of x-y pairs. The software then interpolates linearly between the points. Analytic functions can also be used easily (a logistic or Gompertz function might be used here). Clearly the relationship between market share and appropriability of knowledge in the model, particularly the numerical values, is speculative; they are chosen simply to illustrate the ways in which complex hypotheses about decision making behavior may be represented easily in models of this type.

The feedback loop structure of the revised model is shown in Figure 6. Inspection of the figure

reveals that there are now more complex interactions between the model's feedback loops. The positive feedback loops created by the learning curve are now potentially offset by negative feedback loops created by the process of learning from competitors (loops 4, 5 and 6). A firm that finds itself falling behind can learn from the practices of its rivals and thus close the gap in unit costs, restoring market share, staying in the game – perhaps ultimately using the learning curve to its advantage. The relative strength of the positive experience curve loops and the negative cross-firm learning loops determines the nature of the equilibrium achieved. As seen in the simple model, fully appropriable learning means the positive loops dominate and one firm must drive all others to extinction. If learning were not appropriable, and the time constant for knowledge diffusion were short enough, the negative loops that tend to equalize learning would dominate. Thus, whenever a firm began to develop a lead in experience, and hence a cost advantage, its competitor would rapidly learn from the experience and neutralize the leader's advantage. The industry equilibrium would be an even split of the market among the different competitors. Industry leaders would emerge from time to time as a result of the random component assumed for customer preferences, but such periods of leadership would be short-lived and would not favor any particular firm.

In the full extended model the relative importance of the positive and negative loops varies endogenously as a function of market share, introducing another set of positive feedbacks. As illustrated by loop 7 in Figure 6, the assumption that market dominance allows a firm to prevent rivals from benefitting from its experience creates a positive loop whereby an increase in market share reduces the rate at which other firms can learn, slowing the rate at which the negative learning loops 4-6 can equalize costs, giving the firm still greater opportunity to move ahead on its own learning curve. In contrast to the two extreme cases of complete private appropriability or rapid knowledge diffusion, it is not obvious from inspection how the full model, with this complex nonlinear feedback structure, will behave.

Indeed, simulations of the extended model show a variety of complex paths for the evolution of the industry. Figure 7 shows thirty simulations of the extended model. In most cases, one firm establishes dominance quickly and drives the other to extinction before the losing firm can learn enough from the competitor to close the experience gap and equalize unit costs. In these cases the positive learning curve loops dominate, and the farther behind a firm gets the less it is able to benefit from competitor experience. In other cases the initial leader finds its rival is able to close the gap, equalize market shares, and essentially begin the game again. Figure 8a shows such a case. Firm 1 gains initial advantage, but is not able to prevent firm 2 from learning from its experience. Despite firm 1's market share advantage of nearly two-to-one in year 5, firm 2 eventually wins. Occasionally, the initial leader suddenly loses, after a long period of high market share, as shown in Figure 8b. Here, industry leadership passes between the two firms several times. Around year 18, firm 2 is able to reverse the advantage of firm 1 through learning, and dominate the industry with about 70% market share from years 25 through 40. Nevertheless, firm 1 ultimately emerges the winner. The interesting feature of this simulation is the speed of the ultimate triumph for firm 1 after decades of slow change. In still other simulations, the equilibrating negative loops caused by the exchange of knowledge dominate the differentiating effects of the positive experience curve loops and the two firms remain roughly equal for very long periods of time, as in Figure 8c.

Obviously, though only two firms are treated here for simplicity, the model generalizes readily to N firms, so the interaction of large populations of firms can be studied. Further, one can easily extend the model to include explicit entry and exit; heterogeneity of firm attributes, customers, and technology; more sophisticated representations of decision making; and more sophisticated representations of technology and organizations, including changes in fundamental architectures that may destroy firm competencies (Henderson and Clark 1990, Tushman and Romanelli 1985).

Despite the simplicity of the model, the simulations exhibit a number of key features of evolutionary models. First, the dynamics are strongly path dependent. Second, the behavior is self-organizing: what begins as a market of identical agents rapidly organizes itself into a highly differentiated structure. The particular firm that dominates cannot be predicted in advance, yet the model spontaneously organizes itself into characteristic patterns. Third, the landscape in which the different firms compete against one another is changing as they move through it: as production experience and market share change, so does the strength of the various feedback loops, thus conditioning the future evolution of the market. In the language of feedback control theory and system dynamics, the evolution of the industry endogenously alters the dominant feedback structure of the system. These changes in active feedback structure then feed back to condition the dynamics of the system.

Software and other Resources for Evolutionary System Dynamics Modeling

Over the years, a variety of software packages, books, and professional journals have been

developed specifically for the field of system dynamics. In terms of software, DYNAMO (Pugh 1983), DYSMAP2 (1992), and NDTRAN (Davisson and Uhran 1979) are available for both mainframe and personal computers; Vensim (Eberlein 1991) is available for PCs and some UNIX-based workstations; and STELLA (Richmond and Peterson 1992), I Think, and MicroWorld Creator (1990) are available for the Apple Macintosh.

Basic text books describing the system dynamics method include those by Forrester (1961), Forrester (1968), Goodman (1977), Richardson and Pugh (1981), Roberts et al. (1983), and Richmond and Peterson (1992). Since 1985 the international System Dynamics Society has published a professional journal, the System Dynamics Review, covering the theory and application of system dynamics in a wide range of disciplines.

Conclusions

Recent developments in nonlinear theory, the psychology of decision-making, and experimental economics have joined to form the basis for empirically testable, nonlinear, disequilibrium theories of evolutionary economic dynamics. Advances in the mathematics of nonlinear dynamical systems allow modelers to represent the non-average behavior of individual agents and to portray systems far from equilibrium. Advances in simulation techniques, software, and computer hardware make such capabilities accessible to anyone with a personal computer and knowledge of basic mathematics.

However, evolutionary economics cannot succeed merely as a technical undertaking. If evolutionary approaches are to generate penetrating insights into the behavior of actual economic systems, the tools of modeling must be complemented by appropriate tools of empirical investigation so that theories are grounded in experimental test and field study of economic decision making. Evolutionary models should portray the decision making behavior and heuristics of the people in the system as they exist, warts and all, including explicit attention to the many limitations of cognitive capabilities, the role of habits, emotions, culture, and other bounds on human rationality. Though traditional tools of econometric estimation will continue to be useful, the decision rules used in evolutionary models must be investigated first hand, in the field and laboratory. The work and methods of economic historians and institutionalists, psychologists, sociologists, anthropologists, and others have much to offer in this endeavor.

System dynamics is well suited to the development and testing of evolutionary models. With its historic emphasis on explicit modeling of stocks and flows, nonlinearities, feedback processes, and behavioral decision making, it provides a well-developed body of theory, technique, and examples for modeling disequilibrium dynamics in economic systems. Further, system dynamics practitioners have developed diverse methods for investigating decision making in the field, eliciting the mental models and decision rules people use, and testing the resulting formulations. Modern developments in system dynamics software and pedagogy have so simplified the mechanics of the model-building process that pre-college students are regularly building evolutionary models, firms and government agencies are using such models to help design corporate strategy and public policy, and research into new applications of evolutionary dynamics is growing.

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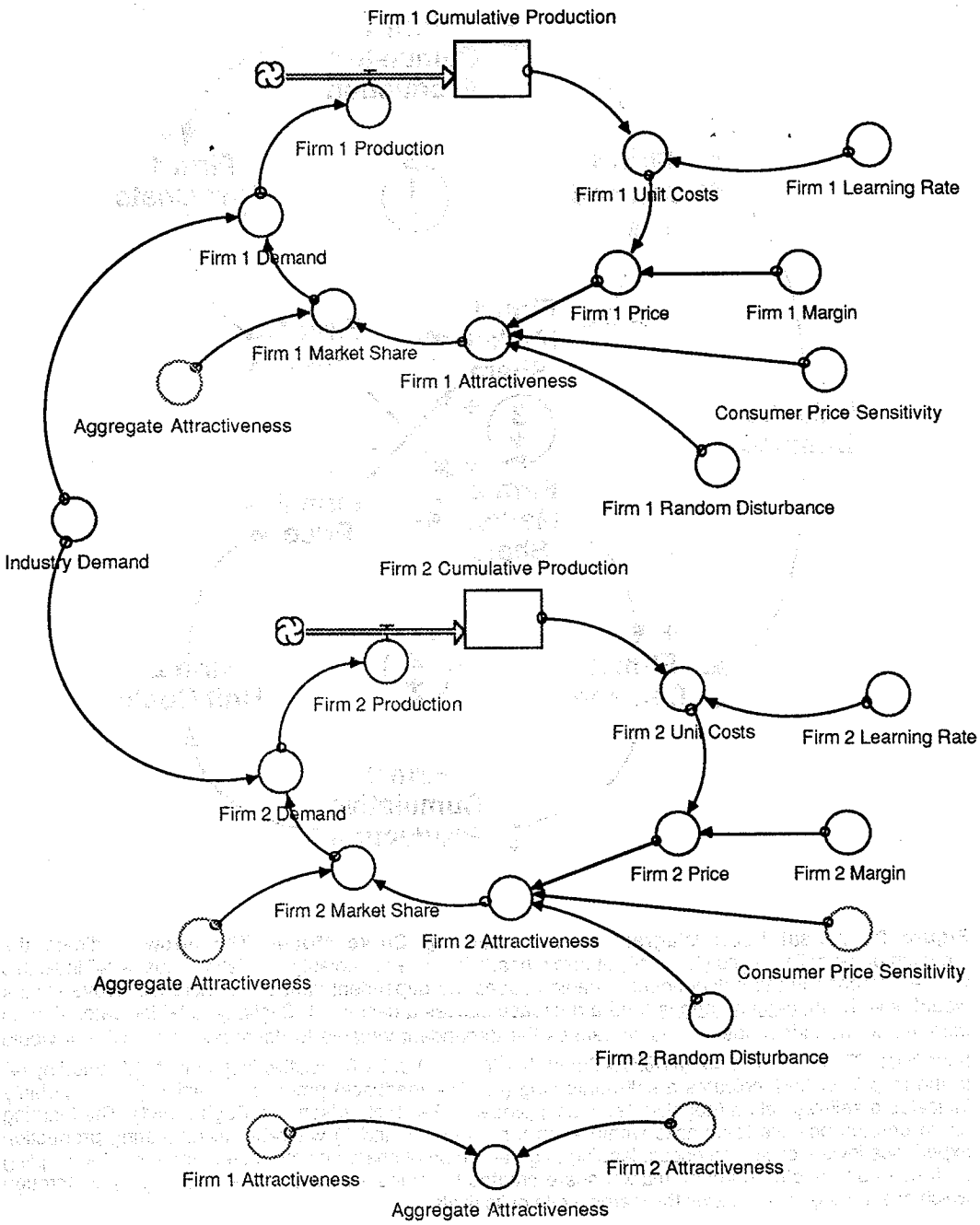

Figure 1: System Dynamics Model of Duopoly Under a Learning Curve. The diagram is reproduced exactly from the simulation model in STELLA. The dashed circles are "ghosts," or copies of variables defined elsewhere in the diagram (to avoid cluttering the diagram with crossed lines).

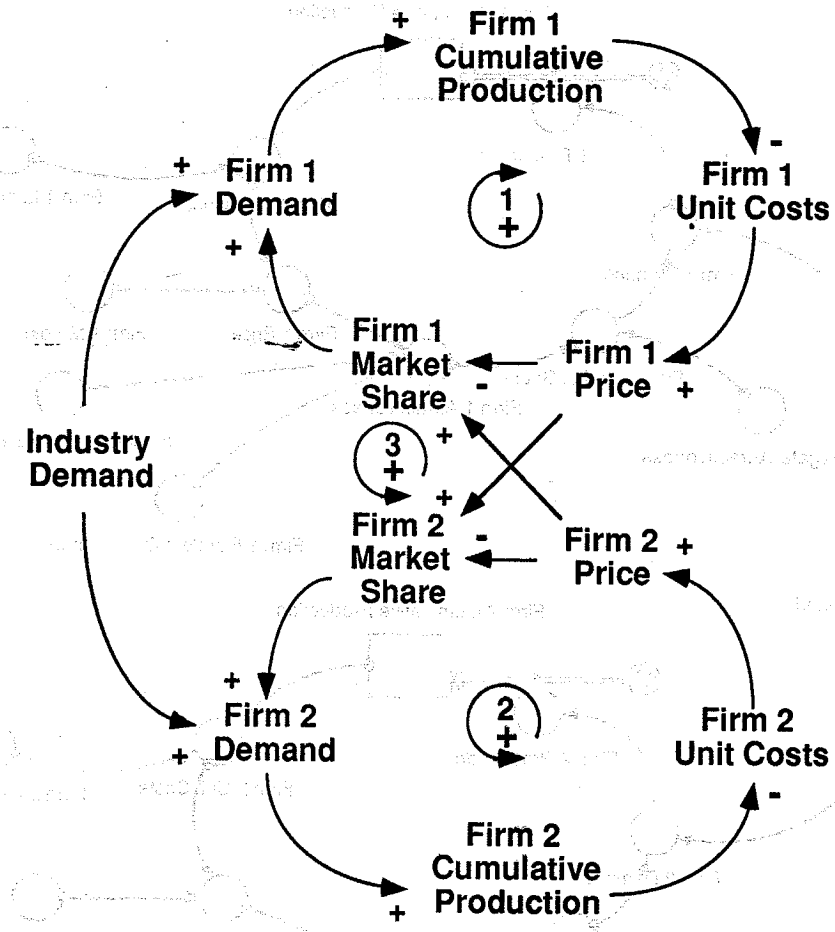

Figure 2: Causal Loop Diagram of the Learning Curve Model. The arrows indicate the direction of causality. Signs ("+" or "-") at arrow heads indicate the polarity of relationships: a "+" indicates that an increase in the independent variable causes the dependent variable to increase above what it would have been, ceteris paribus (and a decrease causes a decrease). Similarly, a "-" indicates that an increase in the independent variable causes the dependent variable to decrease below what it would have been. That is, $X \rightarrow {}^{+}Y \Rightarrow (\partial Y/\partial X) > 0$ and $X \rightarrow {}^{-}Y \Rightarrow (\partial Y/\partial X) < 0$. Positive loop polarity [denoted by (+) in the loop identifier] indicates a self-reinforcing (positive feedback) process. Negative (-) loop polarity indicates a self-regulating (negative feedback) process. See Richardson and Pugh (1981). The learning curve creates positive feedbacks within each firm (loops 1 and 2) whereby accumulating production experience lowers costs and prices, leading to greater market share and still faster learning. The coupling of firms to one another through market share creates the "Figure 8" positive feedback (loop 3) through which one firm's gain also slows the learning rate of its rivals.

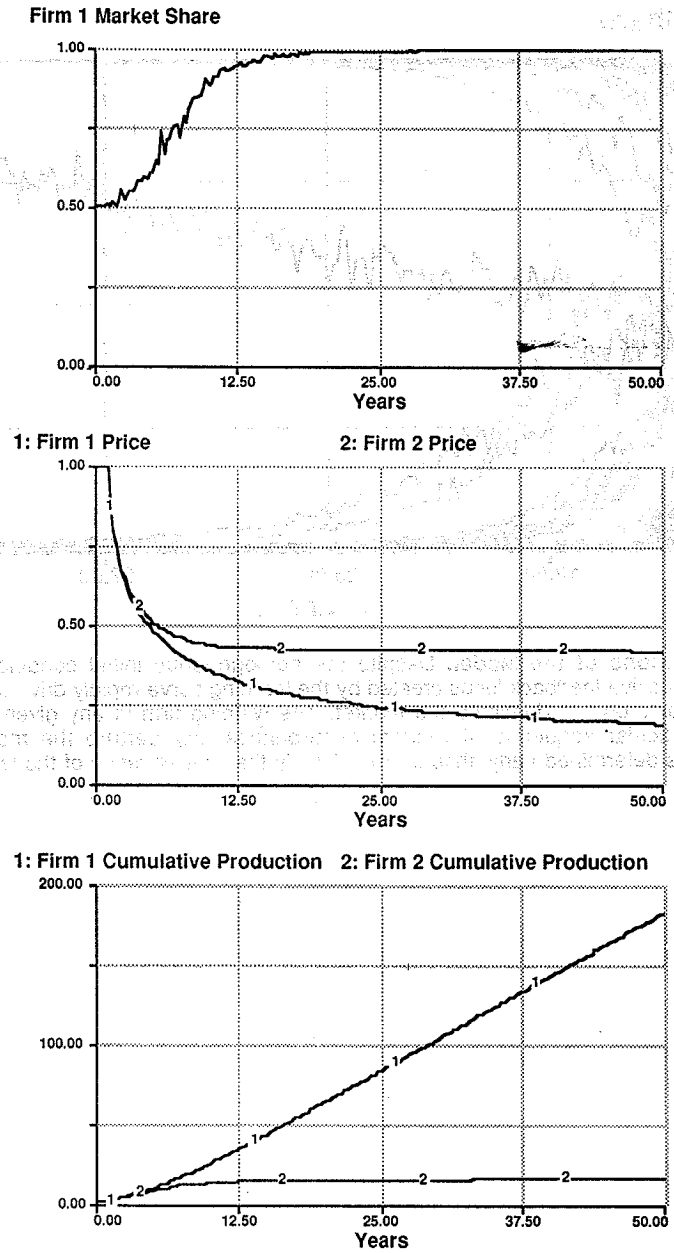

Figure 3: Simulation Run Where Firm 1 "Wins." Small initial differences in cumulative production caused by random disturbances are amplified by the positive feedback loops until firm 1 forces firm 2 completely out of the market, despite equal initial conditions.

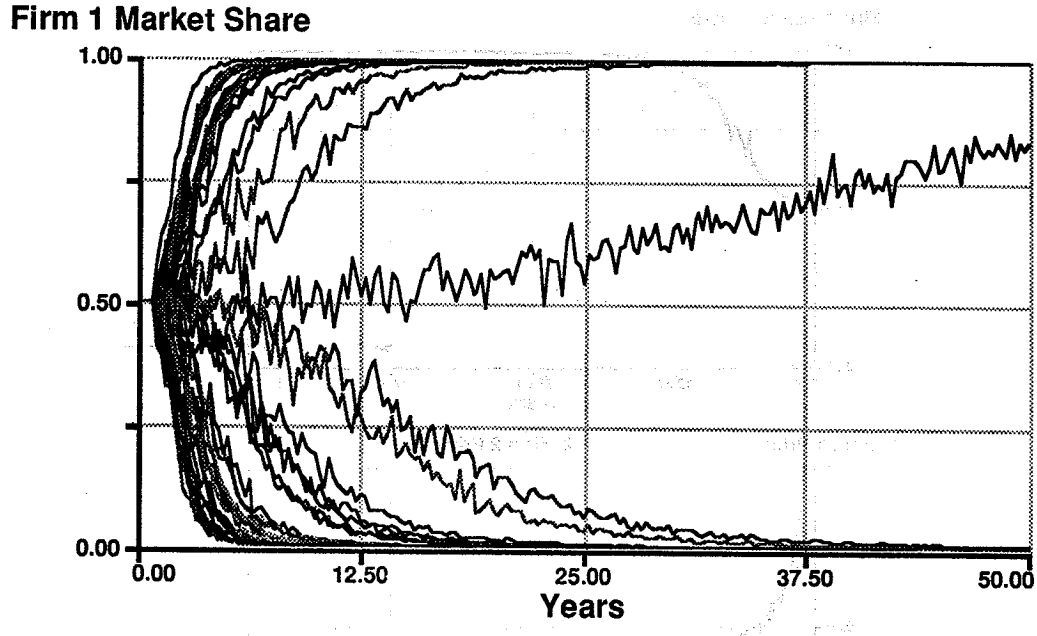

Figure 4: Fifty Simulations of the Model. Despite the homogeneous initial conditions where all firms are identical, the positive feedback loops created by the learning curve rapidly drive one firm out of business while the other grows to dominate the market. The winning firm in any given simulation is determined by the particular sequence of random disturbances that perturb the model. In most simulations the winner is determined early, though occasionally the differentiation of the two firms takes many years.

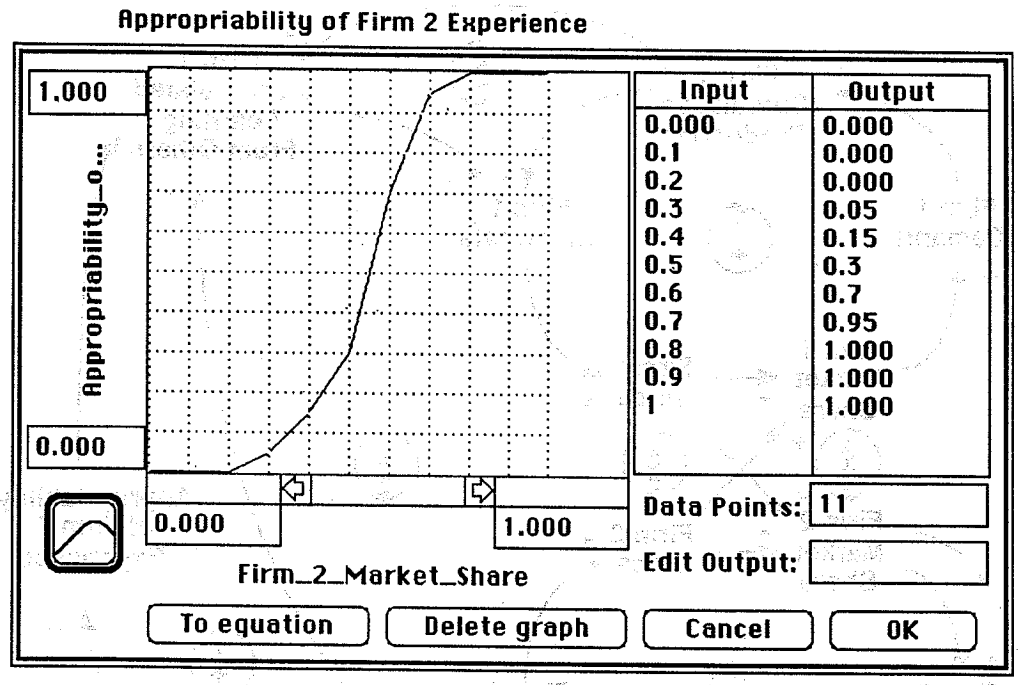

Figure 5. Graphical Function Showing Assumed Dependence of Knowledge Appropriability on Market Share, for Firm 2 (Equation 17). The curve reflects the assumption that the larger firm 2's market share, the more it can appropriate its experience and prevent rivals from benefitting. The software interpolates linearly between the specified points. The user can select any domain and interval for the independent variable, thus controlling the smoothness of the relationship. While analytic expressions can be used to capture such nonlinear functions, the ability to specify arbitrary nonlinearities as look-up tables greatly speeds model development, enhances flexibility, and makes complex nonlinear modeling accessible to students, managers, and others without extensive training in mathematics.

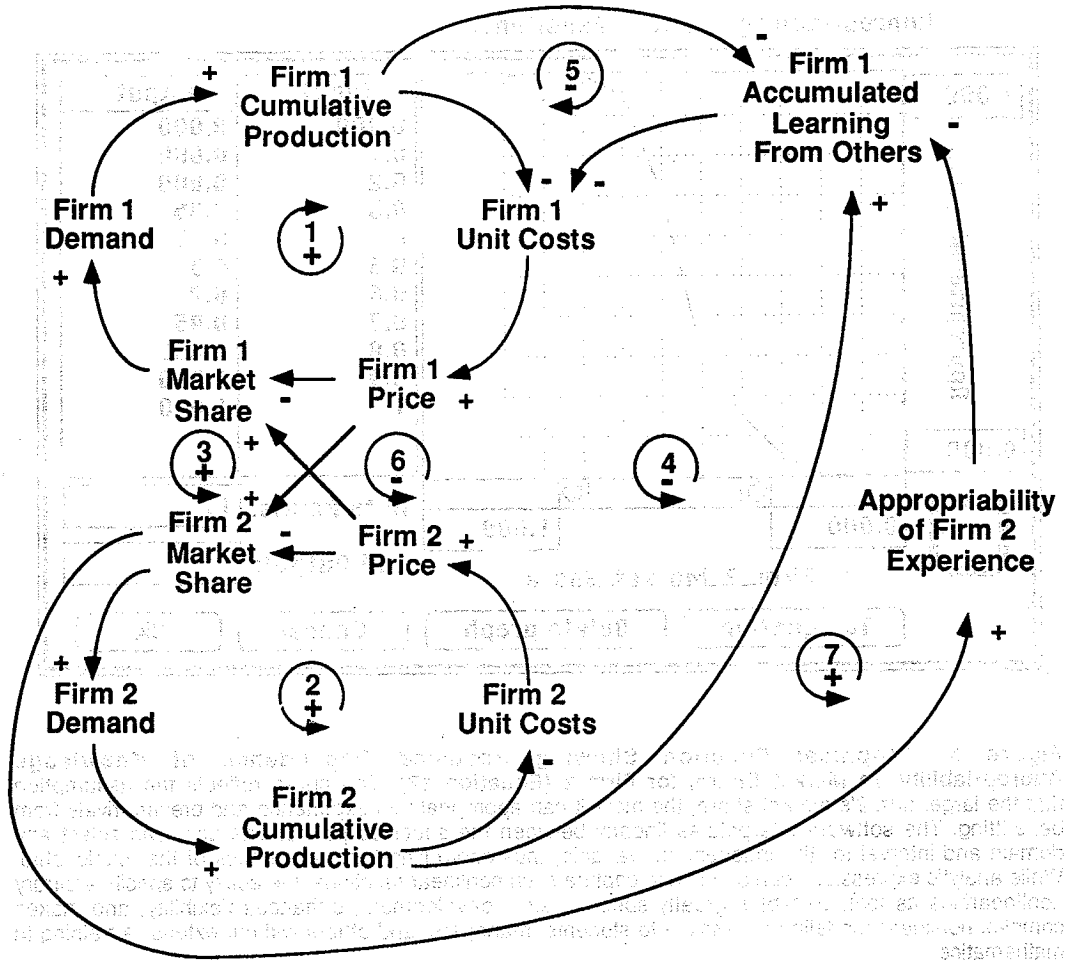

Figure 6: Causal Loop Diagram Showing the Feedback Structure of the Extended Model, in Which Firms Can Benefit From the Accumulated Experience of Their Rivals. For clarity, the structure for inter-firm learning is shown for firm 1 only. The structure of inter-firm learning for firm 2 (not shown) is symmetrical and creates many more loops than are shown in the diagram. Inter-firm learning introduces negative feedback loops that tend to equalize prices (loops 4, 5, 6), while the assumed dependence of knowledge appropriability on market share creates additional positive feedbacks (loop 7).

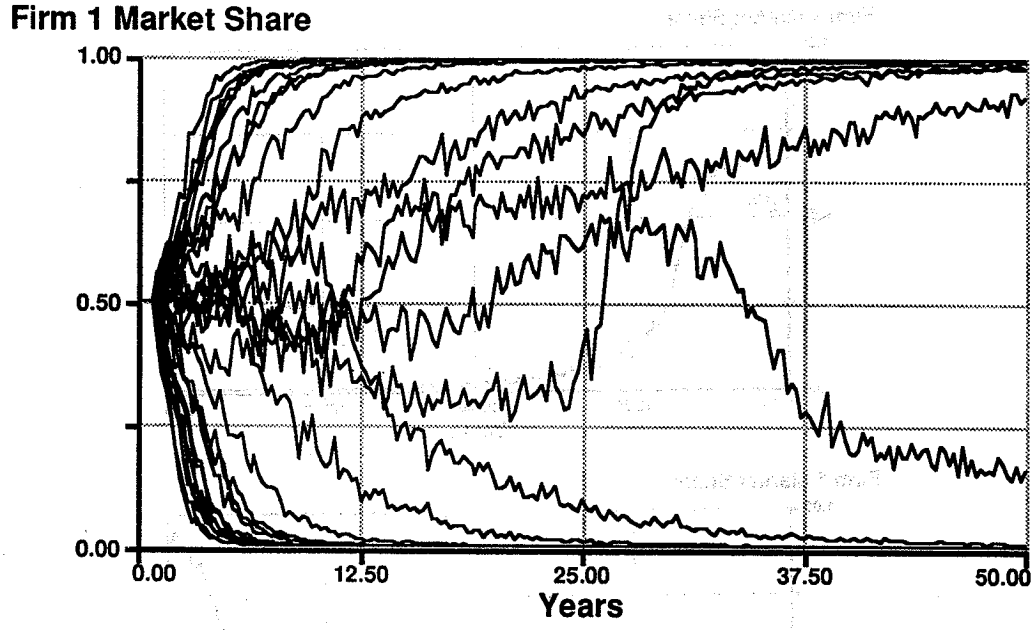

Figure 7. Thirty Simulations of the Extended Model, Showing Many More Complex Paths of Industry Evolution Arising When Firms Can Learn from One Another. Note the cases where market leadership reverses through inter-firm learning. The ultimate winner is often not selected for decades, and long periods of market share dominance no longer guarantee a firm will ultimately triumph.

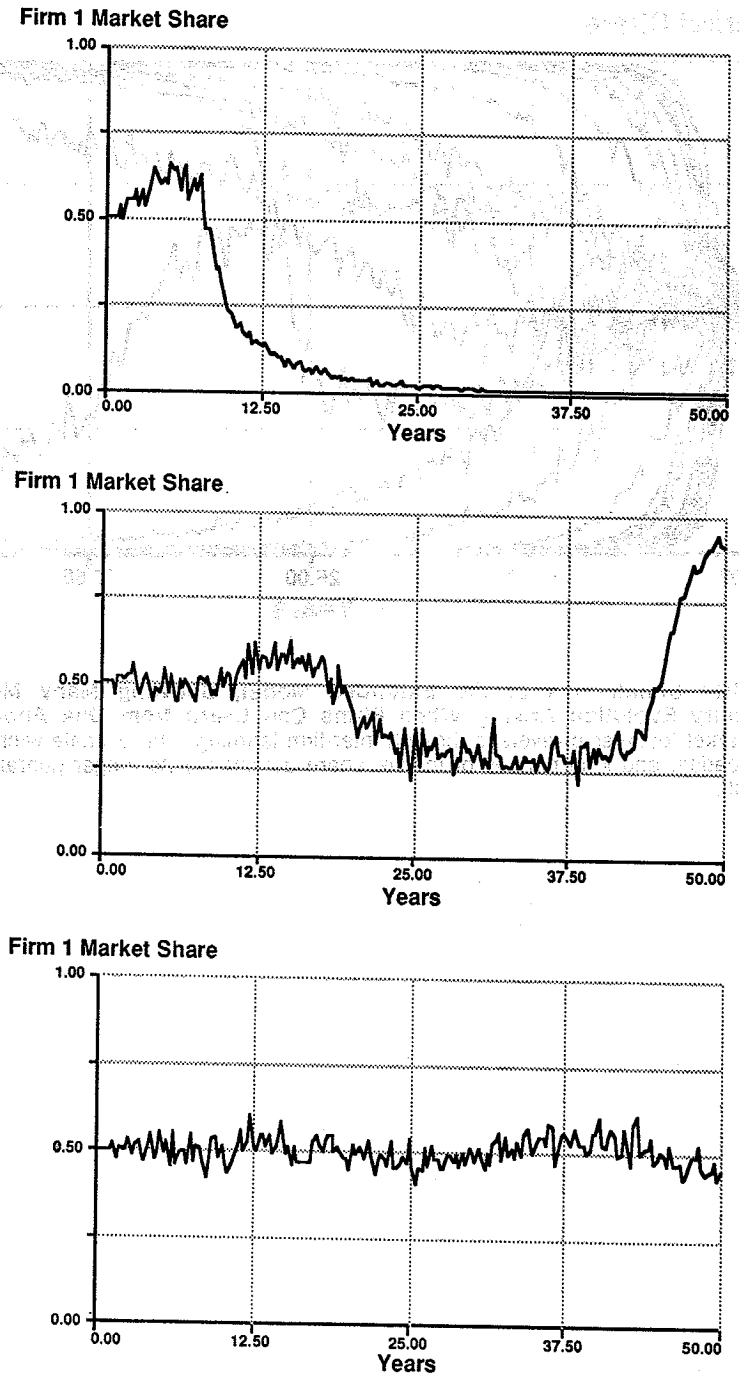

Figure 8. Three Simulations of the Extended Model, Showing the Diversity of Paths of Market Evolution.