Improvement of Badr's Algorithm of Robust Control System

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ABSTRACT:Time–domain robustness of control system is studied in this paper .Badr's algorithm is enhanced by using matrix perturbation theory and convex polyhedron idea,so as to suit the needs of multiple–parameter variations of high–order system.With matrix trace as performance criterion,a new synthetical algorithm of robust control systems is presented,an example based on this algorithm is also given.

KEY WORDS: Control system,Stability robustness,Performance robustness

1. Introduction

Generally,there are two methods on uncertain control system research:adaptive control scheme and robust control scheme.The system based on the former,while the parameters of controller vary with the parameters of plant,has complicated structure and high cost;however the latter,adopting fixed controller,has simple structure and low cost,and it is also easily realized.Consequently,it causes more and more attentions.

The research interest concerning robust control system is generally classified into two perspectives:stability robustness and performance robustness.Stability robustness means that the system remains stable with system parameter variations. The representative works on this respect arecompleted by Horisberger[1],Darision[2] and Yedavalli[3],etc.Performance robustness means that the performance criterion remains efficient with parameter variations of the system .In this respect, Chang and Vinkler's work is more representative.Though robust control system has been of major interest for many years,more attention are paid to the respect of ability robustness.By using the convex polyhedron concept and optimum control theory,Badr[6] combined the two respects ingeniously,and then a synthetical algorithm on robust system is raised.But it is so complicated in calculating high–order,multiple–variable parameter systems,thus it's necessary to get improved.Toward this target,a new synthetical algorithm on robust control systems is proposed in this paper

2. Stability Robustness

2.1. Basic definitions and theorems

A linear time–invariant system is denoted as:

$$\dot{X} = F(v)X \qquad (1)$$

where: 1.) The set of parameter variations v is assumed to be polyhedron of the form:

$$V = (V:\ l \leqslant v \leqslant u\ , u \in R^{q}) \qquad (2)$$

l and u are given constant real vectors.

2.) F(v) is a multiple linear function of parameter vector v, if we fix other components of v, $f_{ij}(v)$ only are linear functions of (v_i).

The vector number of convex polyhedrom is $N = 2^{q}$, where q is the number of variable parameters, the corresponding vectors are defined as:

$$v^{(1)}, v^{(2)}, \ldots, v^{(N)}$$

The corresponding state matrices are:

$$F^{(i)} = F(v^{(i)}),\quad i \in \{1,\ 2\ ,\ldots,\ N\}$$

Lemma (2.1): For a linear control system defined by eqns. (1) and (2), if there exists a real symmetic positive–definite matrix P, such that

$$PF(v^{(i)}) + F^{T}(v^{(i)})P < 0,\ i \in \{1,\ 2,\ \ldots\ N\}$$

then system (1) is asymptotically stable ($\forall v \in V$).

Definition (2.1): For $A = (a_{ij}) \in R^{n \times n}$, if $a_{ij} \geqslant 0$, $i, j \in \{1,\ 2,\ \ldots,\ n\}$, then A is named non–negative matrix (written as $A \gg 0$).

Lemma (2.2): For $A, B \in R^{n \times n}$, if $A \leqslant |A| \leqslant B$, then any eigenvalue of matrix A is less than the maximum eigenvalue r of matrix B, that is

$$|\lambda| \leqslant r$$

Proof : By the theory of matrices, we have,

$$|A^{m}| \leqslant |A|^{m},\ m \in \{1,\ 2,\ \ldots\ \} \qquad (3)$$

If $0 < A < B$, then:

$$0 \leqslant A^{m} \leqslant B^{m} \qquad (4)$$

Combine eqns. (3) and (4), we have

$$|A^{m}| \leqslant |A|^{m} \leqslant |B^{m}|$$

Morever, because $|A| \leqslant |B|$, then

$$||A||_2 \leqslant ||B||_2$$

According to the properties of L_2 norm

$$||A||_2 \leqslant ||\ |A|\ ||_2$$

$$\|A^m\|_2 \leqslant \| \, |A^m| \, \|_2 \leqslant \|B^m\|_2 \qquad (5)$$

$$\|A^m\|_2^{1/m} \leqslant \| \, |A^m| \, \|_2^{1/m} \leqslant \|B^m\|_2^{1/m}$$

When $m \to \infty$, we have

$$\rho(A) \leqslant \rho(|A|) \leqslant \rho(B)$$

Therefore

$$|\lambda| \leqslant r$$

Let the numbers of variable entries of state matrices A and B be respectively q_1 and q_2, then $2^{q_1+q_2}$ subsystems can be formed. The sets of system can be denoted by

$$\dot{X}^{(i)} = A^{(i)}X^{(i)} + B^{(i)}U^{(i)} = (A_0 + \triangle A^{(i)})\, X^{(i)} + B^{(i)}\, U^{(i)}$$

$$i \in \{1,\ 2,\ \ldots, 2^{q_1+q_2}\} \qquad (6)$$

where A_0 is the asymptotically stable matrices of nxn norminal system.

Definition (2.2): Let the disturbance matrix be

$$\triangle A^{(i)} = \beta_i E_i \qquad (7)$$

where E_i are constant, describable matrices, in which the maximum entries are 1, while β_i are uncertain parameters.

Definition (2.3) Let $\triangle$ be n-order matrix, and

$$\triangle_{ij} \equiv 1, when\ A_{0ij} \neq 0$$

From definition (2.2) and (2.3), apparently

$$\triangle_{ij} \geqslant |E_{ij}| \geqslant 0$$

If $\quad s = \max\limits_{i} \beta_i, \qquad i \in \{1,\ 2, \ldots, 2^{q_1+q_2}\}$ then the following theorem can be established.

Theorem (2.1): Let the sets of systems be defined by eqns. (6), if

$$s\triangle \geqslant \triangle A^{(i)} \quad i \in \{1,\ 2, \ldots 2^{q_1+q_2}\}$$

and $P = P^T > 0$, then we have

$$\max_{i=1,2,\ldots 2^{q_1+q_2}} \lambda_{max}\, [P(A_0 + \triangle A^{(i)} - B^{(i)}G) + (A_0 + \triangle A^{(i)} - B^{(i)}G)^T P]$$

$$\leqslant \max_{i=1,2,\ldots 2^{q_1+q_2}} \lambda_{max} [P(A_0 + \varepsilon\triangle - B^{(i)}G) + (A_0 + \varepsilon\triangle - B^{(i)}G)^T P]$$

Where G is feedback gain matrix.

Proof: since $\varepsilon\triangle \geqslant \triangle A^{(i)}$, $i \in \{1, 2, \ldots, 2^{q_1+q_2}\}$

Therefore $A_0 + \varepsilon\triangle - B^{(i)}G \geqslant A_0 + \triangle A^{(i)} - B^{(i)}G$

$$i \in \{1, 2, \ldots 2^{q_1+q_2}\}$$

Moreover,because $P = P^T > 0$,then

$$P(A_0 + \varepsilon\triangle - B^{(i)}G) \geqslant P(A_0 + \triangle A^{(i)} - B^{(i)}G) \qquad (8)$$

and:

$$(A_0 + \varepsilon\triangle - B^{(i)}G)^T P \geqslant (A_0 + \triangle A^{(i)} - B^{(i)}G)^T P \qquad (9)$$

According to Lemma(2.2), we have

$$\max_{i=1,2,\ldots 2^{q_1+q_2}} \lambda_{max} [P(A_0 + \triangle A^{(i)} - B^{(i)}G) + (A_0 + \triangle A^{(i)} - B^{(i)}G)^T P]$$

$$\leqslant \max_{i=1,2,\ldots 2^{q_1+q_2}} \lambda max [P(A_0 + \varepsilon\triangle - B^{(i)}G) + (A_0 + \varepsilon\triangle - B^{(i)}G)^T P]$$

Theorem (2.2):if there exists a real symmetric positive-definite matrix P,such that

$$P(A_0 + \varepsilon\triangle - B^{(i)}G) + (A_0 + \varepsilon\triangle - B^{(i)}G)^T P < 0, i \in \{1, 2, \ldots 2^{q_2}\} \qquad (10)$$

then system $\dot{X}^{(i)} = (A_0 + \varepsilon\triangle)X^{(i)} + B^{(i)}U^{(i)}$ is asymptotically stable.

Corollary (2.1):

$$\lambda_{\substack{max \\ i=1,2,\ldots 2^{q_2}}} [(A_0 + \varepsilon\triangle - B^{(i)}G)^T P + P(A_0 + \varepsilon\triangle - B^{(i)}G)] \leqslant \max_{i=1,2,\ldots 2^{q_2}} \lambda_{max} [(A_0 + \varepsilon\triangle - B^{(i)}G)^T P + P(A_0 + \varepsilon\triangle - B^{(i)}G)]$$

(Proof see [7].)

Theorem (2.3):Assuming that P is a real symmatric positive-difinite matrix,such that

$$\eta_i(p) = \lambda_{max}[P(A_0 + \varepsilon\triangle - B^{(i)}G) + (A_0 + \varepsilon\triangle - B^{(i)}G)^T P],$$

$$i \in \{1, 2, \ldots 2^{q_2}\}$$

ıen $\eta_i(p)$are continuous, convex and differentiable.(Proof see [7]).

Definition (2.4):A function g(p) is

$$g(p) = \max_{i=1,2,\dots,2^{n}} \{\eta_i(p)\} \qquad (11)$$

From theorem (2.3),g(p) has the properties of η_i (p).

Definition (2.5):

$$h(s) = \max_{v \in V(s)} \lambda_{max} \left(PF(v(s)) + F^{T}(v(s))P\right) \qquad (12)$$

2.2 Caculating Programme

According to the definitions and theorems given above,the calculating programme of stability robustness is denoted as Fig.1.

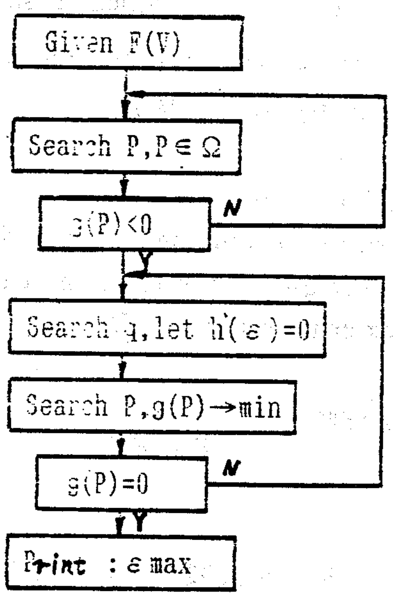

Fig.1.

The convegence property of the above algorithm is guaranted by the following theorem.

Theorem(2.4): The above algorithm gives a series $\{s_q\}$.This series are either definite or indefinite,but there's always a limit $\lim_{q\to\infty} s_q = s_{opt}$.It is impossible for $\tilde{s} > s_{opt}$ that satisfies $\tilde{P}F(v) + F^{T}(v)\tilde{P} < 0, \tilde{P} = \tilde{P}^{T} > 0, v \in V(\tilde{s})$)

(Proof see [1].)

3. Performance Robustness

Let the sets of system be defined as eqns.(6),their weighting performance index criterion is

$$J=\sum_{i=1}^{N}C_iJ_i=\sum_{i=1}^{N}C_iT_r(S_i) \qquad (13)$$

where,C_i are weighting factor. S_i are subject to Lyapunov's Eqns.

$$(A^{(i)}-B^{(i)}G)^TS_i+S_i(A^{(i)}-B^{(i)}G)+Q+G^TRG=0$$

then the performance index criterion has its minimum value.

Theorem (3.1): Let the sets of systems be defined by eqns.(6) and the performance index criterion be defined by eqns.(13) If and only if there exists an initial feedback matrix G_0 ,which is subject to $R_e\lambda[A_0-B^{(i)}G_0]<0$,then there also exists an optimal value G^*, such that J(G) is minimized.(Proof see [6])

4. Robustness Synthetical Algorithm

Combine the two algorithms above,then a synthetical robustness algorithm is founded.This algorithm not only gives the largest stability region,but minimizes the performance index criterion as well.Detailed algorithm see Fig.2.

5. Examples

Consider the following system

$$\dot{X}=AX+BU$$

where:

$$A=\begin{bmatrix}-1.356 & 0.2232 & 0\\ 11.76 & -5.389 & 0\\ 0 & 1 & 0\end{bmatrix},B=\begin{bmatrix}0.1127\\ 1.886\\ 0\end{bmatrix}$$

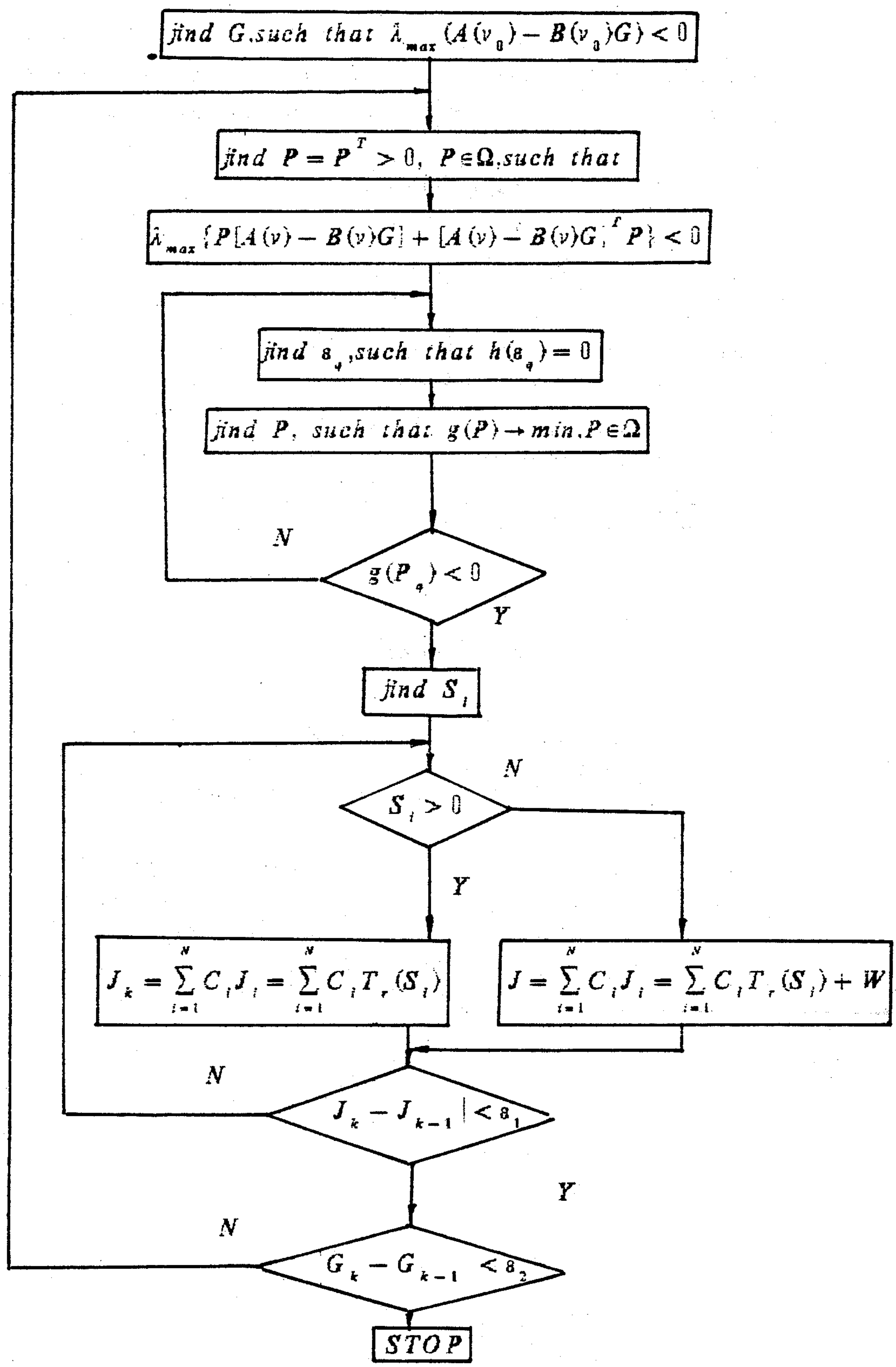
find G, such that λmax (A(v0) − B(v0)G) < 0
find P = P^T > 0, P ∈ Ω, such that
λmax {P[A(v) − B(v)G] + [A(v) − B(v)G]^T P} < 0
find s_q, such that h(s_q) = 0
find P, such that g(P) → min, P ∈ Ω
N
g(P_q) < 0
Y
find S_i
N
S_i > 0
Y
J_k = Σ C_i J_i = Σ C_i T_r(S_i)
J = Σ C_i J_i = Σ C_i T_r(S_i) + W
N
|J_k − J_{k−1}| < ε_1
Y
N
G_k − G_{k−1} < ε_2
STOP

Fig.2.

If using Badr's algorithm, it needs calculating 4096 sub-system states.However,according to the algorithm proposed in this paper,it only needs calculating 8 sub-system ststes.Calculating results are shown in Table 1 and 2.

Table 1 Badr algorithm

Times	C_0	C_i	$\varepsilon_{max} \times 10^2$	$J(G)$	$\triangle J‰$		G	
1	.2	0.0125	3.7003	13.0917	2.219	.80853	.10492	.3163
2	.6	0.00625	3.5545	12.9046	0.758	.70812	.10481	.3162
3	.8	0.00312	3.5219	12.8531	0.356	.70823	.10490	.3162

Table 2 improved Algorithm

Times	C_0	C_i	$\varepsilon_{max} \times 10^2$	$J(G)$	$\triangle J‰$		G	
1	.2	0.2	3.1974	18.3834	47.44	1.0741	.15364	.4740
2	.6	0.1	2.8573	15.7117	23.07	0.8911	.12854	.4062
3	.8	0.05	2.7845	14.0931	10.04	0.8744	.12456	.4032

6. Conclusions

From the calculating results given above,the improved synthetical algorithm proposed is simple and explicit.The calculating work is far less than that of Badr's algorithm,while the time cost is 1 / 7 of the latter.It particularly fits high–order systems with multiple parameter variations,though it exists a little conservatism.

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