BYNAMO:SYSTEM DYNAMICS SIMULATOR FOR BEGINNERS AND EDUCATIONAL USE

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1. THE PURPOSE OF OUR IDEA

It is well known that BASIC language is quite a popular one in the world. So it goes without saying that by using BASIC language, the system dynamics simulation can be provided more easily than DYNAMO does.

This paper describes a simulator for system dynamics, which is implemented by BASIC on Personal Computer PC-9800 series. We call the simulator "BYNAMO". BYNAMO is designed by using many graphic functions of the BASIC language, so as to increase efficiency for programming the system dynamics equations.

It is a strong point of BYNAMO that operations for BYNAMO are simple and easy, so that beginners can study the system dynamics without much experience for programming.

2. FEATURES OF THE SYSTEM

For the purpose of beginners and educational use, BYNAMO has the following seven featuers:

(1) It can be described models for system dynamics by simple sentences based on BASIC language.

(2) It can program SD models by understanding definitions of equations to be described, even if DYNAMO language is not understand.

(3) We can display good presentations, such as graphs, tables for outputs, using many graphic commands.

(4) It has a good efficiency for handling, by using multi-windows.

(5) In order to build models easily, we define user-defined functions in addition to included functions of BASIC language.

(6) We can modify our system so as to fit users, because it can use all of commands, BASIC language.

(7) We can run models immediately, built by BYNAMO without any process, such as compiler.

3. MAIN FRAME OF THE SYSTEM

The system has two main parts: one is used for simulation and building models, another is used for representation of graphs and tables, whose construction is shown in Fig. 1.

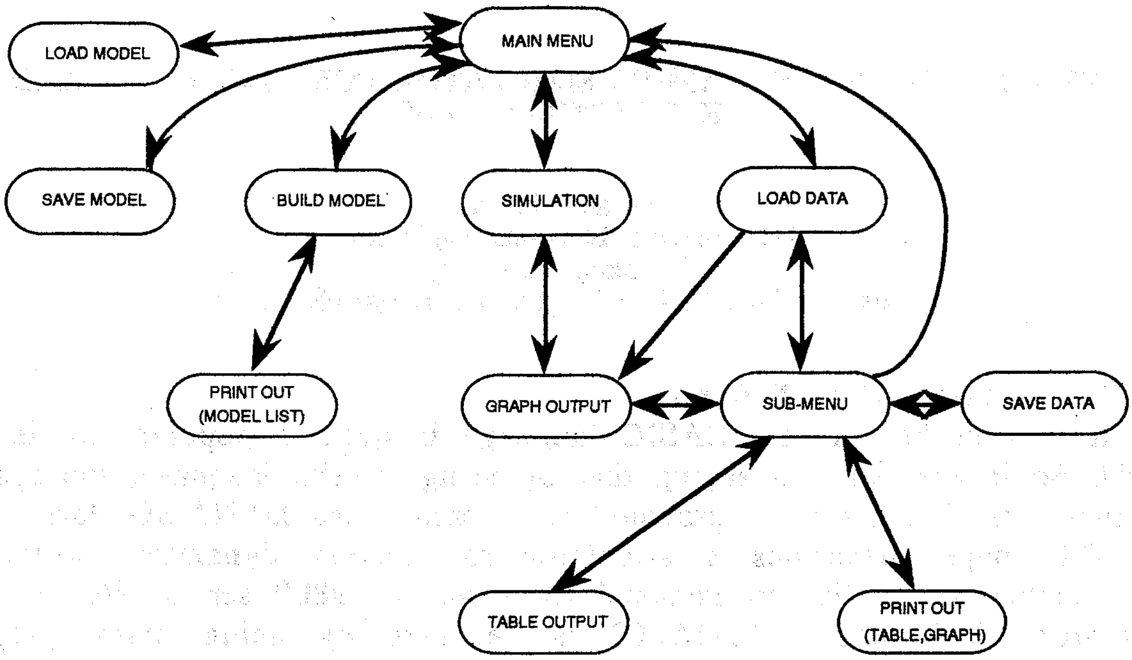

Fig.1 System Configuration for BYNAMO.

4. PROGRAMMING EXAMPLE

The programming language is based on BASIC, and we can easily obtain system dynamics equations. One example, which is known as Kaibab plateau model, is given as follows:

```
2010 *Z.CONST
2020 DP=4000
2030 PP=8000
2040 F=350000!
2050 FCAP=350000!
2060 AREA=800000!
2070 NFPD=1
2080 RF=.2
2090 RST=1905
2100 DGRFT$="-.75/-.50/-.25/0.00/0.12/0.20/0.23/0.24"
2110 DKRT$="0.00/0.20/1.20/3.20/5.40/7.60.8.60/9.30/
             9.80/10.0/10.0"
2120 PGRFT$="-.40/0.00/0.02/.035/.045/0.05/.055"
2130 FRTT$="20.0/8.00/3.00/2.00/1.00"
2140 FCPDT$="0.00/.0.25/0.50/0.75/1.00/1.12/1.20"
2999 RETURN
3000 '*** RATE, AUXILIARY EQUATIONS ***
3010 *Z.RATE
3020 PBR=FNSTP(RF,RST)*PP
3030 FDP=F/DP
3040 FR=FDP/NFPD
3050 FRT=FNTABL(FRTT$,F/FCAP,0,1!,.25)
```

```
3060 GR=(FCAP-F)/FRT
3070 FCPD=FNTABL(FCPDT$,FR,0,1.5,.25)
3080 FC=DP*FCPD
3090 DGRF=FNTABL(DGRFT$,FR,.25,2!,.25)
3100 DNGR=DP*DGRF
3110 DD=DP/AREA
3120 DKR=FNTABL(DKRT$,DD,0,.1,.01)
3130 DPR=DKR*PP
3140 PGRF=FNTABL(PGRFT$,DKR,0,.6,.1)
3150 PNGR=PP*PGRF
3999 RETURN
4000 '*** LEVEL EQUATIONS ***
4010 *Z.LEVEL
4020 F=F+DT*(GR-FC)
4030 DP=DP+DT*(DNGR-DPR)
4040 PP=PP+DT*(PNGR-PBR)
4999 RETURN
5000 '*** TAKING IN VARIABLES ***
5010 *Z.PUSH
5020 '---------------------VARIABLE (1)
5030 Z(1,Z.J%)=PP
5040 '---------------------VARIABLE (2)
5050 Z(2,Z.J%)=F
5060 '---------------------VARIABLE (3)
5070 Z(3,Z.J%)=DP
5999 RETURN
```

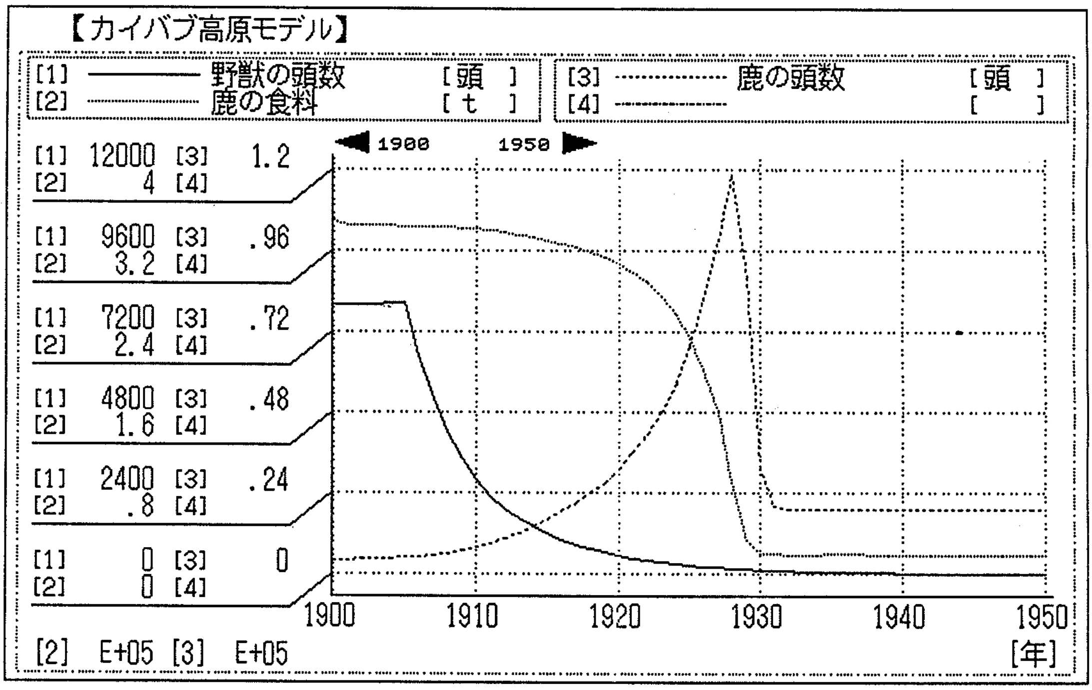

Fig.2 An output example of Kaibab plateau model (comments in Japanese).

5. CONCLUDING REMARKS

Our system, BYNAMO, will provide a suitable tool for beginners. We have had a society for the system dynamics and related problems, which belongs to the Operations Research Society of Japan, which has a meeting every month. Through the meetings, we feel that there are a lot of people who want the computer tool to be executed by personal computers, especially, PC-9800 series. That is reason why, we have developed this tool. We are going to improve BYNAMO to more comfortable tool for beginners and educational use.

2. Stability Robustness

2.1. Basic definitions and theorems

A linear time–invariant system is denoted as:

$$\dot{X} = F(v)X \tag{1}$$

where: 1.) The set of parameter variations v is assumed to be polyhedron of the form:

$$V = (V:\ l \leqslant v \leqslant u\ ,u \in R^{q}) \tag{2}$$

l and u are given constant real vectors.

2.) F(v) is a multiple linear function of parameter vector v, if we fix other components of v, $f_{ij}(v)$ only are linear functions of (v_i).

The vector number of convex polyhedrom is $N = 2^{q}$, where q is the number of variable parameters, the corresponding vectors are defined as:

$$v^{(1)}, v^{(2)}, \ldots, v^{(N)}$$

The corresponding state matrices are:

$$F^{(i)} = F(\ v^{(i)}),\quad i \in \{1,\ 2\ ,\ldots,\ N\}$$

Lemma (2.1): For a linear control system defined by eqns.(1) and (2), if there exists a real symmetic positive–definite matrix P, such that

$$PF(v^{(i)}) + F^{T}(v^{(i)})P < 0,\ i\ \in\ \{1,\ 2,\ \ldots\ N\}$$

then system (1) is asymptotically stable($\forall v \in V$).

Definition (2.1): For $A = (a_{ij}) \in R^{n\times n}$, if $a_{ij} \geqslant 0,\ i,j\ \in \{1,\ 2,\ \ldots,\ n\}$, then A is named non–negative matrix (written as A$\succ$0).

Lemma (2.2): For A,B $\in R^{n\times n}$, if $A \leqslant |A| \leqslant B$, then any eigenvalue of matrix A is less than the maximum eigenvalue r of matrix B, that is

$$|\lambda| \leqslant r$$

Proof: By the theory of matrices, we have,

$$|A^{m}| \leqslant |A|^{m},\ m \in \{1,\ 2,\ \ldots\ \} \tag{3}$$

If 0$\prec$A$\prec$B, then:

$$0 \leqslant A^{m} \leqslant B^{m} \tag{4}$$

Combine eqns.(3) and (4), we have

$$|A^{m}| \leqslant |A|^{m} \leqslant |B^{m}|$$

Morever, because $|A| \leqslant |B|$, then

$$||A||_{2} \leqslant ||B||_{2}$$

According to the properties of L_2 norm

$$||A||_{2} \leqslant ||\ |A|\ ||_{2}$$

$$\left\| A^m \right\|_2 \leqslant \left\| \, |A^m| \, \right\|_2 \leqslant \left\| B^m \right\|_2 \qquad (5)$$

$$\left\| A^m \right\|_2^{1/m} \leqslant \left\| \, |A^m| \, \right\|_2^{1/m} \leqslant \left\| B^m \right\|_2^{1/m}$$

When m→∞,we have

$$\rho(A) \leqslant \rho(|A|) \leqslant \rho(B)$$

Therefore

$$|\lambda| \leqslant r$$

Let the numbers of variable entries of state matrices A and B be respectively q_1 and q_2,then $2^{q_1+q_2}$ subsystems can be formed.The sets of system can be denoted by

$$\dot{X}^{(i)} = A^{(i)}X^{(i)} + B^{(i)}U^{(i)} = (A_0 + \triangle A^{(i)})\ X^{(i)} + B^{(i)}\ U^{(i)}$$

$$i \in \{1,\ 2,\ \ldots,2^{q_1+q_2}\} \qquad (6)$$

where A_0 is the asymptotically stable matrices of nxn norminal system.

Definition (2.2):Let the disturbance matrix be

$$\triangle A^{(i)} = \beta_i E_i \qquad (7)$$

where E_i are constant,describable matrices,in which the maximum entries are 1,while β_i are uncertain parameters.

Definition(2.3) Let $\triangle$ be n−order matrix,and

$$\triangle_{ij} \equiv 1, when\ A_{0ij} \neq 0$$

From definition (2.2) and (2.3),apparently

$$\triangle_{ij} \geqslant |E_{ij}| \geqslant 0$$

If $s = \max_i \beta_i$, $i \in \{1,\ 2,\ \ldots,\ 2^{q_1+q_2}\}$ then the following theorem can be established.

Theorem (2.1):Let the sets of systems be defined by eqns.(6),if

$$s\triangle \geqslant \triangle A^{(i)} \quad i \in \{1,\ 2,\ \ldots\ 2^{q_1+q_2}\}$$

and $P = P^T > 0$,then we have

$$\max_{i=1,2,\ldots 2^{q_1+q_2}} \lambda_{max}\ [P(A_0 + \triangle A^{(i)} - B^{(i)}G) + (A_0 + \triangle A^{(i)} - B^{(i)}G)^T P]$$

$$\leqslant \max_{i=1,2,\ldots 2^{q_1+q_2}} \lambda_{max} [P(A_0 + \varepsilon\triangle - B^{(i)}G) + (A_0 + \varepsilon\triangle - B^{(i)}G)^T P]$$

Where G is feedback gain matrix.

Proof: since $\varepsilon\triangle \geqslant \triangle A^{(i)}$, $i \in \{1, 2, \ldots 2^{q_1+q_2}\}$

Therefore $A_0 + \varepsilon\triangle - B^{(i)}G \geqslant A_0 + \triangle A^{(i)} - B^{(i)}G$

$$i \in \{1, 2, \ldots 2^{q_1+q_2}\}$$

Moreover,because $P = P^T > 0$,then

$$P(A_0 + \varepsilon\triangle - B^{(i)}G) \geqslant P(A_0 + \triangle A^{(i)} - B^{(i)}G) \quad (8)$$

and:

$$(A_0 + \varepsilon\triangle - B^{(i)}G)^T P \geqslant (A_0 + \triangle A^{(i)} - B^{(i)}G)^T P \quad (9)$$

According to Lemma(2.2), we have

$$\max_{i=1,2,\ldots 2^{q_1+q_2}} \lambda_{max} [P(A_0 + \triangle A^{(i)} - B^{(i)}G) + (A_0 + \triangle A^{(i)} - B^{(i)}G)^T P]$$

$$\leqslant \max_{i=1,2,\ldots 2^{q_1+q_2}} \lambda max [P(A_0 + \varepsilon\triangle - B^{(i)}G) + (A_0 + \varepsilon\triangle - B^{(i)}G)^T P]$$

Theorem (2.2):if there exists a real symmetric positive–definite matrix P,such that

$$P(A_0 + \varepsilon\triangle - B^{(i)}G) + (A_0 + \varepsilon\triangle - B^{(i)}G)^T P < 0, i \in \{1, 2, \ldots 2^{q_2}\} \quad (10)$$

then system $\dot{X}^{(i)} = (A_0 + \varepsilon\triangle)X^{(i)} + B^{(i)}U^{(i)}$ is asymptotically stable.

Corollary (2.1):

$$\lambda_{max_{i=1,2,\ldots 2^{q_2}}} [(A_0 + \varepsilon\triangle - B^{(i)}G)^T P + P(A_0 + \varepsilon\triangle - B^{(i)}G)] \leqslant \max_{i=1,2,\ldots 2^{q_2}} \lambda_{max} [(A_0 + \varepsilon\triangle - B^{(i)}G)^T P + P(A_0 + \varepsilon\triangle - B^{(i)}G)]$$

(Proof see [7].)

Theorem (2.3):Assuming that P is a real symmatric positive–difinite matrix,such that

$$\eta_i(p) = \lambda_{max} [P(A_0 + \varepsilon\triangle - B^{(i)}G) + (A_0 + \varepsilon\triangle - B^{(i)}G)^T P],$$

$$i \in \{1, 2, \ldots 2^{q_2}\}$$

ıen $\eta_i(p)$are continuous, convex and differentiable.(Proof see [7]).

Definition (2.4):A function g(p) is

$$g(p) = \max_{i=1,2,\ldots,2^n} \{\eta_i(p)\} \quad (11)$$

From theorem (2.3),g(p) has the properties of η_i (p).

Definition (2.5):

$$h(s) = \max_{v \in V(s)} \lambda_{max} (PF(v(s)) + F^T(v(s))P) \quad (12)$$

2.2 Caculating Programme

According to the definitions and theorems given above,the calculating programme of stability robustness is denoted as Fig.1.

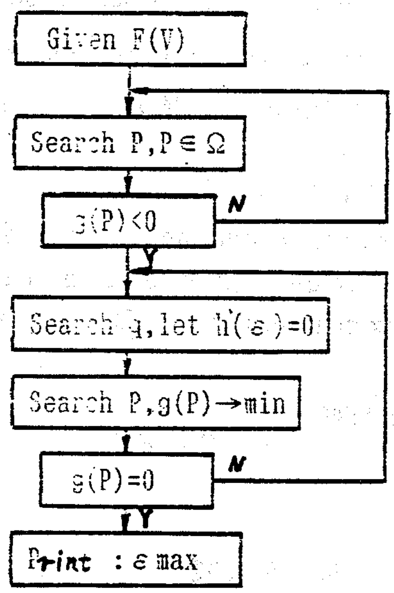

Fig.1.

The convegence property of the above algorithm is guaranted by the following theorem.

Theorem(2.4): The above algorithm gives a series $\{s_q\}$.This series are either definite or indefinite,but there's always a limit $\lim_{q \to \infty} s_q = s_{opt}$.It is impossible for $\tilde{s} > s_{opt}$ that satisfies $\tilde{P}F(v) + F^T(v)\tilde{P} < 0, \tilde{P} = \tilde{P}^T > 0, v \in V(\tilde{s})$)

(Proof see [1].)

3. Performance Robustness

Let the sets of system be defined as eqns.(6),their weighting performance index criterion is

$$J = \sum_{i=1}^{N} C_i J_i = \sum_{i=1}^{N} C_i T_r(S_i) \tag{13}$$

where,C_i are weighting factor. S_i are subject to Lyapunov's Eqns.

$$(A^{(i)} - B^{(i)}G)^T S_i + S_i(A^{(i)} - B^{(i)}G) + Q + G^T RG = 0$$

then the performance index criterion has its minimum value.

Theorem (3.1): Let the sets of systems be defined by eqns.(6) and the performance index criterion be defined by eqns.(13) If and only if there exists an initial feedback matrix G_0 ,which is subject to $R_e \lambda [A^{(i)}_0 - B^{(i)}G_0] < 0$,then there also exists an optimal value G^{*}, such that J(G) is minimized.(Proof see [6])

4. Robustness Synthetical Algorithm

Combine the two algorithms above,then a synthetical robustness algorithm is founded.This algorithm not only gives the largest stability region,but minimizes the performance index criterion as well.Detailed algorithm see Fig.2.

5. Examples

Consider the following system

$$\dot{X} = AX + BU$$

where:

$$A = \begin{bmatrix} -1.356 & 0.2232 & 0 \\ 11.76 & -5.389 & 0 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.1127 \\ 1.886 \\ 0 \end{bmatrix}$$

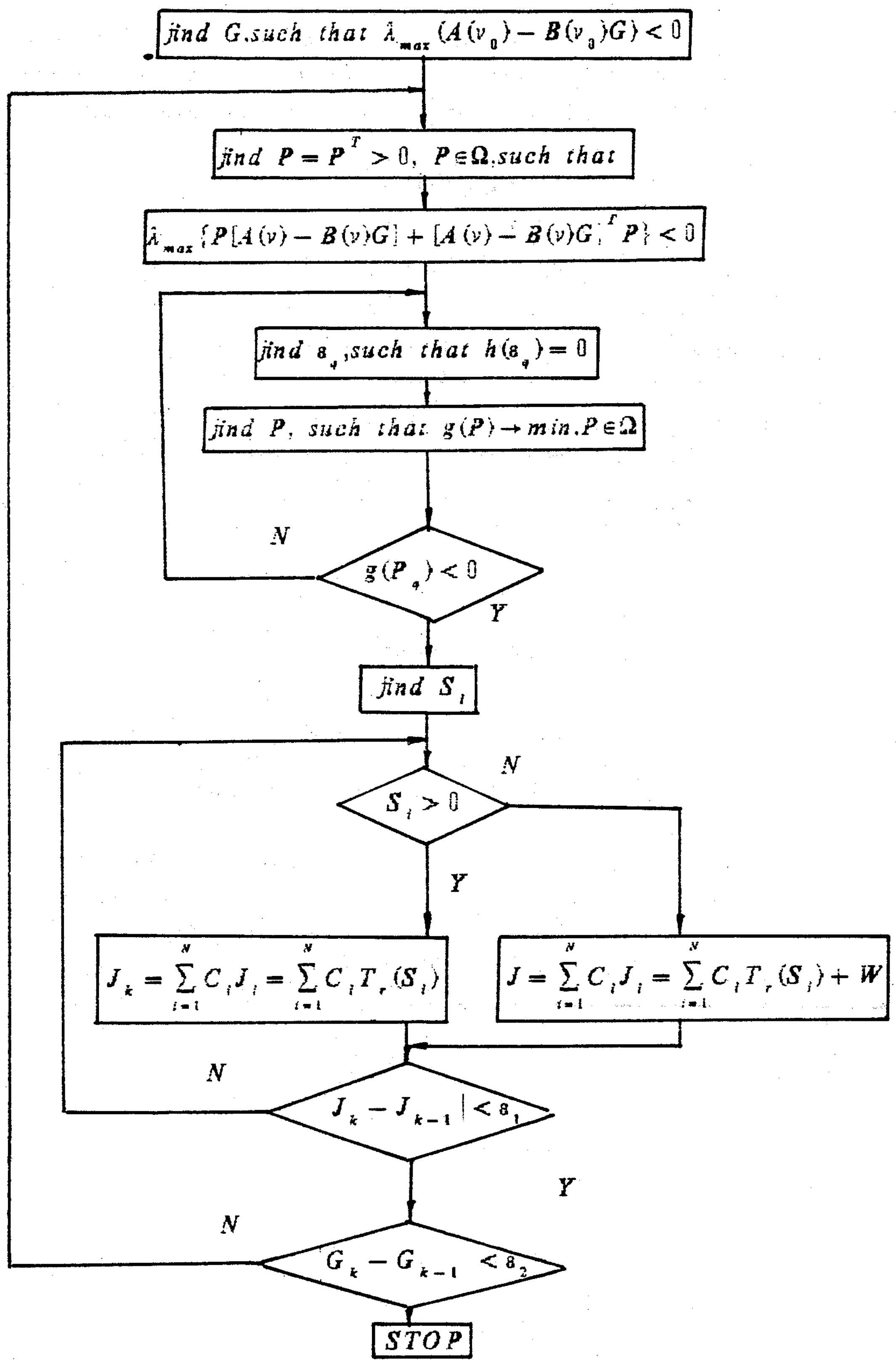
find G,such that λ_max (A(v_0) − B(v_0)G) < 0
find P = P^T > 0, P∈Ω,such that
λ_max{P[A(v) − B(v)G] + [A(v) − B(v)G]^T P} < 0
find ε_q,such that h(ε_q) = 0
find P, such that g(P) → min,P∈Ω
N
g(P_q) < 0
Y
find S_i
N
S_i > 0
Y
J_k = Σ C_i J_i = Σ C_i T_r(S_i)
J = Σ C_i J_i = Σ C_i T_r(S_i) + W
N
|J_k − J_{k−1}| < ε_1
Y
N
G_k − G_{k−1} < ε_2
STOP

Fig.2.

If using Badr's algorithm, it needs calculating 4096 sub–system states.However,according to the algorithm proposed in this paper,it only needs calculating 8 sub–system ststes.Calculating results are shown in Table 1 and 2.

Table 1 Badr algorithm

Times	C_0	C_i	$\varepsilon_{max} \times 10^2$	$J(G)$	$\triangle J$‰		G	
1	.2	0.0125	3.7003	13.0917	2.219	.80853	.10492	.3163
2	.6	0.00625	3.5545	12.9046	0.758	.70812	.10481	.3162
3	.8	0.00312	3.5219	12.8531	0.356	.70820	.10490	.3162

Table 2 improved Algorithm

Times	C_0	C_i	$\varepsilon_{max} \times 10^2$	$J(G)$	$\triangle J$‰		G	
1	.2	0.2	3.1974	18.3834	47.44	1.0741	.15364	.4740
2	.6	0.1	2.8573	15.7117	23.07	0.8911	.12854	.4062
3	.8	0.05	2.7845	14.0931	10.04	0.8744	.12456	.4032