FORECASTS OF NATIONAL FINANCE INCOME AND AMOUNT OF CURRENCY CIRCULATION OF CHINA

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Abstract: This paper presents two approaches to forecast the National Finance Income (NFI) and the Amount of Currency Circulation (ACC) of China. Firstly it uses Combined Hierarchical Periodic Adjustment (CHPA) forecasting approach, which fully considers the periodic characteristics of the stochastic time-varying financial system, to forecast the national finance income, and then it uses Controlled Auto-Regressive models with Multi-Step Recursive (MSR) algothrim to forecast the amount of currency circulation, both have got better results.

1. Introduction

Along with the developments of Chinese economy, the role and position of finance in macro-economy have been more and more important. In which, National Finance Income (NFI) and Amount of Currency Circulation (ACC) are two important economic indexes of the country and the important signatures representing the economic and financial status of the country. Since the economic system is an open complex large-scale dynamic system, there are many disturbances and stochastic factors in it. It is not only relevant to the status of the economic system itself, but also relevant to every aspect of social, economic and political effects. Therefore, the macro forecasts of NFI and ACC become the important aspects in economic forecasting. The scientific forecasts of NFI and ACC will be beneficial to the consequential, stable and coordinated development of national economy. This paper intents to use Combined Hierarchical Periodical Adjustment (CHPA) approach and Controlled Auto-Regressive (CAR) model with time-varying parameters to forecast the NFI and ACC of China.

2. System Analysis and Modelling

In the forecasting of economic systems, the data series can be divided into four parts (An and Gu, 1986): long tendency, stochastic perturbation, seasonal wave and periodic wave, by analyzing the historical data series. Where long tendency means the states of the serie show an obvious tendency of increasing and decreasing in a long term, it can be described by linear and non-linear regression models; stochastic perturbation means the stochastic varying tendency in the serie, it can be described by Box-Jenkins time series models; long tendency and stochastic perturbation togather represent the basic tendency of the serie; seasonal wave means the predicable wave along with the tendency within one year. Periodic wave means the continuous swing along with the tendency over one year.

In a long-term economic system, usually there are some periodic waves caused by various social, political and historical events, and the economic system itself also has some periodicity. So it is necessary to analyze and find the periodic pattern and remove its effects in order to seperate the periodic wave from the time serie and improve the precision of the forecast.

These are called periodic adjustment. At the same time, the effect of stochastic pertubation is also very strong, it needs to remove the stochastic effects to get real tendency of the system, so it is need to set up stochastic process model. Also because of the computation amount is very large, recursive algothrim is needed.

It can be seen that the NFI and ACC of China both have a basic tendency of continuous increasing after the analysis. This shows the continuous development of national economy of China during the forty years. Also it has the effects of disturbances and varies according to certain regulation.

According to the description above, we model the financial system as follows:

$$\mathcal{Y}_T(t,N) = \mathcal{F}[\mathcal{Y}_{T-1}(t,k), \mathcal{X}_T(t), \Delta_T(t), S_T(t), \Omega_T(t), t] \quad (2\text{-}1)$$

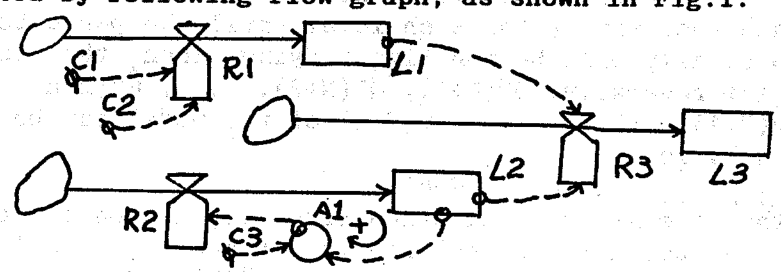
where

$\mathcal{Y}_T(t,N) = [Y(t), Y(t+1), \ldots , Y(t+N-1)]^T$ is the future N-year forecasts.

$\mathcal{Y}_{T-1}(t,k) = [Y(t-k), Y(t-k+1), \ldots , Y(t-1)]^T$ is the past known K-year data.

$\mathcal{X}_T(t)$ is the controlling element.
$\Delta_T(t)$ is the stochastic perturbation element.
$S_T(t)$ is the seasonal effect element.
$\Omega_T(t)$ is the periodical effect element.

From the view of system dynamics (Wang, 1988), the above model can be demonstrated by following flow graph, as shown in Fig.1.



R1: wave rate; R2: control rate; R3: increasing rate;
L1: wave tendency; L2: basic tendency; L3: system output;
A1: controlling input; C1: period coefficent;
C2: season coefficent; C3: stochastic coefficent;

Fig.1: Flow graph of financial system model

The correct analyse of NFI and ACC system can help us to set up the correct model and use the suitable approach to forecast scientifically.

3. Combined Hierarchical Periodic Adjustment forecasting approach

As shown above, it is very important to solve the problems of parameter estimation and prediction and the problems of periodic effects of the system, for the stochastic time-varying system, such as NFI system. The conventional forecasting methods usually use mathematical models with fixed parameters and also do not incorporate the periodic effects, so the forecasting errors will increase more and more along with the time varying. In this paper, we intense to use CHPA approach, which is based on Multi-Level Hierarchical (MLH) prediction method, to solve the problems.

3.1 Multi-Level Hierarchical Prediction Method

MLH prediction method (Han, 1983) is a method about state prediction of time varying parameters. Its primary meaning is to seperate the state prediction of the time-varying system into the prediction of time-varying parametes and the prediction of system states based on the former. The prediction of time-varying parameters will lead to the decreasing of prediction errors.

Suppose the system model is:

$$Y(k) = f[Y_{k-1}, u_k, \theta, k] + v(k) \qquad (3\text{-}1\text{-}1)$$

where $Y_{k-1} = [y(0), y(1), \ldots , y(k-1)]^T$ is the output.
$u_k = [u(0), u(1), \ldots , u(k)]^T$ is the input.
$v(k)$ is the stochastic noise, θ is the parameter set.

The formula of parameter θ variation is:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \emptyset * \nabla_{\theta(k-1)} f[k, \hat{\theta}(k-1)] \{Y(k) - f[Y_{k-1}, u_k, \hat{\theta}(k-1), k]\} / \| \nabla_{\theta(k-1)} f[k, \hat{\theta}(k-1)] \| 2 \qquad (3\text{-}1\text{-}2)$$

where $\nabla_{\theta(k-1)} f[k, \hat{\theta}(k-1)] = \partial f[Y_{k-1}, u_k, \theta, k] / \partial \theta \, |_{\theta = \hat{\theta}(k-1)}$ (3-1-3)
$\emptyset$ is a positive number suitablely selected.

The varying states of parameter $\hat{\theta}$ can be estimated recursively by (3-1-2). If $\hat{\theta}$ is non-time varying, a fixed $\hat{\theta}^*$ will be selected to be the forecasting value of $\hat{\theta}$. If $\hat{\theta}$ is time varying, then the new model will be set up for $\hat{\theta}$ according to its varying pattern.

$$\hat{\theta}(k) = g\, [\hat{\theta}_{k-1}, \emptyset(k), k] \qquad (3\text{-}1\text{-}4)$$

where $\hat{\theta}_{k-1} = [\hat{\theta}(k-1), \hat{\theta}(k-2), \ldots , \hat{\theta}(0)]$ is the parameter set of the new model.

The varying status of $\emptyset$ can be examined by the same process. Taking the same steps to estimate recursively for each level until the parameters of the model which set up finally will be non-time varying. Then the forecasting valuue of θ can be got reversely: $\hat{\theta}^*(N+1)$, $\hat{\theta}^*(N+2)$, ... , $\hat{\theta}^*(N+h)$, and also the prediction of $\hat{y}(N+1)$, $\hat{y}(N+2)$, ... , $\hat{y}(N+h)$ of the model can be got. The k-step prediction $\hat{y}(N+k)$ is:

$$\hat{y}(N+k) = f[Y^*_{N+k-1}, u^*_{N+k}, \hat{\theta}^*(N+k), N+k)] \qquad (3\text{-}1\text{-}5)$$

It can be seen that the k-step prediction error depends on the prediction error of time-varying parameter $\hat{\theta}(N+h)$ and the former prediction errors $y(N+i)-\hat{y}(N+i)$, $i=1,2,\ldots, k-1$. Since the parameter $\hat{\theta}(k)$ has been predicted, the prediction precision has been improved.

3.2 Combined Hierarchical Forecasting

Based on MLH prediction method, Combined Hierarchical forecasting (Li, 1989) divides the system forecasting model into two parts: deterministic tendency model and stochastic error model:

$$Y(t) = Y_1(t) + E(t) \qquad (3\text{-}2\text{-}1)$$

where $Y_1(t) = F[t, x(t), \theta(t)]$ (3-2-2)
is the deterministic tendency part, it can be described by common forecasting models, and the basic tendency of the system forecast can be determined.

$$E(t) = C_1(t)E(t-1) + C_2(t)E(t-2) + \ldots + C_n(t)E(t-n) + v(t) = \sum_{i=1}^{n} C_i(t)E(t-i) + v(t) \qquad (3\text{-}2\text{-}3)$$

is the stochastic error model, it is Auto-Regression (AR) model and represents the forecast of stationary stochastic part, i.e., stochastic error part. The error between tendency part and actual states of the system can be forecasted. $v(t)$ is the final forecasting error of the whole system.

$$\begin{aligned} E(t) &= Y(t) - Y_1(t) \\ &= Y(t) - F[t, x(t), \theta(t)] \\ &= C_1(t)\{Y(t-1) - F[t-1, x(t-1), \theta(t-1)]\} \\ &\quad + C_2(t)\{Y(t-2) - F[t-2, x(t-2), \theta(t-2)]\} \end{aligned}$$

$$+ \dots + C_n(t)\{Y(t-n) - F[t-n, x(t-n), \theta(t-n)]\} + v(t)$$

Therefore,

$$E(t) = \sum_{i=1}^{n} C_i(t)Y(t-i) - \sum_{i=1}^{n} C_i(t)F[t-i, x(t-i), \theta(t-i)] + v(t) \quad (3\text{-}2\text{-}4)$$

$$Y(t) = Y_t(t) + E(t) = F(t, x(t), \theta(t)] + \sum_{i=1}^{n} C_i(t)Y(t-i) - \sum_{i=1}^{n} C_i(t)f[t-i, x(t-i), \theta(t-i)] + v(t) \quad (3\text{-}2\text{-}5)$$

The forecasts of Y (t) and E(t) can got by using MLH prediction method seperately.

The forecast of the whole system can be got by combining the forecasts deterministic tendency model and stochastic error model. The final forecasting results of the whole system will be:

$$\hat{Y}(N+h) = F[N+h, x(N+h), \hat{\theta}(N+h)] + \sum_{i=1}^{n} \hat{C}_i(N+h)\hat{Y}(N+h-i) - \sum_{i=1}^{n} \hat{C}_i(N+h)f[N+h-i, x(N+h-i), \hat{\theta}(N+h-i)] \quad (3\text{-}2\text{-}8)$$

3.3 Combined Hierarchic Periodic Adjustment Approach

CHPA approach (Li, 1990) uses two kinds of periodic adjustment model, i.e., addition model and multiplication model:

$$Y(t) = T(t) + P(t) \quad (3\text{-}3\text{-}1)$$

and

$$Y(t) = T(t) * P(t) \quad (3\text{-}3\text{-}2)$$

where T(t) is the basic tendency of the system, and Y(t) is the periodic wave.

When the wave caused by system periodic effects is the same during the entire period, the addition periodic model can be got by adding the two factors. When the periodic wave of the system is higher when the tendency level is higher and lower when the level is lower, it will be multiplication model.

Periodic wave reflects the periodic effects upon the basic tendency, usually is described by trigonometric functions.

For addition periodic adjustment model:

$$P(t) = A * \sin wt + B * \cos wt \quad (3\text{-}3\text{-}3)$$

where w is the frequency of periodical wave, $w=2\pi/M$, M is the wave period.

When $P(t)>0$, it means the periodic effects caused by the increasing of the sample value, $P(t)<0$, caused by the decreasing of the sample value. $P(t)=0$, it means no periodic effect now. It can be approved that the addition of periodic variation within one period is zero.

For multiplication model:

$$P(t) = A * \sin wt + B * \cos wt + C \quad (3\text{-}3\text{-}4)$$

When $P(t)=C$, it means no periodic effect. $P(t)<C$, means the periodical effects cause the decrease of the sample value. $P(t)>C$, cause the increase of the sample value. The larger the absolute value of the difference between P(t) and C, the stronger the periodic effect. Therefore, the average of periodic variation within one period wil be constant C.

The basic tendency will be:

$$T(t) = F[t, \theta(t)] + \sum_{i=1}^{n} C_i(t)E(t-i) \quad (3\text{-}3\text{-}5)$$

Using CHPA approach to estimate the parameters of T(t) and forecast the tendency, the forecast of the system will be:

$$Y(t+h) = \{f[t+h,\ \theta(t+h)] + \sum_{i=1}^{n} C_i(t+h)E(t+h-i)]$$
$$* [A \sin w(t+h) + B \cos w(t+h) + C] \qquad (3\text{-}3\text{-}6)$$

or
$$Y(t+h) = F[t+h,\ \theta(t+h)] + \sum_{i=1}^{n} C_i(t+h)E(t+h-i)$$
$$+ A \sin w(t+h) + B \cos w(t+h) \qquad (3\text{-}3\text{-}7)$$

Since this kind of periodic adjustment approach not only considers the time-varying characteristics of the system but also the effects of stochastic perturbation, it has better adjustment effect for periodic wave.

3.4. Algorithm for CHPA

Using CHPA approach for forecasting, firstly pre-estimation is got by using system basic tendency. Since there are stochastic parts in the system, it needs to examine the tendency and remove the stochastic parts in order to get the basic tendency of system varying, when the period is not distinct. Fixed parameter linear regression model is taken for the pre-estimation to fix the system because various deterministic tendency models have some similarity to the description of the linear regression model.

$$Y(t) = A' + B' * t + e(t) \qquad (3\text{-}4\text{-}1)$$

where A', B' are pre-estimation parameters.

$$A' = \{\bar{Y}\Sigma t - t\Sigma[t\ Y(t)]\} / [\Sigma t^2 - \bar{t}\Sigma t] \qquad (3\text{-}4\text{-}2)$$
$$B' = \{[t\ Y(t)] - t\Sigma Y(t)\} / [\Sigma t^2 - \bar{t}\Sigma t] \qquad (3\text{-}4\text{-}3)$$

Then, the serie removing the linear tendency will be periodic wave serie which has obvious periodicy.

For addition model:
$$P'(t) = Y(t) - (A' + B' * t) \qquad (3\text{-}4\text{-}4)$$

For multiplication model:
$$P'(t) = Y(t) / (A' + B' * t) \qquad (4\text{-}1\text{-}5)$$

Secondly, frequency analyse is made for P'(t) with obvious periodic wave, the wave period M and frequency w are got.

Thirdly, trigonometric function is fixed for P'(t), and the mathematical model of periodic wave is got.

For addition model:
$$\begin{bmatrix} P'(t-k) \\ P'(t-k+1) \\ \cdots\cdots \\ P'(t-1) \end{bmatrix} = \begin{bmatrix} \sin w(t-k) & \cos w(t-k) \\ \sin w(t-k+1) & \cos w(t-k+1) \\ \cdots & \cdots \\ \sin w(t-1) & \cos w(t-1) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \qquad (3\text{-}4\text{-}6)$$

For multiplication model:
$$\begin{bmatrix} P'(t-k) \\ P'(t-k+1) \\ \cdots\cdots \\ P'(t-1) \end{bmatrix} = \begin{bmatrix} \sin w(t-k) & \cos w(t-k) & 1 \\ \sin w(t-k+1) & \cos w(t-k+1) & 1 \\ \cdots & \cdots & \\ \sin w(t-1) & \cos w(t-1) & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} \qquad (3\text{-}4\text{-}7)$$

LS method is used to get the coefficients A,B,C. When it is ill for LS, Householder or Givens transform is used to get A, B, C, let:

$$U = \begin{bmatrix} \sin w(t-k) & \cos w(t-k) \\ \sin w(t-k+1) & \cos w(t-k+1) \\ \cdots & \cdots \\ \sin w(t-1) & \cos w(t-1) \end{bmatrix} \quad V = \begin{bmatrix} \sin w(t-k) & \cos w(t-k) & 1 \\ \sin w(t-k+1) & \cos w(t-k+1) & 1 \\ \cdots & \cdots & \\ \sin w(t-1) & \cos w(t-1) & 1 \end{bmatrix}$$

$$R = [P'(t-k),\ P'(t-k+1),\ \ldots,\ P'(t-1)]^T$$

Then, for additon model parameters:

$[A,B]^T = [U^T U]^{-1} U^T R$ (3-4-8)

for multiplication model parameters:

$[A,B,C]^T = [V^T V]^{-1} V^T R$ (3-4-9)

From (3-3-3) and (3-3-4), periodical wave serie P(t) can be got, and then P(t) is removed form the sample serie, and mediation time serie T(t) is got with no periodic wave.

For addition model:

$T(t) = Y(t) - P(t)$ (3-4-10)

For multiplication model:

$T(t) = Y(t) / P(t)$ (3-4-11)

For basic tendency T(t), combined hierarchical forecating approach is used to get the forecasting results of the basic tendency of system development, and the effects of periodic wave are added to get the system forecasting results of the system with periodic wave.

$\hat{Y}(N+h) = \hat{T}(N+h) + P(N+h)$ (3-4-12)

and $\hat{Y}(N+h) = \hat{T}(N+h) * P(N+h)$ (3-4-13)

4. Multi-Step Recursive Forecasting Approach

For the ACC system, it is important to consider the strong stochastic effects in it and need to use recursive algorithm to forecast the varying states of the input and output of the system (Li and Shi, 1990).

4.1 Mathematic Model

Suppose the ACC system is a discreat linear Single-Input Single-Output (SISO) system, it can be modelled as followings:

$$y(t) = -\sum_{i=1}^{m} a_i y(t-i) + \sum_{s=1}^{r} \sum_{j=1}^{m} b_{sj} u_s(t-j) + \xi(t), \quad t>1 \qquad (4-1-1)$$

where y(t), $u_s(t)$, s=1, 2, ... , r means the output and input of the system respectively, and y(t) is called state variable and $u_s(t)$ are called controlling variable, $\{\xi(t)\}$ is discreet-time stochastic process. a_i: i=1, 2, ..., m; b_{sj}: s=1, 2, ..., r, j=1, 2, ..., m are parameters and compose the parameter matrix :

$$\theta = \begin{bmatrix} a_1 & b_{11} & \dots & b_{r1} \\ a_2 & b_{12} & \dots & b_{r2} \\ \cdot & \cdot & \cdot & \cdot \\ a_m & b_{1m} & \dots & b_{rm} \end{bmatrix}$$

Also suppose the data matrix at time t is:

$$\varphi^T(t) = \begin{bmatrix} -y(t-1) & -y(t-2) & \dots & -y(t-m) \\ u_1(t-1) & u_1(t-2) & \dots & u_1(t-m) \\ \cdot & \cdot & \cdot & \cdot \\ u_r(t-1) & u_r(t-2) & \dots & u_r(t-m) \end{bmatrix}$$

then (4-1-1) can be denote to

$$y(t) = \varphi^T(t)\,\theta + \xi(t), \quad t>1 \qquad (4-1-2)$$

If m, t are known, then the identification of the system can be simplified to the estimation of unknown parameter θ. Suppose the estimation of θ is $\hat{\theta}$, then:

$$\hat{y}(t) = -\sum_{i=1}^{m} a_i\, y(t-i) + \sum_{s=1}^{r} \sum_{j=1}^{m} b_{sj} u_s(t-j) = \varphi^T(t)\hat{\theta} \qquad (4-1-3)$$

Define the error is:

$$\varepsilon(t) = y(t) - \hat{y}(t) \qquad (4-1-4)$$

Then equation (4-1-3) will be:

$$y(t) = -\sum_{i=1}^{m} a_i\, y(t-i) + \sum_{s=1}^{r} \sum_{j=1}^{m} b_{sj} u_s(t-j) - \sum_{i=1}^{m} a_i(t-i) \tag{4-1-5}$$

If a group of input output data have been known, let

$Y_N = [y(1),\ y(2),\ \ldots\ ,\ y(N)]^T$

$\Phi_N = [\ \ (1),\ \ (2),\ \ldots\ ,\ \ (N)]^T$

Define:

$\xi_N = [\xi(1),\ \xi(2),\ \ldots\ ,\ \xi(N)]^T$

$\varepsilon_N = [\ \ (1),\ \varepsilon(2),\ \ldots\ ,\ \varepsilon(N)]^T$

and the cost function:

$$J_N = 1/N * \sum_{i=1}^{m} \varepsilon^2(t) = 1/N * (Y_N - \Phi_N\theta)^T (Y_N - \Phi_N\theta) \tag{4-1-6}$$

If $\Phi_N^T\Phi_N>0$, then the Least Square (LS) estimation is:

$$\hat{\theta} = (\Phi_N^T\Phi_N)^{-1}\Phi_N^T Y_N \tag{4-1-7}$$

It can be approved that $\hat{\theta}$ is the unbiased eatimation of θ, and when N is adequately large, $\hat{\theta}$ is the consistent estimation of θ.

Because of the strong stochastic effects in ACC system, usually $\xi(t)$ is not a Gussian white noise time serie, therefore $\xi(t)$ can be described by the Auto-Regressive (AR) model of white noise e(t). Then (4-1-1) is denoted as:

$$y(t) = -\sum_{i=1}^{m} a_i y(t-i) + \sum_{s=1}^{r} \sum_{j=1}^{m} b_{sj} u_s(t-j) + \sum_{i=1}^{p} c_i e(t-i) \tag{4-1-8}$$

where p is the AR order of e(t).

From (4-1-8), if θ has been estimated, then:

$$\sum_{i=1}^{p} c_i e(t-i) = \sum_{i=0}^{m} a_i y(t-i) - \sum_{s=1}^{r} \sum_{j=1}^{m} b_{sj} u_s(t-j) \tag{4-1-9}$$

here $a_0 = 1$. (4-1-9) is AR model with order p (i.e., AR(p)). p can be estimated by using AIC or WPC. Using LS or Yule-Walker estimation, the estimation of y(t) can be got.

$$y(t) = -\sum_{i=1}^{m} a_i y(t-i) + \sum_{s=1}^{r} \sum_{j=1}^{m} b_{sj} u_s(t-j) + \sum_{i=1}^{p} c_i e(t-i) \tag{4-1-10}$$

Because the system is time-varying, it is necessary to modify the parameter values continuously. But in the modification process, from (4-1-8), the inverse matrix must be computed, and usually the dimension of the matrix is larger, and also consists of some repetive computation. Therefore it is need to set up a new algothrim which can modify parameters and discrease computation amount at the same time.

4.2 Multi-Step Recursive Algorithm

For adequate large N, if $[\Phi_N^T\Phi_N]^{-1}$ is existed, let

$P_N = [\Phi_N^T\Phi_N]^{-1}$, then from equation (4-1-7), we have:

$$\hat{\theta}_N = P_N \Phi_N^T Y_N \tag{4-2-1}$$

$$\Phi_{N+1} = [\Phi_N^T\ \varphi(N+1)]^T$$

then

$$P_{N+1} = (\Phi_{N+1}^T \Phi_{N+1})^{-1} = P_N - P_N\varphi(N+1)[1 + \varphi^T(N+1)P\,\varphi(N+1)]^{-1} \varphi^T(N+1)P_N \tag{4-2-2}$$

therefore $\hat{\theta}_{N+1} = P_{N+1}\Phi_{N+1}^T Y_{N+1}$

$= [P_N - P_N \varphi(N+1)\varphi^T(N+1)P_N]*[\Phi_N^T Y_N + \varphi(N+1)y(N+1)]/[1+\varphi^T(N+1)P_N\varphi(N+1)]$

$= P_N \Phi_N^T P_N + P_N \varphi(N+1)[Y_{N+1} - \varphi^T(N+1)P_N \varphi(N+1)Y_N]/[1+\varphi^T(N+1)P_N \varphi(N+1)]$ (4-2-3)

let $K_{N+1} = P_N \varphi(N+1)/[1 + \varphi^T(N+1)P_N \varphi(N+1)]$ (4-2-4)

then $\hat{\theta}_{N+1} = \hat{\theta}_N + K_{N+1}[Y_{N+1} - \quad (N+1)\hat{\theta}_N]$ (4-2-5)

From (4-2-5) it can be seen that $\hat{\theta}_{N+1}$ is equal to θ plus modification items $K_{N+1}[Y_{N+1} - \varphi^T(N+1)\theta_N]$, where $\varphi^T(N+1)\theta_N$ means the output forecast. Therefore, $Y_{N+1} - \varphi^T(N+1)\theta_N$ means the output forecasting error. It is the source of parameter modification information. The lager the forecasting error, the larger the modification amount. K_{N+1} is called gain factor meaning the weighted matrix of forecasting error. (4-2-5) is called the Recurssive Least Square (RLS) estimation. In the estimation, initial values ususlly are got from the solution of $\hat{\theta}_N$ and P_N from given samples, then recursive algorithm begins from data N+1. AR(p) model is set up by substituting θ_{N+1} into (4-1-9). From the estimation of p, c , we can get y(N+1). At the same time, the forecasts of controlling variables u (t) are got by using R-AR models. This is the Multi-Step Recurssive algorithm.

4.3 The Order Estimation of Model AR(p)

For the order of model AR(p), the widespread order selection method is A--Information Criterion (AIC), i.e.,

$$AIC(p) = \log[S_p^2\} + 2p/n \qquad (4\text{-}3\text{-}1)$$

where S is the average remainder square error of AR model with order p. But because in financial system the sample number is usually samll, Weak Parameter Criterion (WPC) has better results. Weak papameter means that when the aquare estimation of the parameter is lesser than the double of its variance, the parameter is call weak.

$$WPC(p) = S_p^2 / \sum_{i=1}^{p} (1-2v_i) \qquad (4\text{-}3\text{-}2)$$

where v is the weak parameter depend on parameter estimation method. For LS estimation:

$$v_i = 1/(n + 2 - 2i) \qquad (4\text{-}3\text{-}4)$$

where n is the length of the sample.

It can be approved that the order selection of WPC to model AR(p) is graduately the same with AIC. WPC is consistent with AIC for larger samples, but better for small samples. The main advantage of WPC is that the selected order is strongly dependent on data themselves and is less dependent on estimation method. This is superior to AIC. After many experiments, it is confirmed that WPC has better results for small samples, especially for financial data.

4.4 R-AR Model

For controlling variables in (4-1-1), R-AR model is used to forecast u(t). Consider u(t) is two order stationary process.

$$\tilde{u}(t) = u(t) - \mu(t) \qquad (4\text{-}4\text{-}1)$$

The observation of u(t) is used to fix the polynominal estimation of μ (t).

Define $U = [u(1), u(2), \ldots, u(N)]^T$

$$u(t) = \sum_{i=1}^{m} b_i t^i + \varepsilon(t) \qquad (4\text{-}4\text{-}2)$$

The goal is to solve the LS estimation of parameter B = [b , b , ..., b] to minimize object function

$$J(b) = \sum_{i=1}^{N} \mathcal{E}^2(t) \tag{4-4-3}$$

let $E = [\,\mathcal{E}(1),\ \mathcal{E}(2),\ \dots,\ \mathcal{E}(t)]$

$$T_N = \begin{bmatrix} 1^0 & 1^1 & \dots & 1^m \\ 2^0 & 2^1 & \dots & 2^m \\ . & . & . & . \\ N^0 & N^1 & \dots & N^m \end{bmatrix}$$

then $U = T_N B + E$ (4-4-4)

$$J(b) = E^T E \tag{4-4-5}$$

then the LS estimation will be

$$B = (T_N^T T_N)^{-1} T_N^T U \tag{4-4-6}$$

and also we have $\hat{u}(t) = \sum_{i=1}^{m} b_i t^i$ (4-4-7)

let $w(t) = u(t) - \hat{u}(t)$ (4-4-8)

$W = [w(1),\ w(2),\ \dots,\ w(N)]^T$

Then the AR(p) model of W will be

$$w(t) = a(t) + \sum_{i=1}^{p} w(t-i) \tag{4-4-9}$$

where a(t) is white noise serie with $a(t) \sim N(0,\ \delta_a^2)$

Then let $Z = [w(p+1),\ w(p+2),\ \dots,\ w(N)]^T$

$A = [a(P+1),\ a(P+2),\ \dots,\ a(N)]^T$

$$Z = Z\varphi + A \tag{4-4-10}$$

$$X = \begin{bmatrix} w(p), & w(p-1), & \dots, & w(1) \\ w(p+1), & w(p), & \dots, & w(2) \\ . & . & . & . \\ w(N-1), & w(N-2), & \dots, & W(N-p) \end{bmatrix}$$

$$S(\varphi) = \sum_{t=p+1}^{M} a^2(t) \tag{4-4-11}$$

The LS estimation of φ, δ_a^2, $S(\varphi)$ will be

$$\hat{\varphi} = (X^T X)^{-1} X^T Z \tag{4-4-12}$$

$$\hat{\delta}_a^2 = 1/(N-p) * \sum_{i=1}^{N} a^2(t) \tag{4-4-13}$$

$$S(\varphi) = \sum_{t=p+1}^{N} a^2(t) \tag{4-4-14}$$

where $a(t) = w(t) - \sum_{i=1}^{p} \varnothing(i) w(t-i)$ (4-4-15)

It can be approved that $\hat{\varphi}$ is the generally the unbiased, consistent and minimum variance estimation of φ.

Then the system model of u(t) will be

$$u(t) = \sum_{i=0}^{M} b_i t^i + \sum_{i=1}^{p} \varnothing w(t-i) + a(t) \tag{4-4-16}$$

This is called R-AR model (R-AR(p-m)) where p and m are the order of AR and R (Liu and Li, 1989). Then the forecast of u(t) will be

$$u(t+1) = \sum_{i=0}^{M} b_i (t+1)^i + \sum_{i=1}^{p} \varnothing(i)\ w(t-1) \tag{4-4-17}$$

It can be approved that R-AR model has better forecasting precision in economic forecast and the computation amount of R-AR is lesser than ARIMA.

5. The forecast of NFI and ACC of China

According to the NFI of China from 1952 to 1989, additional CHPA approach is used (Li, 1990). The data from 1952 to 1987 are used to forecast the NFI status of China from 1988 to 1995 (the data from 1988 to 1989 are used for examing the forecasting precision).

The forecasting errors are as follows:

1988: 2.04%; 1989: 0.11%

According to the ACC of China from 1971 to 1987, ACC is defined to the output of the system, i.e., state variable, and wages, agriculture products, commercials and savings are controlling varialbels to be the input of the system. CAR model is set up and R-AR model is used to forecast the controlling variables firstly, let m=1, the estimation of the unknown parameters by MSR are got and also the forecast of ACC (Li and Shi, 1990). The forecasting errors are:

1988: 1.94%; 1989: 1.75%

From above, using CHPA and MSR approach to forecast the NFI and ACC of China, the forecasting results are better and are accord with the varying pattern of the system. At the same time, from the analysis of the forecsts of NFI of China in future years, it can be seen that they have stable increasing tendency. These are accord with the general economic development plan of China. Therefore the forecasting method, models and the results are suitable to the system.

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