COMPUTING THE OPTIMAL PRODUCTION RATES IN A COMPLEX PRODUCTION SYSTEM

Authors : Rafael RUIZ USANO
Adolfo CRESPO MARQUEZ
Dpto. de Organización Industrial y Gestión de Empresas
Escuela Superior de Ingenieros Industriales
UNIVERSIDAD DE SEVILLA
Avda. Reina Mercedes s./n.,
41012 - SEVILLA (SPAIN)

ABSTRACT.

It is very well known that the setting of the optimal production rates in a complex production system is one of the most important and difficult decisions to be taken. This decision making process frequently seems to be an artisan job as it is necessary to take into account many influential factors simultaneously. In this paper we discuss the way of setting the optimal production rates, by applying optimization techniques and system dynamics methodologies. We study the effects of this approach on costs in order to minimize them and obtain good overall results, even for non-financial measurements. Simulation results are presented.

1. INTRODUCTION.

The use of System Dynamics to build simulation models is widely accepted mainly due (Coyle, 1990) to the speed, which one can model, the transparency of influence diagrams and its adecuacy in modelling real situations. A good number of applications have been developed since the appearance of Forrester's models on Industrial Dynamics (Forrester, 1964). At present we can consider the S.D. methodology as a conventional and powerful tool within which a huge scientific and professional community is actively working all over the world in the industrial dynamics field and developing new approaches to improve the results.

In this paper we try to compute the production rates in a production planning system using an optimization model connected with the S.D. model. In section 2, we briefly explain the S.D. model used to reproduce the structure and behaviour of the production planning system. Section 3 shows the optimization model that interacts with the dynamic model. It computes the optimal production rates which are fed into the S.D. model. Section 4 describes the design of the proposed approach and the relationships between both models: S.D. and Optimization. Some simulation results are presented at the end of the paper with a short discussion on the advantages of the proposed mixed approach to modelling the system.

2. THE SYSTEM DYNAMICS MODEL

A basic production-inventory model is presented in Figure_1. In this model there are three levels which determine the future status of the system at any time:

- *Customer's orders backlog: number of orders pending to be shipped at the instant k.*
- *Work in process: manufacturing inventory at the instant k. Depends on the production lead time.*
- *Inventories of finished products at the instant k.*

The demand is taken as an exogenous variable and two policies are developed to calculate the rates and set up the behaviour of the system.

Shipping a unit depletes both the backlog of customers orders and the inventory of finished products. The level of backlog is assumed to be a company's policy depending on the levels of the inventories of finished products.

The most important decision seems to be the selection of the appropriate production rates.

In the model presented, the production rates for the next integration period are evaluated by checking the inventories (INV), demand forecasting (DF), and making sure that the rates computed are within the maximal production capacity (MPC).

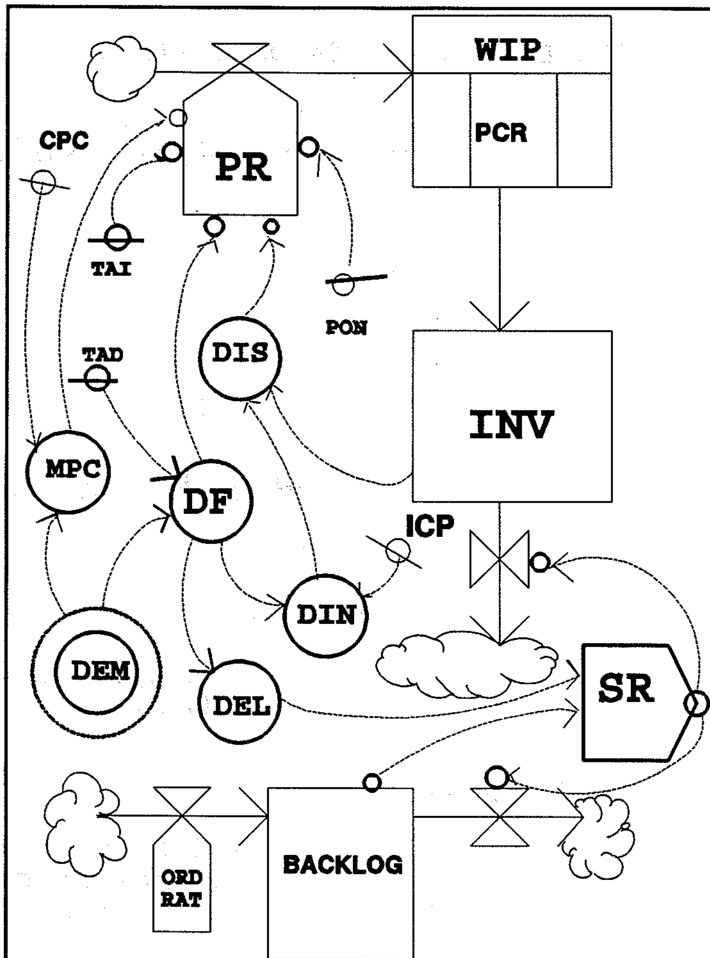

Figure 1. FORRESTER´S DIAGRAM

Ilustr. 1. Forrester's Diagram.

The demand forecasting will establish the desired inventory by considering a minimal inventory cover period (ICP). Obviously, the production rate should be acceptable to match the discrepancies (DIS) between the current and the desired inventory within a required time (TAI).

3.THE OPTIMIZATION MODEL

The production management tries always to coordinate the strategic and the tactical planning. It therefore has to take into account the real cost structure and constraints of different types: financial, economical, technological, etc.

With this modelling criterion, the production rates can be calculated by computing the optimal solution of a linear programming problem, at regular intervals. The objective function of this problem would incorporate the cost structure of the production system (Ruiz Usano, 1990).

The L.P. model is as follows:

$$MAX \sum_{i=1}^{i=N} (C_{i1} X_{i1}^k - C_{i2} X_{i2}^k - C_{i3} X_{i3}^k)$$

s.t.:

$$\begin{array}{lcl} X_{i1}^k & \leq & DF^k \\ X_{i2}^k & \geq & DIN^k \\ X_{i3}^k & \leq & MPC^k \\ X_{i1}^k + X_{i2}^k - X_{i3}^k & = & INV^k \\ X_{i1}^k,\ X_{i2}^k,\ X_{i3}^k & \geq & 0 \end{array}$$

$i = 1, 2, \ldots . N$ (Number of products)
$k = 1, 2, \ldots . n$ (Time)

Subscripts:

1	sales for the next period
2	inventories at the end of the next period.
3	production rates for the next period.

Coefficients of the objective function:

C_{i1}	unit income
C_{i2}	unit inventory holding cost
C_{i3}	unit production cost

Right hand side values:

DF	Demand forecasting
MPC	Maximal production capacity
DIN	Desired inventory
INV	Actual inventory of finished products

4. COMPUTING THE OPTIMAL PRODUCTION RATES.

In most production planning system the calculation of the production rate is made on a heuristic basis (Forrester, 1964; Coyle, 1977) , as follows:

PR = SMOOTHED DEMAND + K(DESIRED VALUE OF LEVEL VARIABLES - CURRENT VALUE)*

Where PR is the Production Rate and the variables are usually inventory, backlog, etc.

The Production Output Rate is normally calculated as a delay of the entering Production Rate above (PR).

Is this a good production rate?. On a heuristic basis the answer could be. Why not?

Can we make some progress setting the production rates in a different way? The answer should be: it depends on what objectives you are trying to achieve. In the context of management criterion the setting of the production rates is one of the most important decision to be made.

On the other hand we have powerful optimization techniques which we often use to calculate the optimal decision variables in a rather isolated context, without taking into account the dynamic and interacting environment of the production planning system.

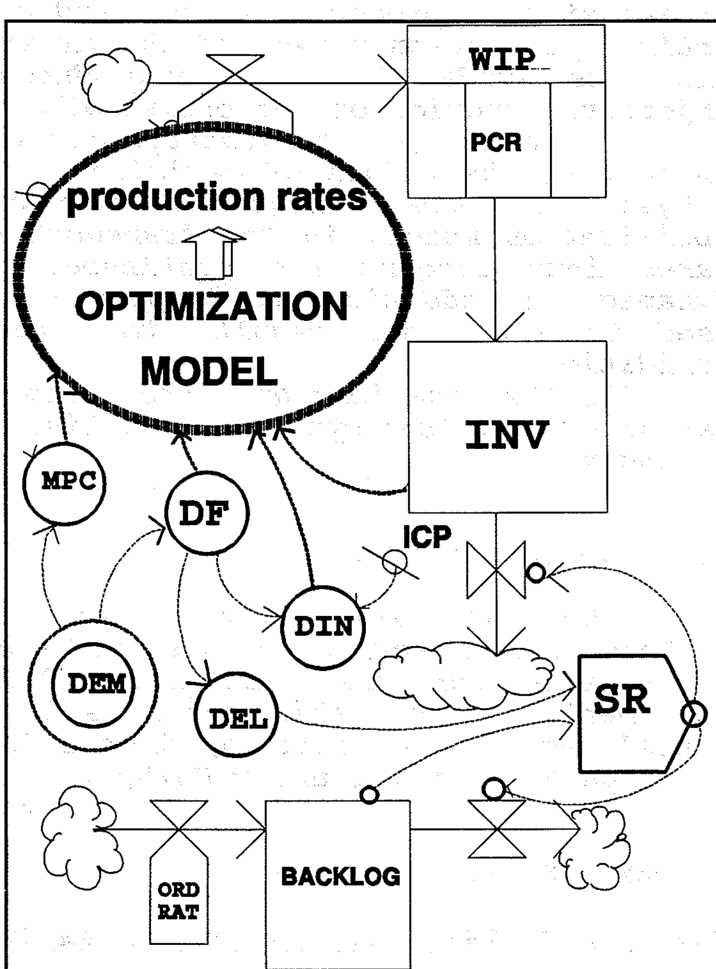

Fig. 2. S.D.- OPTIMIZATION'S DIAGRAM

Figure 2. **S.D. - Optimization's Diagram.**

Therefore, the basic idea is to design an optimization model that computes the optimal production rates , with an objective function given by the system's manager and these are fed into the S.D. model.

The S.D.-Optimization's diagram is shown in Figure 2. As it can be seen a part of the system has been replaced by the optimization model. The arrows entering the optimization model are the values calculated by the S.D. model and fed into the optimization model in order to update the parameters it needs.

5. SIMULATION RESULTS.

We show the results obtained by simulation of S.D.-Optimization approach. Figure 3 and 4 represent respectively the optimal production rates and the inventories for both products in the case study. Figure 5 reproduces the value taken by the objective function of the optimization model

An important advantage of using this approach is the way of setting the values of decision variables. These variables are subject to the management criterion translated into the optimization model, in the framework of the whole system, which takes into account the continuous interaction of the system dynamics. In addition, this approach (Wolstenholme, 1990) can be used to fit model results to actual observations for model validation.

All the equations and the parameter's numerical values can be found in the Dynamo source list of the model provided in the Appendix.

6. ACKNOWLEDGEMENTS.

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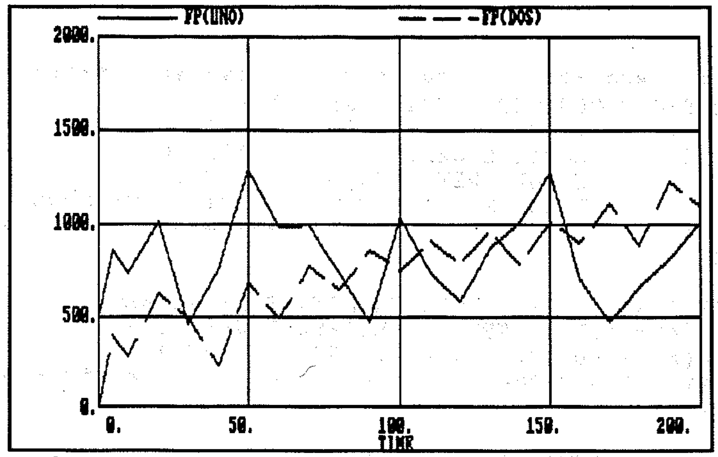


Figure 3. Optimal Production Rates.

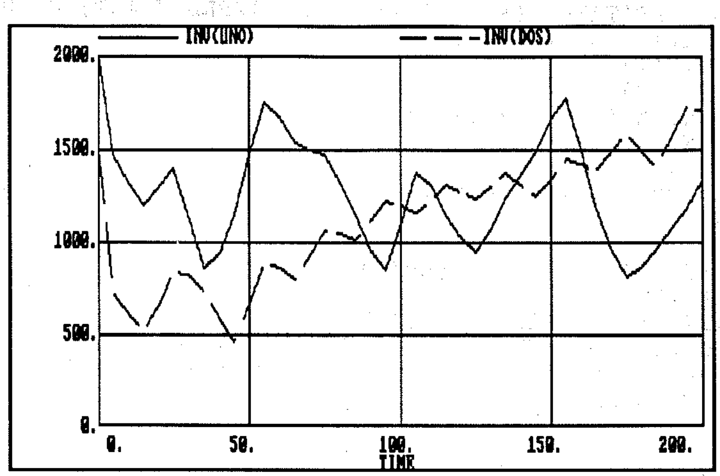
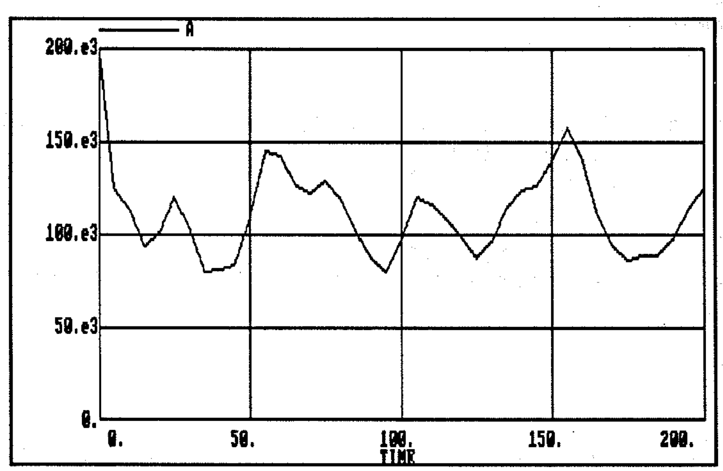


Figure 4. Levels of Inventory.



Ilustr. 5. Objective Function.

8. APPENDIX.

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* ***************************************************************************
*                 BASIC PRODUCTION-INVENTORY MODEL                          *
* ***************************************************************************
FNCTN MAXIMIZ(7,1,9)       ROUTINE CALL FOR OPTIMIZATION
INSERT MODFOR              MODULE WITH FOR' DEFINITIONS
DMNSN VRES(VAR)            RESULTS VECTOR FROM OPTIMIZATION MODEL
* ***************************************************************************
* LEVEL EQUATIONS
*
L CART.K(ART)=CART.J(ART)+(DT)*(FCON.JK(ART)-SUMI.JK(ART))        BACKLOG
L INV.K(ART)=INV.J(ART)+(DT)*(FPT.JK(ART)-SUMI.JK(ART))           INVENTORY
L WIP.K(ART)=WIP.J(ART)+(DT)*(FP.JK(ART)-FPT.JK(ART))             WIP
* ***************************************************************************
* AUXILIARY EQUATIONS
*
A D.K(ART)=TABLE(TDEM(*,ART),TIME.K,0,200,10)                     DEMAND
A PRD.K(ART)=SMOOTH(D.K(ART),TAD)                                 FORECAST
A IDES.K(ART)=PRD.K(ART)*GCI
A RETRASO.K(ART)=TABLE(TETRAS(*,ART),INV.K(ART)/PRD.K(ART),0,10,1)
A MCP.K(ART)=SMOOTH(1.8*D.K(ART),TAC)                             MAX.CAPACITY
A INVAGR.K=SUM(INV.K(*))
A WIPAGR.K=SUM(WIP.K(*))
A DINSAT.K(ART)=CART.K(ART)-SUMI.KL(ART)
A TINV.K(ART)=WIP.K(ART)+INV.K(ART)
A TINVAGR.K=WIPAGR.K+INVAGR.K
* ***************************************************************************
* FLOW EQUATIONS
*
R FPT.KL(ART)=DELAY3(FP.KL(ART),LT(ART))
R FCON.KL(ART)=D.K(ART)
R SUMI.KL(ART)=CART.K(ART)/RETRASO.K(ART)
INSERT MODOPT    MODULE FOR THE MATRIX RIGHTHAND SIDE UPDATE
* ***************************************************************************
* CONSTANTS AND PARAMETERS
*
P TAC=8          SEMANAS
P TAD=3          SEMANAS
P GCI=3          SEMANAS
C LT(*)=0.4,0.4  SEMANAS
P POND=0.8       80 %
P TAI=1.5        SEMANAS
* ***************************************************************************
* TABLES
*
T TDEM(*,DOS)=500,300,600,500,250,650,500,750,650,850,750,900,800,950^
  ,800,1000,900,1100,900,1200,1100
T TDEM(*,UNO)=1000,750,1000,500,750,1250,1000,1000,750,500,1000,750,600,850^
  ,1000,1250,750,500,650,800,1000
T TETRAS(*,UNO)=6,1,1,1,1,1,1,1,1,1,1
T TETRAS(*,DOS)=6,1,1,1,1,1,1,1,1,1,1
* ***************************************************************************
* INITIAL CONDITIONS
*
N CART(ART)=FCON(ART)
N INV(ART)=IINV(ART)
I IINV(*)=2000,1500
N WIP(ART)=D(ART)*LT(ART)
* ***************************************************************************
* SIMULATION SPEC
*
SPEC LENGTH=200/DT=0.1/PRTPER=5/SAVPER=5/TINIC=0/TCAMB=200
SAVE INV,WIP,CART
SAVE D,PRD,IDES,RETRASO,MCP,INVAGR,WIPAGR,DINSAT,TINV,TINVAGR
SAVE FPT,FCON,FP,SUMI
```

```
*    COMPUTING THE OPTIMAL PRODUCTION RATES
* ****************************************************************
     OBJECTIVE FUNCTION COEFFICIENTS
I    IMAT(*,FOB)=0,250,120,-25,-25,-100,-60
     MAX. CAPACITY CONSTRAINTS
I    IMAT(*,R1)=4400,0,0,0,0,-1,0
I    IMAT(*,R2)=2200,0,0,0,0,0,-1
     SALES CONSTRAINTS
I    IMAT(*,R3)=1000,-1,0,0,0,0,0
I    IMAT(*,R4)=500,0,-1,0,0,0,0
     INVENTORY CONSTRAINTS
I    IMAT(*,R5)=2000,0,0,-1,0,0,0
I    IMAT(*,R6)=1500,0,0,0,-1,0,0
     INVENTORY BALANCE EQUATIONS
I    IMAT(*,R7)=2000,-1,0,-1,0,1,0
I    IMAT(*,R8)=1500,0,-1,0,-1,0,1
N    MAT(J,I)=IMAT(J,I)
* ********************************************************
     NUMBER OF VARIABLES SENT TO SIMPLEX L.P. MODEL
P    N=6
     NUMBER OF CONSTRAINTS <=
P    M1=4
     NUMBER OF CONSTRAINTS >=
P    M2=2
     NUMBER OF CONSTRAINTS =
P    M3=2
P    ER=10E-3
* ********************************************************
*    SIMPLEX MATRIX RIGHT HAND SIDE UPDATE
A    MAT.K(OBJ,R1)=MCP.K(UNO)
A    MAT.K(OBJ,R2)=MCP.K(DOS)
A    MAT.K(OBJ,R3)=PRD.K(UNO)
A    MAT.K(OBJ,R4)=PRD.K(DOS)
A    MAT.K(OBJ,R5)=0.5*IDES.K(UNO)
A    MAT.K(OBJ,R6)=0.5*IDES.K(DOS)
A    MAT.K(OBJ,R7)=INV.K(UNO)
A    MAT.K(OBJ,R8)=INV.K(DOS)
* ********************************************************
     CALL TO MAXIMIZE ROUTINE ( A = OBJECTIVE FUNCTION )
A    A.K=MAXIMIZ(VRES.K,MAT.K,N,M1,M2,M3,ER)
     OPTIMAL PRODUCTION RATES
R    FP.KL(UNO)=VRES.K(VA5)
R    FP.KL(DOS)=VRES.K(VA6)

SAVE VRES,MAT,FP,A
```

MODFOR.DYN

```
FOR ART=UNO,DOS                          PRODUCTS
FOR I=FOB,R1,R2,R3,R4,R5,R6,R7,R8        ROWS
FOR J=OBJ,V1,V2,I1,I2,P1,P2              COLUMNS
FOR VAR=VA1,VA2,VA3,VA4,VA5,VA6          SIMPLEX OUTPUT VARIABLES
```