MATHEMATICAL FORMULATION OF SYSTEM PRINCIPLES IN SYSTEM DYNAMICS

Yoshiaki TOYODA and Shizuo MAWATARI
Aoyama Gakuin University
6-16-1 Chitosedai, Setagaya, Tokyo 157, Japan

Almost 35 years have passed since J. W. Forester published his paper "Industrial Dynamics" in 1958, which was the first paper in this field and which later became "system dynamics." While many books and articles in the field raised its methodology, most of them have described models and discussed applications of system dynamics to specific areas. As a result, evaluation of system dynamics has been obscured by inconclusive debate about particular models. The efforts of many practitioners are leading system dynamics to a better understanding and more comprehensive presentation. But, its methodology needs further development and codification for revealing general characteristics of complex systems. Particularly, stronger links are necessary to the control theory and to enhance the system's mathematics.

This paper constructs a mathematical theory for thoroughly and precisely analyzing such general models as produced by system dynamics. First, we formulated mathematically, as the axioms of system dynamics, all principles of systems from which "general" characteristics of complex systems are generated. Secondly, we attempted to adequately express the essential mathematics of system dynamics, based on the axioms mentioned above. That is, we investigated the structural stability and the discontinuity of dynamic behavior of complex systems using the concepts in the Catastrophe Theory. And we mathematically explained some important results described in past articles of system dynamics such as the characteristics of complex systems initiated by Forester. Furthermore, we describe a new theoretical method to elucidate structural characteristics in SD models using concepts of Combinatorial Topology.

1. INTRODUCTION

Nearly 35 years have passed since Forrester (1958) presented the first paper in the field of system dynamics. This paper studied about a system theory which grasps an objective as a system, makes its characteristics clear, designs a policy to control the objective and assesses it. The first interest in this field was in industrial dynamics (ID). Thereafter, it was expanded to a general system such as urban dynamics (UD), world dynamics (WD), natural dynamics (ND), health dynamic (HD), etc. Thus, as an application area this methodology has become much broader, and the field has become to be called system dynamics (SD). Presently SD methodology is also applicable to very broad areas such as problems in education, global environmental change and moon resource exploitation (Yamagiwa et al. 1990), etc.

System dynamics has also been criticized, critics maintain that SD is only a defective simulation technique. These arguments have been continuing since 1968 especially those between Ansoff et al. (1968) and Forester (1968(b)). The criticisms which they have raised has improved methodology, and several projects to produce a more comprehensive presentation of system dynamics methodology have been undertaken. It is hoped that the improvements made in these projects will give not only impetus but also support a more rigorous resolution of the criticisms (Legasto et al. 1980; Randers 1980).

However the real problems of system dynamics methodology have not yet been resolved: "System dynamics literature still does not adequately convey the essential mathematics of the field nor expose the principles which should guide judgment in modeling of systems, nor does it provide an adequate number of examples to be used as guides in system structuring" (Forrester 1968(a)). This paper intends to fundamentally resolve these methodological problems. By formulating concepts and principles in system dynamics mathematically, defining and analyzing theoretically the dynamic and structural characteristics which are the two main pillars in system dynamics, and describing some significant results obtained.

2. NATURE OF PROBLEMS

When formulating a system of SD theory, it is necessary to have a clear-cut system philosophy, understanding of the model's nature, and fundamental characteristics of real world systems as an object of study in SD theory, beforehand. Here we intend to present only the fundamental characteristics of real world systems.

2.1 FUNDAMENTAL ASPECTS OF SYSTEMS

In SD theory, policy design and its assessment are also studied and its real world system is limited to one with the following characteristics.

1) the change of status in the system is controlled by a feedback of information.

2) its dimension is generally large, it has nonlinear, multiple and positive/negative feedback loops, and it has no common measuring scale.

2.2 FUNDAMENTAL CHARACTERISTICS OF SYSTEMS

When studying a system's characteristics, it is convenient to divide them into two fundamental aspects such as dynamic and structural characteristics. Here, dynamic characteristics refers to the diachronic aspects which is related to the dynamic change of a system's status, and structural characteristics which refers to synchronic aspects which in turn is related to the feedback loops of information.

2.3 METHODS FOR THE ANALYSIS OF DYNAMIC CHARACTERISTICS

In chapter 3 of this paper we studied the dynamic behavior of a system. The term, "dynamic behavior of a system", is often used as a fundamental concept in SD methodology. However, this dynamic behavior has not been clarified. It is also said that the proposition: - "dynamic behavior of a system is insensitive to fluctuation of many system parameters and structural variations of equations", is maintained, and therefore, it is not necessary to introduce precise system parameters. However, the meaning of the word, "insensitive", has not been clarified, and the proposition has not been theoretically proved and such phenomena are only noticeable by SD model simulations. Critics of SD methodology are not satisfied by the results above mentioned. By actually constructing and simulating a SD model, the phenomena may be observed, which indicates that the proposition is acceptable. In this paper, using the results from the Catastrophe Theory (Poston 1978) we define the concept of dynamic behavior of a system and mathematically prove the proposition.

2.4 METHODS FOR THE ANALYSIS OF STRUCTURAL CHARACTERISTICS

For the analysis of structural characteristics of SD models using causality loops, flow diagrams, structural equations, etc. have been generally used. However these methods are lacking in objectivity, precision and logicality. For example, in cases where all the feedback loops are analyzed, it is very complicated to enumerate completely all the loops by a total flow diagram, and it is also difficult to find all the loops with divided sectors. In chapter 5, we intend to elucidate fundamental structures in SD models using concepts of combinatorial topology (MacLane 1967).

3. PREPARATION OF CONCEPTS AND SYMBOLS

It is desirable to develop SD theory axiomatically for establishing SD methodology as an acceptable theory. As a starting point of such a development is to compile the all SD principles into a group of axioms. Such an axiom compilation is called an "F-dynamical system," defined as follows.

3.1 F-DYNAMICAL SYSTEM

F-dynamical system is a pack D(t) which satisfies the axiom 1,2,3 and 4 described below:

$D(t)=[n, L(t), M, S, F, G, \Phi]$

where $t \in R_+=[0,\infty)$ and

(1) n is an integer, which is called a system dimension.

(2) $L(t)=\{L_1(t),L_2(t),\ldots,L_n(t)\}$ is a set consisting of n sets $L_i(t)$. Here each $L_i(t)$ is called a level and depends upon $t \in R_+$.

(3) $M=\{m_1,m_2,\ldots,m_n\}$ is a set consisting of n set-functions $m_i:B(L_i(t)) \rightarrow R$. Here each m_i is a measurement.

(4) $S=\{s_1,s_2,\ldots,s_n\}$ is a set consisting of n real numbers s_i. Here each s_i is called an initial value of the level $L_i(t)$.

(5) $F=\{f_1,f_2,\ldots,f_n\}$ is a set consisting of n functions $f_i:R_+ \times U \rightarrow R_+$. Here each f_i is called an input rate and U is an open subset of the Euclidian space R^n.

(6) $G=\{g_1,g_2,\ldots,g_n\}$ is a set consisting of n functions $g_i:R_+ \times U \rightarrow R_+$. Here each g_i called an output rate.

(7) $\Phi=(\phi_{jk})$ is a (l,m)-matrix, whose element $\phi_{jk}:R_+ \times U_j \rightarrow R$ (j=1,2,...,l: k=1,2,...,m: $m \geq 2n$) is not more than a "third class function" which is called a structural equation. U_j is an open subset of the Euclidian space $R^{(j-1)+n}$.

[AXIOM 1] (LADDER AXIOM)

For any $t \in R_+$ and $x=(x_1,x_2,\ldots,x_n) \in U$,

$y_{0i}=x_i$ (i=1,2,...,n), $y_0=(y_{01},y_{02},\ldots,y_{0n})$,

$y_{jk}=\phi_{jk}(t,y_0,y_1,\ldots,y_{j-1})$ (j=1,2,...,l: k=1,2,...,m),

$y_j=(y_{j1},y_{j2},\ldots,y_{jm})$ (j=1,2,...,l)

are defined and the following relationship is satisfied.

$$y_{lk}=\begin{cases} f_i(t,x) & \text{if } k=2i-1, \\ g_i(t,x) & \text{if } k=2i \end{cases} \quad (1 \leq k \leq 2n).$$

[AXIOM 2] (FEEDBACK AXIOM)

Set <i,j>=0 if the difference $f_j(t,x_1,x_2,\ldots,x_n)-g_j(t,x_1,x_2,\ldots,x_n)$ is always constant with regards to the variable x_i and <i,j>=1 if otherwise. For any i,j where <i,j>=1, there are the natural numbers $j_1,j_2,\ldots j_a$ which are less than or equal to n and satisfy the following relationship:

$\langle i,j\rangle\cdot\langle j,j_1\rangle\cdot\langle j_1,j_2\rangle\cdot\langle j_2,j_3\rangle\cdots\langle j_{a-1},j_a\rangle\cdot\langle j_a,i\rangle=1.$

[AXIOM 3] (INTEGRATION AXIOM)

For any $t \in R_+$, the following relationship is satisfied:

$m_i(L_i(t))=s_i+\int_0^t\{f_i(u,|L|(u))-g_i(u,|L|(u))\}du$ (i=1,2,...,n)

where $|L|(t)=(m_1(L_1(t)),m_2(L_2(t)),\ldots,m_n(L_n(t)))$.

[AXIOM 4] (BOUNDED AXIOM)

The difference $f_i(t,x)-g_i(t,x)$ and $m_i(L_i(t))$ (i=1,2,...,n) are bound on the definition domain.

In the above axioms, $\Phi=(\phi_{jk})$ is called a structure matrix of D(t). In axiom 2, the n degree matrix $A=(a_{ij})$ where $a_{ij}=<i,j>$ is called an adjoining matrix of Φ. When $a_{ij}=1$, a cross-linkage (i,j) is said to exist, and when i=j, a self-linkage said to exist.

3.2 F-DYNAMICAL SYSTEM AND SD PRINCIPLES

The principles of system dynamics which are under consideration in this paper include all of the 31 principles enumerated in Forester's publication (1968(c)). An SD model is considered one that conforms to all of these 31 principles.

Nonnegative real number $t \in R_+$ in the F-dynamical system (F-d.s.) represents time. If we consider a real system to be momentarily changing, a level set $L_i(t)$ consists of elements determining the status of a system. Each element m_i in a measurement set M represents its dimension and a method numerizing its corresponding level. The measuring principles of SD described in Forester's publication are as follows.

1) Definition of system boundary. Necessary constants, variables and functions have to be defined in its model as self-contained terms for representing causal relations among dynamic behavior of variables in the system under consideration. This corresponds to the definition of L, M, S, F, G, Φ in F-d.s.

2) Organization of feedback loops. A casual relationship between variables dynamic behavior has to be formulated in loop-form which consist of chains of structural equations. This corresponds to the ladder axiom and the feedback axiom.

3) Classification of variables in the SD model into levels, rates and auxiliary variables and definition of those functions and their mutual relationship. The level represents an accumulation of activities in the system. The input/output rate represents a variable to control the level variation. Auxiliary variables lay between composite functions into which these rates are resolved. These variables correspond to y_{jk} of the integration axiom and the ladder axiom in F-d.s.

4) Elucidation of decision making framework. A decision making framework has to be formulated into a rate equation, regarding the framework as a "behavior revising difference" between objective and actual values. a)Objective values, b)observed values, c)the difference between these values and its revised behavior have already been abstracted by structural equation ϕ_{jk} in F-d.s.

Furthermore, each level and rate can never reach positive/negative infinity. This corresponds to the bounded axiom. To compute all values of variables in the model constructed with an F-d.s. framework, all variables of the model should be kept discrete. Thus, the integration values in the integral axiom may be computed with the Euler formula in numerical analysis. The discrete model obtained here is a so-called SD model and programmed using computer languages such as DYNAMO, CSMP, etc.

4. ANALYSIS OF DYNAMIC BEHAVIORS IN F-DYNAMICAL SYSTEMS

Concepts playing important roles in SD methodology such as the characteristics of dynamic behavior in a system, the insensitivity of a system against changes of parameters and structural equations, etc. are not clearly formed in usual SD theory. This fact points to the immaturity of the present SD methodology. It is necessary to construct the mathematical representation of such concepts at the creation of a system abiding by a strict SD theory.

4.1 DYNAMIC BEHAVIOR AND ITS FORM

In the integration axiom of F-d.s., let $x_i(t)=m_i(L_i(t))$ $(i=1,2,\ldots,n)$ and $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))$, then

$$dx_i/dt=f_i(t,x)-g_i(t,x),\quad x_i(0)=s_i \quad (i=1,2,\ldots,n) \tag{1}$$

As the values of variables y_{jk} are decided by (t,x(t)) and the ladder axiom, each status of the system is determined by an orbit x(t) in differential equation (1). The shape of the graphical expression of each variable based on the results of simulation of a SD model can then be said to be a kind of "form" in the Catastrophe Theory with orbit x(t) or its component functions $x_i(t)$, therefore using information obtained from the theory, the characteristics of system's dynamical behavior in F-d.s. can be mathematically discussed.

4.2 MATHEMATICAL DEFINITION OF FORMS IN F-DYNAMICAL SYSTEMS

The orbit x(t) determined by the differential equation (1) varies according to the initial value of s_i and changes of structural equation. A mathematical space including all of the changes is defined as follows. Let r be a nonnegative integer and $C^r(t_1,t_2)$ be the whole mapping $f:[t_1,t_2] \rightarrow R$ which is continuously differentiable r times. Suppose that Whitney topology (W^r topology) is applied to $C^r(t_1,t_2)$. The orbit x(t), if limited over the closed interval $[t_1,t_2]$ always belongs to $C^r(t_1,t_2)$. Suppose two of the orbits x(t), y(t) given over the closed interval $[t_1,t_2]$ and if two topological isometry h and h' exist and the following diagram (Figure 1) is commutative, x(t) and y(t) are defined to have the same forms over $[t_1,t_2]$.

$$\begin{tikzcd}
[t_1, t_2] \arrow[r, "x(t)"] \arrow[d, "h"'] & R^n \arrow[d, "h'"] \\
[t_1, t_2] \arrow[r, "y(t)"] & R^n
\end{tikzcd}$$

Figure 1 Map Diagram

4.3 MATHEMATICAL DEFINITION OF INSENSITIVITY

When in SD methodology the dynamic behavior of a system is insensitive against changes of parameters and structure equations, this indicates that the difference between graphical shapes of before and after changes explained in equation (1) is small. In terms of the Catastrophe Theory regarding the forms mentioned above, the form is considered structurally stable, and is mathematically defined as follows.

The orbit x(t) in equation (1) is structurally stable directly above and below $[t_1,t_2]$ indicates that N(x) exists in the neighborhood of x(t) based on the Whitney topology W^r and that all mappings belonging to N(x) has the same form as x(t). According to this definition the proposition of SD methodology described above indicates that the orbit x(t) in F-d.s. is also structurally stable. This definition regarding form and structural stability is also applied to the component function $x_i(t)$ of x(t).

Based on this definition, the main results obtained in this chapter are as follows (Mawatari 1983).

[THEOREM 4.1]

The orbit x(t) in F-d.s. is structurally stable over any finite closed interval with no singular point. Furthermore, the condition that x(t) singular points do not exist, is generic. //

[THEOREM 4.2]

Each level curve $x_i(t)$ in F-d.s. is structurally stable over any finite closed interval where $x_i(t)$ is a Morse function. Furthermore, the condition that $x_i(t)$ singular points do not exist, is generic. //

[THEOREM 4.3]

Assuming that the orbit x(t) in F-d.s. is defined over finite closed interval $[t_1,t_2]$ with no singular point. Then the orbit y(t) obtained by varying each structure equation ϕ_{jk} in a sufficiently small neighborhood of x(t) based on Whitney topology W^r produces the same form as the original orbit x(t). //

[THEOREM 4.4]

Assume that a level curve $x_i(t)$ in F-d.s. is defined over a finite closed interval $[t_1,t_2]$ and is a Morse function. Then a level curve $y_i(t)$ obtained by varying each structure equation ϕ_{jk} in a sufficiently small neighborhood of $x_i(t)$ based on W^2 topology produces also the same forms as the original level curve $x_i(t)$. //

4.4 SENSITIVE PARAMETERS

A parameter p_j that is sensitive to a level curve $x_i(t)$, is indicated as, that parameter vector p with components p_j belongs to a bifurcation set B of mapping $x_i:R_+ \times R^n \rightarrow R$. If p_0 is a sensitive parameter, the form of a level curve $x_i(t,p)$ continuity changes drastically in the case of letting the parameter p_0 vary in the neighborhood of p_0. This is a so called Catastrophe occurrence.

4.5 DYNAMIC CHARACTERISTICS OF A COMPLEX SYSTEM

Forrester explains about dynamic characteristics of complex system in SD methodology in his publication as follows: -

"Complex systems have many important behavior characteristics that we must understand if we expect to design systems with better behavior. Complex systems: (1) are counter intuitive; (2) are remarkably insensitive to changes in many system parameters; (3) stubbornly resist policy changes; (4) contain influential pressure points, often in unexpected places, from which forces will radiate to alter system balance; (5) counteract and compensate for externally applied corrective efforts by reducing the corresponding internally generated action (the corrective program is largely absorbed in replacing lost internal action); (6) often react to a policy change in the long run in a way opposite to how they react in short run; (7) tend toward low performance." (Forrester 1969)

Also the above behavior characteristics are presented by using natural languages (in the above case; English) or a simulation using an SD model, but has never been strictly proved theoretically. Furthermore these expression formats are very ambiguous. And these ambiguous characteristics directly influence SD methodology. In the following we intend to elucidate how to explain these characteristics based on the result of this chapter.

(1) it is obviously clear by axiomatizing SD principles as F-d.s. in chapter 3. Now, the object of SD study is established as F-d.s. This dynamical system is high dimensional and nonlinear, and has multiple and dipolar feedback loops. It is certain that behavior of a dynamic system with such fundamental characteristics are beyond our intuitive insight.

(2) it has been mathematically proved that after insensitivity is defined as the structural stability of a form.

Policy in (3) presents a rule representing how to use the available information for behavioral determination, and consists of structure equations and parameters, therefore change of policy is a change of structural equations and parameters, and the stubborn resistance is, in other words, insensitivity.

Then, as in (2), the structural stability of (3) is clearly explained in section 4.3.

Pressure points in (4) means where behavior is sensitive to the system. Then, (4) points out that there may exist a few points which are sensitive owing to the change of structural equations and parameters. Sensitivity of parameters is described as the Catastrophe occurrence in section 4.4. Structural equations are closely related to characteristics so called generic. That is, characteristics that the orbit doesn't have singular points or the level curve is of the Morse function is generic and structurally stable as proved in the theorem 4.1 and 4.2. Therefore, in many cases a small changes of structural equations are insensitive, however, in the ingeneric cases the structural stability is not always held. This means that in a very few cases it is sensitive.

In (5), if a modification program is given to a system from outside, at least one of structure equations causes alternations. These alternations give further alternations in structure equations, and finally in level equations by the ladder axiom. Then, the first alternations are effected less in time and magnitude by the integration axiom. Next, the consequences reaches other levels by the feedback axiom. Here, broader structural equations are given effect by the ladder axiom. The consequence of a modification program given from outside sometimes propagates the creation of a feedback loop described in 2.3. A detailed consideration will be presented in the following chapter where the working of feedback loops is studied.

(6) is related closely to the integral axiom and the bounded axiom in F-d.s. Because dynamic behaviors of the integrated function:

$$H_i(u)=f_i(u,x(u))-g_i(u,x(u)) \qquad (u \in R_+,\ i=1,2,\ldots,n) \qquad (2)$$

is bonded by axiom 4, the dynamic behaviors between long and short term projects are different. When a policy in SD methodology is changed at a time u_0, the sign of $H_i(u)$ sometimes changes or its absolute value greatly varies after a specified time passes. Otherwise the change of policy is small. As the SD model is structurally stable, such a little change has often no meaning. After all, in the case that the change is significant, the dynamic behavior of an orbit obtained by the integration of each $H_i(u)$ is noticeably different between long term and short term projects. This is a mathematical explanation of (6).

(7) is caused by the results that controllable variables are changed incorrectly by intuitive judgment concerning (1) and by the short term improvement concerning (6). To avoid a bad policy design the control analysis should be done under free consideration of the structural stability of the form, the high and low development, the occurrence of Catastrophe, the effect of feedback loops, etc. The assertions in SD described above in simple expression formation by Forrester can be now strictly proved.

5. ANALYSIS OF STRUCTURE CHARACTERISTICS IN F-DYNAMICAL SYSTEM

A mathematical area which regards an objective as a polyhedron and pursues its topological nature using "complex" obtained by decomposing the objective into "simplex" is the combinatorial topology (MacLane 1967). This paper uses some of the essential concepts in this area, for example, simplex, oriented simplex, simplical decomposition, complex, subcomplex, connected complex, polyhedron, s-chain group, s-cycle and Betti number. Here we show a geometrical expression of an SD model and some important results obtained using these concepts (Mawatari 1985).

5.1 GEOMETRICAL EXPRESSION OF SD MODEL

Set t at a certain time and define a sequence a_i with y_{jk} in the ladder axiom.

$a_{mj+k}=y_{jk}$ $(j=0,1,\ldots,l;\ k=1,2,\ldots,m)$ where $y_{0k}=0$ $(n<k\leq m)$ (3)

For each a_{mj+k}, let V(mj+k) be a set of its essential independent variables. For each a_{mj+k}, let S(mj+k) be the simplex that is defined as the set obtained by arranging elements of V(mj+k) in order of their subscripts, the last element of which is a_{mj+k}. Here a_{mj+k} is called a main vertex of S(mj+k).

[PROPOSITION 5.1]

Let $K_1=\{S(i):\dim S(i)\geq 1,\ 1\leq i\leq J\}$ where $J=m(l+1)$.

and $K(n,l,m)=\{S(i), \text{face of } S(i):S(i)\in K_1\}$ (4)

Then K(n,l,m) is the complex. //

Then SD terminologies correspond well to terminologies in the geometrical expression, as follows: -

Terminology in SD	Terminology in the geometrical expression
real system	intricate and high dimensional geometrical space
system analysis	simplical decomposition
SD model	complex, polyhedron
structure equation	simplex
feedback loop	1-cycle
valid SD model	complex and polyhedron well holding characteristics of the space under consideration

If we geometrically construct a process of analyzing a real system and construct the SD model using the terminology above, that process is one which decomposes the intricate and high dimensional geometrical space into the simplex, and construct the complexes properly maintaining "original space characteristics." This means to create a structural analysis method which is intuitively easy to be understood and is not ambiguous. The main results obtained are as follows.

Let subscripts of the vertex of a simplex S(i) which belong to K_1 in the equation (4), which are arranged in order be $b_{i0},b_{i1},\ldots,b_{ik}$ where k depends upon i, indicated as k_i if necessary. Define the sets as follows.

$S(i,j)=(b_{ij}b_{ik})$ $(0\leq j\leq k_i-1)$

$L_1=\{S(i,j)\ :\ S(i)\in K_1\}$

$L(n,l,m)=\{S(i,j), \text{vertexes of } S(i,j):S(i,j)\in L_1\}$.

Then, L(n,l,m) is a subcomplex of K(n,l,m) and is a geometrical expression of the total flow diagram under consideration.

Consider 1-chain group G(L,1) consisting of all 1-chains with integer coefficients of L(n,l,m). A propagation of variation of variables in SD model is expressed using 1-chains with integer coefficients as follows: -

$C_1=S(i_1,i_2)+S(i_2,i_3)+\ldots+S(i_{s-1},i_s)$ (5)

If $i_s=i_1$, then equation (5) represents a feedback loop. This is a 1-cycle in G(L,1). If 0-simplex s L(n,l,m) which does not correspond to a level joining a positive direction with the vertex corresponding to any level, a vertex s is called normal. Otherwise it is called abnormal. To define the following sets.

$L_2=\{b,(ab):a,b\in L(n,l,m),$ b is the main vertex and abnormal.$\}$

$L_3=L(n,l,m)-L_2$

$L_4=L_3\cup\{-S(i,j):S(i,j)\in L_3\}$ (6)

If the set L_4 is a connected complex, L(n,l,m) is called normal. Otherwise it is called abnormal.

[THEOREM 5.2]

Necessary and sufficient conditions, where L(n,l,m) is the normal complex, is that a finite sequence $j_1,j_2,\ldots,j_r$ exists, which satisfies the following conditions (1) and (2).

(1) $\{j_1,j_2,\ldots,j_r\}=\{1,2,\ldots,n\}$

(2) $<j_1,j_2>\cdot<j_2,j_3>\cdots<j_{r-1},j_r>=1$. // (7)

When applying any policy change to an SD model its change is represented as a change of parameters and the structure equations in an SD model. This change only effects some levels, it also effects all the other variables (level, rate, auxiliary equation,etc.) based on theorem 5.2. As a result a policy change it effects all variables sooner or later.

To investigate structural characteristics, first it may be necessary to enumerate all the feedback loops in an SD model and analyze these effects. Then, some contrivance is necessary to analyze skillfully the effect over all feedback loops. But it is considered difficult because there may be many feedback loops in an SD model. This problem can be solved by computing the Betti numbers and linearly independent 1-cycles of the complex L(n,l,m).

[PREPOSITION 5.3]

Let L(n,l,m) be the normal complex. And let a 0-simplex and 1-simplex of L_3 be α and β respectively. Then the Betti numbers $N(_3,0,p)$ and $N(L_3,1,p)$ of L_3 are represented as follows.

$N(L_3,0,p)=1$, $N(L_3,1,p)=1-\alpha+\beta$. // (8)

In the following, if L(n,l,m) is the normal complex its corresponding SD model is to be called normal, and if otherwise, abnormal.

[THEOREM 5.4]

Let L(n,l,m) be a general expression of a total flow diagram in a normal SD model, and let F(n,l,m) be the whole of geometrical expression of feedback loops in this SD model and r be the Betti number $N(L_3,1,p)$ in the equation (8). Then, linearly independent cycles $C_1,C_2,\ldots,C_r$ of F(n,l,m) exist and any element "C" of F(n,l,m) be the total sum of $C_1,C_2,\ldots,C_r$. //

According to theorem 5.4 it is shown that all feedback loops in a normal SD model are decided by the Betti number $r=N(L_3,1,2)$ and r linearly independent 1-cycle in L_3. It is shown here that all feedback loops are represented as a sum of fundamental feedback loops. The number of these loops is determined by the Betti number. Therefore, the policy controlling the system under consideration may be analyzed by introducing these fundamental feedback loops. Hence, the structure analysis method in SD methodology may be concluded as follows.

[THE ALGORITHM FOR STRUCTURE ANALYSIS OF AN SD MODEL]

(1) The construction of a complex K(n,l,m) which is a geometrical expression of an SD model.

(2) The construction of a subcomplex L(n,l,m) of K(n,l,m) which is a geometrical expression of a total flow diagram of an SD model.

(3) The construction of a subcomplex L_3 which excludes abnormal vertexes in L(n,l,m).

(4) To investigate whether L_3 is connected. If L_3 is connected, execute (5) and (6) in its present state. If L_3 is not connected, all components of L_3 should execute (5) and (6) as separate components. Here we describe the case where L_3 is connected.

(5) To compute the Betti number $r=N(L_3,1,2)$ of L_3 and obtain linearly independent 1-cycles $C_1,C_2,\ldots,C_r$. (These 1-cycles are called a basis of feedback loops.)

(6) To analyze various properties (such as sign, sensitivity, position, etc.) of feedback loops and their mutual relationship for 1-cycles $C_1,C_2,\ldots,C_r$.

6. CONCLUSION

In this paper, it is shown that the two main areas of concern in SD methodology is the dynamic characteristics and structure characteristics of a system, that can be strictly analyzed in a mathematical framework of F-d.s.

Here system analysis based on F-d.s. leaves much more room for progress. When analyzing a system using optimal coordination in hierarchical formulation discussed by Mesarovic(1970), the knowledge in this area can be applied to an SD model as F-d.s. Furthermore, if regarding F-d.s. as a kind of nonlinear dynamics and using the theory of "nonlinear dynamics and chaos" by Thompson, et al. (1986) in F-d.s., an assessment of policies can be discussed theoretically. Hence most of the criticisms on SD methodology, for example that SD cannot be called a system theory, are essentially nullified. It is also possible to deduce some important facets which could not be deduced from traditional system theories.

REFERENCES

Ansoff, H. J., Slevin, D. P. 1968. An Application of Industrial Dynamics. *Management Science*. 14(7): 384-397

Forrester, J. R. 1958. Industrial Dynamics. *Harvard Business Review*. (July-August): 37-66

Forrester, J. W. 1968(a). Industrial Dynamics --- After the First Decade. *Management Science*. 14(7): 398-415

Forrester, J. W. 1968(b). Industrial Dynamics --- A Response to Ansoff and Slevin. *Management Science*. 14(9): 601-618

Forrester, J. W. 1968(c). *Principles of Systems*. Massachusetts: Wright-Allen Press, Inc.

Forrester, J. W. 1969. *Urban Dynamics*. Massachusetts:The MIT Press

Legasto, JR., Forrester, J. W., Lyneis, J. M. 1980. *System Dynamics*. New York: North-Holland

MacLane, S. 1967. *Homology*. Berlin: Springer-Verlag

Mawatari, S. 1983. Theoretical Analysis of Dynamic Behaviors in System Dynamics. *J. Operations Research Society of Japan*. 26(3): 253-277 (In Japanese)

Mawatari, S. 1985. Structural Analysis in System Dynamics. *Trans. the Society of Instrument and Control Engineers*. 21(3): 262-269. (In Japanese)

Mesarovic, A. 1970. *Hierarchy System Formulation and Optimal Coordination*.

Poston, T., Stewart, I. 1978. *Catastrophe Theory and Its Application*. London: Pitman

Randers, J. ed. 1980. *Elements of the System Dynamics Method*. Massachusetts: The MIT Press

Thompson, J. M. T., Stewart, H. B. 1986. *Nonlinear Dynamics and Chaos*. New York:Wiley & Sons

Yamagiwa, Y., Kuriki, K. 1990. The Earth Ecological and Economic System and the Moon Resource Exploitation. *J. the Japan Society for Aeronautical and Space Science*. 38(441): 516-523. (In Japanese)