# SIMPLE QUANTUM CHAOTIC MODEL FOR FLUCTUATING DISSIPATIVE SYSTEMS

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## ABSTRACT

Energy dissipative systems are considered through a general approach. The one direction non-steady state equation for mass, heat and momentum transport shows that the energy used by the system could be considered through an *"energy dissipation function"*, comparable to the *"wave function"* used in Quantum Mechanics. A complex time scale is proposed. This permits to consider the fluctuations as being in a time scale which is different from our classical one. The non commutation of operators of the basic equation introduces quantification which supports the use of finite different equations instead of a differential equation. A discretized Chaotic Process is proposed as a model for actual systems. The example of a fluidized bed shows that quantum considerations through a ground dynamic state and an excited state could support the above proposal which is in agreement with the actual qualitative behaviour. The Chaotic Process can be put in agreement with the thermodynamics based principle when comparing the minimum energy dissipation of the actual system.

## INTRODUCTION

Dissipative systems show the possibility of fluctuations and turbulent behaviors. Energy dissipation is not sufficient but it is a necessary input: systems are not workable without energy.

Chaos is recognized as possibly being at the border of classical mechanics and quantum mechanics (BERRY, 1987). For example atoms coupled to their field can be considered as dissipative systems in the same way as classical energy dissipative systems such as chemical reactions (PRIGOGINE, 1988). Chaos is identified in quantum mechanics systems such as atoms of hydrogen (DELANDE, 1989)

Time could be considered in different scales: thermodynamic, internal, biological; it can be also considered as an operator (PRIGOGINE, 1980, 1988). Whether time is reversible or not is now higly questionned (COVENEY,1989) and a turbulent time concept has emerged (MULLIN 1989).

Many studies were conducted on complex energy dissipative systems but few attempts, except for chemical reactions, were made in the field of chemical engineering. This paper is a follow up of some dispersed studies on unit operations such as sieve plates (MORA, 1976, 1978; BES, 1982,1985), liquid-liquid extraction column ( BES, 1986) or fluidized beds (CHEBHOUNI, 1985).

These systems such as fluidized beds show very complex behaviors but the basic equations are still unable to follow this complexity. Empirical equations are generally used by adding identified parameters from experiments. Design and scaling up are possible but only in narrow ranges of operating parameters. One of the problems for these systems is still to get models that are able to represent qualitatively the overall complex behaviour and particularly the possibilities of several regimes by means of simple general equations.

## ENERGY DISSIPATION EQUATION

A general one direction non-steady state equation for the three basic transport phenomena, i.e, mass, heat and momentum is given by:

$$K\frac{\partial^2 a}{\partial z^2} = \frac{\partial a}{\partial t} \qquad (1)$$

where $a^2$ is the flux density multiplied by the driving force. For the particular case of linearity or simple non linear models, $a^2$ is the entropy source. This varies in the same way as the available energy which dissipates into heat due to irreversible processes.

"a" can be called *"Dissipative Activity Indicator"* (BES, 1982) but in this paper naming it as *"Energy Dissipation Function"* seems more appropriate.

**FEIGENBAUM PROCESS**

A simplified form of equation (1) is :

$$K\frac{da}{dt} = (a_o - a) \qquad (2)$$

This equation can be split into two equations:

$$K\frac{da}{dt} = a_1 - a \qquad (3)$$

$$K\frac{da}{dt} = a_2 \qquad (4)$$

The corresponding finite difference equations are:

$$a(t+1) = a_1 T + a(1-T) \qquad (5)$$

$$a(t+1) = a + a_2 T \qquad (6)$$

T is linked to the time step for discretization.

Equations (4) and (5) define two straight lines, which can be used in an alogorithm to define a chaotic process (Figure 2).

Each of the two branches may be representative of two dynamic states of the system.

The increasing value of "a" is an *"excited state"*, the decreasing value being the *"ground dynamic state"*.

**OPERATORS**

In equation (2) the state of the system is defined by "a". This is obtained from:

$$a = a_o - K\frac{da}{dt} \qquad (7)$$

Two operators act on "a":

$$\hat{S} \text{ such that } \hat{S}(a) = a_o$$

$$\hat{K} \text{ such that } \hat{K}(a) = -K\frac{da}{dt} \qquad (8)$$

The resulting operator is $\hat{H} = \hat{S} + \hat{K}$ (9)

The physical state given by "a" is an eigen function of $\hat{H}$, the eigen value being 1.

It is possible to show that the commutator is not zero:

$$C_o = \hat{K}\hat{S} - \hat{S}\hat{K} = a_o \qquad (10)$$

"$a_o$" represents the experimental cause of fluctuations as $a_o^2$ is the external energy supplied to the system.

Equations (3) and(4) represent two operators, i.e, one for each of the two branches, the commutators is:

$$C_o = a_2 T^2$$

were $a_2T^2$ is the parameter governing the chaotic process.

Quantum mechanics justifies the Heisenberg's uncertainty principle from the non commutation of two operators of the Schrödinger equation. Following this idea we may assume that different equations considering two dynamic distinct states (as equation (5) and (6)) may provide a mathematical representation of fluctuating physical systems. The "quanta" could be $a_o$ or $a_2T^2$.

## TIME SCALES

Different time scales have been already proposed in order to take into account complex systems in many scientific fields: thermodynamics, biology, hydrodynamics, quantum physics, astronomy, biology, chemistry, chemical engineering, etc...

**a) Thermodynamic time, internal time**

This may be defined as : dt'= s dt or dt' = a dt. In this case the linear equation (2) becomes a quadratic one fully comparable to the logistic function classically used as illustration of chaotic behaviors.

**b) Universal Time**

A universal time may be defined through a number of events: number of earth revolution, number of quartz vibration, number of atomic disintegration, etc... This latter definition is used to verify the Theory of Relativity.

**c) Complex Time**

Complex time is used in astrophysics based on two components, the real and imaginary parts of a complex number.

**d) Turbulent Time**

The concept of turbulent time is now proposed in order to take into account the complexity of hydrodynamic turbulent systems.

**e) Continuous and discretized time**

Time evolution of "a" can be calculated from the analytical integration of equation (2) or from computation of an associated difference equation for different steps of time. The set of numerical values of "a" is a Poincare map. If one wants to get the same results from the two ways, different time scales, dt' and dt should be used. One condition for getting the same values of "a" at the same time is:

$$dt'' = k\ dt + \delta w$$

age "t" will be such that

$$t = N\ kdt + N\delta w = N''\ dt''$$

showing two components, one directly linked to a usual time step (dt), the other linked to a number (Nw). The two components suggest that time be considered as a complex function.

**f) Imaginary time**

The basic equation of quantum mechanics is derived from equations of type (1).

$$\frac{-k}{2m\partial z^2} - i\frac{\partial \varphi}{\partial t} \qquad (11)$$

$\varphi$ is the wave function. Complex notation has been introduced in order to get periodic solutions as equation (1) is unable to do that.

Periodic regimes are now recognized as first steps leading to chaos. It seems reasonable to consider equations of type (11) as being the starting point to seek more realistic models.

## COMPLEX ENERGY DISSIPATION FUNCTION

The *"Energy dissipation function"* may be compared to the *"Wave Function"* if we use a complex time scale: dt = i $dt_i$, so that

$$-\frac{K\partial a}{\partial z^2} = i\frac{\partial a}{\partial t_i} \qquad (12)$$

In this case "a" should be a complex:

$$a = \alpha + i\beta \qquad (13)$$

The module is

$$aa* = \alpha^2 + \beta^2 \qquad (14)$$

and

$$a + a* = \alpha \qquad (15)$$

For linear basic transport phenomena: mass, heat and momentum, aa* will be the entropy source: $\sigma$.

## TIME RELATIVITY

Fluctuating or turbulent system are simply characterized by time averaged values, i.e, temperature, velocity, composition, so that:

$$a = \bar{a} + a' \qquad (16)$$

One of the input parameters is the time averaged value of the energy provided to the system: $\bar{e}$. This parameter is easily obtainable from experiments. e includes energy required to steady state and fluctuations, given by:

$$\bar{e} = \bar{a}^2 + \bar{a}'^2 \qquad (17)$$

However this equality is not true for an instantaneous value as:

$$\bar{e} \neq (\bar{a} + a')^2 \qquad (18)$$

In order to get similar equality as equation (17), we may use two time scales so that

$$\bar{a}^2\ \Delta\theta^2 = \bar{e}\Delta t^2 - a'^2\Delta t^2 \qquad (19)$$

$$\text{or} \quad \frac{\bar{a}^2}{\bar{e}}\Delta\theta^2 - \Delta t^2\left(1 - \frac{a'^2}{\bar{e}}\right) \qquad (20)$$

$$or \quad \Delta t'^2 = \Delta t^2\left(\frac{1-a'^2}{\overline{e}^2}\right) \qquad (21)$$

So that

$$\Delta t' = \Delta t\sqrt{1-\frac{a'^2}{\overline{e}^2}} \qquad (22)$$

$\Delta t'$ is a thermodynamic or internal time.

Equation (22) should be compared to the well known Time Relativistic equation

$$t' = t\sqrt{1-\frac{v^2}{c^2}} \qquad (23)$$

From equation (22) it appears that the additional energy dissipated through fluctuations modifies the internal time scale in the same manner as velocity does it to moving system. A consequence is: *"fluctuations decrease the internal time"*.

COMPLEX THERMODYNAMIC TIME

Equation (19) is the module of a complex time:

$$\overline{a}^2\Delta\theta^2 + a'^2\Delta t^2 = \overline{e}\Delta t^2 \qquad (24)$$

$$or \quad \Delta t^2 - \frac{\overline{a}^2}{\overline{e}}\Delta O^2 + \frac{a'^2}{\overline{e}}\Delta t^2 \qquad (25)$$

so that

$$\Delta t^2 = \Delta\mathcal{E} \cdot \Delta\mathcal{E}^* \qquad (26)$$

with

$$\Delta\mathcal{E} = \Delta t' + i\,\frac{a'}{\overline{e}^2}\,\Delta t \qquad (27)$$

Coming back to the introduction of "a" as a complex function (equation 13), we can identify:

$\alpha$ as the averaged value of a:$\overline{a}$

ß as the fluctuation of a'

so that

$$a = \overline{a} + ia' \qquad (28)$$

Consequently we may consider that a turbulent system as operating in two different time scales:

a) our time scale for the averaged value,
b) its own internal time scale for fluctuations.

According to this development, the non-steady state classical diffusion type model seems relevant for the averaged values while *"Schrödinger"* type equations would be relevant for fluctuations.

The classical modelling and the quantum modelling could be considered as coexisting at the same time through two time scales. As a result of this, we may consider that chaotic systems can run their course in a time scale which we are not able to synchronize with our usual time scale. The difficulties to understand the behaviour of these systems may derive from this particularity.

The signal giving the dissipation versus time is a *"broken"* curve. This could be characterized by the number of minima and maxima, i.e, singular points, which is one of the various means for measuring the degree of *"complexity"* of the *"folded"* curve. This leads to the concept of *"fractal"*, or *"geometric entropy"*. Equation 19 gives in fact the module of a vector which is one element of the broken curve that is directly derived from experimental curves (Fig. 1).

## FLUIDIZED BED

The basic phenomena in a fluidized bed correspond to a solid particle maintained in a rising flow (gas for example). The bed (1 to 20 m height and 0.1 to 2 m diameter) could be composed of sand, glass bead, pellets of catalyst etc. The basic equation comes from fluid mechanics:

$$m\frac{dv}{dt} = K(V_g-v)+mg \qquad (29)$$

This can be modified as follows, using the minimum velocity for fluidization:

$$K\frac{dv}{dt} = (V-v) \qquad (30)$$

as the energy supplied to the bed by the fluid rises, the kinetic energy of particles rises up to a certain level, then the particles falls and rises and falls again etc... Kinetic and potential energies acquired from the fluid when rising are transformed into heat by shocks with other particles and when falling. The kinetic energy equation may be written as:

$$K\frac{da}{dt} = (a_o-a) \qquad (31)$$

It is not possible to derive experimentally and theoretically from models, or equations the exact paths and states of one particle due to very complex interactions between particles and fluid. It is easier to consider two states:

State *"one"*: the particle is moving up.

State *"two"*: the particle is moving down (falling).

The steady state: hydrodynamic equilibrium, i.e, a strictly non moving particle sustained with a zero velocity in the moving fluid, does not have any meaning in a fluidized bed. The concept of two dynamic states is more realistic. State *"one"* is the excited state. Each of the two states may represent a branch of Chaotic Processes defined in equations (5) and (6).

Thus, it is possible to get a very simple model for the actual fluctuating with several possibilities depending on the value of "a": i.e, the energy supplied to the bed. The various regimes are those of the Chaotic process. These can be put qualitatively in agreement with the actual observed regimes. Each of these regimes corresponds to a different energy excitation state: i.e, value of "e through the fluid flow rate supplied to the equipment.

The main characteristic of a fluidized bed is given by the curve correlating pressure drop versus the liquid flow rate. This displays a flat part: fluidization in between two rising parts: fixed bed an transportation (Fig. 3). These two extreme branches may be compared to the branches of the computerized Choatic Process (Fig. 2).

Following this fact, *"Fluidization"* can be presented as a mixing, along time, of two well defined dynamic states represented by classical equations. Fluidization is a succession of transitions between the two states. One transition accumulates energy while the other dissipates this energy into heat. This may be compared to energy excitation of atoms or molecules.

The fluidization characteristic curve may be transformed into an energy curve showing that fluidization corresponds to a minimum of energy dissipation (Figure 4) in agreement with the *"Curie Prigogine"* principle. Following this principle, a physical system behaves such that it dissipates a minimum of energy per unit of time and unit of volume.

It is possible to show that the numerical Chaotic Process corresponds to a minimum of the average value of "$\bar{a}$" i.e. the steady state corresponding to convergence is for a higher value of "a" than the computer time averaged value. The fluctuating process could be considered as *"natural"*, for actual systems.

## CONCLUSION

A simple model is proposed in order to take into account, at least qualitatively, the complex behavior of turbulent dissipative systems. In this view, a Numerical Chaotic Process is assumed to be a means to find appropriate models. This paper is an attempt to justify this assumption. Strong analogies exist with the basic equations of Quantum Mechanics leading to analogy with the Theory of Relativity of time.

Some of these analogies may be partly justified by the behaviors of some widely used classical engineering systems, such as fluidized beds.

From the proposed model it appears that one of the difficulties for appraising turbulent systems may be that they run their course in a *"complex"* time scale. One component of this *"complex"* time scale built from a chaotic process is for the time being impossible to synchronize with our usual time scale.

The time relativity seems to be in agreement with our common sense: systems under fluctuant energy stress age faster than the same system under stabilized stress.

The problem now is to find the relevance of this kind of modelling when quantitatively compared with actual systems and how this model can be effectively used to go further in comprehending the turbulent systems?

## NOTATIONS

| | |
|---|---|
| $a$ | dissipative activity Indicator, $(J/s\ m^3)^{1/2}$ |
| $a'$ | turbulent component of a |
| $\bar{a}$ | time averaged value of a |
| $a_o$ | a for external system |
| $a_1$ | component 1 of $a_o$ |
| $a_2$ | component 2 of $a_o$ |
| $a^*$ | conjugate of a |
| $C_o$ | commutator (-) |
| $c$ | velocity of light in vacuum (m/s) |
| $e$ | energy supplied to the system $(J/m^3 s)$ |
| $G$ | volumetric flow rate of the fludizing fluid $(m^3/s)$ |
| $k$ | constant of the Schrödinger equation (unit defined by the equation) |
| $k'$ | drag coefficient (kg/s) |
| $\underline{K}$ | coefficient (s) |
| $\hat{K}$ | operator (-) |
| $k_o$ | time coefficient (-) |
| $m$ | mass (kg) |
| $n$ | number (-) |
| $P$ | pressure drop $(P_a)$ |
| $s$ | entropy source $(J/K\ m^3/s)$ |

| | |
|---|---|
| t | time (s) |
| t' | time (s) |
| t" | time (s) |
| T | dimensionless time (-) |
| v | velocity (m/s) |
| V | fluid useful velocity for fluidization (m/s) |
| $V_g$ | fluid velocity |
| z | position (-) |
| w | constant (s) |
| θ | time (s) (m/s) |
| φ | wave function (-) |

REFERENCES

**BERRY M.** (1987), Quantum Physics on the Edge of Chaos, New Scientist, 116 (1987): 44-47.

**BES R.S.,MORA J.C.**, (1982), Modélisation Entropique et Simulation du Comportement d'un Système. Dissipatif Complexe, Proceedings Int. AMSE Conf. Modelling and Simulation, S1: 216-220,Paris, ed Mesnard G. Paris

**BES R.S., MORA J.C.**, (1991), Quantum Mechanics, Chaos, Energy Dissipation and Time Fluctuating Systems. AIT Report, Feb. 1990.

**BES R.S, MOLINIER J.,MORA J.C.**, (1986), Thermodynamic Modelling of Backmixing Effect in a Mechanical Agitated Extraction Column, Int. Solvent Extraction Conf. Munchen, Sept 11-16, H 11 Po

**CHEHBOUNI A., GABSI S., BES R.S., MORA J.C.** (1985) Modélisation des Fluctuations dans un Lit Fluidisé par un Processus d'Intéraction Dissipative, Proceedings, 4th Journées Européennes de Fluidisation, Toulouse, Sept 18-20

**COVERNEY P.V.**, (1989), L'Irréversibilité du Temps, La Recherche, 20 (207): 190-198

**COVERNEY P.** (1990), Chaos, Entropy and Arrow of Time, New Scientist, September 1990, vol. 127, No. 1736, pp. 39-42.

**DELANDE D., GAY J.C.,BOMMIER A.**,(1989) L'Atome d'Hydrogène en Champ Magnétique Intense: un Modèle de Chaos Quantique, Le Courrier du CNRS, 74: 49-55

**GLEICK J.**, (1987), Chaos, New York, The Viking Press

**HAO BAI-LIN**, (1990), Chaos II, Singapore, World Scientific

**HAWKING S.**, (1988), A Brief History of Time, New York, Bantam Book

**MORA J.C., BUGAREL R.**, (1976), Application de la Thermodynamique des Transformations Irréversibles en Génie Chimique-Modélisation d'un Plateau de Rectification de Grand Diamètre, Entropie, 69: 5-20

**MORA J.C.,(1978)**, Mass Transfer and Backmixing for a Diphasic Flow Apparatus; the Problem of Experimental Determination of the Coefficients: Int. CHISA'78 Congress, Praha, Tchekoslovakia, August.

**MULLIN T. (1989)**, Turbulent Times for Fluids, New Scientist 124, (1690): 52-59

**PRIGOGINE I**, (1980) Physique, Temps et Devenir, Paris, eds Masson

**PRIGOGINE I, STENGERS I.**, (1988), Entre Temps et Eternite, Paris, eds Fayard

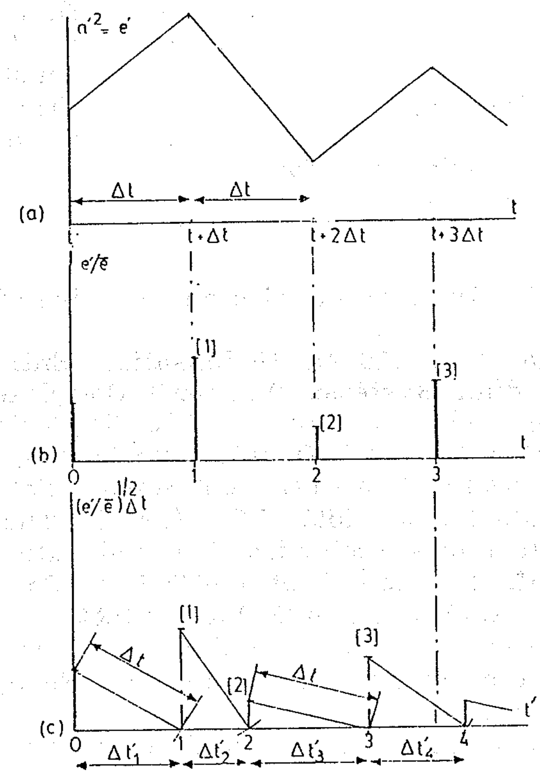

FIGURE 1: Thermodynamic Complex Time
a: signal from a datalogger or from a computerized chaotic process (datalogger or computer time scale)
b: signal to be used for the generation of the internal time scale
c: transformation of the computer time scale into an internal time scale.
Three events (fluctuations) occur in the usual time scale (t), figure (a), while four events occur in the internal time scale (t'), figure (c)

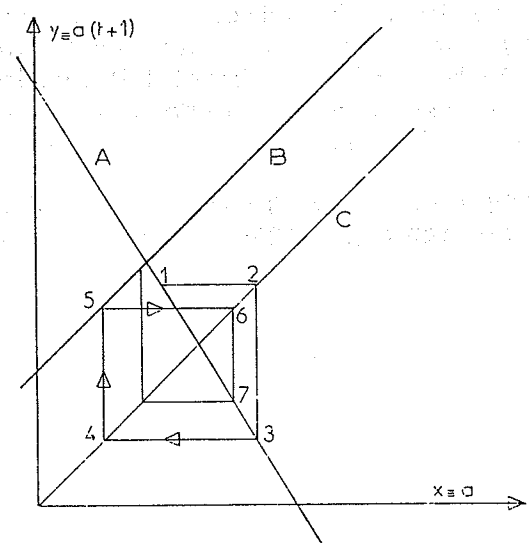


FIGURE 2: Chaotic Process

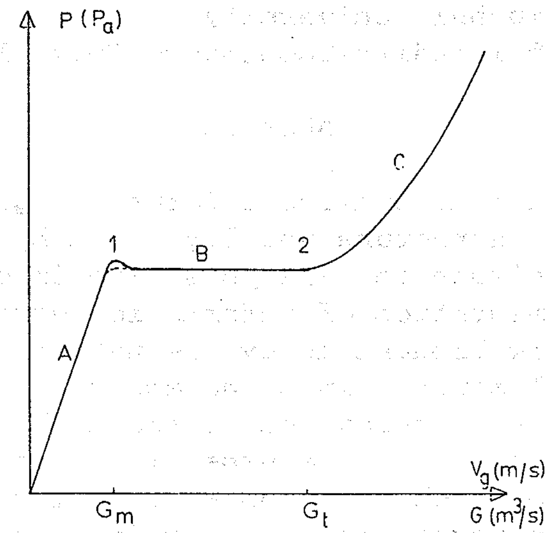


**FIGURE 3:** Fluidization Characteristic curve
Pressure drop versus fluid volumetric flow rate or velocity
A: fixed bed, B: Fluidization, C: Transportation
1: Homogeneous fluidisation

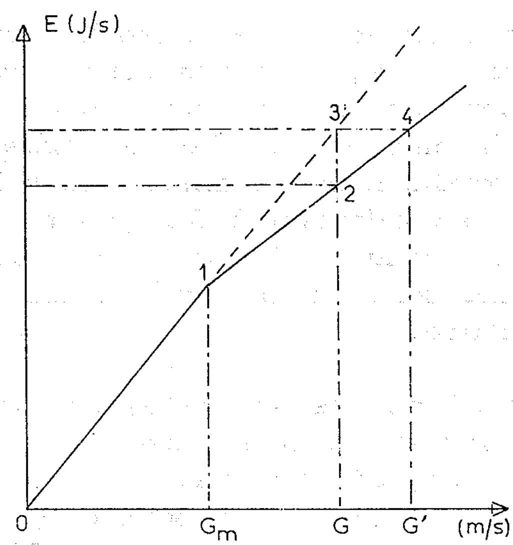


**FIGURE 4:** Energy dissipation curve for a fluidized bed
Mechanical Energy dissipated into heat versus fluid flow rate
1: minimum of fluidization 2: actual energy comsunption
3: hypothetical energy consumption in fixed bed regime
4: with the same energy consumption the fluidized bed allows a higher flow rate passing through than a fixed bed: G' instead of G