FUZZY EVALUATION ON THE VALIDITY OF SYSTEM DYNAMICS MODELS

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Introduction

How to evaluate the validity of the system dynamics models, which is a subject of interest to us. Generally speaking, it is necessary to test the model extensively from different angles, so that people can understand and believe in that model all through, this is so called the validity of the model. Of course, this shouldn't be all right to evaluate the validity of the whole model only depend on a single test. Hence, the problem is how to evaluate completely the validity of the model through multiple tests. In practice, this is a problem about the synthetical evaluation. The synthetical evaluation can use the approach to find the sum,namely keep the score for the result of each test and then count up them. It can also use the weighted average method. But they aren't suitable to evaluate the validity of the models. Because the validity is a fuzzy conception. Only using a simple fraction to evaluate the validity of the system dynamics model is not precise. Therefore, it probably is a better method to evaluate the validity of the model synthetically using the fuzzy sets theory. This paper discusses the multilevel evaluation system and fuzzy synthetical evaluation method for the validity of the system dynamics model, and gives a computation example at last. The example proves that fuzzy synthetical evaluation can give satisfactory results.

Multilevel Evaluation System

Analysing the treatises on the validity or confidence of the system dynamics models, it consider that the validity of the model can be evaluated more objectively through following 11 tests, these are:

(1) Structure verification test;
(2) Parameters verification test;
(3) Extreme conditions test;
(4) Boundary moderation test;
(5) Dimension consistency test;
(6) Behaviour reproduction test;
(7) Behaviour abnormality test;
(8) Behaviour sensitivity test;
(9) System improvement test;
(10) Behaviour transformation test;

(11) Strategy sensitivity test.
Above mentioned 11 tests can be divided into three groups;(1)-(5) is called the structure confidence, (6)-(8) the behaviour confidence and (9)-(11) the strategy confidence.
It is possible to synthesize these confidences in order to evaluating the validity of the model as a whole. For this reason we pose a three level evaluation system of the validity of the system dynamics model, as shown Fig.1.

Mathematical Description of Fuzzy Evaluation

The mathematical description of the fuzzy synthetical evaluation

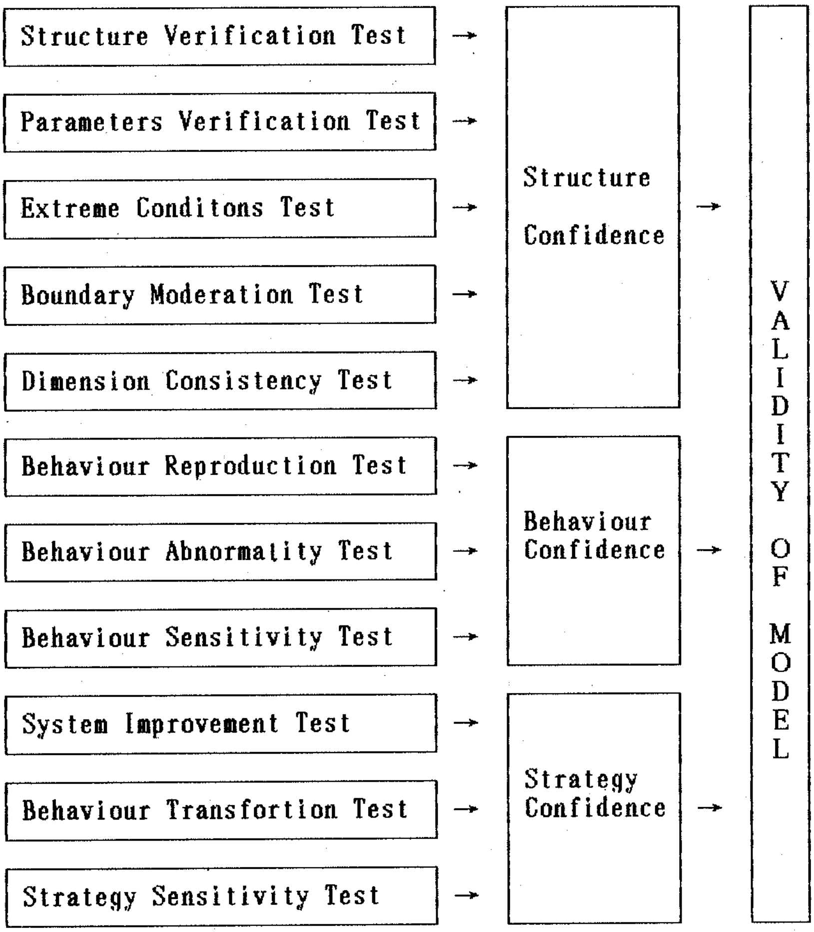

Figure 1. 3-level evaluation system of the validity of the system dynamics model

as follows;
Suppose the factors set

$$U=\{u_1\ \ u_2\ \cdots\ u_n\} \tag{1}$$

and the evaluation set

$$V=\{v_1\ \ v_2\ \cdots\ v_m\} \tag{2}$$

Each element in the evaluation set correspond to the grade of the evaluation standard for the validity of the model, such as "very good", "good", "not bad", "bad" and "very bad".
In the fuzzy sets, the characteristic function of the elements is called the membership function which is to take value out of an interval [0, 1] continuously. The membership function can take the normal distribution or other type.
When we apply the fuzzy evaluation method to a system dynamics model, the fuzzy evaluation of factor i in the factors set U can be described as a fuzzy subset on the evaluation set V

$$\underset{\sim}{R}_i=\{r_{i1}\ \ r_{i2}\ \cdots\ r_{im}\} \tag{3}$$

If there are n factors, then there is a fuzzy matrix

$$\underset{\sim}{R}=\begin{bmatrix}\underset{\sim}{R}_1\\ \underset{\sim}{R}_2\\ \cdot\\ \cdot\\ \underset{\sim}{R}_n\end{bmatrix}=\begin{bmatrix}r_{11} & r_{12} & \cdots & r_{1m}\\ r_{21} & r_{22} & \cdots & r_{2m}\\ \cdot & \cdot & & \cdot\\ \cdot & \cdot & & \cdot\\ r_{n1} & r_{n2} & \cdots & r_{nm}\end{bmatrix} \tag{4}$$

Where $\underset{\sim}{R}$ is called the single factor fuzzy evaluation matrix.
The allocation of the weights for the factors is a fuzzy subset $\underset{\sim}{A}$ on the factors set U

$$\underset{\sim}{A}=\{a_1\ \ a_2\ \cdots\ a_n\} \tag{5}$$

and have

$$\sum_{i=1}^{n} a_i=1 \tag{6}$$

The result of the synthetical evaluation is a fuzzy matrix

$$\underset{\sim}{B}=\underset{\sim}{A}\cdot\underset{\sim}{R} \tag{7}$$

Where the sigh $\cdot$ means the multiplying operation of the fuzzy matrix. In more detail, $\underset{\sim}{R}$ is a fuzzy relation between the factors set U and the evaluation set V. The fuzzy synthetical evaluation is to find the fuzzy subset $\underset{\sim}{B}$ on the evaluation set V in the given weighted conditions through the fuzzy mapping $\underset{\sim}{R}$. The steps of the multiplying operation for the fuzzy matrix are similar to the common matrix operation, namely

$$b_{ij}=\max_k \min\ [a_{ik},\ r_{kj}]=\bigvee_k [a_{ik}\wedge r_{kj}] \tag{8}$$

Where the sigh $\wedge$ and $\vee$ represents drawing minimum and maxmum operation respectively.

Steps of Fuzzy Synthetical Evaluation

The steps of the fuzzy synthetical evaluation for the validity of the model as follows:

(1) Give the testing data of the model;
(2) Let the evaluation grade of the valitidy;
(3) Let the grade standard of each test;
(4) Calculate the testing accuracy;
(5) Calculate the membership r_{ij} for evaluating each test;
(6) Constitute the fuzzy matrix $\underset{\sim}{R}$;
(7) Let the weighted matrix $\underset{\sim}{A}$;
(8) Find the evaluation result $\underset{\sim}{B}$.

According as the result of the first level evaluation, then the fuzzy evaluation in second and third level are determined by the same method.

The computer flow chart for three level fuzzy synthetical evaluation is shown in Fig. 2.

An Example

The result of three level fuzzy evaluation for the validity of the system dynamics model to study the scientific and technical investment in a region as follows:

(1) First level evaluation

$\underset{\sim}{B}^1 = (0.2574\quad 0.7602\quad 0.0420\quad 0.0010\quad 0.0000)$
$\underset{\sim}{B}^2 = (0.4721\quad 0.4506\quad 0.1410\quad 0.0000\quad 0.0000)$
$\underset{\sim}{B}^3 = (0.2880\quad 0.7403\quad 0.0132\quad 0.0001\quad 0.0000)$
$\underset{\sim}{B}^4 = (0.1512\quad 0.7811\quad 0.1032\quad 0.0001\quad 0.0000)$
$\underset{\sim}{B}^5 = (0.3576\quad 0.6257\quad 0.0540\quad 0.0002\quad 0.0000)$
$\underset{\sim}{B}^6 = (0.1041\quad 0.9211\quad 0.0350\quad 0.0010\quad 0.0000)$
$\underset{\sim}{B}^7 = (0.2501\quad 0.7604\quad 0.0420\quad 0.0001\quad 0.0000)$
$\underset{\sim}{B}^8 = (0.2073\quad 0.8203\quad 0.0135\quad 0.0020\quad 0.0000)$
$\underset{\sim}{B}^9 = (0.3512\quad 0.6844\quad 0.0240\quad 0.0010\quad 0.0000)$
$\underset{\sim}{B}^{10} = (0.3427\quad 0.7015\quad 0.0150\quad 0.0001\quad 0.0000)$
$\underset{\sim}{B}^{11} = (0.3054\quad 0.7502\quad 0.0120\quad 0.0000\quad 0.0000)$

(2) Second level evaluation

$\underset{\sim}{B}_1 = \underset{\sim}{A}_1 \cdot \underset{\sim}{R}_1$
$= (0.215\quad 0.385\quad 0.15\quad 0.15\quad 0.10)(\underset{\sim}{B}^1\ \underset{\sim}{B}^2\ \underset{\sim}{B}^3\ \underset{\sim}{B}^4\ \underset{\sim}{B}^5)^T$
$= (0.3388\quad 0.6277\quad 0.0872\quad 0.0003\quad 0.0000)$

$\underset{\sim}{B}_2 = \underset{\sim}{A}_2 \cdot \underset{\sim}{R}_2$
$= (0.35\quad 0.35\quad 0.30)(\underset{\sim}{B}^6\ \underset{\sim}{B}^7\ \underset{\sim}{B}^8)^T$
$= (0.1861\quad 0.7946\quad 0.0311\quad 0.0012\quad 0.0000)$

$\underset{\sim}{B}_3 = \underset{\sim}{A}_3 \cdot \underset{\sim}{R}_3$
$= (0.35\quad 0.35\quad 0.30)(\underset{\sim}{B}^9\ \underset{\sim}{B}^{10}\ \underset{\sim}{B}^{11})^T$
$= (0.3345\quad 0.7101\quad 0.0173\quad 0.0004\quad 0.0000)$

(3) Third level evaluation

$\underset{\sim}{B} = \underset{\sim}{A} \cdot \underset{\sim}{R}$
$= (0.375\quad 0.375\quad 0.25)\ (\underset{\sim}{B}_1\ \underset{\sim}{B}_2\ \underset{\sim}{B}_3)^T$
$= (0.2805\quad 0.7109\quad 0.0497\quad 0.0006\quad 0.0000)$

Because

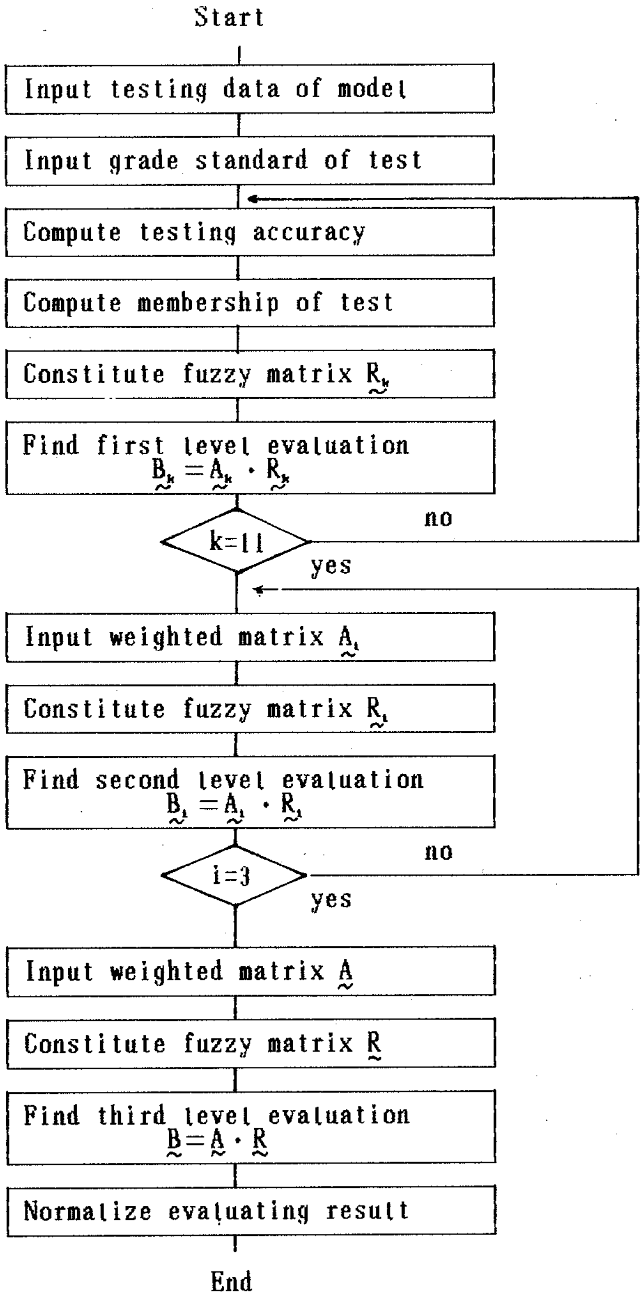

Figure 2 The computer flow chart of 3-level fuzzy evaluation

$$\sum_{j=1}^{m} b_j = 0.2805+0.7109+0.0497+0.0006+0.0000=1.0417$$

$\underset{\sim}{B}$ ought to be normalize

$$\underset{\sim}{B}^* = (0.2693\ \ 0.6824\ \ 0.0477\ \ 0.0006\ \ 0.000)$$

The computing rsult indicaties that validity of the model is satisfactory. From fuzzy conceptation this model is 26.93 percent belong to "very good" standard and 68.24 persent belong to "good" standard.

Conclusion

The multilevel fuzzy synthetical evaluation raised in this paper is a complete and convincing method for evaluating the validity of the system dynamics models.The computation indicates that method is easy to operate on the computer. In addition, this method can be discover that harmful factors existed in structure, behaviour and strategy of the model through the fuzzy evaluation. Therefore it provides the possibility to overcome the weaknesses of the model.

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