THE TIME DELAY AND OSCILLATION OF ECONOMIC SYSTEM

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ABSTRACT

The time delay θ and capital life-time β of economic system must be considered in discussing the system's dynamic behaviour. In this paper a model of economic system with time lag has been proposed to deal with time delay and life-time. The core of this model is investment decision equation, in which the transfer function $W_1(Z)=\sum_{j=0}^{r} b_j Z^{-j} \Big/ (1+\sum_{i=1}^{m} a_i Z^{-i})$ has been introduced. By selecting appropriate coefficients a_i and b_j the desired dynamic behaviour can be obtained. The impulse response $h(t)$ of the system is defined by using numerical solution of its characteristic equation. The numerical solutions for proportional and proportional-plus-integral control system with 60 different sets of θ & β has been calculated. According to the calculating results we use multiple regression analysis to get the regression equation between the critical oscillation parameters (period, amplification coefficient and amplitude) and time delay & life-time. It is convenient to apply these regression equations for choosing parameters.

INTRODUCTION

In practice, there is a definite lapse of time for carrying out each decision. For example, an industrial investment is beginning at time t and the effect of this investment (output) appears at time $t+\tau$, τ is the dead time of the system. The system with time delay is concerned early in automatics,[1-2] the Polish academican O. Lange had introduced a pure time delay component in discussing the Kaleski Business cycle[3]. Similar pure time delay component is encountered in researching the output price and supply-demand relation of farm products, in this case the time delay is often one year. The characteristic equation of the system including a pure time delay component is a transcendental equation, so it is difficult to find out the stability boundary when the order of characteristic equation is high.

The another problem in economic system concerns with the life-time of capital. For example, a machine has its operating life, the product based on old technology may be replaced by new technology, this period will be shorten with the advance in society and technology. Such economical phenomena can also be treated using corresponding time delay component.

THE MODEL OF ECONOMIC SYSTEM WITH DEAD TIME

The market economy is directly connected with goods, assuming the demand is decreased with the increase of its price.

$$d(p)=d_0-ap \tag{1}$$

and the supply $S\ (p)$ is increased with the increase of its price.

$$s(p)=s_0+ap \tag{2}$$

In the condition $d(p)=S(p)$, the equilibrium price p_e can be found out

$$p_e=\frac{d_0-S_0}{a+b} \tag{3}$$

s_0, d_0, a & b are the factors independent of price and such factors are varied from year to year, so the equilibrium price fluctuates. Assuming T is the sampled-period of economic system, the present price at $t=n\mathrm{T}$ is $p\ (n)$, and $p(n-i)$ represent corresponding price just before i years. Assuming $\hat{p}\ (n)$ represents the dummy price, $\hat{p}\ (n)$ is related with $p(n)$ by following difference equation

$$\hat{p}(n)+\sum_{i=1}^{m}a_i\hat{p}(\mathrm{n}-i)=\sum_{j=0}^{r}b_jp(\mathrm{n}-\mathrm{j}) \tag{4}$$

if a_i and b_j are known, the variation of $\hat{p}$(n) can be calculated from variation of $p(n)$, based on $\hat{p}(n)$ the production is arranged. Because of the time delay $\tau=\theta\mathrm{T}$, the production plan arranged at $n\mathrm{T}$ can't produce new supply for the market before time $(n+\theta)\mathrm{T}$, so the supply of goods

$$\mathrm{S}(n+\theta)=\mathrm{S}_0+b\hat{p}\ (n) \tag{5}$$

The demand is determined by present prices

$$d(n+\theta)=d_0-ap(n+\theta) \tag{6}$$

Assuming $s=d$ in equilibrium condition, the dynamic equation of price is found out

$$p(\mathrm{n}+\theta)=-\rho\hat{p}(n)+f(n) \tag{7}$$

where $\quad \rho=\frac{b}{a},\quad f(n)=\frac{d_0-s_0}{a}$

After the Z-transformation has been effected, (7) becomes

$$\mathrm{P(Z)}=-\rho\mathrm{Z}^{-\theta}\hat{\mathrm{P}}(\mathrm{Z})+\mathrm{Z}^{-\theta}\,\mathrm{F(Z)}+\sum_{i=0}^{\theta-1}p(i)\mathrm{Z}^{-i} \tag{8}$$

where P(Z),$\hat{\mathrm{P}}$ (Z),F(Z) are the Z-transform of the discrete function $p(n)$,$\hat{p}(n)$, & $f(n)$, respectively. The Z-transform of eq. (4) is found to be

$$\hat{\mathrm{P}}\ (\mathrm{Z})=\frac{\sum_{j=0}^{r}b_jZ^{-j}}{1+\sum_{i=1}^{m}a_i\mathrm{Z}^{-i}}\mathrm{P(Z)} \tag{9}$$

define $\quad \mathrm{W(Z)}=\dfrac{\sum_{j=0}^{r}b_jZ^{-j}}{1+\sum_{i=1}^{m}a_iZ^{-i}} \qquad (10)$

combining (8), (9) and (10) yields

$$[1+\rho\mathrm{W(Z)Z}^{-\theta}]\mathrm{P(Z)}=\mathrm{Z}^{-\theta}\mathrm{F(Z)}+\sum_{i=0}^{\theta-1}p(i)Z^{-i} \tag{11}$$

$$\Phi(\mathrm{Z})=[1+\rho\mathrm{W(Z)Z}^{-\theta}]^{-1} \tag{12}$$

The form of Φ(Z) is determined if a_i, b_j and ρ is given. Then eq. (12) can be written as

$$\mathrm{P(Z)}=\Phi(\mathrm{Z})\mathrm{Z}^{-\theta}\mathrm{F(Z)}+\Phi(\mathrm{Z})\sum_{i=0}^{\theta-1}p(i)\mathrm{Z}^{-i} \tag{13}$$

The first term at the right-hand side of (13) represents the effect of $f(n)$ on the price, $f(n)$ influences the supply and demand but is independent of price; the second term represents the effect of initial condition $p(i)$ on price $p(n)$. If the form of Φ (Z) is known, the F(Z) and $\sum_{i=0}^{\theta-1} p(i)Z^{-i}$ is given, then $P(Z)$ is determined. To determine a given system is stable or not, the characteristic equation of system must be considered at first. From (11) the characteristic equation of system is

$$1+\rho W(\lambda)\lambda^{-\theta}=0 \tag{14}$$

combining (10) and (14) yields

$$\lambda^{\theta+r}+\sum_{i=1}^{m} a_i\lambda^{\theta+r-i}+\rho\sum_{j=0}^{r} b_j\lambda^{r-j}=0 \tag{15}$$

the highest order of λ is $\theta+r$, so there are $(\theta+r)$ characteristic roots. If the characteristic equation has any roots out-side the unit circle, the system will be unstable. If some of the roots lie on the unit circle, the system will be critical stable. Now discuss some special cases.

(A) Original cobweb model

Given $r=0$, $m=0$, $b_0=1$, assuming the time delay equals to the sampled period, i. e. $\theta=\tau/T=1$, $f(n)=(d_0-s_0)/a$. Putting these data in eq. (15), the characteristic equation of Cobweb model is found out

$$\lambda+\rho=0 \tag{16}$$

combining (10), (12) and (13) yields

$$P(Z)=\left(\frac{d_0-s_0}{a}\right)\frac{Z^{-1}}{1-Z^{-1}}\cdot\frac{1}{1+Z^{-1}}+\frac{p(o)}{1+Z^{-1}} \tag{17}$$

Using the inverse Z-transform, from eq. (17) it is found that

$$P(n)=Z^{-1}[P(Z)]=(-\rho)^n p(o)+[1-(-\rho)^n]\frac{d_0-s_0}{a+b} \tag{18}$$

where $p(o)$is the initial value of price at $t=0$. It is obvious that if the absolute value of ρ is less than 1, the system is stable, and the price $p(n)$ converges to the equilibrium value $(d_0-s_0)/(a+b)$ during $n\to\infty$. On the other side, if $|\rho|=1$, the system becomes critical stable and oscillation with period 2T and constant amplitude will be generated. If $|\rho|>1$, the price curve will diverge. So the Cobweb model is a simplest example in our model.

(B) D. G. Luenberger model

The observed period of pork price oscillation is not a fixed value 2T. D. G. Luenberger observed that the price fluctuation is characterized by a cycle with 4 years period. In order to explain this phenomena, he proposed that the sampled-period T = 0.5 year. In this case, the time delay $\tau=2T$, i.e. $\theta=2$. The average price of past 5 years (including the present year) is taken to arrange the production, this corresponds $r=4$, $b_0=b_1=b_2=b_3=b_4=1/5, a_i=0$,substituting these data into eq. (15), we get

$$\lambda^6+\rho(\lambda^4+\lambda^3+\lambda^2+\lambda+1)/5=0 \tag{19}$$

the critical stable condition of eq. (19) is $\rho=2.07$, the corresponding principal root is $\lambda=\sqrt{2}/2\pm j\sqrt{2}/2=e^{j\pi/4}$, its module equals 1, the oscillation period

$$CT=\frac{2\pi}{\pi/4}T=8T=4 \text{ years}$$

This model only explains the phenomena that the period of American pork price oscillation is 4 years, but the model can't make any suggestion about how to improve the price oscillation.

(C) PID control

There are three parameters may be set in PID control, so the root distribution of characteristic equation can be changed in a wider range, which will result in better dynamic behaviour, For PID control, eq. (10) becomes

$$W(z)=K_p+K_i\frac{1}{1-z^{-1}}+K_d(1-z^{-1}) \tag{20}$$

where K_p,K_i & K_d represent proportional gain, integral gain & derivative gain respectively. The values of these gains can be selected. Comparing eq. (20) and (10) it is known that $a_1=-1$, $b_0=K_p+K_d+K_i$, $b_1=-(K_p+2K_d)$, $b_2=K_d$, i.e. $m=1$, $r=2$. For example, studying D.G. Luenberger pork price model, $\theta=2$, taking eq. (20) as $W(z)$, substitute these data into eq. (15) and define $\rho=1$. (because K_p ,K_i ,K_d ,can be arbitrary selected, $\rho=1$ does not lose the generality) Thus, the characteristic equation

$$\lambda^4-\lambda^3+(K_p+K_i+K_d)\lambda^2-(K_p+2K_d)\lambda+K_d=0 \tag{21}$$

Because three parameters K_p, K_i & K_d can be arbitrary selected, the loci of three roots among four characteristic roots can be set as desired. Assuming the four characteristic roots are λ_1, λ_2, λ_3 & λ_4, utilization of eq. (21) yield

$$\lambda_1+\lambda_2+\lambda_3+\lambda_4=1$$

Assuming λ_1 is the principal root, its module is largest. $\lambda_2=\alpha_2\lambda_1$, $\lambda_3=\alpha_3\lambda_1$, $\lambda_4=\alpha_4\lambda_1$ and $|\alpha_i|<1$ $(i=2,3,4)$

$$\lambda_1=\frac{1}{1+\alpha_2+\alpha_3+\alpha_4}$$

For demonstrating this concept, assuming all 4 roots are real roots and $\alpha_2=0.2$, $\alpha_3=0.3$, $\alpha_4=0.5$ yield $\lambda_1=0.5$, $\lambda_2=0.1$, $\lambda_3=0.15$ and $\lambda_4=0.25$, then using relationship between the roots and coefficient find out $K_p=0.245$, $K_i=0.096875$, $K_d=0.001875$. Therefore the system not only is stable, but also has desired dynamic behaviour.

In this example $\theta=2$, in order to attain the desired distribution of characteristic roots the simpler form of $W(z)$ (comparing with PID control) may be chosen

$$W(z)=\frac{b_0}{1+a_1z^{-1}} \tag{22}$$

where coefficient b_0, a_1 may be arbitrarily chosen, so defining $\rho=1$ does not lose the generality, from (15) yield

$$\lambda^2+a_1\lambda+b_0=0 \tag{23}$$

By selecting appropriate a_1, b_0, the desired distribution of two characteristic roots can be obtained, therefore the control law represented by eq. (22) is simpler and more effective than PID control. Furthermore, it is convenient to apply cybernetics optimization theory for determining the values of a_1, b_j.

THE DYNAMIC MODEL OF SUPPLY–DEMAND DISEQUILIBRIUM

Defining $d(n)$,$s(n)$ & $e(n)$ represent demand, supply and the difference between demand and supply at $t=nT$ respectively. Then

$$e(n)=d(n)-S(n) \tag{24}$$

Assuming $e(n)$ or stock (i.e. the integral of $e(t)$) is observable, it is obvious that

$e(n)$ contains the information of price, so the signal $e(n)$ may be used to construct the dummy price $\hat{p}(n)$ model.

$$\hat{p}(n)+\sum_{i=1}^{m} a_i \hat{p}(n-i)=\sum_{j=0}^{\tau} b_j e(n-j) \tag{25}$$

Assuming the time delay of production process is also $\tau = \theta T$, therefore the supply s is still defined by (5). Defining $W(z)=\hat{P}(z)/E(z)$, from (25) yield

$$W(z)=\sum_{j=0}^{\tau} b_j z^{-j}/(1+\sum_{i=1}^{m} a_i z^{-i}) \tag{26}$$

After the z-transformation has been effected, (5) becomes

$$S(z)=Z^{-\theta}s_0(Z)+\sum_{i=0}^{\theta-1} S(i)Z^{-i}+bZ^{-\theta}\hat{P}(z) \tag{27}$$

Combining (24)~(27), the flow chart shown in Fig. 1 may be plotted, W(Z) in Fig. 1 is represented by (26) and b can be regarded as open loop gain. Using the relationship between open loop and close loop and close loop transfer function yield

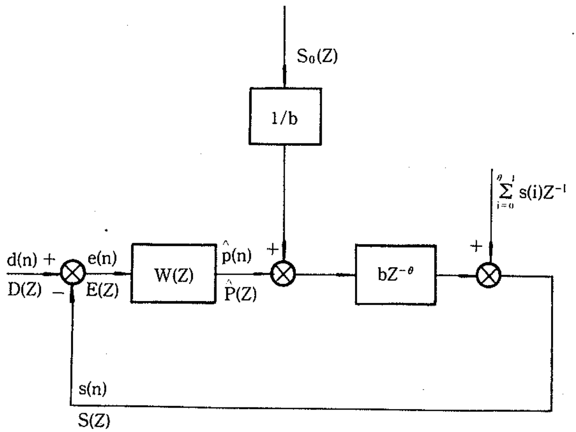

Fig. 1 The flow chart for model of economic system with time delay

$$S(z)=\Phi(z)D(z)+\Phi_\varepsilon(Z)Z^{-\theta}S_0(Z)+\Phi_\varepsilon(z)\sum_{i=0}^{\theta-1} S(i)Z^{-i} \tag{28}$$

where

$$\Phi(z)=\frac{bZ^{-\theta}W(z)}{1+bZ^{-\theta}W(z)} \tag{29}$$

$$\Phi_\varepsilon(z)=\frac{1}{1+bZ^{-\theta}W(z)} \tag{30}$$

The characteristic equation of the system

$$1+bW(\lambda)\lambda^{-\theta}=0 \tag{31}$$

It is obvious, if the form of W (z) is defined, b & θ is given, the $\Phi_\varepsilon(z)$ and $\Phi(z)$ will be determined, Form (28) it is known supply $S(z)$ is comprised of three parts: the main term is determined by demand $D(z)$; the second term is regarded as disturbance of the system; the third term is the influence of initial condition.

THE CAPITAL LIFE–TIME

Slightly changing the economic meaning of the variable in Fig. 1, which can be used to deal with the life-time of capital. For clarification, we define the variable in Fig. 2 as followings: $e(n) = d(n) - s(n)$ is the difference between demand and supply, $B(n)$——the planned investment. Assuming $e(n)$ and $b(n)$ is connected by following difference equation

$$B(n)+\sum_{i=1}^{m} a_i B(n-i)=\sum_{j=0}^{r} b_j e(n-j) \tag{32}$$

we call eq. (32) the planned investment decision equation, the corresponding transfer function of which is

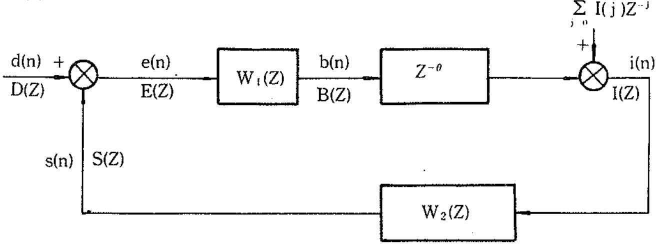

Fig. 2 The flow chart of capital life-time

$$W_1(z)=\frac{B(Z)}{E(z)}=\sum_{j=0}^{r} b_j Z^{-j}\Big/\Big(1+\sum_{i=1}^{m} a_i Z^{-i}\Big) \tag{33}$$

this form is coincided with $W(z)$ in Fig. 1. In literature there are several cases in discussing investment decision. First, the present $e(n)$ determines the present planned investment, i.e. $B(n)=b_0 e(n)$, corresponding to $W_1(z)=b_0$. Second using stock to determine the planned investment. i. e. $W_1(z)=\frac{b_0}{1-Z^{-1}}$. Third, using weighted sum of the stock and $e(n)$ to determine the planned investment, i. e. PI control, corresponding to $W_1(z)=\frac{b_0+b_1 Z^{-1}}{1-Z^{-1}}$. Fourth, $W_1(Z)=\frac{b_0+b_1 Z^{-1}+b_2 Z^{-2}}{1-Z^{-1}}$ is corresponding to PID control. All the cases mentioned above can be regarded as the the simple special examples in our model. Assuming the planned investment $B(n)$ is decided at $t=nT$, the production capability is formed at $t=(n+\theta)T$ and let I represents the investment of formed production capability, then

$$I(n+\theta)=B(n) \tag{34}$$

Using the Z-transform

$$I(Z)=Z^{-\theta}B(Z)+\sum_{j=0}^{\theta-1} I(j)Z^{-j} \tag{35}$$

Assuming at $t=jT$ the effective investment $I(j)$ contributes to supply products, after elapsing interval βT the effect of investment disappears, which means the machine has been scrapped or the product fails in market competition and is forced to stop production. So the life-time of capital $I(j)$ is βT. This process can be represented by the difference of two step functions, as shown in Fig. 3. The corresponding z-transform of the difference between the two step functions is

$$I(j)\left[\frac{Z^{-j}}{1-Z^{-1}}-\frac{Z^{-(j+\beta)}}{1-Z^{-1}}\right]=I(j)Z^{-j}\sum_{i=0}^{\beta-1} Z^{-i} \tag{36}$$

Summing all investments at jT $(j=0,1,2,\ldots)$ yield the supply S

$$S(Z)=I(Z)\Big(K\sum_{i=0}^{\beta-1} Z^{-i}\Big)=I(Z)W_2(Z) \tag{37}$$

where $$W_2(Z)=K\sum_{i=0}^{\beta-1}Z^{-i} \tag{38}$$

K——effect of investment.

If the dynamic response of investment effect during rising and decay period must be considered, assuming the dynamic response is an exponential time function with time constant τ_1 and τ_2, the transfer function will be rewritten

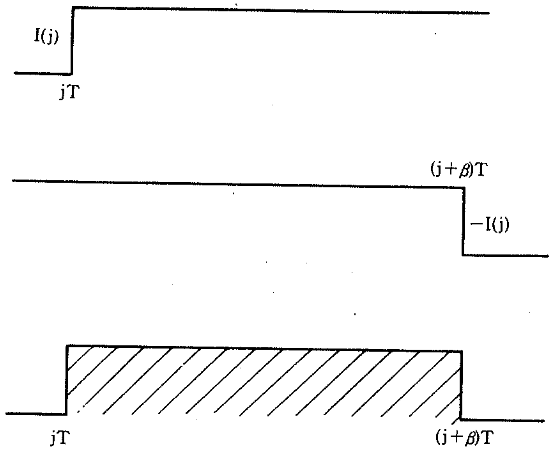

Fig. 3 The life-time of capital represented by two step functions

$$W_2(Z)=K\left(\sum_{i=0}^{\beta-1}Z^{-i}-\frac{1}{1-aZ^{-1}}+\frac{Z^{-\beta}}{1-bZ^{-1}}\right) \tag{39}$$

where $a=e^{-1/\tau_1}$, $b=e^{-1/\tau_2}$.

The $Z^{-\theta}$ component and $W_2(Z)$ component are determined by the feature of control object, which is characterized by the value of θ and β. The main task of the model is that in order to attain the desired dynamic behaviour of economic system, the appropriate form of $W_1(Z)$ is selected according the various given value θ and β. For this purpose discuss the characteristic equation

$$1+\sum_{i=1}^{m}a_i\lambda^{-i}+(\sum_{j=0}^{r}b_j\lambda^{-j})K(\sum_{i=\theta}^{\theta+\beta}\lambda^{-i})=0 \tag{40}$$

In general, selected m is less than $(\gamma+\theta+\beta)$, so there are $(\gamma+\theta+\beta)$ characteristic roots. Assuming $m=\theta+\beta$, $a_i(i=1,2,\dots\theta+\beta)$ and $b_j(1,2,\dots\gamma)$, in total $(\theta+\beta+\gamma)$ coefficients, can be arbitrarily selected, so $(\gamma+\theta+\beta)$ characteristic roots attain the desired distributions, and hence the system obtains the desired behaviour. Assuming $\gamma=0$, $m=\theta+\beta$, there are only $(\theta+\beta)$ characteristic roots, in this case $(\theta+\beta)$ coefficients a_i may be arbitrarily chosen. In practical system, the value of θ and β may be comparative large, for example, T=1year, θT=5years, β T=20years, in order to attain the desired roots distribution it is necessary to choose 25 coefficients, so microcomputer must be used in many practical researches.

STUDY ON NUMERICAL SOLUTION OF MODEL'S CRITICAL STABLE CONDITION AND MULTIPLE REGRESSION ANALYSIS OF THE RESULTS

In this paper we first study the numerical solution of the following two cases:

the planned investment is determined by the difference between demand and supply or stock, the corresponding characteristic equation of which are

$$1+K\sum_{i=\theta}^{\theta+\beta}\lambda^{-i}=0 \tag{41}$$

and

$$1-Z^{-1}+K\sum_{i=\theta}^{\theta+\beta}\lambda^{-i}=0 \tag{42}$$

The critical stable condition

$$K_{cr}=f_1(\theta,\beta) \tag{43}$$

$$CT=f_2(\theta,\beta) \tag{44}$$

$$AM=f_3(\theta,\beta) \tag{45}$$

where K_{cr}——critical amplification coefficient or gain margin (the coefficient of investment effect);

CT——period of critical oscillation;

AM——amplitude of critical oscillation.

must be found out by numerical method in these two cases. If these functions are known then the K_{cr} may be calculated at given θ and β and the acceptance of the system's behaviour may be initially assessed. Therefore this is very useful in practice. In order to find out the numerical solution, the input is the unit-delta function and the output of the system is impulse transient function $h(n)$, for system characterized by eq. (42)

$$h(n)-h(n-1)+K\sum_{i=\theta}^{\theta+\beta}h(n-i)=0 \tag{46}$$

and $h(0)=1$

Using Z-transform

$$h(n)=Z^{-1}[H(z)]=Z^{-1}\left[\frac{1}{1-Z^{-1}+K\sum_{i=\theta}^{\theta+\beta}Z^{-i}}\right] \tag{47}$$

The power-series expansion of H(Z) is the quotient of the long-division process. So given K, using long-division process yield the variation of $h(n)$ and which is plotted on graph by microconpucomputer. When the variation of $h(n)$ forms an oscillation with constant amplitude, (in numerical calculation 4 significant digit is equal) as shown in Fig. 4, the parameter's value of the system in this case are the critical

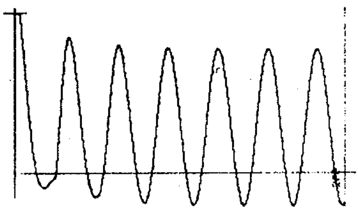

Fig. 4 The critical oscillation for PID control system with $\theta=5$, $\beta=20$

stable condition, and then K_{cr}, CT and AM can be obtained. The computer program of long-division is universal for characteristic equation (41), (42) and PID control system, but the amount of calculation work is comparative large, so more computer time is needed. In order to save computer time the recursive program is used to calculate $h(n)$. Using recursive program to make numerical calculation of American pork price model, i.e. given $\theta=2, \beta=4$ in eq. (41), the obtained result $K_{cr}=0.414$ and period equals to 8T, which fully coincides with analytical solution in section Ⅱ of this paper. The AM=0.415 can't be obtained in analytical solution. Using numerical calculation, 26 sets of θ, β for eq. (41) and 34 sets of θ, β for eq. (42) have been calculated and the critical parameter of stability boundary have been found out. For convenience in practical application, using multiple regression analysis program on microcomputer[5-6] the regression equation is found out from numerical calculating results. The regression equations are listed in table 1 and 2 with t statistics in parentheses. Statistical data indicate that there is significant evidence that CT, K_{cr}, AM and θ, β are related and the fit of these equations are quite good. Using equations listed in Table 1 to calculate American pork price model, substituting $\theta=2$ and $\beta=4$ into eq. (48), (49), (50), (51) gives CT=7.96214, K_{cr} = .46101, the positive amplitude AM^+ = .42569 and the negative amplitude AM^- = −.43356, which closely approximate to the direct numerical calculation result by recursive program. But using regression equations, the calculation is very simple.

TABLE 1 Regression equations of critical oscillation parameters for eq. (41)

regression equation	eq. No.	corretation coefficient R	F statistics	standard error σ
$CT=-.54982+.96149\beta+2.3330\theta$ (34.22) (18.93)	(48)	.9924	747.59	.9171
$K_{cr}=.85569-.14927\beta^{0.8}+.035938\beta^{1.2}-.065930\theta$ (−8.02) (7.14) (−11.94)	(49)	.9559	77.64	.041045
$AM^+=.91357-.22196\beta^{0.8}+.050786\beta^{1.2}-.041536\theta$ (−8.68) (7.34) (−5.48)	(50)	.9470	63.70	.056345
$AM^-=-.95044+.21380\beta^{0.8}-.053436\beta^{1.2}+.075396\theta$ (9.34) (−8.63) (11.11)	(51)	.9538	73.96	.050439

TABLE 2. Regression equation of critical oscillation parameters for eq. (42)

regression equation	eq. No.	corretation coefficient R	F statistics	standard error σ
$CT=-2.3896+2.0649\beta+4.2239\theta$ (360.6) (168.6)	(52)	.9999	72715.0	.22296
$lnK_{cr}=-.46823-.32166\beta+6.6212*10^{-3}\beta^2-.20797\theta$ (−16.59) (8.28) (−9.62)	(53)	.9982	415.9	.19167
$AM=.93051-.054127\,ln\beta+.10757\,ln\theta$ (−14.24) (19.37)	(54)	.9780	338.5	.018828

In order to study the variation of critical parameters for PID control system, the numerical calculation have been made for following characteristic equation

$$(1-\lambda^{-1})+\bar{K}(1+b_1\lambda^{-1}+b_2\lambda^{-2})\sum_{i=\theta}^{\theta+\beta}\lambda^{-i}=0 \qquad (48)$$

where

$$\bar{K}=K(1+K_i+K_d);$$

$$b_1=\frac{(1+2K_d)}{1+K_i+K_d};$$

$$b_2=K\frac{K_d}{1+K_i+K_d};$$

and

$$K_p=1$$

The calculating results are listed in Tab.3. Comparing the calculating results, the system's gain margin K_{cr} may increase 25% by appropriate choosing K_a and K_i. The time delay has strong destablizing effect, and when θ increases, the period of oscillation also increases and the life-time of capital has stablizing effect.

The data in Table 3 indicate that the gain margin K_{cr} of PID control system is the same order for the P control system, so the regression equations can also be used to calculate the critical parameters of PID control system for reference. If the PID

TABLE 3. Numerical calculating results for PID control system

No.	θ	β	K_p	K_d	K_i	b_1	b_2	K_{cr}	CT	AM'	AM⁻
1	5	20	1	0	0	−1	0	.1294	30	.7656	−.2269
2	5	20	1	.9	.01	−1.465969	.4712	.1294	damped oscillation		
3	5	20	1	.9	.05	−1.435898	.4615385	.1294	undamped oscillation		
4	5	20	1	1.4	.01	−1.576763	.5809128	.1294*1.25	28	.7226	−3.660
5	0	20	1	1.4	.01	−1.576763	.5809128	.1294*10	decay curve		
6	10	20	1	1.4	.01	−1.576763	.5809128	.1294*.632	37	.9100	−.2591
7	10	15	1	1.4	.01	−1.576763	.5809128	.1294*.7033	32	.9588	−.2230

control still can't satisfy the system's dynamic behaviour requirements, more coefficients a_i, b_j in eq. (33) may be chosen to form a new transfer function for achieving desired dynamic behaviour.

CONCLUSIONS

This paper is concerned with the analysis of dynamic behaviour for economic system with time delay and capital life-time, calculating results indicate the model proposed by authors is appropriate for this purpose. Introducing investment decision equation to improve the dynamic behaviour is satisfactory. This model offers clear physical concept and is convenient for numerical calculation, the microcomputer is a suitable tool for study such problem. The method proposed in this paper can be applied to analyze economic system's dynamic behaviour, for example, economic long wave, dynamic input-output analysis, price fluctuation, business cycle, etc. Using regression equations between the critical oscillation parameters and θ & β, the appropriate coefficients of investment decision equation can be selected to obtain desired dynamic behaviour. The model was described by ordinary linear difference equations which are time invariant. In the furture the stochastic variable and nonlinearity will be introduced in the model. The time-varying system will also be concerned. The state vector and state-space approach will be used to describe the new model.

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