Dynamic Scheduling of Flexible Manufacturing Systems

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ABSTRACT

Mid-volume, mid-variety operations characterize flexible manufacturing systems (FMS) or job-shops found in most factories. Profitability of FMS depends upon effective scheduling of material flow, machine use, staffing, and buffer capacities. Many systems adjust to changes in demand and equipment failure in the long term. In the short term, however, large changes may occur in inventories, staffing requirements, and machine utilization. In general, these large changes reduce production efficiency and profits. An approach is demonstrated for attenuating or eliminating changes or swings in a system when there occurs some abrupt change. Delays and delay parameters in the system model are adjusted, subject to practical constraints, to produce a smooth and rapid transition after the change. A simple econometric model is used for illustration. A symbolic and algebraic manipulation language is required to implement the approach.

INTRODUCTION

Mid-volume, mid-variety operations characterize flexible manufacturing systems (FMS) on job-shops found in most factories. Profitability of FMS depends upon effective scheduling of material flow, machine use, staffing, and buffer capacities. Many systems adjust to changes in demand and equipment failure in the long term. In the short term, however, large changes may occur in inventories, staffing requirements, and machine utilization. In general, these large changes reduce production efficiency and profits. An approach is demonstrated for attenuating or eliminating changes or swings in a system when there occurs some abrupt change. The linear fourth order distribution-sales model (Forrester, 1961, p. 396) was used to demonstrate the validity of this approach. The system is described generally by differential equations (approximated by difference equation) which incorporate inherent delays, such as pipeline or transportation delays, processing delays, and any general waiting. These delays have a lower bound but can be adjusted upward; some are similar to feedforward and feedback gains in automatic control system. In our approach, the continuous system differential equations are expressed in terms of transition matrices so that they can be put into discrete form. The entire system is z-transformed and the characteristic equation is obtained algebraically by the computer. A symbolic and algebraic manipulation language is used. The delays are adjusted to produce the smoothest and fastest transition when a change occurs. The roots of the characteristic equation are placed as close as possible to $0 < z < 1$.

THE MODEL

The model is the fourth order system of differential equations (Forrester, 1961, p. 396) where the differential equations are approximated by difference equations. We use similar nomenclature for the variables but with the more usual notation for difference equations.

$$IAR(t + \Delta t) = IAR(t) + \Delta t[UOD(t)/DUD - UOR(t)/DUR]$$

$$UOD(t + \Delta t) = UOD(t) + \Delta t[PDR(t) - UOD(t)/DUD]$$

$$UOR(t + \Delta t) = UOR(t)[120 - UOR(t)/DUR] \qquad (1)$$

$$RSR(t + \Delta t) = RSR(t) + \Delta t[(120 - RSR(t))/DRR]$$

$$PDR(t) = RSR(t)[1 + AIR/DIR] - IAR(t)/DIR$$

This is an illutrative example which describes the transient behavior of

IAR - Inventory At Retail
UOD - Unfilled Orders at the Distributor
UOR - Unfilled Orders at Retail
RSR - Averaged Weekly Sales
PDR - Purchasing Rate Decision

when a 20% step increase in product demand occurs. The desired transient behavior or time response is fast and orderly without swings in inventory or purchasing rate decisions. Oscillatory behavior occurs, however, for the delay parameters DIR, DUD, DRR, DUR selected by Forrester. AIR = 3 weeks for all cases.

METHODOLOGY

There are at least two very general approaches that can be used to reduce or eliminate this oscillatory behavior. One approach uses optimal control or Pontryagin's Minimum Principle to minimize these swings given discrete (i.e., 5% increments) purchasing rate decisions. The author has treated a similar problem in mechanical design with considerable success (Maday, 1978, p. 187). This approach, appropriate for large scale systems, will be treated in a subsequent paper. The second approach, more suited for smaller systems, uses concepts from classical control and modern control theory. It is described here.

From a feedback control point of view the dynamic system in equation (1) can be considered in several ways:

1) As a system of continuous differential equations where Δt is sufficiently small that Euler numerical integration can abe used for simulation. This is Forrester's approach. Simulations are made for delay parameters selected by Forrester.

2) As completely discrete system where Δt is not small. Solution methods for difference equations are used here. Delay parameters are determined from z-plane analysis. A general treatment can include different values of Δt for each state variable. A detailed analysis will be presented.

3) As a general sampled-data system consisting of differential equations and difference equations. One or more of state or decision variables may be discrete. This approach, which depends upon transition matrix representa-

tion of the continuous state variables, is the most interesting and the most difficult to implement. It, too, is described in detail.

Case 1 - Completely Continuous Representation Forrester solves the sytem of equations using Euler integration with 0.5 week computing intervals. That is, inventories, orders, and purchasing rate decisions are updated semi-weekly. The results of these and subsequent simulations are illustrated in Figures 1 and 2 for Inventory At Retail and Purchasing Rate Decision. This is Case 1, for which DUR = 1.0 week and DIR = DRR = DUD = 2 weeks.

Many systems can be treated appropriately this way but there are also cases where decisions are made at longer intervals or where supplies arrive (or depart) as almost discrete events. The completely continuous approach does not simulate these occurences accurately.

Case 2 - Completely Discrete Representation The system of governing equations may be regarded strictly as difference equations, i.e., they are not approximations to differential equations. The delay interval is not necessarily small compared to other time constants in the system. Different delays may be associated with each state variable. In this case, the delay parameters DUD, DIR, DRR, and DUR are constant but not restricted to Forrester's suggested values. They are regarded as adjustable upwards so that information is not transferred at a rate greater than in Forrester's simulation. This feature marks an important departure from other approaches because it means that information utilization is delayed deliberately until it is appropriately used. The flow of information is controlled.

The system of equations is z-transformed to give the characteristic equation

$$(z - 1 + \frac{\Delta t}{DUR})(z - 1 + \frac{\Delta t}{DRR})[z^2 + z(\frac{\Delta t}{DUD} - 2) + \frac{\Delta t}{DUD}\frac{\Delta t}{DIR} - \frac{\Delta t}{DUD} + 1] = 0 \qquad (2)$$

where it is possible to adjust the delay parameters and Δt to place the roots of this equation to obtain the required time response. This is generally referred to as pole assignment. Here the poles are assigned to z = 0 (in the z-plane) by setting

Δt = 4 weeks   DUR = DRR = 4 weeks
DUD = 2 weeks   DIR = 8 weeks

Simulation results are illustrated in Figures 1 and 2 as Case 2A.

As the first step in the recognition that there may be multiple time scales in the system (because some variables are continuous and others are discrete) simulations were made for the system operating with two values of Δt. The delay Δt for UOD (unfilled orders at the distributor) and IAR (inventory desired at retail) remains at 4 weeks. Two other delays for RSR (average sales/week) and UOR (unfilled orders at retail) are used; 1 1/3 weeks for Case 2B and 0.5 week for Case 2C. This is referred to as multi-rate sampling in sampled data systems. For a delay of 2 weeks for RSR and UOD (and 4 weeks for IAR and UOD) the solution converged to the wrong values. This suggests a phenomenon similar to the Nyquist frequency, i.e., one that is twice the highest frequency in the system.

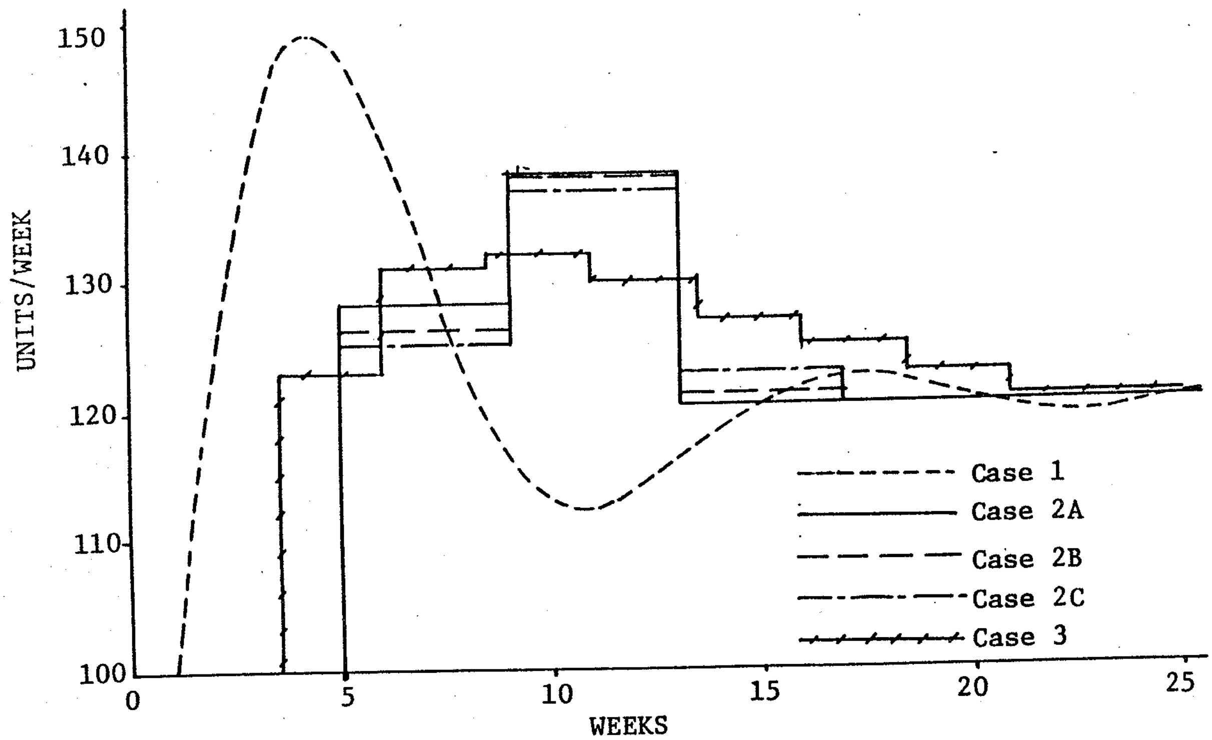

Figure 1. PURCHASING RATE DECISION - 20% Increase in Demand at 1 Week From 100 to 120 Units/week

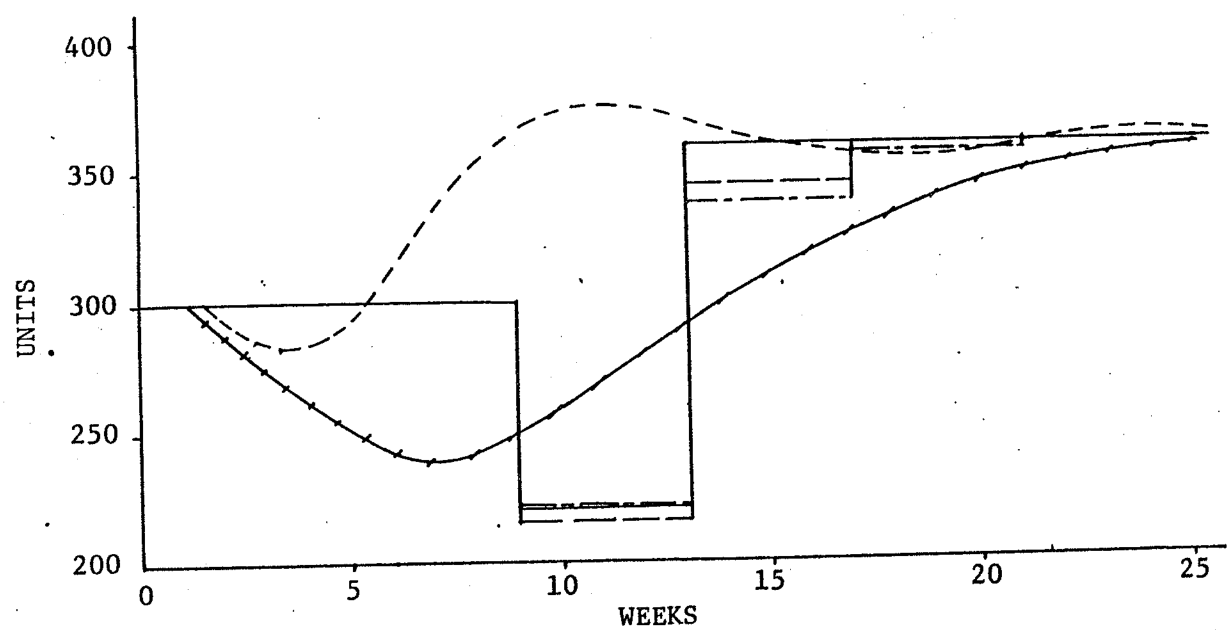

Figure 2. INVENTORY AT RETAIL - Inventory Required at Retail is Three Weeks Supply - 20% Increase in Demand at 1 Week From 100 to 120 Units/Week

Case 3 - Continuous/Discrete Representation It has been noted that some of the variables are primarily continuous while others are discrete in character. For the system of equations (1), the four state variables are considered continuous while the purchasing rate decision PDR is taken to be discrete and revised only at intervals T. The system can be represented in the state variable format

$$\dot{\underset{\sim}{x}} = \underset{\sim}{A}\underset{\sim}{x} + \underset{\sim}{B}\underset{\sim}{u} + \underset{\sim}{C}\underset{\sim}{r}$$

or

$$\begin{bmatrix} \dot{IAR} \\ \dot{UOD} \\ \dot{UOR} \\ \dot{RSR} \end{bmatrix} = \begin{bmatrix} 0 & DUD^{-1} & -DUR^{-1} & 0 \\ 0 & -DUD^{-1} & 0 & 0 \\ 0 & 0 & -DUR^{-1} & 0 \\ 0 & 0 & 0 & -DRR^{-1} \end{bmatrix} \begin{bmatrix} IAR \\ UOD \\ UOR \\ RSR \end{bmatrix} + \begin{bmatrix} 0 \\ PDR \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 120 \\ 120/DRR \end{bmatrix} \qquad (3)$$

where $\underset{\sim}{B} = \underset{\sim}{C} = \underset{\sim}{I}$ and PDR is piecewise constant. The delay parameters are to be selected to give smooth response without oscillations. Pole assignment, however, is hampered by the discrete/continuous character of the system. There is a question of whether to use the s-plane or the z-plane for pole assignment. This question can be resolved by representing the continuous part of the system by difference equations using transition matrices. This representation gives the exact values of the state at the sampling intervals T. It is analagous to using a flashing light to observe the system and use only those values when the light flashes. Thus, for an update or sampling interval T

$$\underset{\sim}{x}(t + T) = \underset{\sim}{\Phi}(T)x(t) + \underset{\sim}{\theta}\,(T)\,[u(t) + r(t)] \qquad (4)$$

where

$$\underset{\sim}{\Phi}(T) = L^{-1}[(s\underset{\sim}{I}-\underset{\sim}{A})^{-1}] = \begin{bmatrix} 1 & 1-\exp(-T/DUD) & -1 + \exp(-T/DUR) & 0 \\ 0 & \exp(-T/DUD) & 0 & 0 \\ 0 & 0 & \exp(-T/DUR) & 0 \\ 0 & 0 & 0 & \exp(-T/DRR) \end{bmatrix}$$

and

$$\underset{\sim}{\theta}\,(T) = \int_0^T \underset{\sim}{\Phi}(T - \tau)d\tau$$

This discrete form of the system equations, consistent with $\underset{\sim}{x}$ continuous and $\underset{\sim}{u}$ and $\underset{\sim}{r}$ discrete is

$$R(t + T) = IAR(t) + [1-\exp(-T/DUD)]UOD(t) - [1-\exp(-T/DUR)]UOR(t)$$
$$+ \{T-DUD[1-\exp(-T/DUD)]\}\{[AIR \cdot RSR(t) - IAR(t)]/DIR + RSR(t)\}$$
$$- 120\{T-DUR[1-\exp(-T/DUR)]\}$$
$$UOD(t + T) = [\exp(-T/DUD)]UOD(t) + \{T-DUD[1-\exp(-T/DUD)]\} \cdot \{[AIR \cdot RSR(t) - IAR(t)]/DIR + RSR(t)\} \quad (5)$$
$$UOR(t + T) = [\exp(-T/DUR)]UOR(t) + 120 \cdot DUR[1-\exp(-T/DUR)]$$
$$RSR(t + T) = [\exp(-T/DRR]RSR(t) + 120[1-\exp(-T/DRR)]$$

The system is z-transformed and produces the resulting characteristic equation

$$(z - a_1)(z - a_2)(z^2 + b_1 z + b_2) = 0 \quad (6)$$

where

$$a_1 = \exp(-T/DUR)$$
$$a_2 = \exp(-T/DRR)$$
$$b_1 = -1 + T/DIR - DUD/DIR + [\exp(-T/DUD)](DUD/DIR-1)$$
$$b_2 = \{T - DUD[1-\exp(-T/DUD)]\}[1-2 \exp(-T/DUD)]/DIR + \exp(-T/DUD)$$

The time response of the system can be tailored by pole assignment. Rapid, non-oscillatory behavior can be obtained by setting the poles along the real axis between $z = 0$ and $z = 1$, as close as possible to $z = 0$. The poles a_1 and a_2 are limited by the decision interval T and the delay parameters DUR and DRR. Repeated poles are associated with the coefficients b_1 and b_2. One acceptable set of parameters is

DUR = 1.000 weeks	DUD = 2.178 weeks	T = 2.5 weeks
DRR = 2.000 weeks	DIR = 8.652 weeks	

The update interval for purchasing rate decisions is 2.5 weeks. This corresponds approximately to $a_1 = 0.082$, $a_2 = 0.287$, $b_1 = 1.2$, $b_2 = .36$. Thus the repeated poles are at $z = 0.6$.

DISCUSSION

A linear fourth order system has been simulated numerically. Whether the system is regarded as discrete, continuous, or hybrid, the difference in the results is dramatic. An important conclusion that can be drawn from these results is that the transfer of information may have to be restricted in order to promote smooth and orderly transient behavior following some discrete event or disturbance.

Numerical simulations for Case 3 established the robustness of the system with respect to the update interval and the delay parameters. This is reasonable since the system poles are no greater than 0.6 which is sufficiently distant from the unit circle.

REFERENCES

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NOMENCLATURE

AIR - Proportionality constant between inventory and average sales at retail, weeks

DIR - Delay in inventory (and pipeline) adjustment at retail, weeks

DRR - Delay in smoothing requisitions at retail, weeks

DUD - Average delay in unfilled orders at Distributor, weeks

DUR - Average delay in unfilled orders at Retail, weeks