STRUCTURE-BEHAVIOR RELATIONS FOR CERTAIN NONLINEAR, SECOND-ORDER SYSTEMS

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Summary

An experiment was conducted using DYNAMO simulation to gain an understanding of the relation between the structure and behavior for a well-defined family of nonlinear, second-order systems. The result of the empirical investigation was 1) a taxonomy of structures--a categorization of the structures that give rise to all of the possible behavior modes; and 2) a set of observations and precepts--simply stated guidelines gleaned from the taxonomy that relate structure and behavior.

Structures Investigated

The infinite set of second-order structures is reduced to a well-defined finite subset by imposing the following restrictions:

1. Parameters and auxilliaries are collapsed into the rate equations;
2. Exogenous inputs, delays, and all special functions are excluded;
3. One net rate affects each level, and each level is initialized to zero.

These restrictions collapse the structures to two levels, two rates and the four equations that link each level to each rate (constituting the major loop and two minor loops). The structures become explicitly defined by restricting the four level-to-rate equations to the eight monotonic table functions, shown in figure 1 in "counterconcave" pairs.

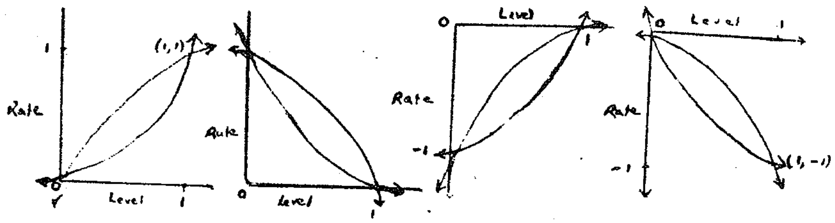

Fig. 1. Eight level-to-rate functions shown in counterconcave pairs

The rate equations are then formed by pairwise linear coupling of these tables functions. (The resulting rate equations are, however, neither monotonic nor linear.) Defined in this manner there are $\sum_{i=2}^{4} 8^i = 4,672$ structures.

Behavior Modes

The behaviors arising from both levels of all the structures are classified into behavior modes. These modes can be defined rigorously by the slope and curvature of the time-varying functions that describe the behaviors, and similar behaviors are defined as all those that belong to the same mode. Thus, for example, the two behavior curves of figure 2 are classified in the overshoot and decline behavior mode because both curves are first increasing and concave down, then decreasing and concave down, and finally decreasing and concave up.

Fig. 2. Example of similar overshoot and decline behaviors

With this classification scheme, all of the structures generate behaviors that fall into 3 modes of stable behavior: 1) Asymptotic (exponential growth and decline to a limit, and sigmoidal growth and decline); 2) overshoot and decline; and 3) oscillatory (sustained and damped); and 3 modes of unstable behavior: 1) Exponential growth and collapse, 2) Overshoot and collapse, and 3) Explosive oscillation.

A random sample of 412 or 9% of the defined structures and behaviors were classified by this taxonomy. Within each behavior mode, the taxonomy was subdivided according to whether the structure contained zero, one, or two minor loops. If one or both of the levels exhibited a particular behavior, the structure was classified by that behavior mode. When each of the two levels exhibited different behavior modes, then the structure was included in the taxonomy twice.

Results

From this classification procedure eight precepts were developed which held for all the structures observed; however, the validity of the precepts is assured only for these structures. The utility of these precepts, then, is not as a substitute for computer simulation to analyze models, but rather as a means for students and practitioners of systems dynamics to develop an intuitive understanding of the underlying forces that drive simple, nonlinear systems.

Second-Order Systems without Minor Loops

Precept 1: The system oscillates when the major loop is negative and the zero and second derivatives of each level-to-rate function are opposite in sign. Only sustained oscillation is possible, and it occurs in both levels simultaneously.

Precept 2: Systems in this category achieve stability only through simple asymptotic growth.

Precept 3: A structure that exhibits growth to a limit in one of its levels maintains that behavior mode when the level-to-rate function of that level only is replaced by its counterconcave function.

Second-Order Systems with One or Two Minor Loops

Precept 4: A system with one minor loop oscillates when the major loop is negative and the zero and second derivative of the positive level-to-rate function of this loop are of opposite sign. This oscillation is damped (in both levels simultaneously) if the minor loop is negative and the level-to-rate function of this loop is concave up. The oscillation explodes if the minor loop is positive and its level-to-rate function is concave down. Sustained oscillation can never occur.

Precept 5: For systems with one minor loop, the steady-state behavior of a level that initially overshoots depends on the polarity of the minor loop: if the minor loop is negative, the overshoot in that level is followed by decline; if the minor loops is positive, the overshoot is followed by collapse. In either case, if both levels overshoot, they then either both decline or both collapse.

Precept 6: If one level of a system with two minor loops exhibits damped oscillation, the other level always exhibits a stable behavior mode.

Precept 7: Exponential growth is not limited to structures with positive major loops; growth to a limit is not limited to structures with negative major loops.

Precept 8: In structures with two minor loops, sustained oscillation can occur only if the two level-to-rate functions affecting each rate are counterconcave to each other and of the same curvature.