SYSTEM DYNAMICS AND INPUT-OUTPUT ANALYSIS

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Abstract

Input-output analysis for an "open" system relates production rates for various sectors of an economy to stipulated final demands. However, it is well known that the conventional dynamic analysis usually does not yield results which approach smoothly to those of the static analysis. In this work, the dynamic analysis is cast into the form of a system dynamics model. A modification of the rule which governs sector production rates is introduced so that stable results are obtained which do approach those of the usual static input-output analysis. The system equations are further modified to incorporate time-lagged stock indices and damping in the production rate rule. Prices are handled throughout as in conventional input-output analysis.

Introduction

Input-output analysis, as formulated by Wassily Leontief [1], is a valuable tool in the analysis of economic systems. In an "open" system, the production rates for the sectors of an economy may be determined in terms of the stipulated exogenous final demands for the products. A set of first-order, differential equations expresses the conservation of goods. Goods produced by a particular sector go to: (1) inputs needed by the productive sectors for production purposes, (2) stocks of goods held by the productive sectors, and (3) the stipulated final demands for the goods:

$$-B\dot{Y} + (1-A)\ Y = X \qquad\qquad (\text{and } Y_N = A1\ Y)\ . \qquad (1)$$

The economy is resolved into N sectors, with sector N the "labor" or "final demand" sector; this sector furnishes the labor utilized by the other sectors and consumes the final products. The production rates for the N-1 productive sectors are given by the column vector Y, with $\dot{Y}$ denoting the time derivatives of the production rates. Y_N is the rate at which labor is supplied by sector N. The exogenous final demands are the N-1 elements of the column vector X. The r-s element of the structural matrix A is the ratio of the input of goods furnished to sector s by sector r to the output of goods produced by sector s. A1 is a row vector, with N-1 elements, that is defined similarly except it is a measure of the labor supplied to sector s by sector N. The r-s element of the stock coefficient matrix B is the ratio of the stock of goods produced by sector r being held by sector s to the output of sector s.

Another set of conservation equations applies to the values of the goods. Money received by one sector for its output must balance payments made by that sector to other sectors for inputs plus payments of wages to labor,

$$P = P_N\ (1-A^{*}-z\ B^{*})^{-1}\ A1^{*}\ . \qquad (2)$$

The prices for goods produced by the productive sectors are given by the N-1 elements of the column vector P, P_N is the exogenously stipulated wage rate, and z is a diagonal matrix with elements $z_{rr}=\dot{Y}_r/Y_r$. The notation E^* denotes the transpose of the matrix, or column vector, E; E^{-1} denotes the inverse of the square matrix E.

In this exposition, many simplifications are made in the interest of improved clarity of the essential points. It is assumed that the labor sector does not itself employ labor or accumulate stocks, and stock coefficients will be assumed time independent. Often, the simplifications can be removed without damage to the analysis, albeit with some added complexity in the formulation.

The static solutions to these equations, corresponding to $\dot{Y}=0$, have probably proved of most utility in the analysis of economic systems. For the static case

$$Y = (1-A)^{-1} X \qquad \text{(and } Y_N = A1\ Y\text{)} \tag{3a}$$

$$P = P_N\ (1-A^*)^{-1}\ A1^*\ . \tag{3b}$$

It is well known that solutions to the dynamic equations (1) are likely to diverge [2]. Hence, the dynamic analysis is usually not helpful either in the determination of the existence of an equilibrium state, to which the static solutions would apply, or in tracing the time evolution of a system from arbitrarily stipulated initial conditions. The dynamic equations do demonstrate the existence of an unstable equilibrium; viz. if the initial state coincides with the static solutions, the system remains in that state. Of course, a perturbation of the system would upset the equilibrium.

In view of the generality of the conceptual nature of input-output analysis and its close relationship with conventional economics parameters, it is an attractive goal to cast the analysis into system dynamics form, as developed by Jay Forrester [3]. The intent is to do this with as little disturbance of the conventional input-output analysis as possible. The resultant, basic system dynamics formulation can then be used as the point of departure for such refinements as are indicated for modelling a specific system. It may prove advantageous to view the same system from the points of view of both methodologies, with the expectation that some features of the system may be more transparent in one formulation, while other features may be clearer in the other formulation.

Modified Input-Output Analysis

In order to achieve a satisfactory system dynamics formulation of the input-output analysis, two matters must be attended to. The dynamical equations (1) are now phrased in terms of production rates, rather than proper system levels, and the equations should have stable solutions which approach the equilibrium state described by the static equations (3). The former matter can be handled through a change in notation, which introduces the stocks as the appropriate system levels, while the latter matter will necessitate a change in the structure of the system.

In the conventional formulation, the stock of goods "r" (i.e. goods produced by sector r) held by sector s, S_{rs}, is related to the production rate of sector s according to $S_{rs}=B_{rs}Y_s$. The further development of this modified analysis is facilitated by introduction of dimensionless stock indices for the N-1 productive sectors, denoted as the elements of the column vector S. New stock coefficients, which play the role of the previous B coefficients, but which are not equal to them, are introduced according to

$$S_{rs} = b_{rs} S_s . \qquad (4)$$

The stock indices will have a "desired" value, which may be chosen arbitrarily, and these desired values of S are denoted by the column vector R. For example, one may choose all elements of R equal to unity. The choice of R affects the values of the b coefficients according to

$$S_{rs}(\text{desired}) = b_{rs} R_s . \qquad (5)$$

A connection between the conventional B and the new b coefficients may be useful. To do this, we can associate the desired stock levels with the equilibrium state and express the B coefficients as $S_{rs}(\text{desired})=B_{rs}Y^e_s$, where Y^e is the solution of the static equations (3). Then, one has $b_{rs}=B_{rs}Y^e_s/R_s$, and also $b_{rs}/b_{r's}=B_{rs}/B_{r's}$.

Although the notation now departs from the usual input-output nomenclature, the substance of the analysis has undergone no change. In order to clarify the problem of stability of the solutions, it can prove helpful to examine a trivial, conventional dynamic input-output model in which all sectors are uncoupled from one another; this can be accomplished by making the A and B matrices diagonal. The influence diagram for a typical productive sector is shown in Fig. 1, with attention given to the upper quantities on the diagram. The control rule gives positive feedback, and solutions of the equations may be expected to diverge exponentially with time.

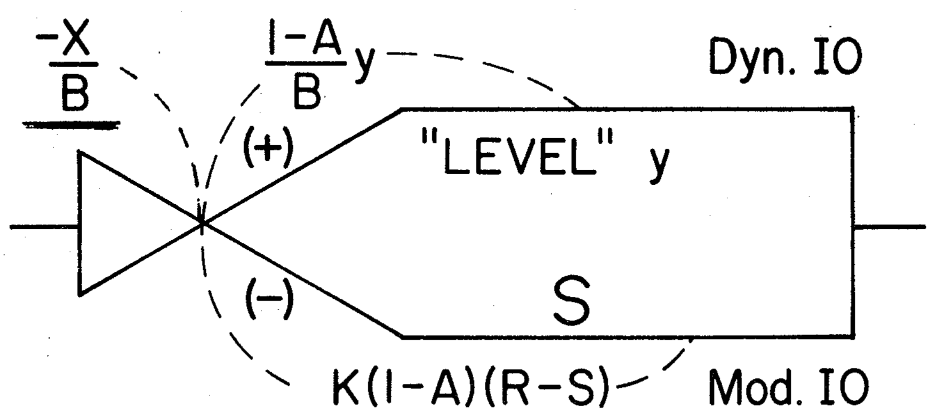

Fig. 1. Influence diagrams for model with uncoupled production sectors. Quantities on the upper part of the diagram refer to conventional input-output analysis. Quantities on the lower part of the diagram refer to the modified analysis.

A modified control rule will be introduced. The sense of the modified rule is that the production rate for a sector has a norm, which is attuned to what would be equilibrium conditions if that equilibrium currently existed, and then adjusted to departures of the various stock levels from their desired values:

$$Y = Y^e + b\ K\ (R-S)\ . \qquad (6)$$

The elements of the diagonal matrix K are the "recovery constants" for the proportional control rule. The intent is to introduce a simple, reasonable control rule and regard this rule not as an inviolable component of the modified formulation but as a base point for whatever rule seems appropriate to the specific system under study. The corresponding influence diagram is depicted in Fig. 1, with attention to the lower quantities on the diagram. The feedback is now negative, and solutions for the uncoupled case are given by

$$S_r = R_r + \text{constant EXP}[-K_{rr}(1-A_{rr})t]\ , \qquad (7)$$

where t denotes the time variable. A smooth approach to the equilibrium state, where S=R, occurs.

The change in notation to the stock indices S and the introduction of an essential modification in the production rate control rule yield the system equations

$$\dot{S} = b^{-1}\ (1-A)\ b\ K\ (R-S) \qquad (8a)$$

$$Y_N = A1\ Y \qquad (8b)$$

$$P = P_N\ (1-A^*-Z\ b^*)^{-1}\ A1^*\ . \qquad (8c)$$

Z is a diagonal matrix with elements $Z_{rr}=\dot{S}_r/Y_r$. At equilibrium, R=S and $Y=Y^e$, where Y^e has been stipulated as the solution to the static equations, viz. $Y^e=(1-A)^{-1}X$.

Another, redundant price equation can be derived. Its validity depends upon the validity of the other system equations. Although the equation yields no new information, it can be helpful as a check on internal consistency of solutions during a computational procedure,

$$P^{*} X - P_N Y_N = 0 . \tag{9}$$

Lagged Stock Indices and Damping

It is straightforward to incorporate some added features into the basic system dynamics formulation. As it stands, the analysis can accommodate certain parameters which depend on values of other system variables. The structural matrices A and A1, the wage rate P_N, the final demands X, and the stock recovery constants K may be related to other variables without alteration of the system equations. Additional terms must be incorporated in some of the equations in order to accommodate other changes, including utilization of labor by the labor sector, the holding of stocks of goods by the labor sector, and time variation of the stock coefficients b.

Often, it is deemed necessary to incorporate time-lagged variables into a model in order to account for inevitable delays in the gathering of data, the making of decisions, and the implementation of the decisions. The inclusion of delays can have a profound effect on the qualitative behavior of the model. A model which evidences good stability in the absence of delays may demonstrate an oscillatory behavior when delays are included. Because of the expected important role of delays in model behavior, some attention will be given here to their inclusion in the basic system dynamics formulation.

The control rule for sector production rates can be altered to include time-lagged values of the stock indices, denoted as the column vector S'. These lagged values are related to the stock indices S according to a first-order exponential delay. Furthermore, in order to provide a mechanism that can be adjusted to cope with the expected oscillatory tendency, derivative feedback, or damping, will also be incorporated into the control rule [4, 5, 6]. The new control rule, to replace Eq. (6), is then

$$Y = Y^e + b\ K\ (R-S') - b\ D\ \dot{S}' \ . \tag{10}$$

The time-lagged stock indices obey

$$\dot{S}' = L\ (S-S') \ . \tag{11}$$

The elements of the diagonal matrix D are the damping coefficients, and the elements of the diagonal matrix L are the delay factors, which govern the lag of the S' indices relative to the prompt stock indices, S. The first of the system equation (8a) is now replaced by

$$\dot{S} = b^{-1}\ (1-A)\ [b\ K\ (R-S') - b\ D\ \dot{S}'] \ . \tag{12}$$

In addition to variables and parameters common to both the conventional and modified onput-output analyses, the additional parameters K (stock recovery constants), L (delay factors), and D (damping coefficients) have been introduced. These additional parameters are rather removed from the usual economics data. The effects these additional parameters are intended to simulate must be ascribed to a complex series of decisions and actions. Accordingly, it will probably be difficult to empirically determine values in the same sense as one can do for the A and B coefficients. A combination of a feeling for the working of an actual system coupled with test studies of the simulation model will probably be necessary in order to arrive at reasonable values.

Model Behavior

In order to provide examples of characteristic behavior of the basic system dynamics formulation of the input-output analysis, a simple, test system of three coupled sectors, two productive sectors and the labor or final demand sector, has been studied. All parameters are held constant.

The system was coded as a conventional system-dynamics computer algorithm, although in this most simple form, analytic solutions are possible. The

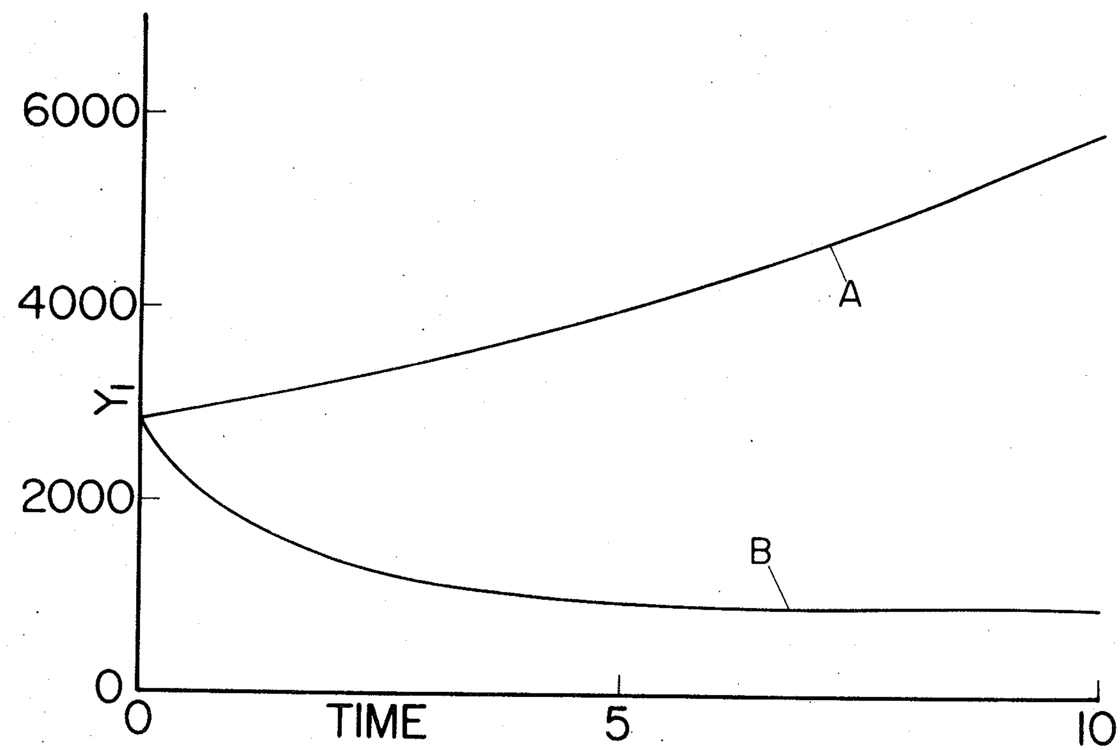

Fig. 2. Behavior of production rate of sector #1 for a test model. Curve A is for a conventional dynamic input-output model, and curve B is for the modified model.

behavior of a conventional dynamic input-output model and the modified model are compared in Fig. 2, for a case where the initial stock levels are lower than the desired values. As suggested by the previous consideration of the model with uncoupled sectors, the conventional model diverges from and the modified model converges smoothly to the equilibrium state.

In Fig. 3, the effects of the time-lagged stock indices and damping are demonstrated. Curve A was obtained from a model which included time-lagged stock indices but with the L parameters set to large values so that the behavior is similar to the simpler model with neither delay nor damping.

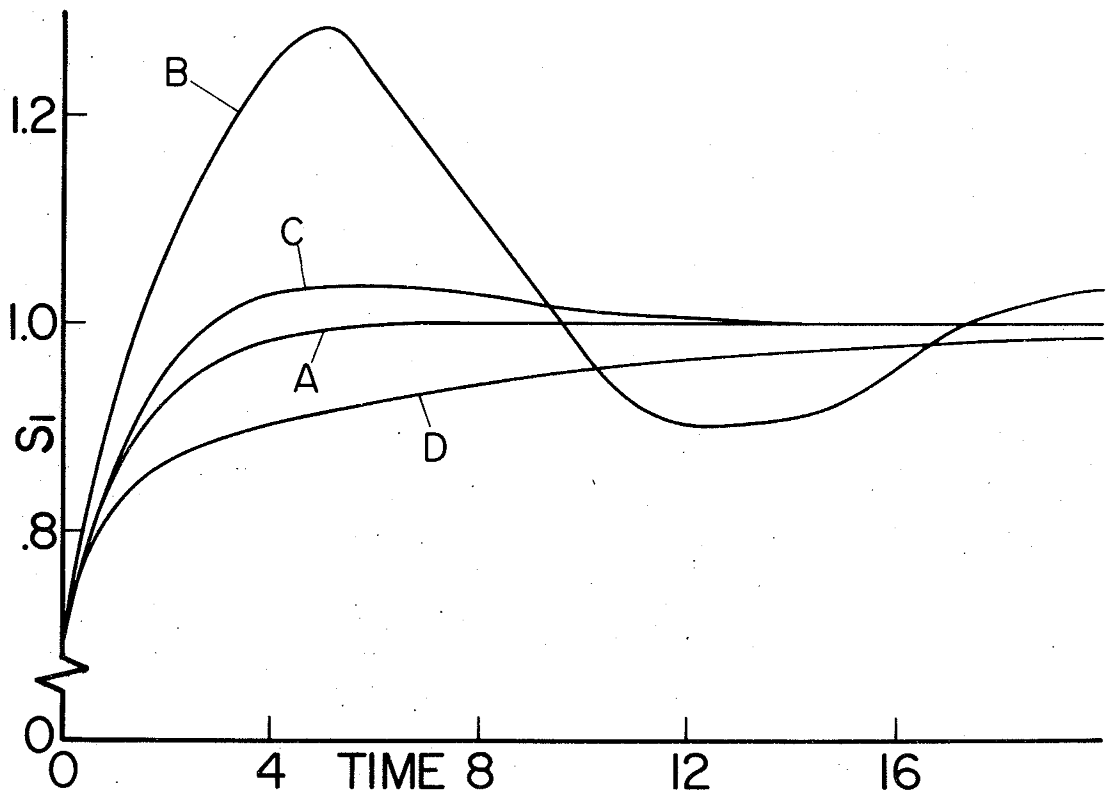

Fig. 3. Behavior of the stock index for sector #1 for the modified model with inclusion of time-lagged stock indices and damping. In curve A, the effects of delay and damping are negligible. Curve B includes delay effects but no damping. Curves C and D include both delay effects and damping.

In curve B, delay effects lead to oscillation, and the damping coefficients are set to zero. The heavily damped nature of the oscillations is due to the presence of "natural" damping in the system equations, i.e. terms which persist when D=0. In curve C, enough damping has been included to largely control the oscillatory tendency. More damping is added in curve D.

Concluding Remarks

An input-output problem can be converted into an equivalent system dynamics model. The model tends to be quite stable, provided a modified but reasonable production control rule is substituted for the usual rule of conventional dynamic input-output analysis. With the model in system dynamics format, additional features that have proven useful in the modelling of specific systems can be incorporated as desired. The system equations have been provided for inclusion of time-lagged stock variables and damping in the production rate control rule.

References

[1] Wassily Leontief, Input-Output Economics (Oxford University Press, New York, 1966).

[2] R. G. D. Allen, Mathematical Economics (Macmillan, London, 2nd edition 1959).

[3] Jay Forrester, Industrial Dynamics (M.I.T. Press, Cambridge, Massachusetts, 1961).

[4] Doug Tengdin, "P-I-D Control in System Dynamics Models," Plexus-System Dynamics News, Vol. 1, pp. 8-11, November 1980.

[5] Charles Braden, "Decision Procedure to Minimize Marginal Production Cost in a System Dynamics Model," Dynamica, Vol. 5, pp. 24-34, Autumn 1978.

[6] R. G. Coyle, Management System Dynamics (John Wiley & Sons, London, 1977).

FOR SIMULATION MODELS

attachment to:
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1. ACCESS TO MODEL:

Name of Model: INPUT-OUTPUT III

Name and current address of the senior technical person responsible for the model's construction: Charles Braden, School of Physics Ga. Inst. of Technology, Atlanta, Ga. 30332

Who funded the model development? institution (Ga. Tech)

In what language is the program written? BASIC

On what computer system is the model currently implemented? CDC Cyber 74/6400

What is the maximum memory required to store and execute the program? about $35{,}000_8$ (10 levels, depends only slightly on # of levels) "loader" not included

What is the length of time required for one typical run of the model? about 0.4 sec/iterative "step" for 10 levels (strong dependence on # of levels); 400 sec for 10 year run

Is there a detailed user's manual for the model? no

2. PURPOSE OF THE MODEL:

For what individual or institution was the model designed? Charles Braden

What were the basic variables included in the model?

levels are stocks of goods held by N sectors of an economic system
sector #N is labor (or final demand)

Over what time period is the model supposed to provide useful information on real world behavior? 1 - 50 years

Was the model intended to serve as the basis of:

an academic exercise designed to test the implications of a set of assumptions or to see if a specific theory would explain historical behavior ______

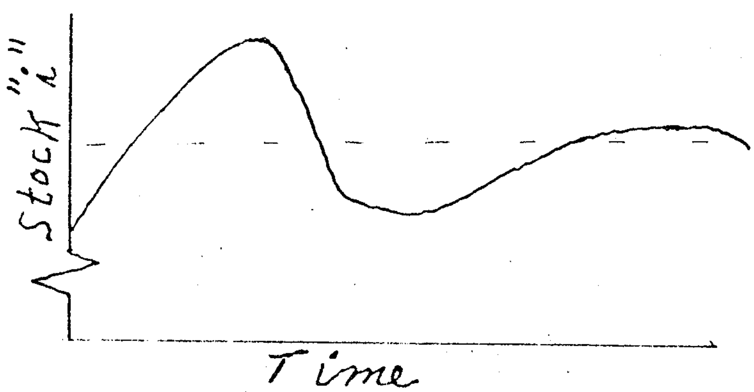
communication with others about the nature and implications of an important set of interactions, exemplify methodology for conversion of input-output model into system dynamics model x

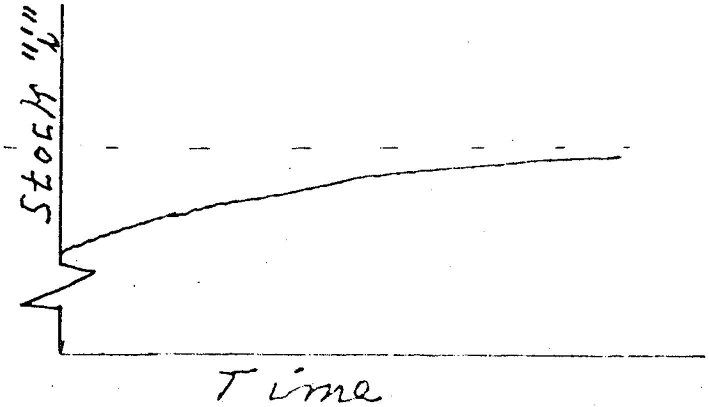
projecting the general behavioral tendencies of the real system ______

predicting the value of some system element(s) at some future point in time ______

3. MODEL SPECIFICATION AND THEORETICAL JUSTIFICATION:

Provide two diagrams illustrating the extreme behavior modes exhibited by the major model elements:





If they are not included in the body of the paper indicate where the reader may find:

a model boundary diagram that indicates the important endogenous, exogenous and excluded variables — noted in paper

a causal influence diagram, a flow diagram, the computer program and definitions of the program elements — available, in part, upon request

Is the model composed of:

simultaneous equations x

difference or differential equations x

procedural instructions ____

Is the model deterministic x or stochastic ____

continuous x or discrete ____

4. DATA ACQUISITION

What were the primary sources for the data and theories incorporated in the model?

Data only exemplary, test data run to date

Theory Leontief dynamic input-output analysis for "open" system, conventional system dynamics methodology

What percent of the coefficients of the model were obtained from:

measurements of physical systems ____

inference from social survey data ____

econometric analyses ____

expert judgment ____

the analyst's intuition ____

What was the general quality of the data? exemplary, test data only

5. PARAMETER ESTIMATION n.a. (test data only)

If they are not given in the publication, where may the reader obtain detailed information on the data transformations, statistical techniques, data acquisition procedures, and results of the tests of fit and significance used in building and analyzing the model? ____

6. MODEL PERFORMANCE AND TESTING n.a. (test data only)

Over what period was the model's behavior compared with historical data? ____

What other tests were employed to gauge the confidence deserved by the model? ____

Where may the reader obtain a detailed discussion of the prediction errors and the dynamic properties of the model? ______

______

7. APPLICATIONS

What other reports are based upon the model? ______

______

Name any analysts outside the parent group that have implemented the model on another computer system. ______

______

List any reports or publications that may have resulted from an evaluation of the model by an outside source. ______

______

Has any decision maker responded to the recommendations derived from the model? ______

______

Will there be any further modifications or documentation of the model? ______

Where may information on these be obtained? ______

XX modifications planned in pricing structure of model (currently, conventional input-output analysis is used); a more detailed description, with additional exemplary runs, is then planned