Role of Stock constraint in single-stock single-inflow two-outflow System Dynamics model structures

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Abstract

The building blocks of a System Dynamics (SD) model are the stocks and its associated flows. A single-stock model with one inflow and two outflows is one of the many structures used in SD modelling. Typically, a stock assumes homogeneity of its contents in all aspects. However, there are instances of single stock models where there are qualitative differences between the two outflows, for example, Recovered population vs Deaths. This paper analyses validity of such model structures and shows that these model representations are incorrect. Outflows in these cases do not apply a stock constraint and mixes entities of two different qualities in the outflows. The model also underestimates the actual stock value. The study uses an Agent Based Model to arrive at the above conclusions.

1. Introduction

The building blocks of a System Dynamics (SD) model are the stocks and its associated flows. Additional objects namely variables, lookup functions and constants are used to regulate the flows in and out of the stocks.

Arrangement of stocks and flows and other variables in a model is dependent upon the modelling objectives. This paper covers one such SD structure where there is a single stock with one inflow and two outflows. This generic structure is shown below:

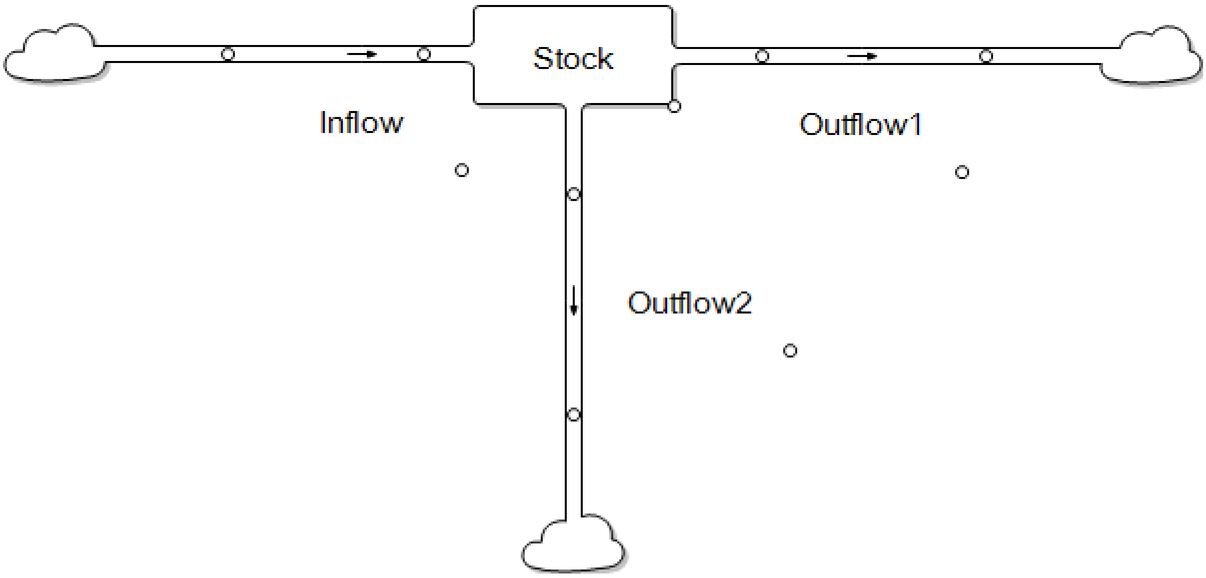

Fig 1: Generic Single-Stock Single-Inflow Two-Outflow model

A recent example the author has come across of this structure was within a Covid pandemic model. In this model, inflow represented new infections, outflow # 1 represented Recoveries and outflow # 2 represented Deaths.

In SD, the stock represents an aggregate of elements such as water or people or products. All elements within the stock are therefore assumed to possess the same properties except for the actual time it takes to clear the stock. For example, if a stock represents a population, then all entities (people) within the stock are assumed to have uniform qualities. Even if there are qualitative differences, say like language used by people, this difference is omitted when they are represented by a single stock. The modeler therefore does not consider language as a characteristic significant enough to have a material impact on the model outcome. Additionally, information about the order in which entities enter the stock is also lost. All entities are perfectly mixed. If one or more outflows are provided for the stock, then any entity within the stock can take any one of the two outflow channels.

This brings into question the validity of policies developed based on the above structure where stock outflows are decided based on a certain attribute of the stock contents. Is it correct to generate 'Recoveries' and 'Deaths' outflows from a single stock? This paper attempts to determine validity of such a model structure.

2. Time Delay or 'fraction-of-stock' based outflows

The Time Delay or 'fraction-of-stock' based outflow policy is one in which the outflows from a stock are determined based on a certain fractional outflow rate. The below diagram shows the policy applied to model infections.

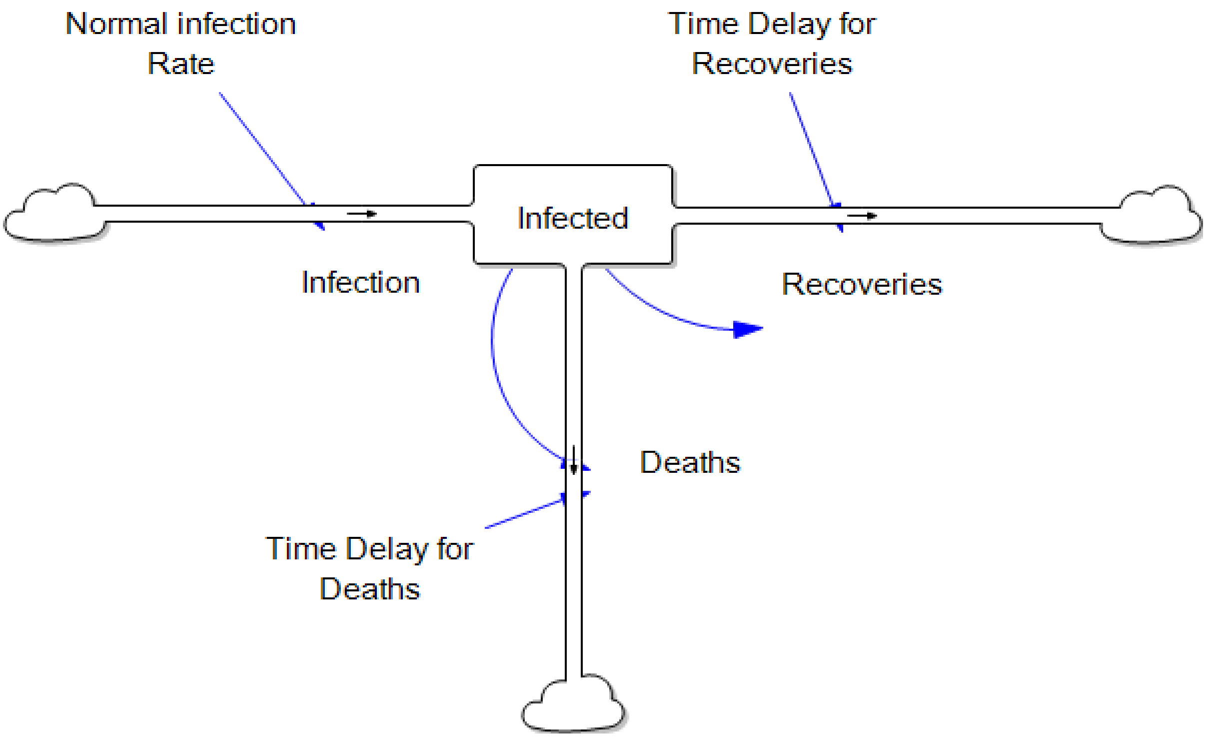

Fig 2: Model 1 - Single-Stock Single-Inflow Two-Outflow model with outflows determined by independent Time Delays

New infections occur at a certain fixed rate. New infections flow into the 'Infected' stock. Some of the infected people die while the remaining recover according to the time delay for death and recoveries, respectively. This model is simulated based on following settings:

Variable / Parameter	Value / Equation	Variable / Parameter	Value / Equation	Variable / Parameter	Value / Equation

Time Unit:	Day	Time Step	0.125	Final Time	100
Normal Infection Rate (People/Day)	100	Time Delay for Recoveries (Day)	5	Time Delay for Deaths (Day)	20
Infected (People)	Initial Value = 0	Infection (People/Day)	= Normal Infection Rate	Recoveries (People/Day)	=Infected / Time Delay for Recoveries
Deaths (People/Day)	=Infected / Time Delay for Deaths				

Table 1: Model 1 Settings

The equations show that outflows are given by the following general equation:

Outflow = Stock / Time Delay

But 1 / Time Delay is nothing but fractional outflow rate. This is the reason for giving this section's heading as Time Delay or 'fraction-of-stock' based outflow.

The output of the simulation is shown in the below graph:

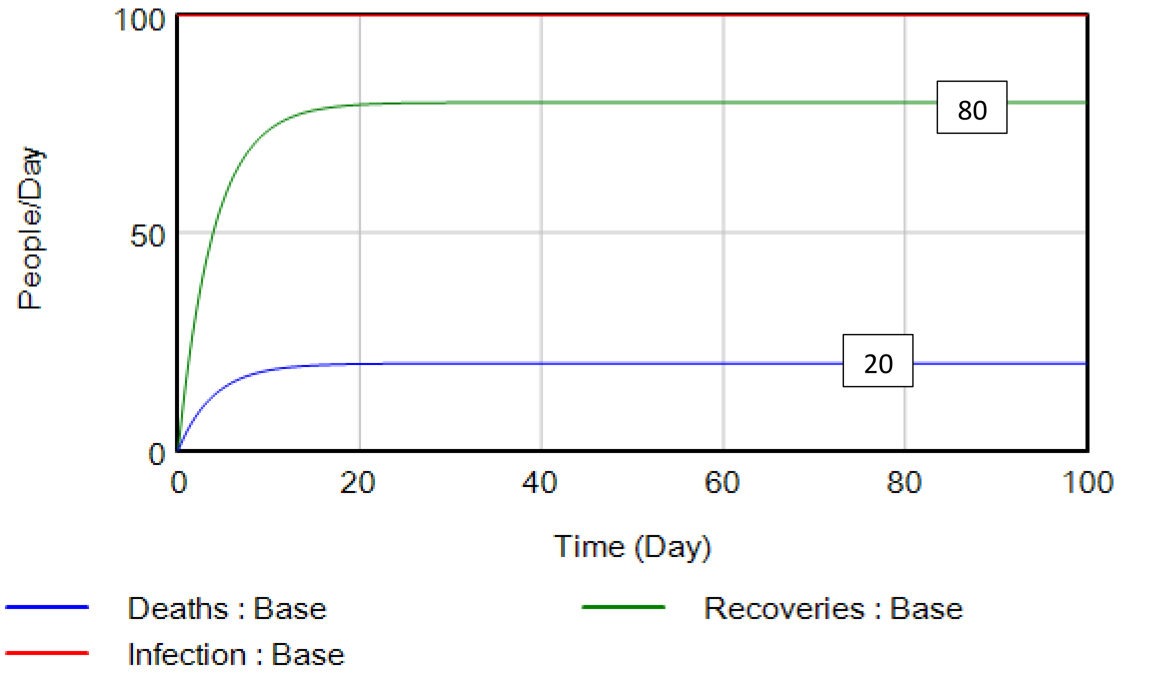

Fig 3: Model 1 outflows

In steady state, sum of outflows must match inflow. Therefore, based on the above graph, 80% of inflows constitute people who will eventually recover and 20% constitute people who will die. Now assume the same inflow but with higher time to death for example, say due to better ICU facilities. Let this new time delay for death be 25. The new outcome from this model is shown below:

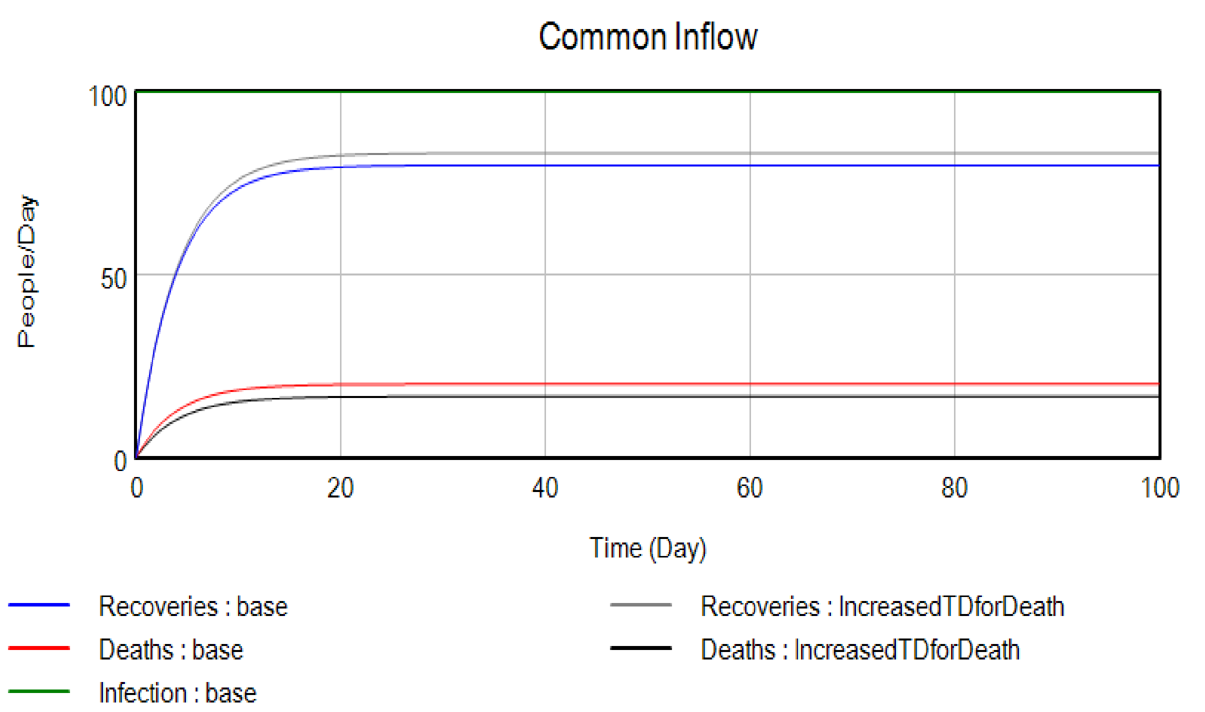

Fig 4: Model 1 outflows with two settings for Time Delay for Death

With a higher time delay for deaths, the steady state deaths has decreased and steady state recoveries has increased compared to the base case.

However, this is not possible for a given proportion of serious cases and recovery cases in the inflow. This aspect can be shown by splitting the inflow into two as shown in the below model:

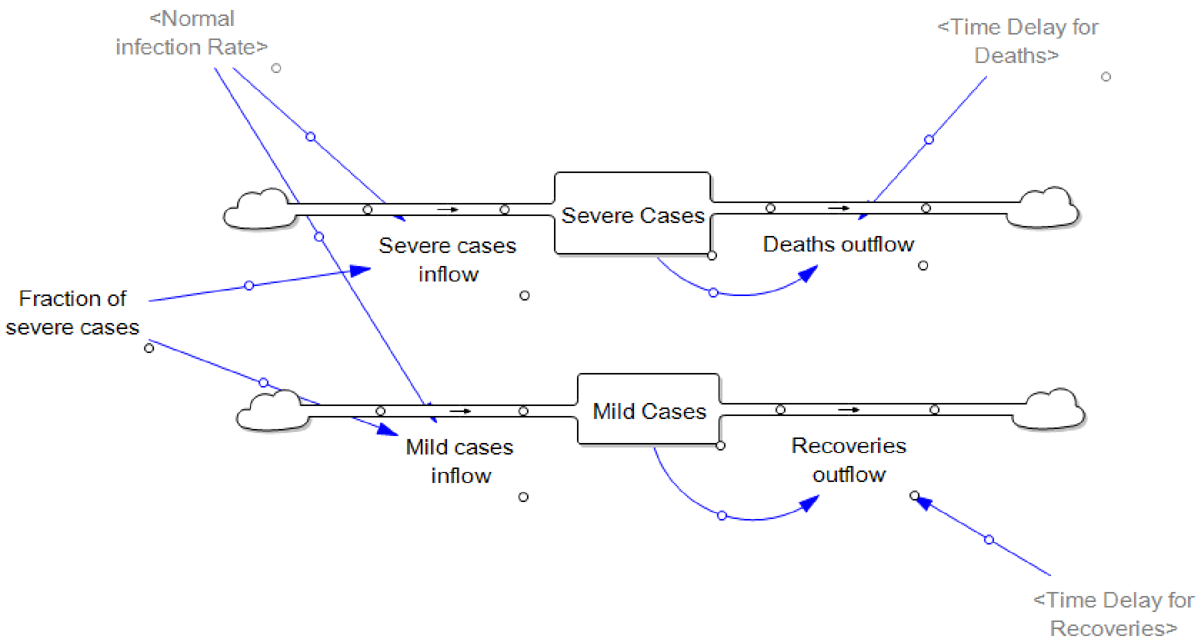

Fig 5: Model 2 – Two Stock model

The fraction of severe cases is taken as 0.2 so that the steady state inflow is 80 mild cases per day and 20 severe cases per day, same as the base case scenario discussed above.

The inflows and outflows of this model are given below:

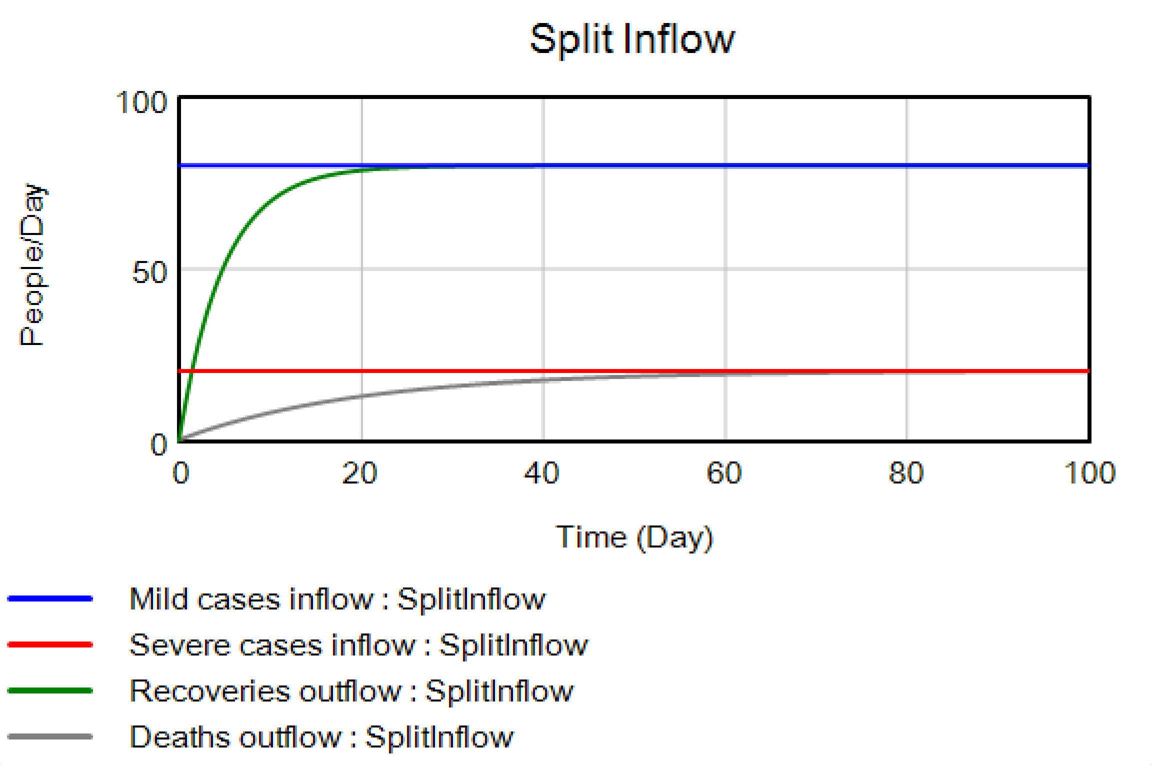

Fig 6: Model 2 outflows

Now, the time for deaths is increased from 20 days to 25 days. The result is as shown below:

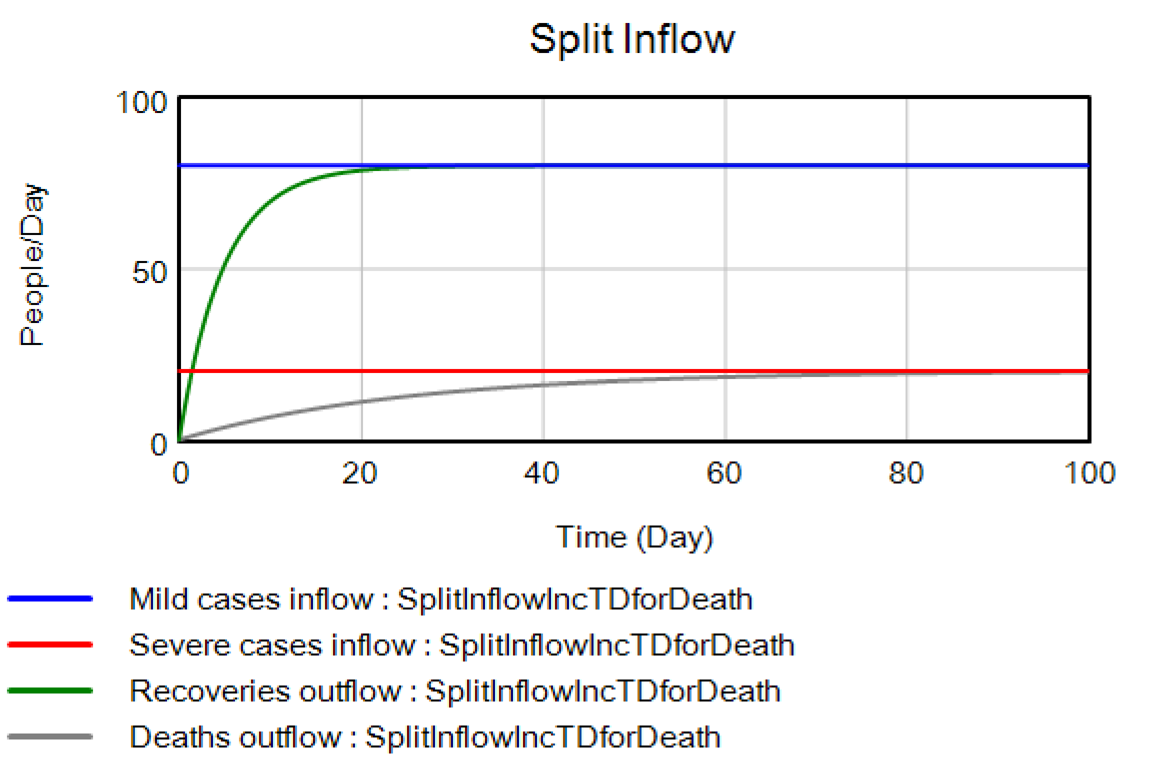

Fig 7 : Model 2 outflows with two settings for Time Delay for Death

The steady state recoveries and deaths remain the same even with increased time delay for deaths. This is in contrast with the original model where the flow rates varied based on variation in time delay.

<u>Comparison of Stocks</u>

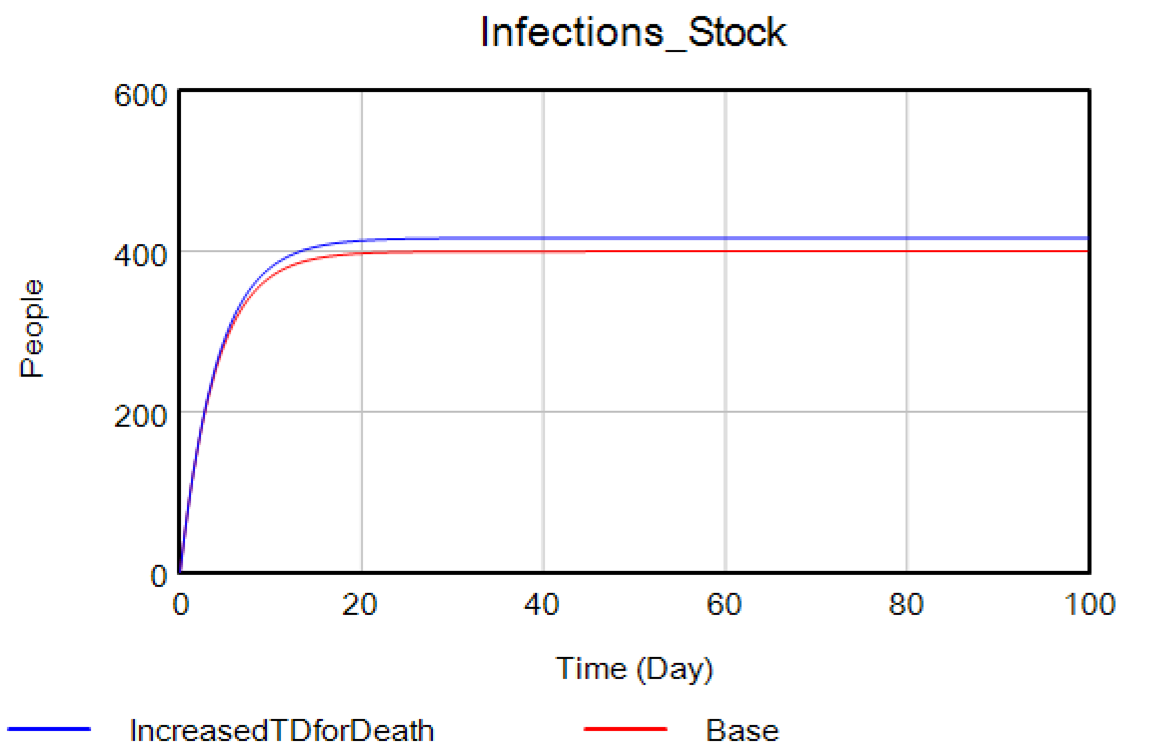

Fig 8: Model 1 Stocks

With a single stock of infected population, the steady state value of total infected cases is 400. This includes both mild cases and severe cases. This value increases slightly above 400 when the time delay for death increases from 20 to 25 days.

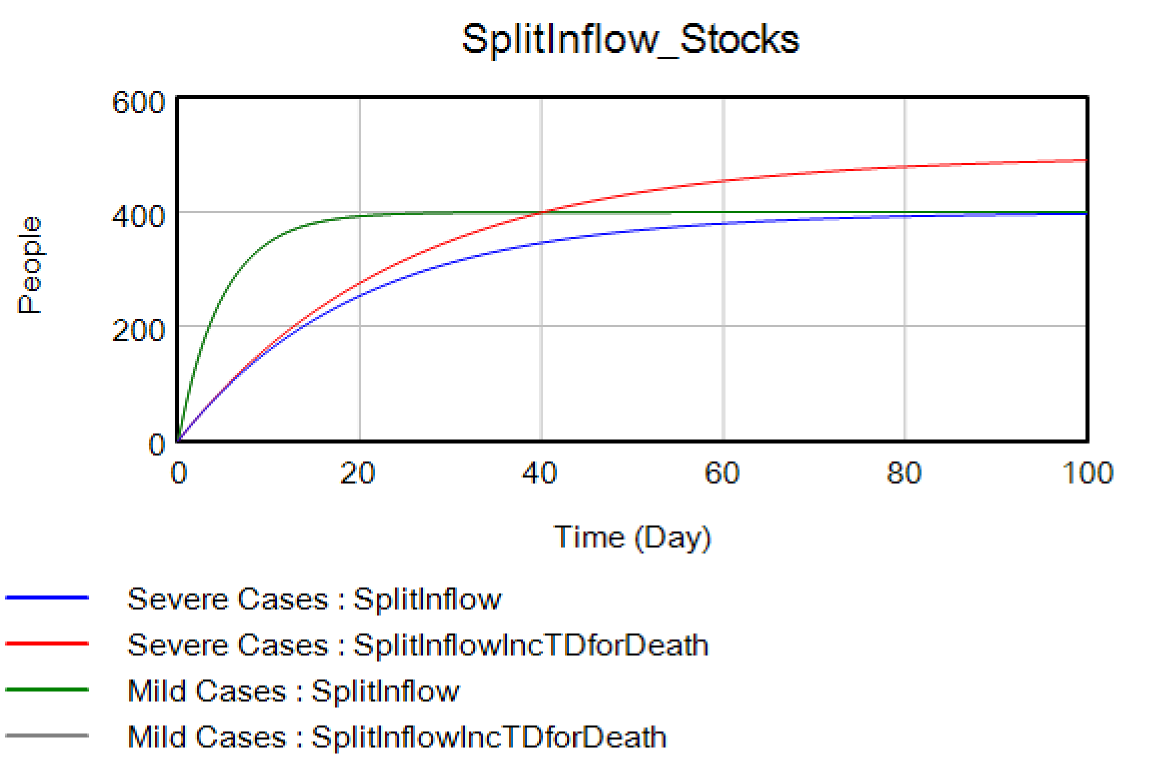

Fig 9: Model 2 Stocks

On the other hand, in the case of split inflows, stock of mild cases and stock of severe cases both have equilibrium value of 400 cases (i.e. total of 800 infected people) (Fig 9). The stock of mild cases remains the same with increase in time delay for deaths whereas the stock of severe cases increases to about 450.

Clearly the use of a single stock to capture mild and severe cases significantly underestimates the total stock of infected people in the system at any given time.

System Dynamics models are often used to represent the continuous time version of real-life discrete processes and activities. An Agent Based Model operates on discrete time and hence can be used as a useful reference for making comparison with corresponding System Dynamics model.

The following image provides an overview of an Agent Based Model that corresponds to the model described above.

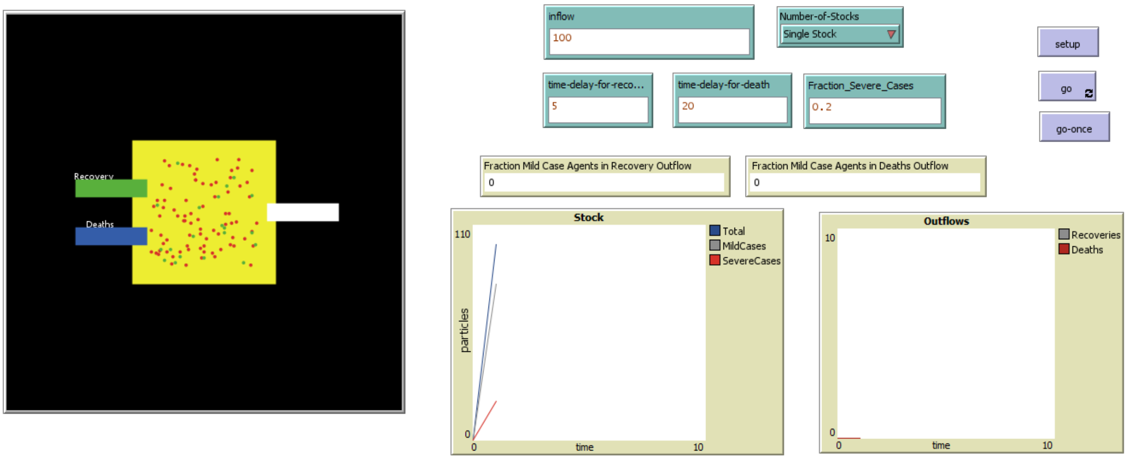

Fig 10: Overview of NetLogo Agent Based model

This model contains two types of agents. First type of agents are those people who contract mild infection and eventually recover. The other type are people who contract severe infection and die. The number of new infected per day (day = 'tick' in the model) is taken as 100. Out of this 100, 80 people eventually recover and 20 will die. The average time delay for recovery is 5 days and average time delay for death is 20 days.

In line with the single stock SD model described above, the single stock simulation in this agent-based model draws a certain number of agents at random (irrespective of agent type) from the stock and these are routed to each of the outflows. The number of agents through the Recovery outflow is equal to the fraction 1 / time-delay-for-recovery times the number of agents in the stock. Similarly, number through the Deaths outflow is equal to the fraction 1 / time-delay-for-death times the number of agents in the stock. This agent-based model replicates the single stock System Dynamics model.

The following charts show output for single stock case simulated for 150 days (ticks):

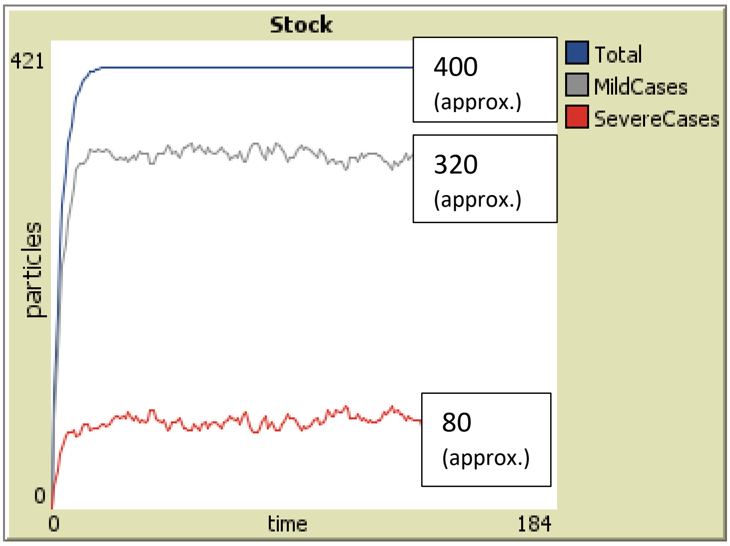

Fig 11: Agent Based Model - Stocks (Single Stock case)

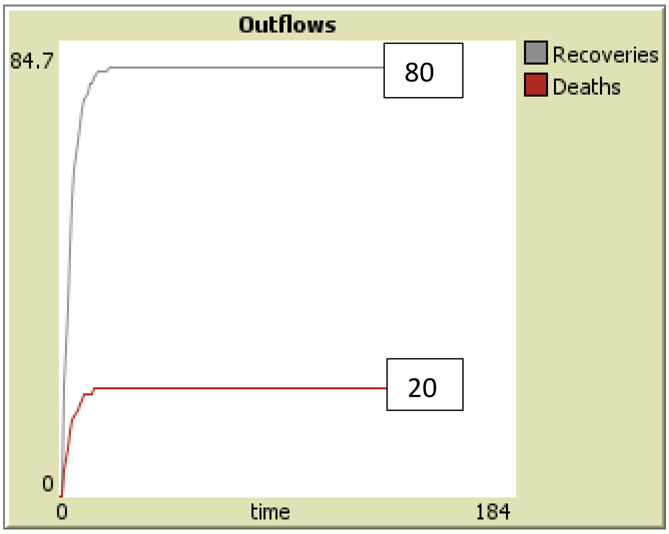

Fig 12: Agent Based Model - Outflows (Single Stock Case)

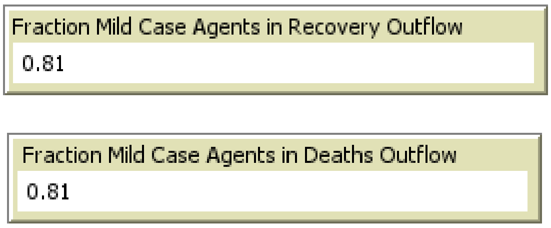

Fig 13: Fraction of Mild Case agents in the Recovery Outflow (top) and Fraction of Mild Case agents in the Deaths outflow (bottom) (Single Stock Case)

The equilibrium stock is about 400 and the stock of population with mild cases and severe cases are about 320 and 80, respectively. The proportion 320:80 is equal to 80:20 which is the proportion of mild to severe cases in the inflow. The outflows are 80 and 20 people / day, respectively, and sum up to the inflow rate. These findings are in line with the single stock SD model.

However, what is interesting is the fact that composition of outflows is not 100% recovered population or 100% deaths. The screenshot of output monitor ‘Fraction Mild Case Agents in Recovery Outflow’ shows a seven-day moving average value of 0.81. Similarly, the ‘Fraction Mild Case Agents in Deaths Outflow’ has a seven-day moving average value of 0.81. That is, about 20% of flow through the designated ‘Recoveries’ outflow are deaths and 80% of flow through the designated ‘Deaths’ outflow are recoveries. That is, each outflow has an agent proportion equal to the inflow agent proportion.

Therefore, even though there is an explicit separation provided for Recovery and Death outflows, the model does not make a distinction between these two agent types. The agent type constraint is not affecting the outflows. This is erroneous. A person with mild case will eventually recover and hence cannot flow out through an outflow designated for deaths.

The single stock SD model cannot show this level of detail. But the agent-based model shows the underlying process happening in the SD model. Essentially, the single-stock single-inflow two-outflow model is not suitable for a situation where the entities in a stock are not homogenous.

The two-stock agent based model is similar to the single-stock model except for the condition that the outflow through Recovery stream is restricted to mild cases only. Similarly, deaths happen only to people suffering severe infection. The output of this simulation is shown below:

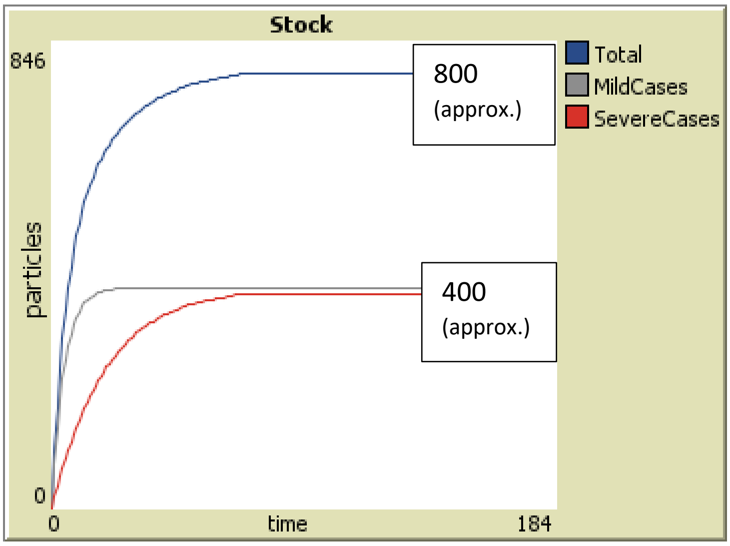

Fig 14: Agent Based Model Stocks (Two-Stock case)

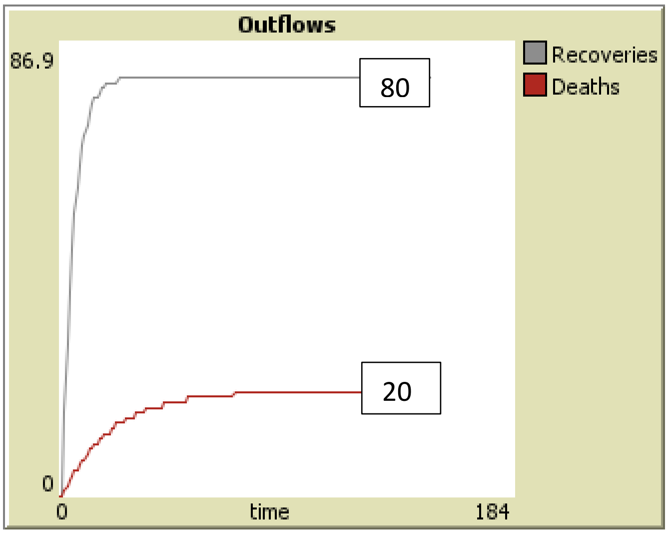

Fig 15: Agent Based Model Outflows (Two-Stock Case)

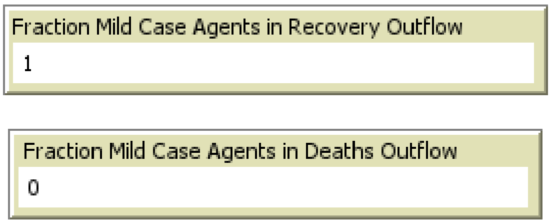

Fig 16: Fraction of Mild Case agents in the Recovery Outflow (top) and Fraction of Mild Case agents in the Deaths outflow (below) (Two-Stock Case)

The total equilibrium stock in this case is about 800 and the stock of mild cases and severe cases are approximately 400 each. The outflow rates are 80 and 20 respectively, which is same as the inflow proportions. These values are equivalent to the values obtained with the two stock SD model.

The ‘Fraction Mild Case Agents in Recovery Outflow’ now has a value of 1 which shows that only recovered population (i.e. mild cases) flow through the designated Recovery Outflow. The ‘Fraction Mild Case Agents in Deaths Outflow’ has a value of 0. This shows that only death cases flow through the designated Death outflows.

The model yields a higher equilibrium stock due to the constraint that stocks cannot be mixed.

The above discussion shows that it is incorrect to use single-stock single-inflow two-outflow structure whenever there is a qualitative difference between the entities in the stock and when this qualitative difference has a bearing on the model outcome. A question that can be asked to verify this is:

“Is it meaningful for the entity flowing out through outflow # 1 to flow out through outflow #2?”

For example, the above question returns ‘yes’ when the stock entity is plain water molecules or grains of wheat stored in a bin. There is no qualitative difference between a grain of wheat flowing out through outflow 1 and outflow 2. Same is the case with a water molecule. But this question returns a negative answer in case of a recovered person as it is meaningless for a recovered person to flow through an outflow designated for deaths.

3. Conclusion

Single-Stock single-inflow split-outflow models are a natural extension of our perception of how an inflow is split into two outflows.

This paper shows that splitting the inflow based only on ‘time delay’ (or fraction outflow from a stock as 1 / time delay is nothing but fraction outflow) can be erroneous whenever there exists a qualitative difference in the entities within a stock and this difference has a bearing on the model outcome. In such a scenario, to ensure validity of the model, there must be a constraint in place such that a particular type of entity can exit only through a particular outflow. A single stock structure does not enforce such a constraint. So, in effect, an entity within the stock can flow through either of the two outflow paths decided solely based on underlying outflow probabilities. Agent Based model of single stock structure has shown that outflows in this case contains both types of entities. This results in incorrect values for outflow rates as well as underestimation of the stock values.

References

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