MODEL DESCRIPTION: A system dynamics study about the effect of ventilation on the spread of COVID-19 in indoor spaces (model description only)

Daniel A. Sepúlveda-Estay[1], Christian M. Sørup[1], and Thorkild I. A. Sørensen[2]

[1]*Dept. of Improvement, Copenhagen University Hospital, Copenhagen, Denmark*
[2]*Dept. of Public Health, University of Copenhagen, Denmark*

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1 Summary

The model presented in this paper represents the process through which a group of people become infected by a pathogen present in the air, within a confined, indoor space. The process represented in the model is the infection process that occurs when infectious particles exhaled by infectious people are inhaled by susceptible people. More specifically, and beyond standard SEIR models, the model represents 1) the number of viral particles produced by infected people as a distribution of particles along four particle size groups, 2) the evolution of these particle size distributions over time, 3) the concentration of these viral particles within an enclosed space as a number of infectious "units" called quanta, and 4) the inhalation and infection process in susceptible people.

The model is described in this document according to the main stocks chosen for the model: a) the people in section 2 and b) the quanta in two ways presented in sections 3 and 3.3.

2 Accumulation of people

The model presented in this paper is based on an epidemiological model of pathogen transmission considering a number of *Susceptible* `(S)` people in an enclosed space that become *Exposed* `(E)` through contact with an airborne pathogen (virions) at a certain rate $\frac{dS}{dt}$, dependent upon the number of *Infected* `(I)` people. The system dynamics (SD) representation of the basic SIR model (given by equation 1) is shown in Figure 1.

$$\frac{dS}{dt} = -\beta(t)SI \tag{1}$$

All `E` are considered asymptomatic ill and are no longer in the population of `S` nor part of `I`, as the incubation time is assumed longer than the simulation time window.

The rate at which `S` become `E` in the traditional SEIR model is proportional to the number of `S` and `I` with a proportionality constant *beta*, the transmission rate. Our

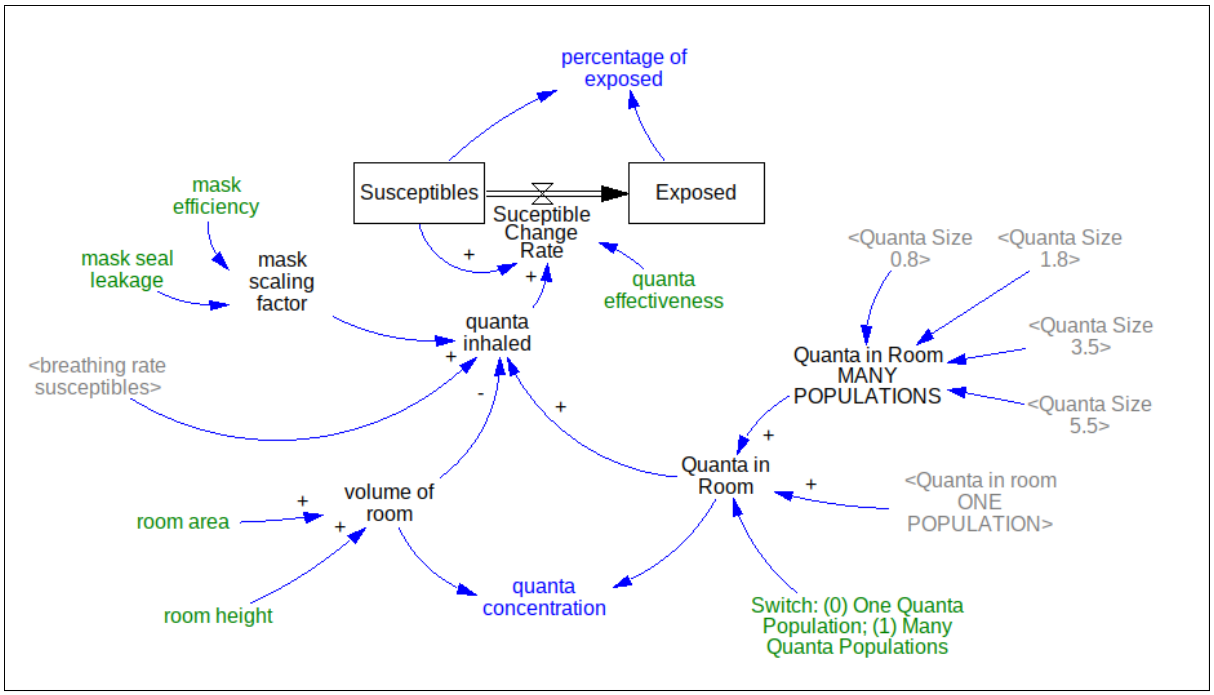

Figure 1: Model representation of infection

model dives into the mechanics behind *beta* by representing this rate as a result of `S` being exposed to enough viral particles to cause an infection.

As the number of inhaled viral particles needed to infect a susceptible individual differs between pathogens, the concept of *quanta* was proposed to reflect similar effects from different pathogens [3]. In order to represent the mechanics of infection using quanta, our model proposes the use of a structure that keeps track of the quanta number and its evolution over time.

The next sections describe the model structure that keeps track of the viral particles through the initial use of one quanta population. Afterwards, we attempt to split the quanta population according to droplet sizes. As can be seen on Figure 1, the user can switch between a single or multiple quanta populations.

3 Accumulation of a single quanta population

Epidemiological theory states that `Susceptibles` can become `Exposed` by coming into contact with a sufficient number of infectious viral particles [3]. The amount required depends upon specific characteristics of the pathogen, such as its infectivity. Wells proposed *quanta* as the number of viral particles necessary to Expose 62,5% of the population that come into contact with it [2].

The quanta accumulation in the room N_q reflects the mass balance between the quanta production and loss rate. The quanta production stems from the infected population I for a specific activity level j, P_q^j, and the quanta loss rate L_q due to 1) ventilation effects L_q^v, 2) air filtration L_q^f, 3) sedimentation L_q^s and 4) virion quanta deactivation L_q^d, as per equations (2) and (3).

$$N_q(t) = \int_0^t (P_q^j - L_q) \tag{2}$$

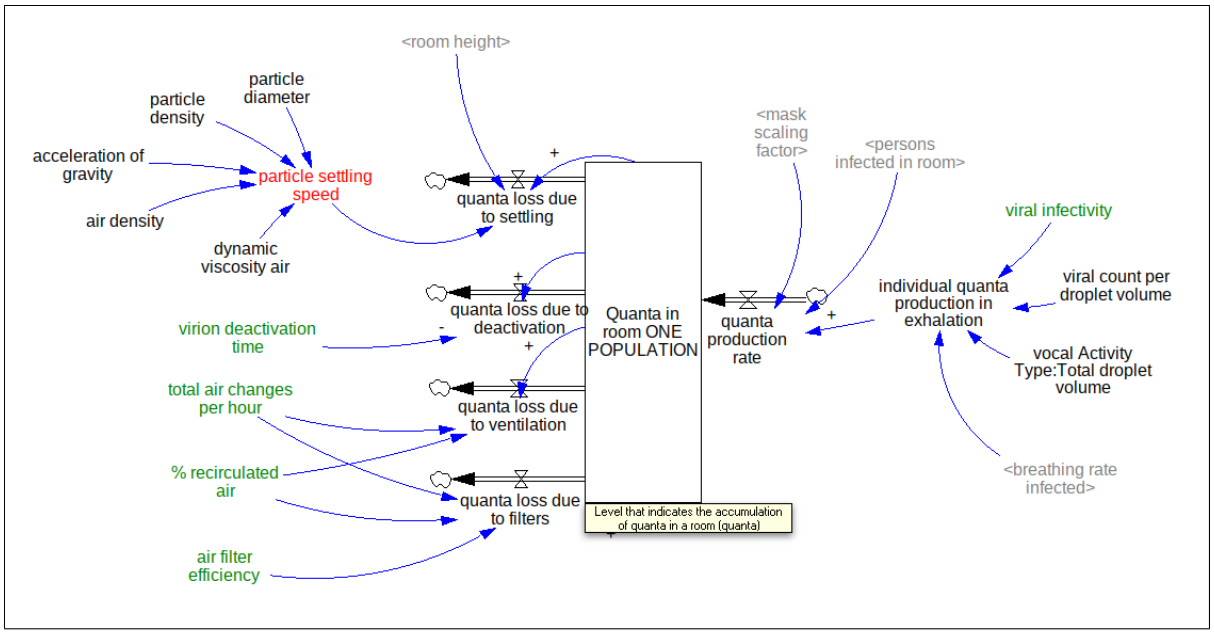

Figure 2: Model representation of One population of quanta

$$L_q = L_q^v + L_q^f + L_q^s + L_q^d \tag{3}$$

Quanta emitted after exhalation consist of viral particles contained inside water droplets. These droplets have a range of diameters that vary depending on the type of activity and the activity level that is being carried out by the `I` in the room. The mechanics for this effect is described in subsection 3.3 about the evolution of the quanta in the room.

The system dynamics stock and flow representation of this causal mechanism is presented in Figure 2. Based on this representation, the next sections will discuss quanta production and loss accordingly.

3.1 Quanta production

For a specific activity level j, the quanta production rate P_q^j is dependent upon 1) infectivity of the viral particles c_i, 2) the viral content of liquid droplets c_v, 3) the breathing rate Q_b and 4) the total volume of liquid droplets in the exhalation, considering the range of different droplet volumes present i, for a specific activity level. This is reflected in equation (4)

$$P_q^j = c_v \cdot c_i \cdot Q_b \cdot \int_{i=0}^{\infty} (N_{i,j} \cdot V_i)\, di \tag{4}$$

3.2 Quanta loss

The removal of quanta in a room can happen through four different means. First, virion quanta can be removed due to ventilation effects L_q^v as a result of new air brought into the room. As seen in equation 5, this quanta loss rate by ventilation is determined by the number of air changes per hour ACH, the existing accumulation of quanta in the room N_q and the fraction of the air that is not recirculated.

$$L_q^v = N_q \cdot ACH \cdot \frac{Q}{Q_f + Q} \tag{5}$$

Second, quanta can be caught through a ventilation system's filters L_q^f and is given by the fraction of air filtered for each air change and the filter performance p_f. This is given by equation 6.

$$L_q^f = N_q \cdot ACH \cdot \frac{Q_f}{Q_f + Q} \cdot p_f \tag{6}$$

Third, susceptible people can avoid inhaling quanta due to particle settling L_q^s. Particle settling includes settling velocity of the particles v_s (given by Stoke's Law) and the surface area perpendicular to that velocity, which in this case is the room floor A as per equation 7.

$$L_q^s = N_q \cdot v_s \cdot A \tag{7}$$

Finally, the quanta lost due to deactivation L_q^d follows the deactivation time τ_d, as shown in equation 8.

$$L_q^d = \frac{N_q}{\tau_d} \tag{8}$$

The number of quanta therefore are determined by the mass-balance equation, as shown in equation 9 a detailed version of equation 2.

$$\frac{dN_q}{dt} = P_q - N_q \cdot (ACH \cdot (1 - \frac{Q_f}{Q_f + Q}) + ACH \cdot p_f \cdot \frac{Q_f}{Q_f + Q} + v_s \cdot A + \frac{1}{\tau_d}) \tag{9}$$

3.3 Quanta Evolution

Our initial model assumed that all droplets emitted during exhalation had the same size. However, viral particles exhaled by the `Infected` are contained in droplets with a distribution of diameters which is a function of the type of vocal activity being carried out by the exhaling `Infected` (breathing, un-modulated vocalization, whispered counting or voiced counting), and on the physical activity being carried out by the `Infected` at the time of the vocal activity (resting, standing, light exercise, moderate exercise, and heavy exercise) [1].

This distribution of particle diameters is represented in our model as the accumulation of several distinct quanta populations in contrast to having only one stock N_q. As per Buonanno et al, [1], four sub-populations of quanta are chosen which accumulate droplets with sizes of 0.8, 1.8, 3.5 and 5.5 microns (shown in Figure 3).

Each of these quanta accumulation sub-populations $[i]$ has a production rate based upon the individual quanta production during exhalation P_q considering physical activity and vocal activity type, the mask scaling factor p_m and the number of persons infected in the room I, leading to Equation 10 and Figure 3.

$$P_q^{[i]} = P_q \cdot p_m \cdot I \cdot f^i \tag{10}$$

with f^i as the factor corresponding to vocal activity and physical activity being carried out by the Infected I at the time of exhalation.

Additionally, the initial distribution of emitted droplet sizes is also dynamic and will adjust to environmental conditions such as relative humidity and temperature. This change in diameter will affect the accumulation of quanta in the air as, for example, smaller droplets take longer to settle on surfaces as they fall slower due to gravitational force and are subject to convection currents in the room for example.

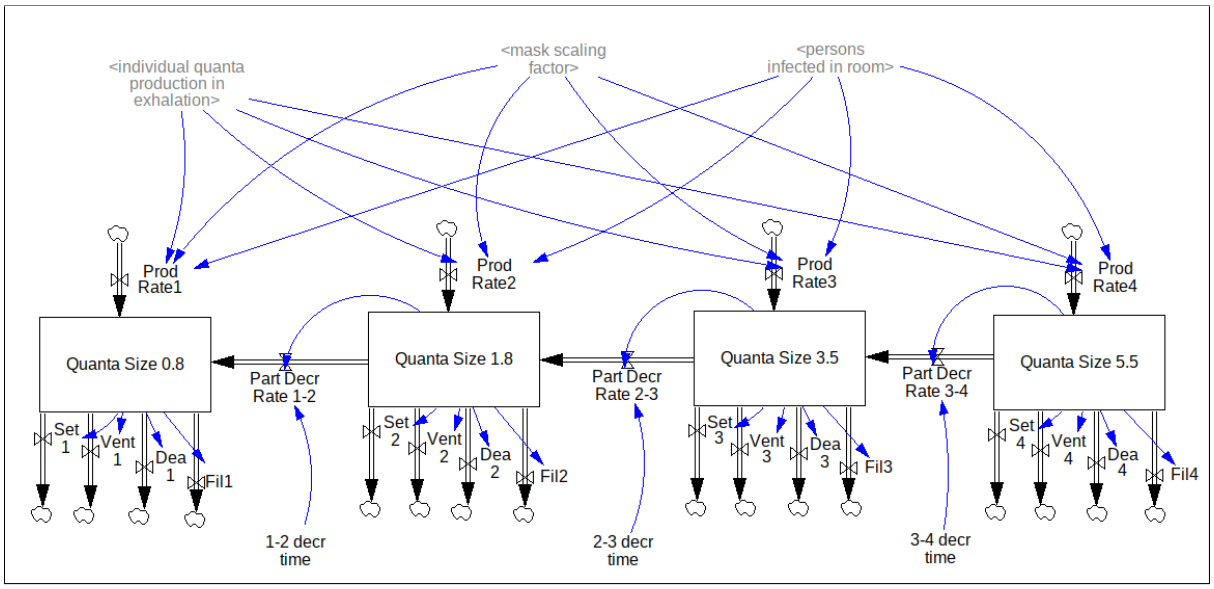

Figure 3: Model representation of four quanta populations

References

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[3] Edward Anthony Nardell. Wells Revisited: Infectious Particles vs. Quanta of Mycobacterium tuberculosis Infection-Don't Get Them Confused. *Mycobacterial diseases*, 6(5):3, 2016.

Appendix

Table 1: Endogenous variables (ordered alphabetically)

Variable Name	**Symbol**	**Unit**	**Type**
Exposed	E	*people*	Stock
Individual quanta production	P_q	*quanta/hour*	Auxiliary
Mask Scaling Factor	p_m	*dimensionless*	Auxiliary
Particle settling speed		m/s	Auxiliary
Quanta in Room	N_q	*quanta*	Stock
Quanta Inhaled		*quanta/hour*	Auxiliary
Quanta loss due to deactivation	L_q^d	*quanta/hour*	Rate
Quanta loss due to filters	L_q^f	*quanta/hour*	Rate
Quanta loss due to settling	L_q^s	*quanta/hour*	Rate
Quanta loss due to ventilation	L_q^v	*quanta/hour*	Rate
Susceptible	S	*people*	Stock
Susceptible change rate	SCR	*people/hour*	Rate
Total quanta loss	L_q	*quanta/hour*	Rate
Volume of Room		$meter^3$	Auxiliary

Table 2: Exogenous variables (ordered alphabetically)

Variable Name	**Symbol**	**Unit**	**Value**
Acceleration of gravity		m/s^2	9.81
Air changes per hour	ACH	$1/hour$	
Air density	ρ_a	kg/m^3	
Air filter efficiency		$dimensionless$	
Dynamic viscosity of air	η_a	$kg/(m*s)$	
Infected persons in the room	I	$people$	
Mask Efficiency		$dimensionless$	0 - 1
Mask Seal Leakage		$dimensionless$	0 - 1
Particle density	ρ_f	kg/m^3	
Particle diameter		m	
Quanta effectiveness		$1/quanta$	0.625 [1]
Recirculated air		$dimensionless$	0 - 1
Room Area		$meter^2$	25 - 120
Room Height		$meter$	2.5 - 4.5
Susceptible change rate	SCR	$people/hour$	Rate
Viral count per droplet volume		$virion/m_f^3$	
Viral infectivity	c_v	$dimensionless$	0.01 - 0.15
Virion deactivation time	τ_d	$hour$	
Volume of Room		$meter^3$	Auxiliary

Table 3: Output Variables (ordered alphabetically)

Variable Name	**Symbol**	**Unit**
Percentage of Exposed	$E\%$	%
Quanta concentration	$[Q]$	$quanta/meter^3$