Appendix to Accompany the Article

Dynamic Supply Chains with Endogenous Allocation

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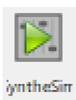
Model Availability

Accompanying the main article and this appendix are the full models available as .mdl files, along with supporting data files to illustrate the how specific analyses were run and figures generated. The .mdl files can be open and run using Vensim software, developed by Ventana Systems, Inc. A free version of the Vensim software for personal use, along with a standalone model viewer, is available from Ventana Systems, Inc.

Ventana Systems, Inc provides detailed documentation on the Vensim software, including how to manipulate and examine specific formulations. However, the reader may quickly explore the influence of parameter choices on the model via the SyntheSim mode on the main Dashboard view of the model. This can be accessed by pressing the corresponding button in the top toolbar of the software as seen below:



For the supporting Food Supply Chain model, the .mdl file is divided into two views, an overview Dashboard, and a view of the full model itself. Different views can be access via the buttons in each view, or via the view menu. Examples of these two views (but not the entirety of these views) are provided below.

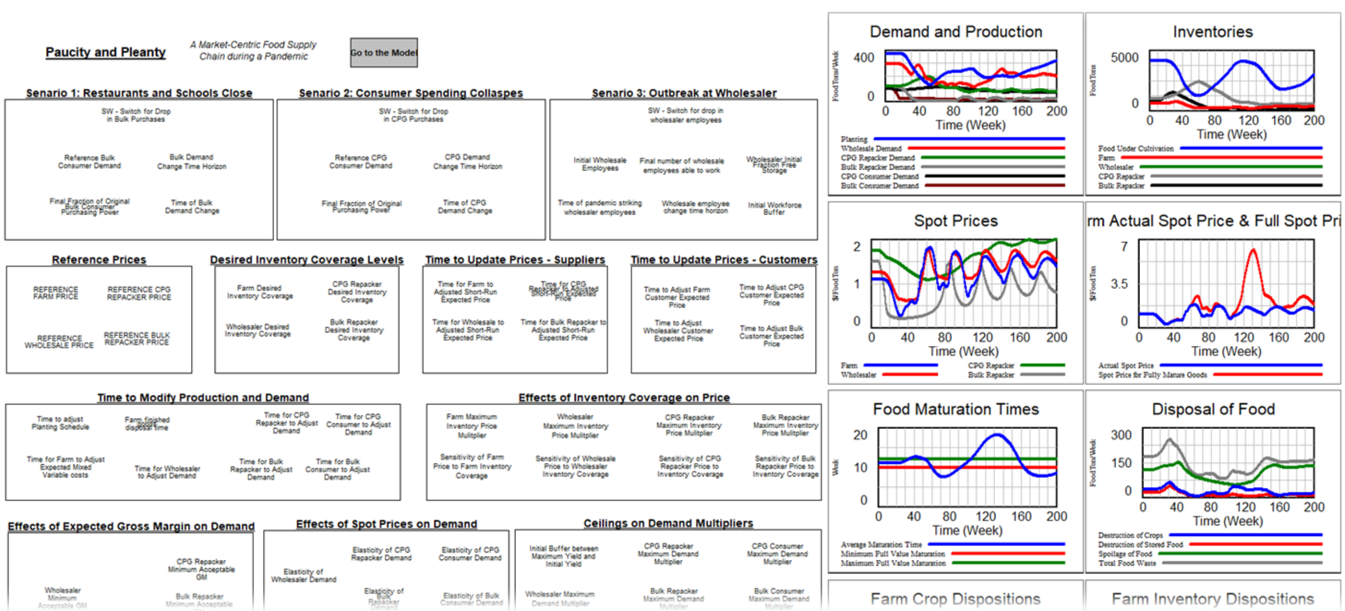


Figure S1: Example of the Dashboard View of the Food Supply Chain Model

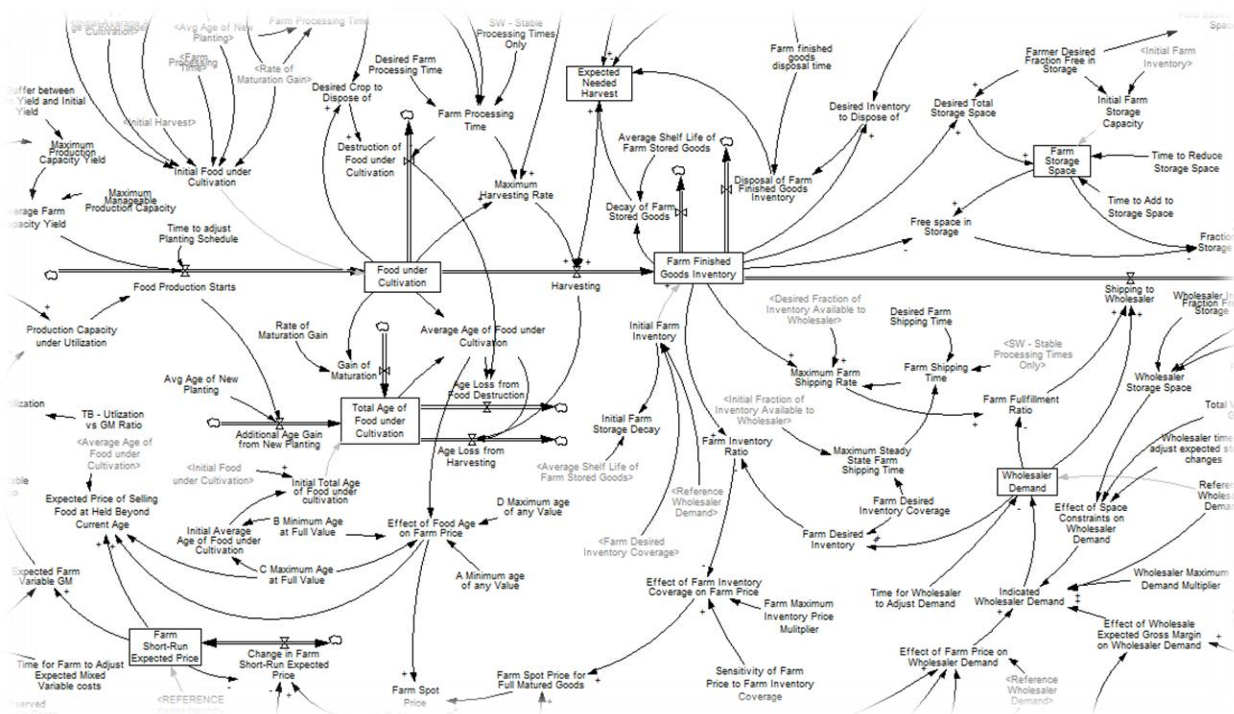


Figure S2: Detail of Aggregate Framework Embedded in Full Model View of the Food Supply Chain Model

As a note, this model view is presented in its entirely and largely has no hidden structure or hidden causal connections. The focus of this article is not necessarily the details of the dynamics of the food supply chain modeled here. Rather this model is used as an illustrative example of the logistic choice model in a larger context. The model is still provided here for the interested reader and allows for the reader to investigate in detail how the frameworks developed in the main article are practically applied as subcomponents in a larger model.

For the model illustrating the details of the aggregate and vintaging framework, the presentation is designed to allow for comparison of the outputs of both frameworks when subjected to the same inputs. The .mdl file is divided into several views, most notably an overview Dashboard, and detail views of each framework. Different views can be access via the buttons in each view, or via the view menu. Examples of these views (but not the entirety of these views) are provided below.

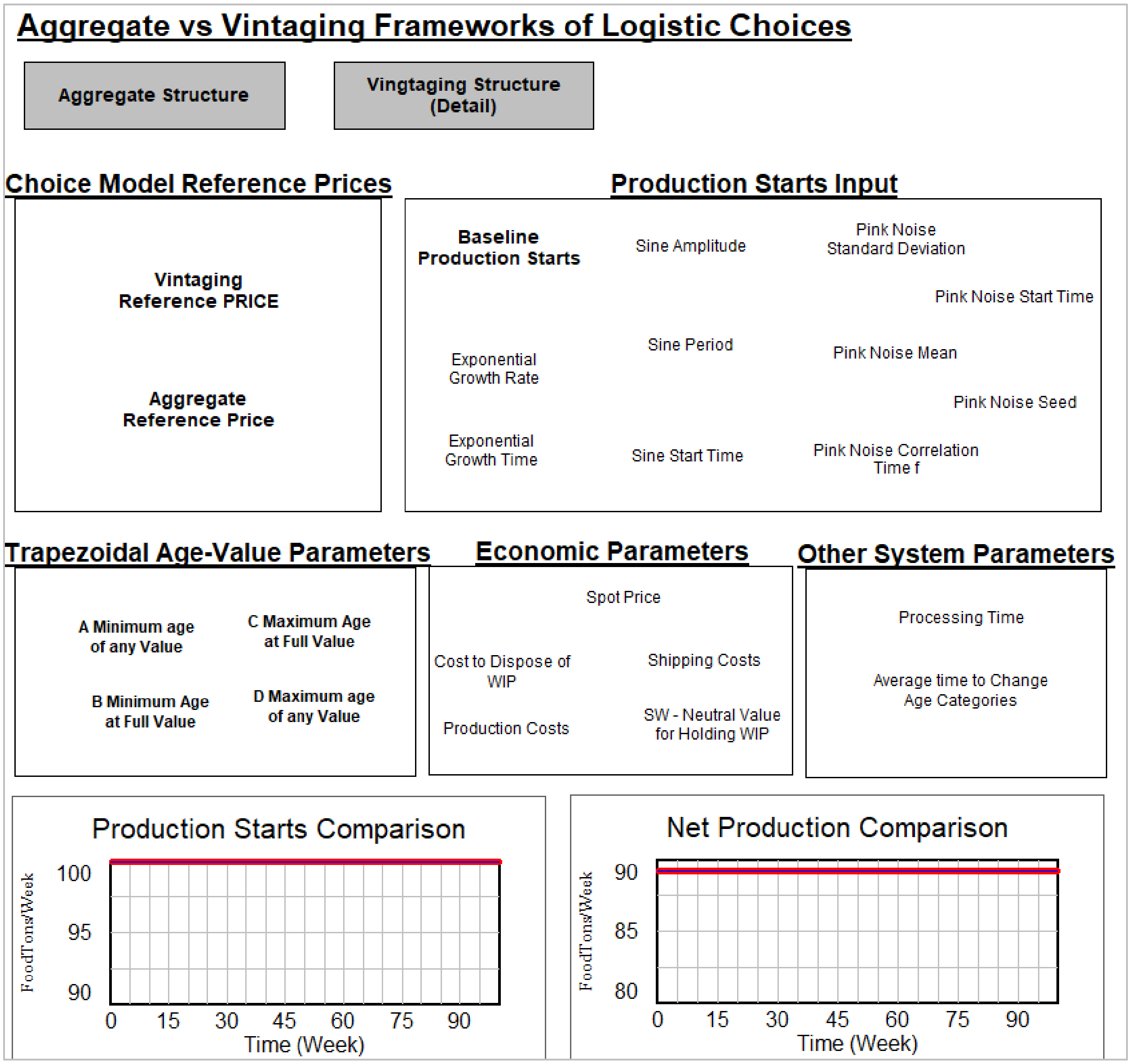


Figure S3: Partial Example of the Dashboard View of the Framework Comparison Model in Vensim

Note that the vintaging framework is presented for 10 age cohorts, and this is fixed by design. This is to make the presentation of the framework direct and easy to interpret without the need for subscripting or array formulations. This presentation can be greatly simplified via array approaches, but doing so hides the underlying interplay of choices in the vintaging structure. However, the cost of this choice is a highly cluttered display of the fully connected model, along with difficulty in adjusting the number of cohorts. This fully connected view is present in the .mdl Vensim file, but the author encourages readers to focus on the detailed view of the beginning and end of the illustrative vintaging chain for ease of understanding.

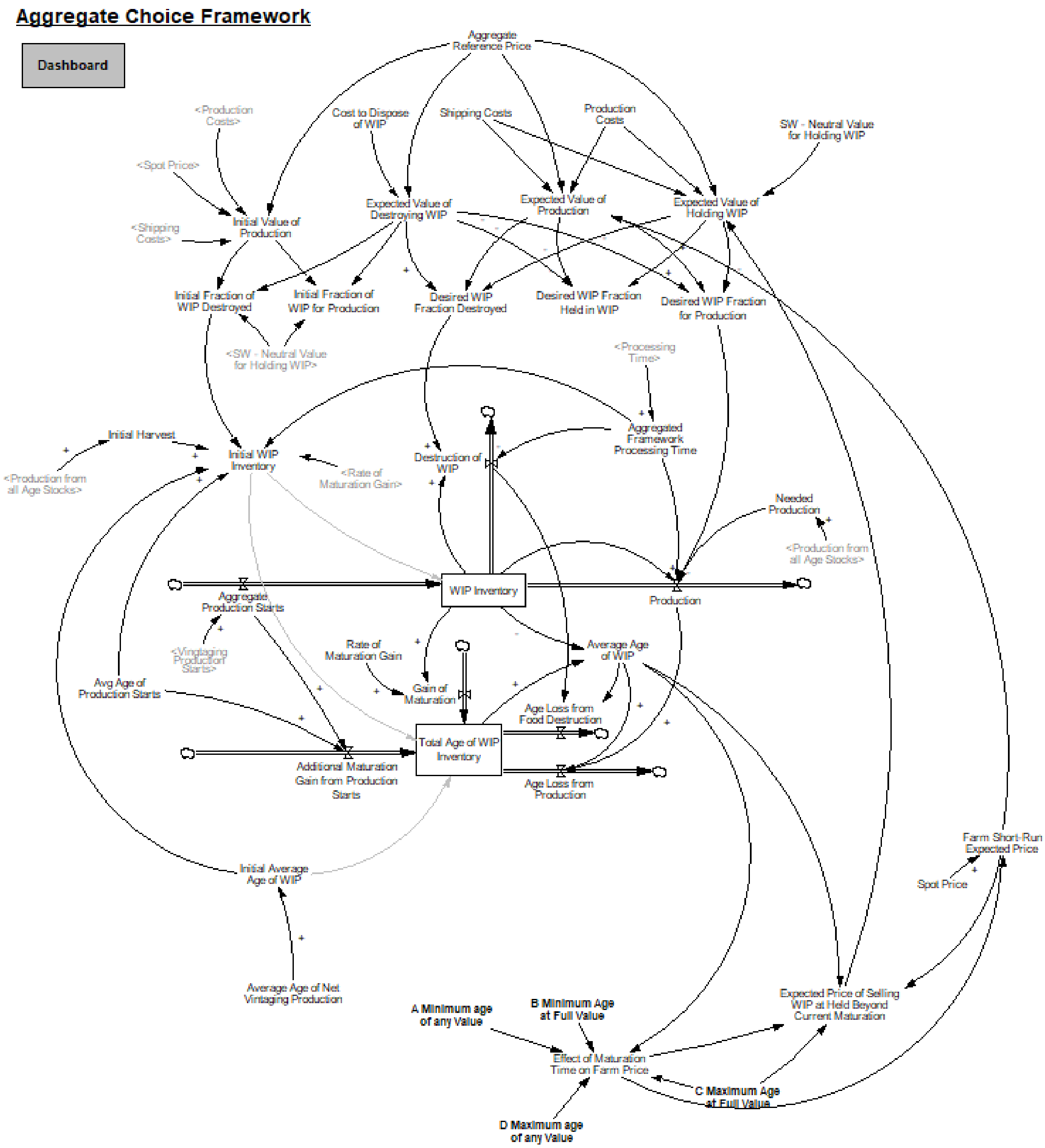


Figure S4: Detail of Aggregate Framework View in the Framework Comparison Model

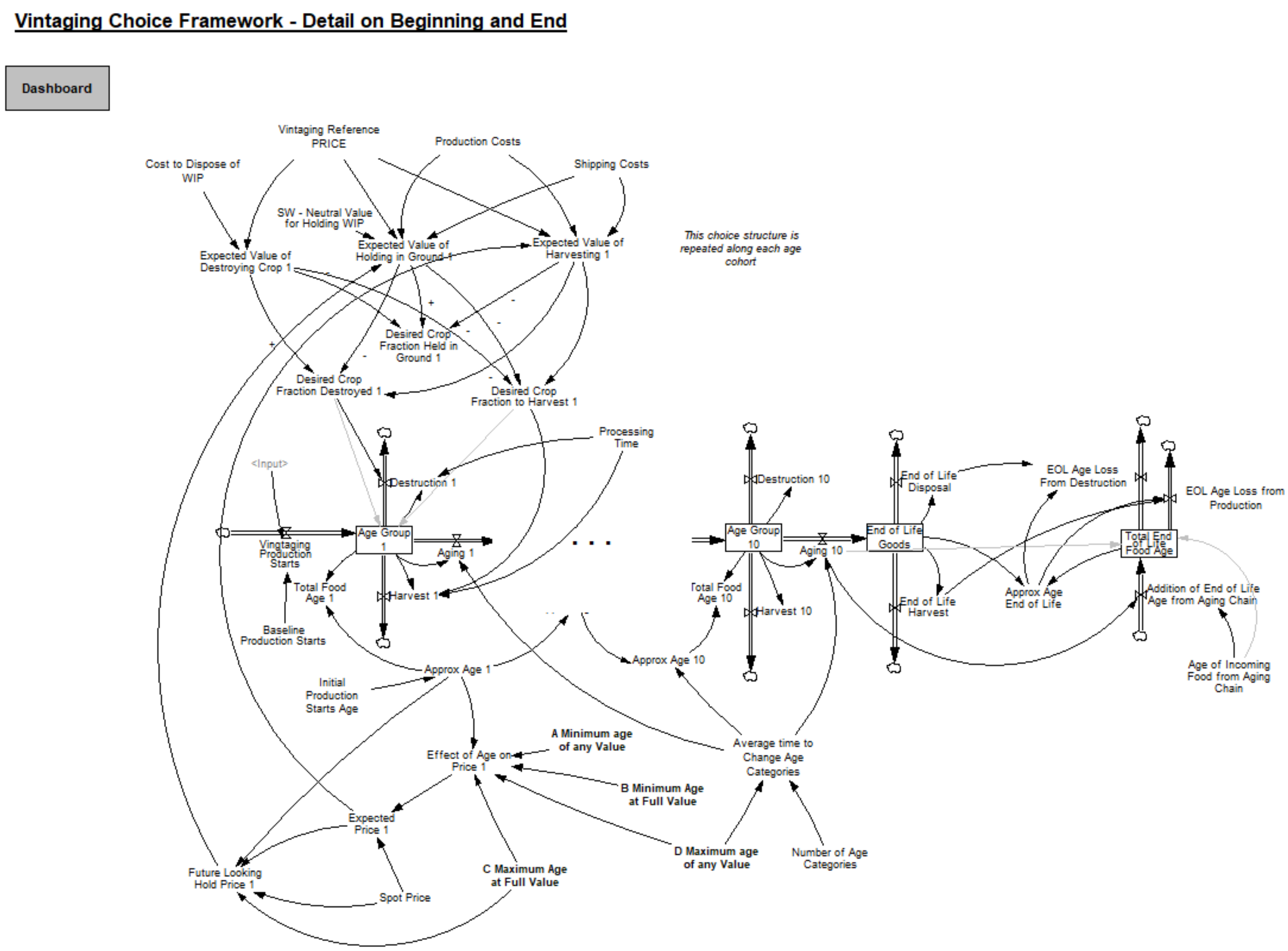
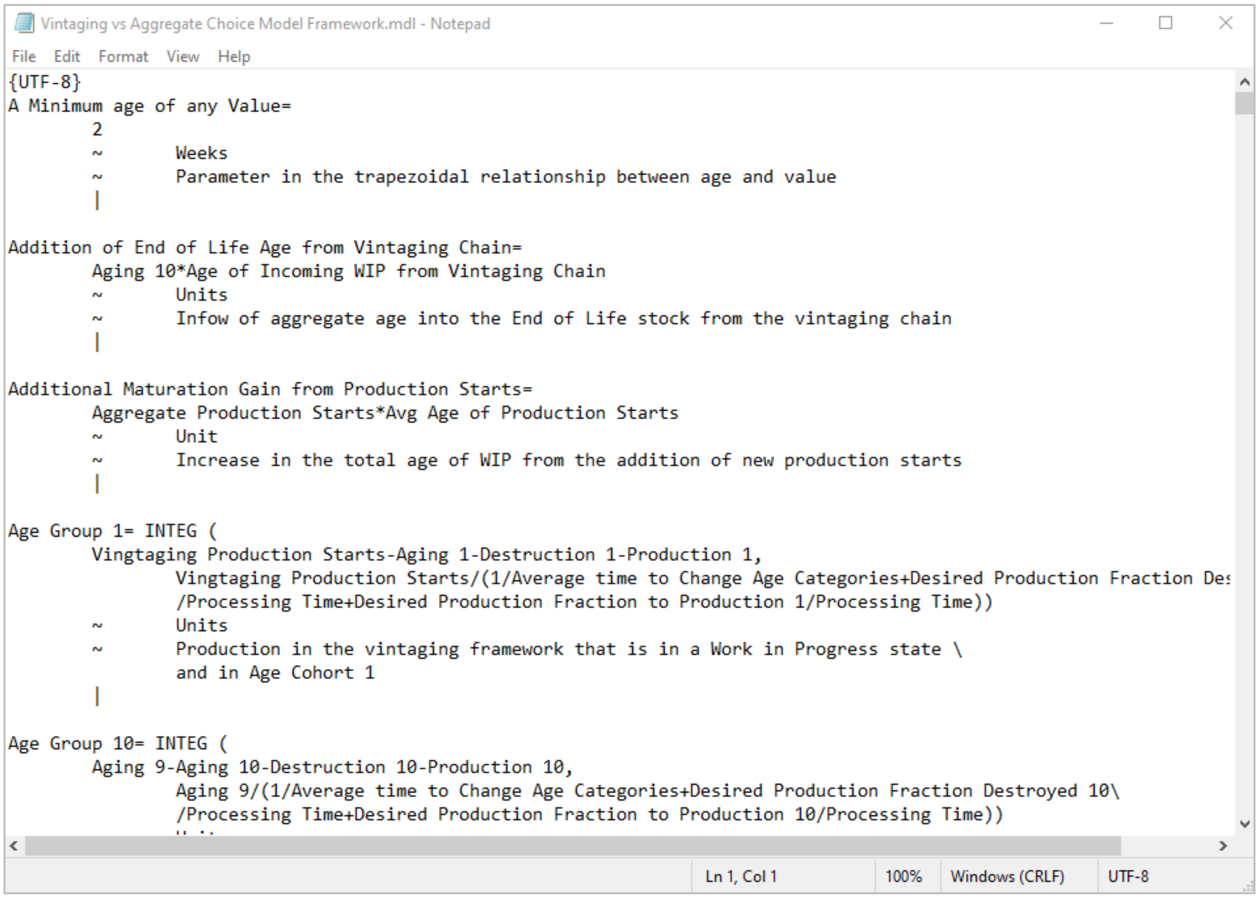


Figure S5: Detail of Vintaging Framework View in the Framework Comparison Model

Furthermore, the .mdl files provided may be opened in any program that is able to read UTF-8 encoding and the formulations directly viewed in plaintext. Examples of program that can open the .mdl file for direct viewing in plaintext include Notepad in the Windows operating system and Textpad in the Macintosh operating system. An example of this view of the model file is seen in Figure S6.



```
{UTF-8}
A Minimum age of any Value=
	2
	~	Weeks
	~	Parameter in the trapezoidal relationship between age and value
	|

Addition of End of Life Age from Vintaging Chain=
	Aging 10*Age of Incoming WIP from Vintaging Chain
	~	Units
	~	Infow of aggregate age into the End of Life stock from the vintaging chain
	|

Additional Maturation Gain from Production Starts=
	Aggregate Production Starts*Avg Age of Production Starts
	~	Unit
	~	Increase in the total age of WIP from the addition of new production starts
	|

Age Group 1= INTEG (
	Vingtaging Production Starts-Aging 1-Destruction 1-Production 1,
		Vingtaging Production Starts/(1/Average time to Change Age Categories+Desired Production Fraction Des
		/Processing Time+Desired Production Fraction to Production 1/Processing Time))
	~	Units
	~	Production in the vintaging framework that is in a Work in Progress state \
		and in Age Cohort 1
	|

Age Group 10= INTEG (
	Aging 9-Aging 10-Destruction 10-Production 10,
		Aging 9/(1/Average time to Change Age Categories+Desired Production Fraction Destroyed 10\
		/Processing Time+Desired Production Fraction to Production 10/Processing Time))
```

Figure S6: Example of Viewing the Supporting .mdl File in Notepad on Windows

The models developed for this article are also fully documented utilizing the SDM-Doc tool described in (Martinez-Moyano, 2012). The output from this documentation tool is available alongside the .mdl files.

Formulation Details for the Supporting Food Supply Chain Model

In the main article, a model of a bifurcated supply chain of a commodity food product was presented as an illustrative example of the use of the frameworks developed in a larger model. The sections below provide additional detail on the development of that food supply chain model, focusing on details that are not necessary to illustrate the frameworks developed in the main article, but are still of interest in the dynamics in this overarching system. As a note, some portions of the explanatory text from the main article are repeated below where needed to create a self-contained description of this larger model.

The example model described below explores the application of this modeling framework to a hypothetical bifurcated food supply chain consisting of the following entities:

- A farmer, who is responsible for making decisions about how much to plant each time period and how to manage her harvest
- A wholesaler firm, which receives raw and unprocessed foodstuff from the farmer, and perhaps does some minimum value-added work to the food
- Two different packaging processors
  - A CPG (consumer packaged goods) processor that received good from the wholesaler and does extensive value-added rework to the food, packaging it in smaller consumer friendly forms for sale to the end consumer at some outlet like a grocery store
  - A Bulk processor that receives goods from the wholesaler and does minor repacking of the food for sale directly to larger consumers like restaurants, governments, or schools.
- The end consumers, which include demand for both CPG and Bulk packaged food

A general visual representation of these players, and the physical flow of food, is shown in Figure S7.

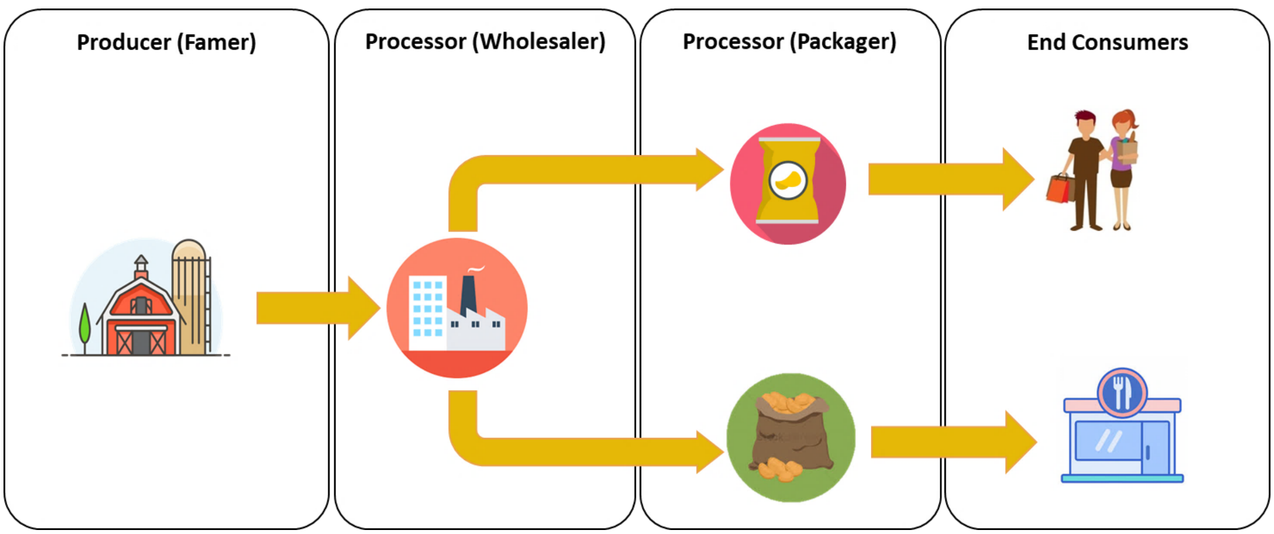


Figure S7. Visual Representation of the example food chain

Defining the Market

While there may be multiple ways to construct the interplay of supply and demand that ultimately forms the spot price at each interface point in the market illustrated in Figure S7, the definition of this marketing being based on a commodity product implies the use of inventory-sensitive spot pricing (Chen et al., 2009; Sterman, 2000; Whelan & Forrester, 1996).

The core of this economic model is two balancing loops across each entity in the supply chain, with spot pricing driving either demand or supply.

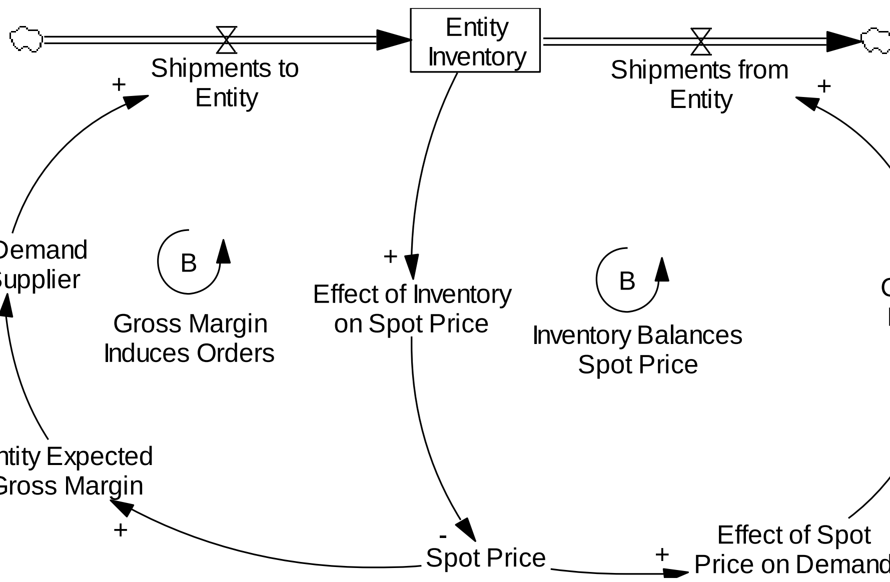


Figure S8. Core Two Balancing Loops Inventory-Based Spot Prices

However, the above entity may exist in a chain of upstream and downstream entities, each ordering from their suppliers and selling to their own customers. This effects the 'Expected Gross Margin' and introduces another balancing loop. Additionally, the spot price is fundamentally anchored to what the market expects it to be, and this introduces a reinforcing loop around the spot price and the expected prices. These two new loops, in the context of the larger supply chain, are seen below:

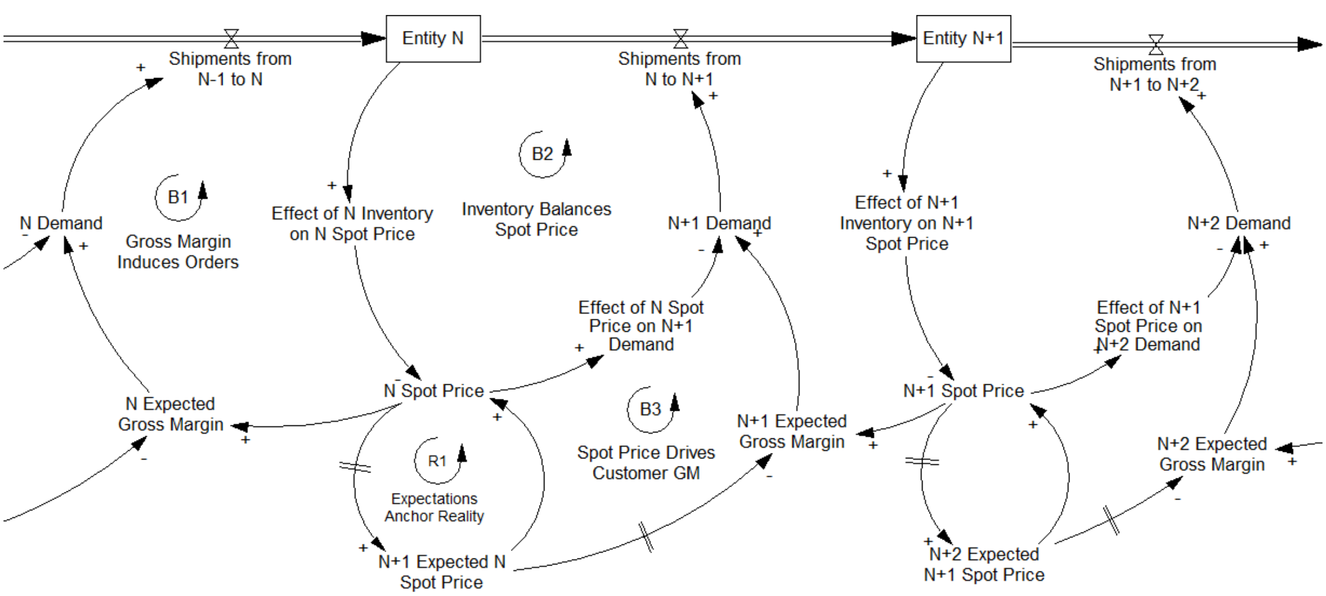


Figure S9. Ordering and Price Setting is Nested in Larger Interconnected Supply Chain

Effect of Inventory Coverage on Price

One of the key features of the pricing model visually summarized above is the effect of inventory coverage on pricing. In net, a model will capture the downward sloping relationship between additional inventory (beyond a set inventory coverage goal) and the price offered by the firm holding that inventory.

$$Effect\ of\ [Entity]\ Inentory\ Coverage\ on\ [Entity]Price = [Entity]Inventory\ Ratio^{-Sensitivty} \quad (1)$$

The *sensitivity* is a parameter that determines how much the price will raise or lower given a change in inventory coverage. As formulated here, *sensitivity* is assumed to be a positive value, with higher values corresponding to increasingly concave response functions

Another feature explored in the above formulation is a 'cap' on the maximum multiplier that inventory coverage could have on price. I.e., if inventory coverage approaches 0 (there is no inventory to sell), then the effect on the price will approach infinity. In reality this does not happen as increase in spot prices drives down demand from downstream customers, preventing the final marginal units of inventory from ever being sold in practice.

Effect of Expected Gross Margin on Demand

A concept of expected gross margin can be used to influence production in the case of the Farm, and demand in the case of all other entities in the supply chain, with increases in expected Gross Margin assumed to induce greater production or demand.

There may be multiple methods of incorporating this relationship here, including truncated sigmoidal functions and directly applying table functions. For this example, consider a simple truncated linear representation that meets the following criteria:

1. Passes through the point of (1,1) on a normalized scale
2. Is truncated at an upper maximum multiple on demand
    a. This assumes that it is infeasible for an entity will ever request more than some multiple of its reference demand at any expected future profit level
    b. This could be due to a number of possible factors not explicitly modeled such as storage space constraints, transportation limitations, or risk of spoilage).
3. Is truncated at a lower level of demand

a. In other words, it is bounded at a minimum acceptable gross margin, which could be greater than 0%

b. Paratactically this means the line passes through the point of *(Minimum Normalized GM, 0)*

Given points 1 and 3 above, the slope of the line is defined fully by the specification of the minimum acceptable gross margin at which any demand or production will exist, and the definitions of the reference gross margin and corresponding reference demands. Examples of what this curve looks like can be seen in Figure S10 below.

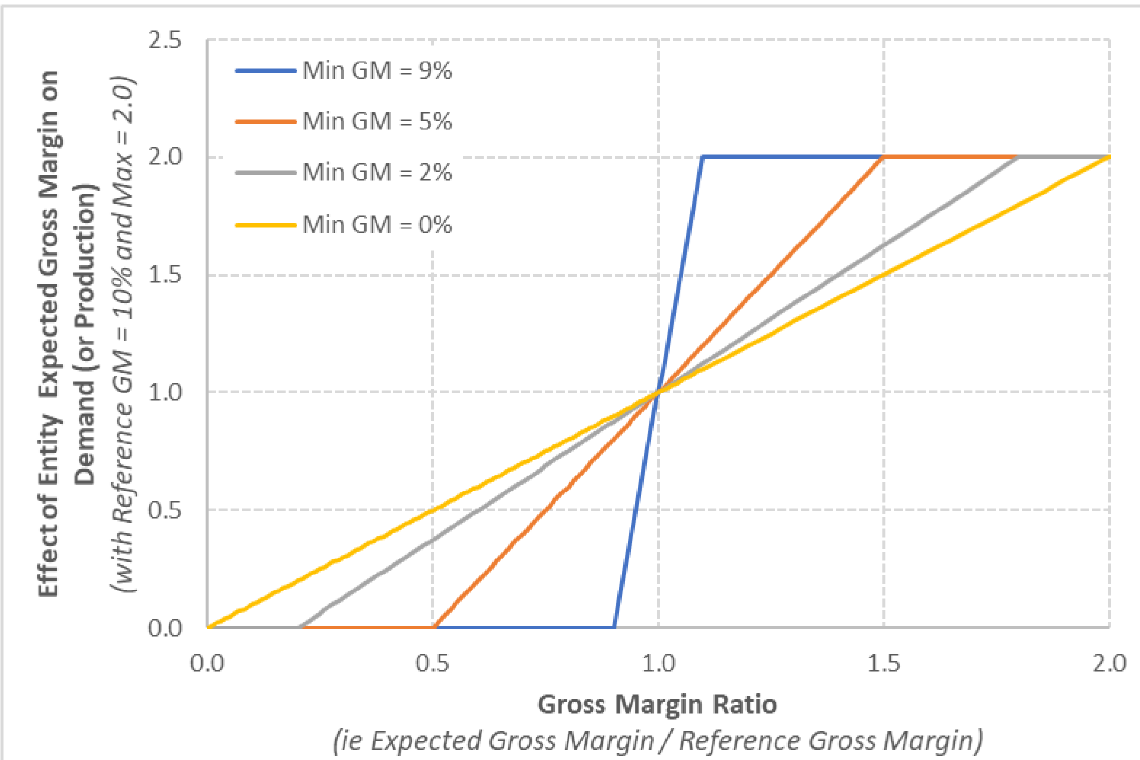


Figure S10. Examples of the Formulation of Demand versus Expected Gross Margin

It should be emphasized that this curve is based on the *expected* gross margin to influence demand, which is in turn can be based on smoothed perceptions of previous prices that the entity has experienced.

Effect of Spot Prices on Demand

The above effect on demand due to expected gross margin does have some element of sensitivity to cost built in from the definition of gross margin. However, it is purposely done in relationship an expected gross margin based on a smoothed view of previous prices (both costs for goods bought and the prices at which they were later sold).

To affect demand based on the instantaneous spot price experienced by each entity, consider a linearly decreasing relationship that captures decreasing demand with increasing prices, with the slope of that relationship affected by some elasticity of demand. The functional form of this expression is seen below:

$$\begin{aligned} &Effect\ of\ Price\ on\ Demand \\ &\quad = MIN\left(Maximum\ Multiplier, MAX\left(0, 1 + Demand\ Curve\ Slope \right.\right. \\ &\quad \left.\left. * \frac{Price - Reference\ Price}{Reference\ Demand}\right)\right) \end{aligned} \tag{2}$$

Where:

$$Demand\ Curve\ Slope = \frac{-Reference\ Demand * Reference\ Elasticity}{Reference\ Price} \tag{3}$$

An example of the shape of this function for various values of elasticity are seen below, where *e* refers to the Reference Elasticity in expression (3) above.

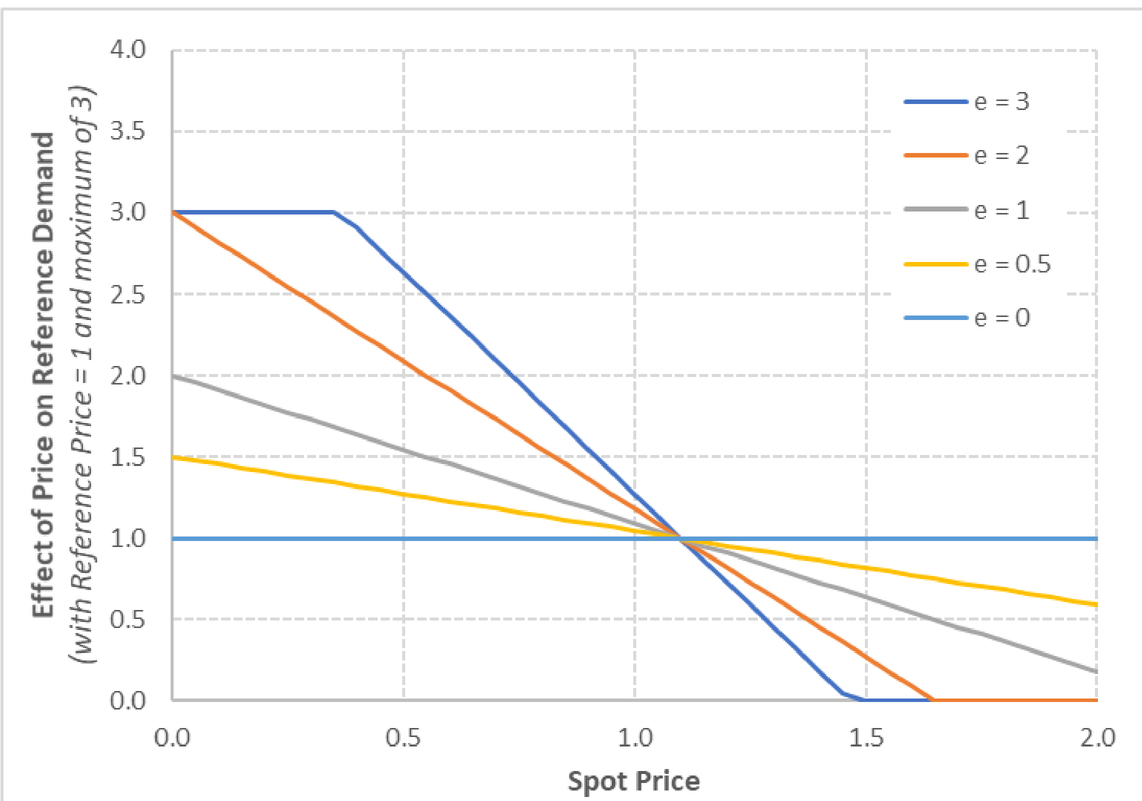


Figure S11. Examples of the Formulation of Demand versus Spot Price

Here I purposely use the spot price to determine the effect of this instantaneous demand. This is purposeful designed to be immediate, in contrast to the effect from expected gross margin which is based on a smoothed concept of both prices and costs.

Combined together, the relationships described in the economic market for this commodity food in which the new modeling framework presented here can be applied.

Production Starts and Capacity Management

The farm considers two different conceptualizations of profitability: the incremental profitability of an additional unit of production (utilizing just the variable costs of production), and the expected profitability from expanding production capacity (utilizing a fully allocated cost of production).

As discussed in other System Dynamics models of commodity markets (notably Chapter 20 of (Sterman, 2000)) utilization is a function of expected gross margin. Furthermore, utilization of existing capacity is unlikely to be at 100% when averaged across all pieces of owned capacity unless at very high levels of expected profitability. The exact shape of this relationship will vary by industry and even by individual producer or individual piece of owned unit of production capacity. To qualitatively capture this behavior, consider a function which approximates a curve approaching the CDF of a collection of different land (capacity) at different utilization depending on local factors. One such curve, and the one utilized in this example is shown in Figure S12.

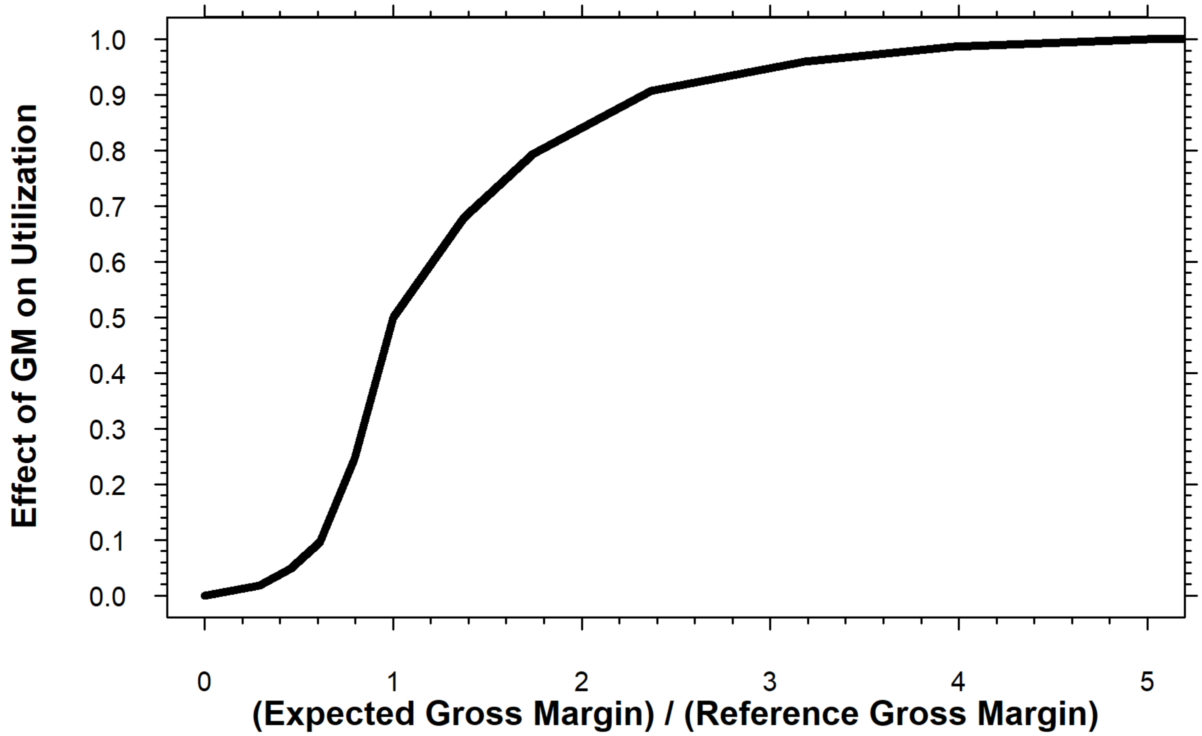


Figure S12. Farm Capacity Utilization versus Expected Gross Margin

As a note, in this example this relationship utilizes the 'Farm Short-Run Expected Price' which is the spot price smoothed over a short time range. The farm considers the price smoothed over a much longer time range when making capacity change decisions. Here, this long-run expected price and a *fully allocated* unit cost is combined to create a fully allocated gross margin from incremental land. This expected gross margin determines the effect on desired capacity (here arable farm that the farmer has access to).

Increased expected profitability, here formulated as expected gross margin, will not only put pressure on the producer to increase utilization of owned capacity, but also possibly invest in new capacity so long as that incremental unit of new capacity is expected to be profitable. A functional form for this relationship is given below.

$$Effect\ of\ Expected\ GM\ on\ Desired\ Capacity = \max(0, \min(Maximum\ Increase, (Slope\ of\ Effect) * Fully\ Allocated\ GM + 1)) \quad (4)$$

The *maximum increase* provides a limit on the gain from the reinforcing loop that forms between *Desired Capacity* and *Arable Land*, preventing the farm from trying to acquire infinite new land in a small period. The outward maximum function prevents the farm from possessing negative land. The figure below shows examples of the shape of this function:

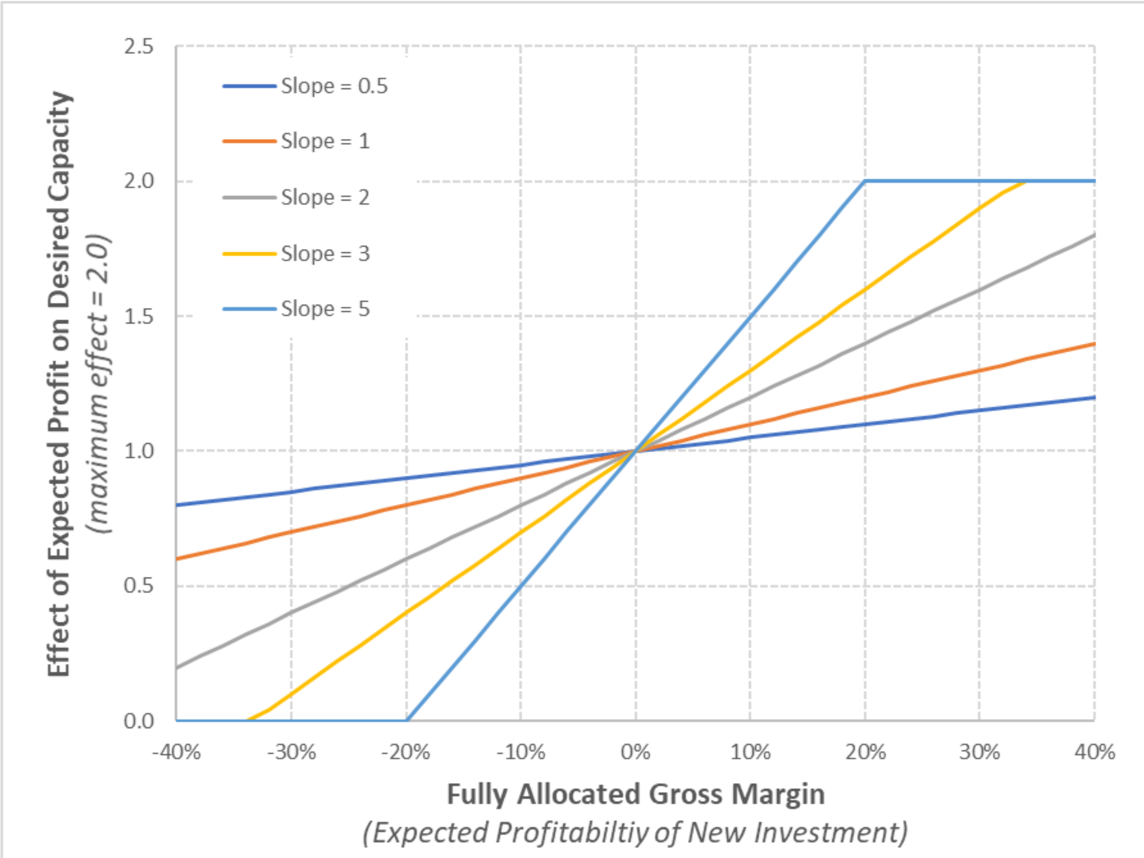


Figure S13. Effect of Expected Profitability of New Capacity on Desired Capacity (land acquisition)

Yield is Decreasing with Additional Arable Land

To a first approximation, one could consider the net incremental productivity added by acquiring new land constant and fixed. However that creates a scenario in which the farm is able to infinitely expand so long as the gross margin is justified (i.e. the additional operating expenses of the land are covered by the new production). A more realistic model though would capture that the net productivity would decrease with additional land under management. In other words, there is likely *decreasing returns to scale* with respect to land and productivity.

While there are multiple ways to model this relationship, here a linear decay model is used where the farm has a known maximum amount of land that they could get any yield out of. Figure S14 below shows the shape of this new function, which I ultimately used in the model for both the reasons named above and simplicity.

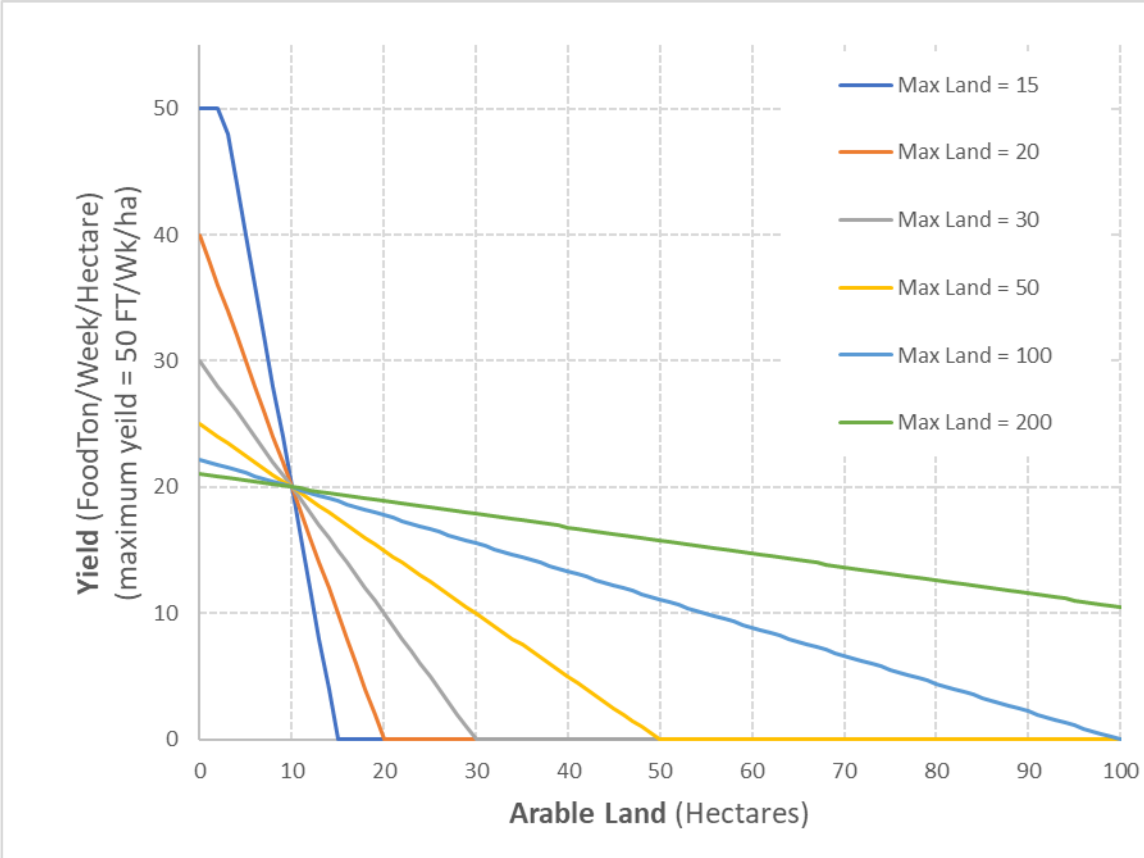


Figure S14. Linearly Decaying Relationship between Yield and Arable Land

Discounting the Farm Spot Price based on Maturation

As discussed above, the relationship between the age of the goods being produced and the value they derive in the marketplace is ultimately context-specific can vary depending on the product under development and how the market as a whole values that product as function of the development or maturation time. For this example, consider a relationship of the same trapezoidal functional form as that described in expression (7) and illustrated in Figure S15. Furthermore, the example below utilizes the single aggregate stock of work-in-progress inventory (here called Food under Cultivation) instead of a more granular vintaging framework as described above.

The quantification of the opportunity cost capturing the tradeoff between time that a unit of potential inventory (here a unit of food) spends under production (or growing) versus the amount of economic value the producer (here farmer) can expect to get from its eventual sale is explored in more detail in the sections below.

$$Effect\ of\ Age\ on\ Price = f(Average\ Age\ of\ Food\ Under\ Culitvation) \tag{5}$$

$$Farm\ Spot\ Price = Effect\ of\ Age\ on\ Price \\ =*\ Farm\ Spot\ Price\ for\ Full\ Mature\ Goods \tag{6}$$

In the above, the 'Farm Spot Price for Full Mature Goods' is defined via the method described in expression (1), and is a function of the inventory coverage of the farm.

Quantifying the Age-Value Relationship

As discussed in more detail in the main article, this relationship that defines 'Effect of Age on Price' is context-specific can vary depending on the product under development and how the market as a whole values that product as function of the development or maturation time.

For this model of a supply chain of a commodity food product, this relationship can be summarized as first having a low value that rises until it reaches a peak of full value at an ideal maturation time, and then declines as it sits in the field either further maturing past its prime or even decaying.

To capture the above dynamics, a table function could be employed but for simplicity consider a trapezoidal relationship between crop value and age (or maturation time). This relationship utilizes four parameters to capture when a crop first has any economic value, the range over which it has full economic value, and the age above which it again has no economic value.

$$f\ (t) = \begin{cases} 0 & t \le a \\ \left(\frac{1}{b-a}\right)t - \left(\frac{a}{b-a}\right) & a < t \le b \\ 1 & b < t \le c \\ \left(\frac{1}{c-d}\right)t - \left(\frac{d}{c-d}\right) & c < t \le d \\ 0 & t > d \end{cases} \tag{7}$$

Figure S15 provides a visual summary of expression (7)

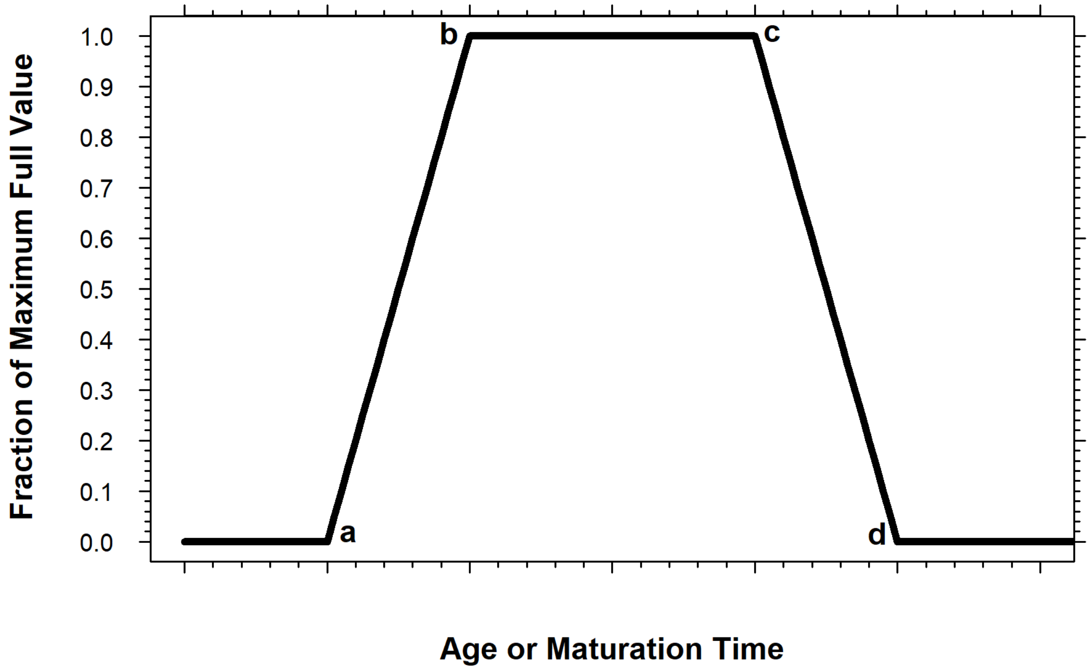


Figure S15. Example of Trapezoidal Function Discounting the Value of Crops based on Maturation

Note that the expression above assumes a linear change from minimum to maximum value, and constant maximum value between points *c* and *d*. More general trapezoidal shapes are possible that do not necessarily have these features (for example see (Dorp & Kotz, 2003)) and may be more appropriate in specific contexts, but this formulation is sufficient here.

Under the aggregate model framework, which is used in this example food supply chain model, the value of the entire stock of work-in-progress inventory is derived by the formulation above. If the vintaging model was used, it would be applied to each cohort of ages.

A Multinomial Logistic model of Crop Dispositions

The section immediately below largely restates material in the main article. It is repeated here to allow for a largely self-contained narrative of the development of the food supply chain model.

In this food supply chain model we consider that the farm has three choices to make with respect to crops that are maturing in the field: 1) Harvest and move into storage (for immediate or later sell to the wholesaler), 2) Keep in the ground to continue to mature (or decay), or 3) Dig up and destroy.

From the point of view of a single producer, each of these dispositions are binary (a farm cannot simultaneously destroy, harvest, and continue to cultivate a single unit of food). Under a model of a single producer, this economic decision becomes a straight-forward assessment of the expected value of each disposition route (for example weighing the of the costs of shipping and storing food versus the costs of destroying it, offset by the value that would come from selling if it were sold). However, for a larger model of a system of producers, it is more appropriate to utilize a *multinomial logistic model*, to represent the probability of a farmer choosing any of the above three options. In this example and across many farms, this becomes the proportion of the total crop that is delegated to each of these three routes.

For some relative economic value π_i for choice X_i, the probability of choosing X_I is given by the expression below:

$$P(X_i) = \frac{e^{\beta \pi_i}}{\sum_{I=1}^{N} e^{\beta \pi_I}} \tag{8}$$

In the above, β is the weight the producer places on the concept of economic value. Under a full logistic model that we could fit to observed data, this becomes a free parameter. Here, we have no observed data, but rather a conceptual model. Thus, to simplify the model overall, we can fix values of β to be the inverse of some reference price for the producer (e.g., the price at which a farm sells its goods under normal steady state conditions). This has the advantage of allowing the relative values of each choice, π_i, to be expressed in terms of prices and monetary values, while allowing the expression above to properly reduce to a dimensionless probability.

$$\beta_i = \frac{1}{reference\ price}\ \forall i \tag{9}$$

How the farm derives the relative values of each of the choices is a matter of modeling freedom and should ideally be based in observations of how real farms value these choices. Given that those observations were not made here, I rely on evidence from the supporting readings and my own assumptions. The advantage of the logistic model is that changing these assumptions only changes the relative value of each choice, and thus the relative proportion of the crop delegated to each option, but not the underlying model.

Valuing Crop Dispositions

As discussed above, how the producer (here the farm) derives the relative values of each of the choices is a matter of modeling freedom and should ideally be based in observations of how real produces value these choices. The advantage of the logistic model is that changing these assumptions only changes the relative value of each choice, and thus the relative proportion of the crop delegated to each option, but not the underlying model.

This is the most straight forward valuation in the model and is simply the cost of destroying the food. The act of digging up and destroying food is not considered ‘free’ and has a cost assigned to it in the model as an exogenous parameter. This could be expanded by applying a ‘mental resistance’ or ‘sunk cost fallacy price’ to further discourage the disposal of food, if evidence supports it. As a note, as modeled here, the value of disposing of food is *always* negative. While the other options can be more negative, even if they are strictly positive, some portion of the crop is nevertheless destroyed each period under the multinomial logistic model.

$$\pi_{destroying\ crop} = -Farm\ Cost\ to\ Dispose\ of\ Crops \tag{10}$$

If the farm is to harvest and store the crop in the ground, they would do so under the expectation that they would receive their current expected price for the goods, less the costs for harvesting, less the eventual costs for shipping.

$$\pi_{Harvesting} = Farm\ Short\ Run\ Expected\ Price - Farm\ Harvesting\ Costs - Farm\ Shipping\ Costs \tag{11}$$

Combined with the above logistic model, this gives a fraction of the crop that could be made available, at most, for shipping.

The farm is harvesting the crop, equivalent to the ‘Production Rate’ flow in the more generic model developed in the main article, is based on both the *expected* future need of food to fulfill demand from the wholesaler and anticipated spoilage or loss in storage.

$$Expected\ Needed\ Harvest = SMOOTH(Wholesaler\ Demand + Disposal\ of\ Stored\ Food, Time\ to\ Update\ Expected\ Harvest) \tag{12}$$

If the farm were fully willing to meet wholesaler demand and replace any goods previously destroyed or spoiled in storage, the above alone would move the goods from planting through to harvest. However, the value of the food in the ground available to be harvest is limited by the logistic model described above. Thus, expression for Production from the main article can be recast into this example context as seen in expression (13) below.

$$Harvesting = \min\left(1, \frac{Maximum\ Harvesting\ Rate}{Expected\ Needed\ Harvest}\right) * Expected\ Needed\ Harvest \tag{13}$$

The actual amount of food left in the ground is ultimately defined by how much food is destroyed and how much food is harvest in each period of time. However, the probability that a farm will choose to destroy, or harvest is also dependent on how the farm values keeping food in the ground. There are two possible ways to capture the value of leaving work-in-progress (here food under cultivation) alone to continue to age, both of which are explained below:

The first option is both the easiest conceptually, and perhaps the most robust because it introduces the least number of additional assumptions: that keeping the harvest in the ground has null value. In many logistic models, there is a 'null choice' or simply a choice of zero value, often used to represent not making a choice at all (e.g., between a red car and a blue car I choose to not buy a car today). For this model, the relative value of holding crops is 0.

$$\text{Option 1: } \pi_{holding\ crops\ to\ mature} = 0 \tag{14}$$

The other option is more behaviorally complex, but perhaps more realistic. Here, the farmer is assumed to be forward looking, anticipating getting the maximum value from her crop that could be expected.

$$\text{Option 2: } \quad \pi_{holding\ crops\ to\ mature} = Farm\ Future\ Looking\ Expected\ Price - Farm\ Harvesting\ Costs - Farm\ Shipping\ Costs \tag{15}$$

Under this model, the farmer is assumed to know the shape of the trapezoidal relationship between age and value discussed above and can expect the maximum fraction of the value of her crop if the maturation time is lower than the ideal maturation time, but nothing better than the current value for maturation times higher than the ideal value.

$$Farm\ Future\ Looking\ Expected\ Price = \begin{cases} Farm\ Short\ Run\ Expected\ Price & if\ \mathrm{T_{maturation}} < \mathrm{T_{ideal}} \\ Farm\ Short\ Run\ Expected\ Price * Effect\ of\ Age\ on\ Price & o.w. \end{cases} \tag{16}$$

Ultimately, the choice of option 2, generally, causes the farm to reserve more crops each period, as the value of the food is viewed higher than null.

Valuing Inventory Dispositions

While the model development immediately above has focused on the valuation and inventory disposition decisions of work-in-progress production, it can be readily applied as well to finished goods inventory in storage as well. Again, the farm has three choices: 1) Make inventory available for the wholesaler, 2) keep finished goods in storage, or 3) Destroy goods. As with the work-in-progress inventory, a multinomial logistic function is used, normalized with β values all chosen to be the inverse of a farm reference price.

As with the previous sector, the value of destroying finished goods can be assumed to be some simple value. It is possible to expand this valuation by considering how destroying inventory frees storage space, which is similar to the previous valuation method used in prior versions of this model. Rather than complicate the valuation here, those consideration are rolled into the valuation of holding goods.

$$\pi_{destroying\ finished\ goods} = -Farm\ Cost\ to\ Destroy\ Inventory \tag{17}$$

The value of making inventory available to ship is simply the current spot price, less the cost of shipping those goods. Note that the current spot price is affected by the maturation of the crops as described above.

$$\pi_{Shipping} = Farm\ Spot\ Price - Farm\ Shipping\ Costs \tag{18}$$

As with the choice to hold crops to further mature, there are two ways to look at the valuation of holding inventory rather than destroying or shipping it, either with a null value or with a more forward-looking model of valuation.

$$\text{Option 1: } \pi_{Hold\ Inventory} = 0 \tag{19}$$

For the forward-looking estimation, consider the that the opportunity cost of storing an additional unit of food for an additional unit of time increases with finite storage space, and the only feasible method of storing additional units of food when storage is full is to acquire additional space at some costs. This is captured in the relationship below:

$$\begin{aligned} &Cost\ to\ Hold\ Based\ on\ Free\ Space \\ &\quad = Farm\ Holding\ Costs + Fraction\ of\ Storage\ Full \\ &\quad * (Farm\ Holding\ Costs + Costs\ to\ Acquire\ Storage) \end{aligned} \tag{20}$$

Furthermore, by hold the finished goods, the farm must be expecting not the current spot price, but some future estimate of the price for their goods. Combined together, this gives the following alternative option for valuing holding inventory in this model:

$$\begin{aligned} \text{Option 2: } \lambda_{Hold\ Inventory} = &\ Farm\ Short\ Run\ Expected\ Price - \\ &\ Cost\ to\ Hold\ Based\ on\ Free\ Space \end{aligned} \tag{21}$$

Either of the two options of valuation above presuppose a decision to acquire storage space if full. Thus, we can consider that the farm has a desired total storage space that is approximately equal to the actual farm inventory, with perhaps an additional allowance for free space for comfort or other purposes.

$$Desired\ Storage\ Space = \frac{Farm\ Inventory}{1 - Farmer\ Desired\ Fraction\ Free\ in\ Storage} \tag{22}$$

The farm will then actively work to adjust the actual storage space to the desired storage space, though perhaps in an asymmetric manner. Specifically, I hypothesize that the farm will be quick to add space but slow to divest of it.

$$Storage\ Space = SMOOTH\left(Desired\ Storage\ Space, \begin{cases} Time\ to\ Add\ to\ Storage\ Space & if\ Desired\ Storage\ Space > Farm\ Space \\ Time\ to\ Reduce\ Storage\ Space & o.w. \end{cases}\right) \tag{23}$$

Alternate Combined Color Figures From the Main Article

For the sake of presentation in print, the figures in the main article are all presented without color. Also to avoid excessively cluttered diagrams, figures of production, demand, and prices were often split into two sections (such as ‘upstream’ and ‘downstream’, or ‘producer’ and ‘non-producer’).

Below are figures exported directly from the Vensim .mdl model viewer and based on the same datasets as those used for the main article.

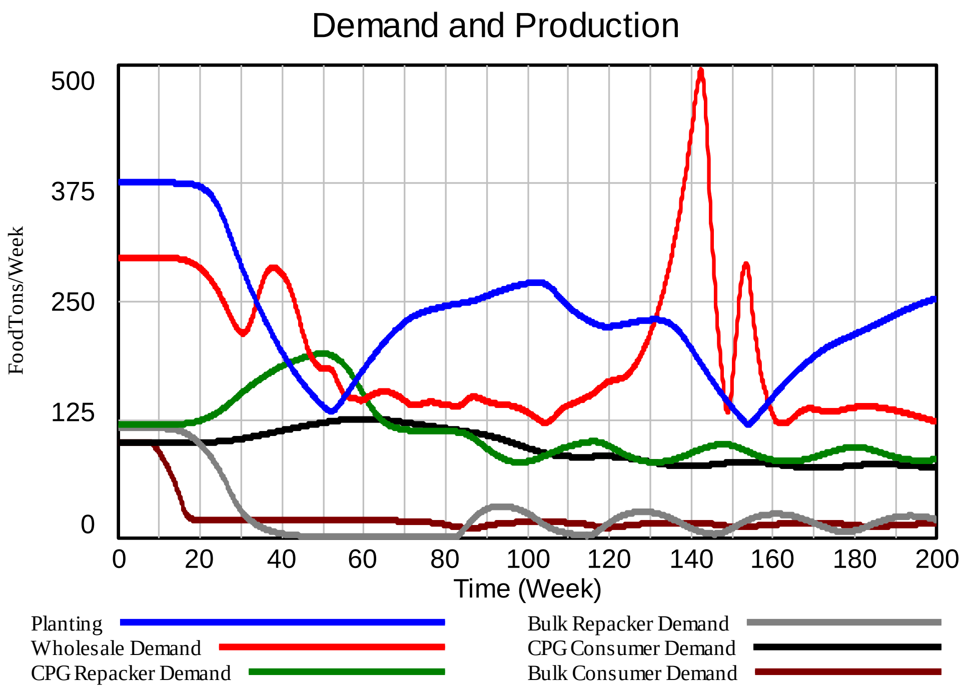


Figure S16. Drop in Bulk Purchasing Power – Demand and Production

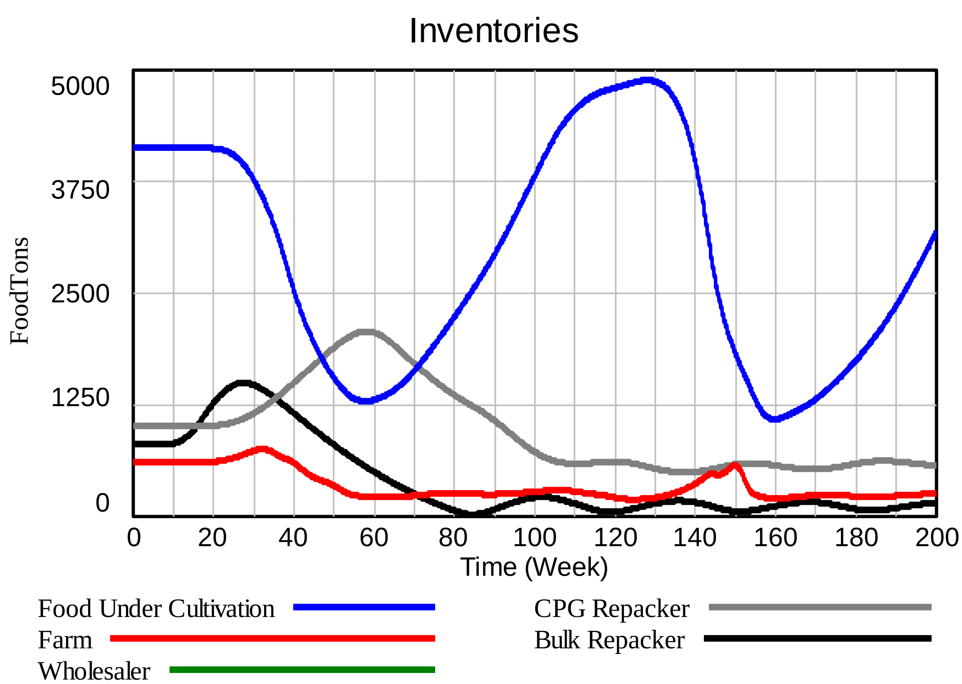


Figure S17. Drop in Bulk Purchasing Power – Inventories and Available Supply

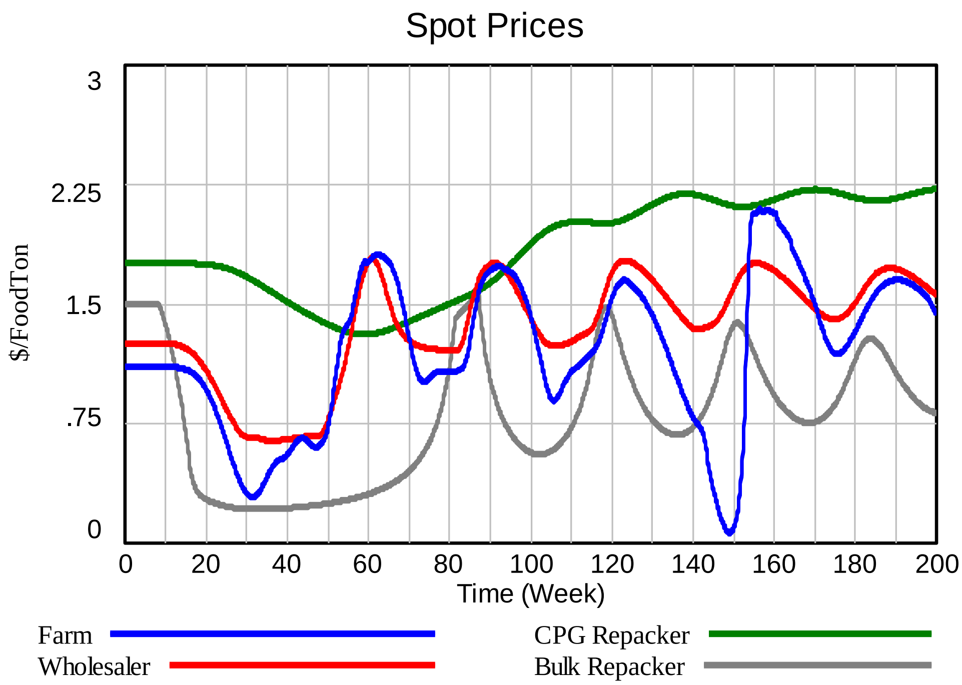


Figure S18. Drop in Bulk Purchasing Power – Spot Prices

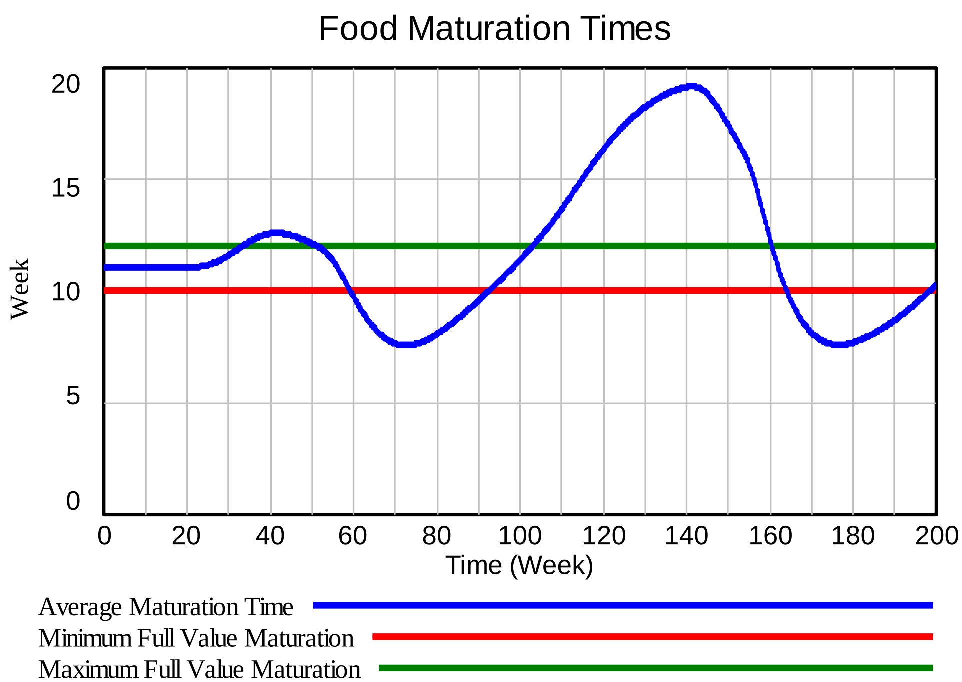


Figure S19. Drop in Bulk Purchasing Power – Age of Food Under Cultivation

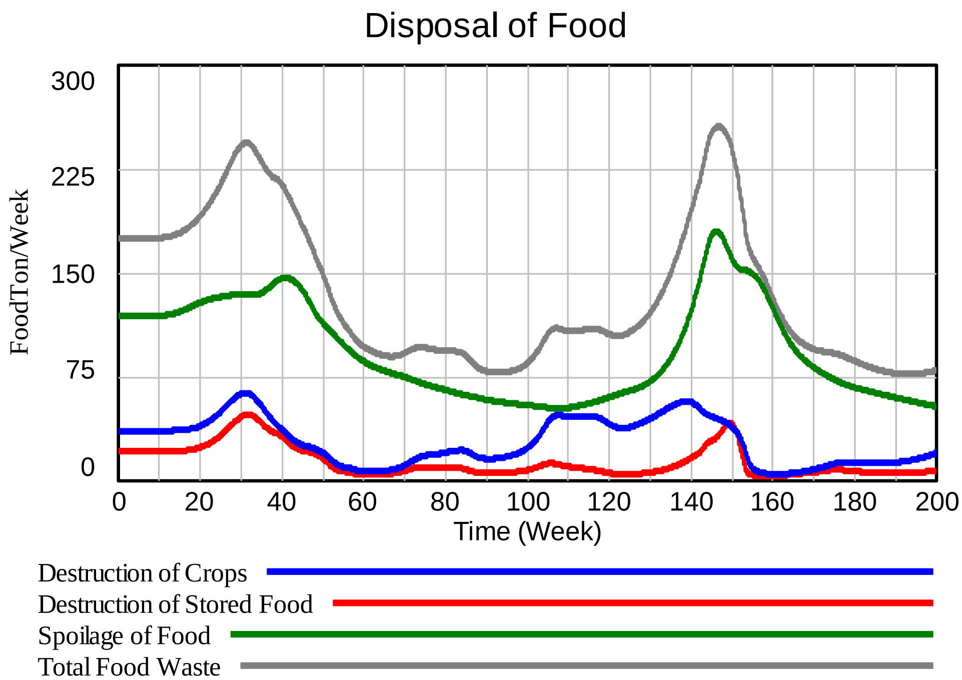


Figure S20. Drop in Bulk Purchasing Power – Disposal and Destruction of Food

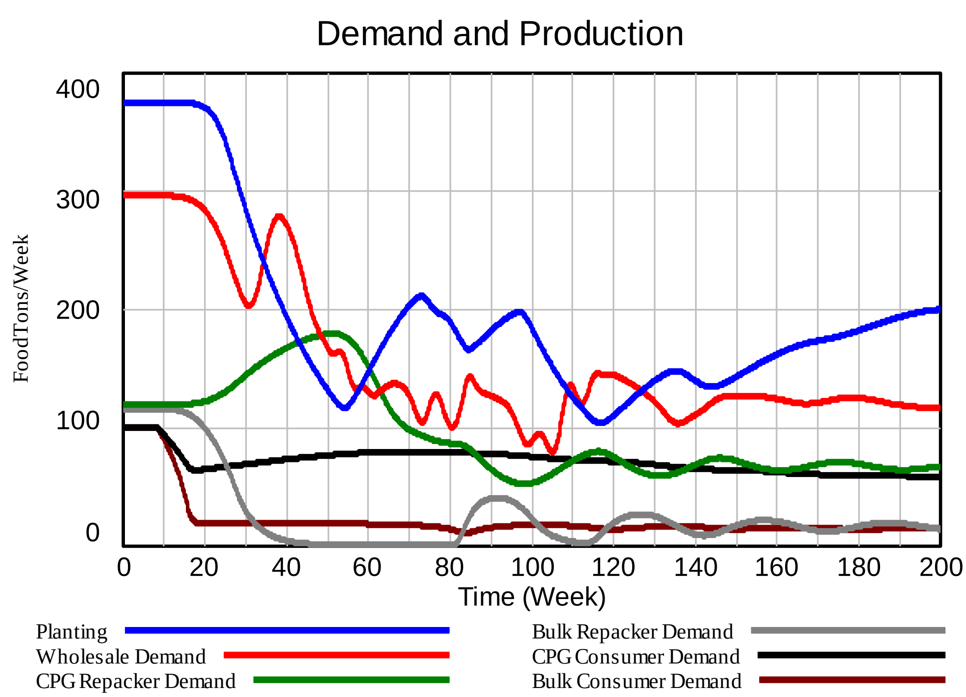


Figure S21. Drop in Bulk Purchasing *and* Consumer Purchasing Power – Demand

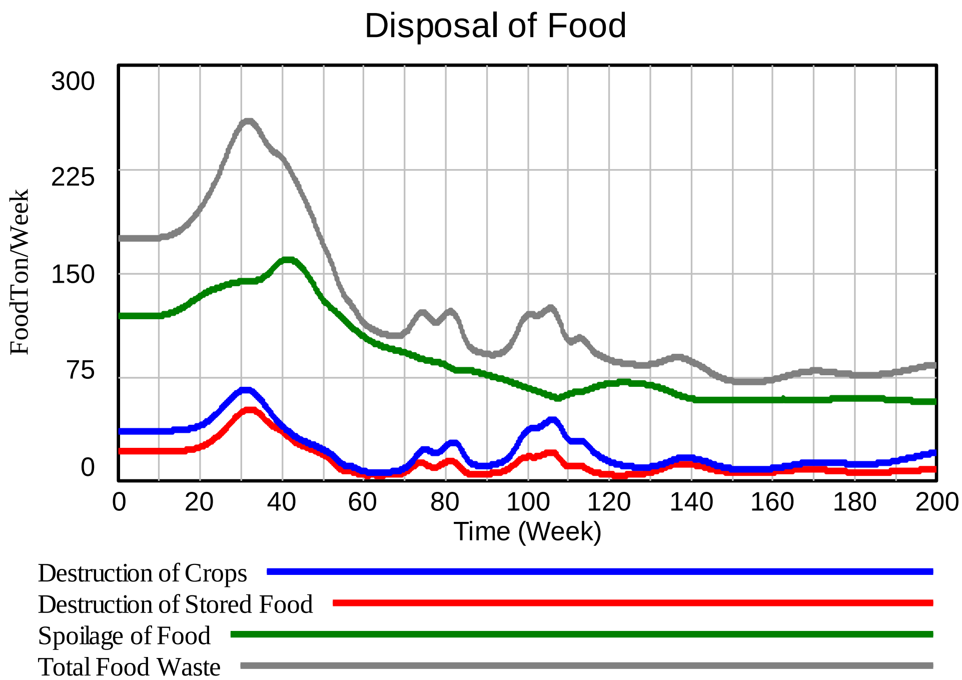


Figure S22. Drop in Bulk Purchasing *and* Consumer Purchasing Power – Disposal and Destruction

Reference to the Appendix

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