Salvador Dali: surrealism - realism feedback (dark reproduction)

Shedding Light on the Harvesting Control Rules in Abstract Bioeconomic Models

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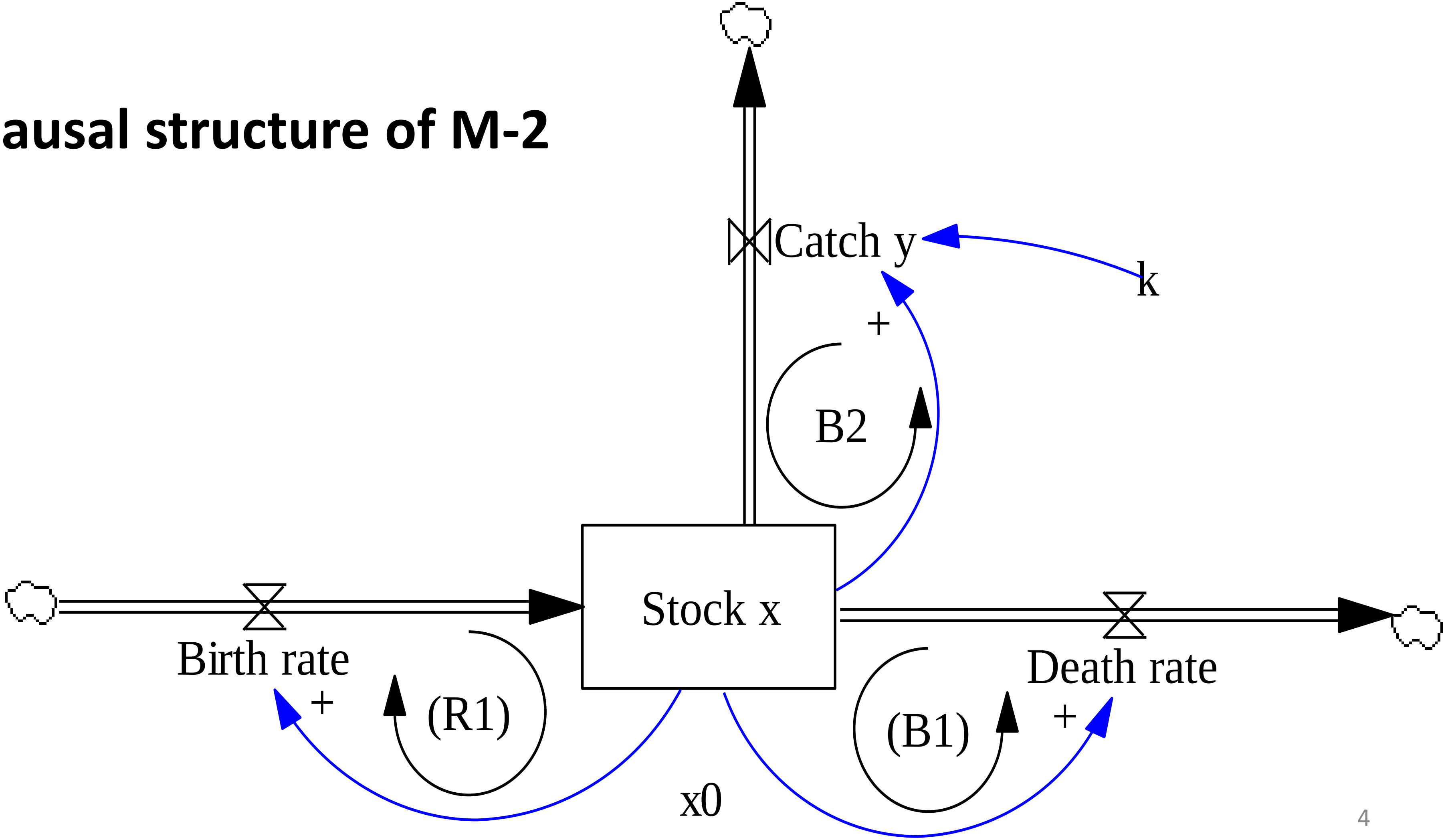
Causal structure of M-2



Properties of M-2 and M-3

Model	Catch y	Stable node x_1	MSY at
M-2	kx	$1-k>0$	$k_s = 0.5$
M-3	mx^2	$1/(1+m) > 0$	$m_s = 1$

$$\dot{x} = x(1-x) - y$$

MSY $y_s = c_s = 0.25, x_s = 0.5$

Mismanagement in M-4 with catch *c* = const can destroy structural stability present in M-1, M-2 and M-3

A birth of the aggravation mode results from the transition from dominant negative feedback $x \xrightarrow{-} \hat{x} \rightarrow \dot{x}$ to dominant positive feedback $x \rightarrow \hat{x} \rightarrow \dot{x}$ at a tipping point, when the sign of

$$\frac{d\hat{x}}{dx} = -1 + \frac{c}{x^2} < 0$$

turns into its opposite. Quite dramatically

$$\frac{d\hat{x}}{dx} \rightarrow \infty \,\mathrm{for}\, x \rightarrow 0.$$

Collapse for catch *c* > MSY in M-4

Proposition 4. Stationary state(s)
for $c = c_s = 0.25\ x_s = 0.5$;
for $c < c_s\ \ x_{1,2} = 0.5 \pm \sqrt{c_s - c}$.
Lower stationary state x_2 is unstable node,
higher stationary state x_1 is stable node.
Proposition 6. Let $c > c_s$.
There is no stationary state.
Exhaustion $x = 0$ occurs.
For ex., if $x_0 = 0.52$, $c = 0.2501\ T_2 \approx 266$.

Collapse for catch c < MSY in M-4

- **Proposition 5. Let $0 < c < c_s$, $x_0 < x_2 < x_s$. There is monotonous decrease in biomass to full exhaustion.**

For ex., if $c = 0.2499$, $x_0 = 0.46 < x_2 = 0.49 < x_1 = 0.51$.
Time left until stock x will be totally depleted $T_1 \approx 24$.

MSY c_s = 0.25 for x_s = 0.5 is critical in M-4

Proposition 7. For $c = c_s$ there is saddle-node bifurcation. The saddle is unstable for $x < x_s$ and stable for $x > x_s$. Depletion of the stock for $x < x_s$ goes on the hyperbolic curve.

Complete extermination of the stock at

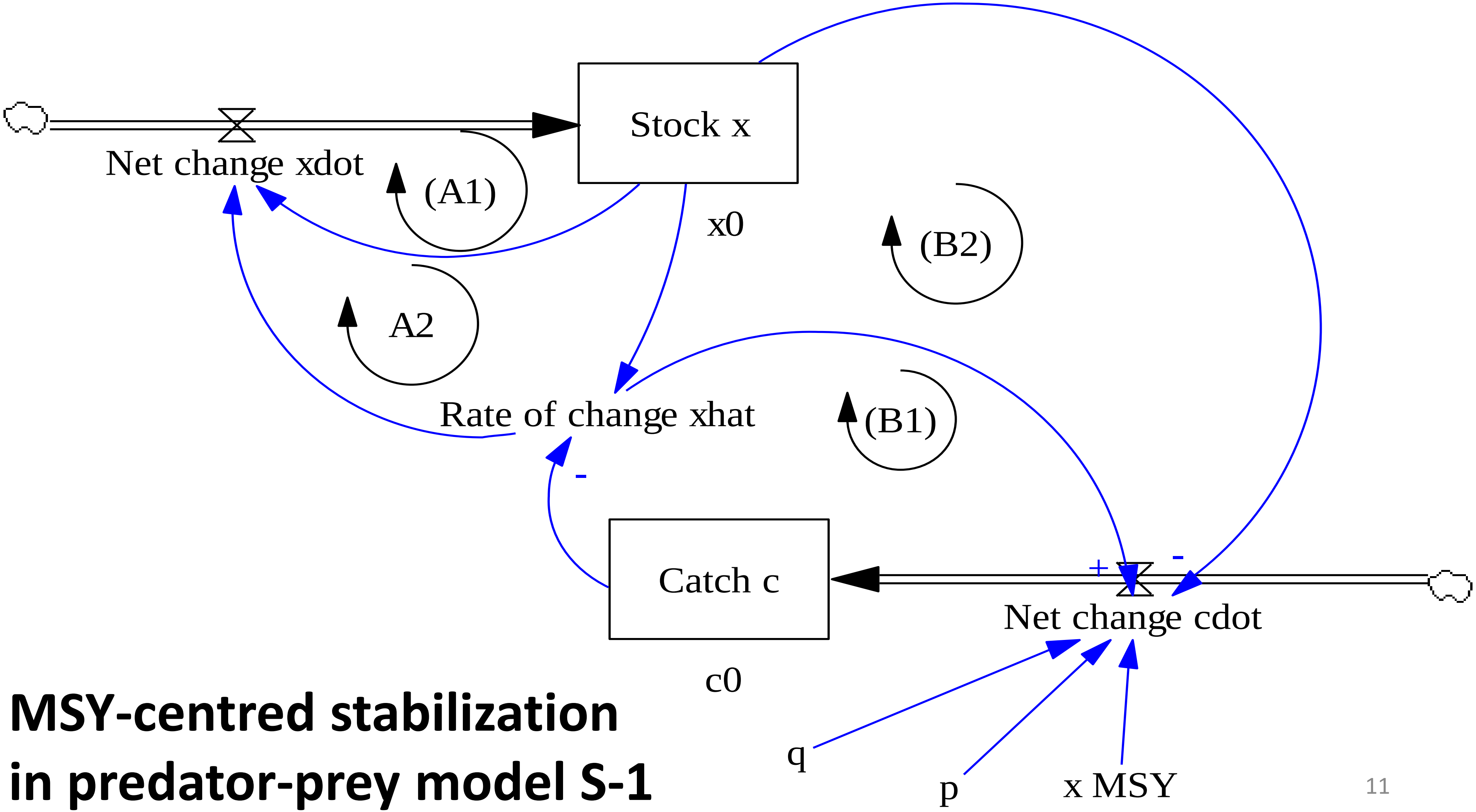
$$T_3 = 2x_0 \frac{1}{0.5 - x_0}.$$

For ex., for $x_0 = 0.2$ $T_3 = 1.33$.

Effects of exponential HCRs on fishery time frame for the same x_0 = 1 and c = 0.2501 in M-4 and M-5

Model	Catch y	Catch growth rate γ	Time left until full depletion
M-4	$y = c =$ const	0	310
M-5	$y = ce^{\gamma t}$	0.02	13
M-5	$y = ce^{\gamma t}$	–0.02	Infinity♠

♠ Fish stock recovers after the initial plunge to a high sustainable level after catch *y* becomes lower than the natural net increment $x - x^2$.



MSY-centred stabilization in predator-prey model S-1

Four feedback loops in S-1

Descendant from M-3

A1 of length 1

$$x \xrightarrow{\quad A \quad} \dot{x}$$

A2 of length 2

$$x \xrightarrow{\quad A \quad} \hat{x} \rightarrow \dot{x}$$

New

B1 of length 2

$$c \xrightarrow{\quad N \quad} \hat{x} \rightarrow \dot{c}$$

B2 of length 4

$$x \rightarrow \dot{c} \rightarrow c \xrightarrow{\quad N \quad} \hat{x} \rightarrow \dot{x}$$

Only negative partial derivative and partial derivative with alternating algebraic sign are explicitly shown with *N* and *A*, respectively.

The predator-prey system of two ODEs with tamed hyperbolic element *c/x* for *p* < 0, *q* > 0

$$\dot{x} = f(x) = x - x^2 - c$$

$$\dot{c} = p(x_s - x) + q\hat{x} =$$

$$= p(x_s - x) + q(1 - x - \frac{c}{x})$$

S-1 has non-trivial stationary state $E_s = (x_s, c_s)$

- *Proposition 10.*

(a) Stationary state E_s is locally asymptotically stable.

(b) E_s is stable node if $0 > p \geq -q^2$;

E_s is stable focus if $p < -q^2 < 0$.

In both cases, it is hyperbolic.

Policy optimization in S-1

Optimization criterion is grasped as cumulative catch *c* under penalty $\delta < 0$ for negative *c*

$$\mathrm{Max}\left(\int_0^T c\,dt + \int_0^T \delta\,dt \right)$$

for the two given ODEs

with $z_0 = (x_0, c_0),\ p = -q^2, T = 10$

initially: $0 \le c_0 = 0 \le 1$ and $1 \le q = 1 \le 3$.

Average results of sustainable HCRs over years 0–10 (k = 0.5 in M-2, m = 1 in M-3, optimal c_0 and p in S-1)

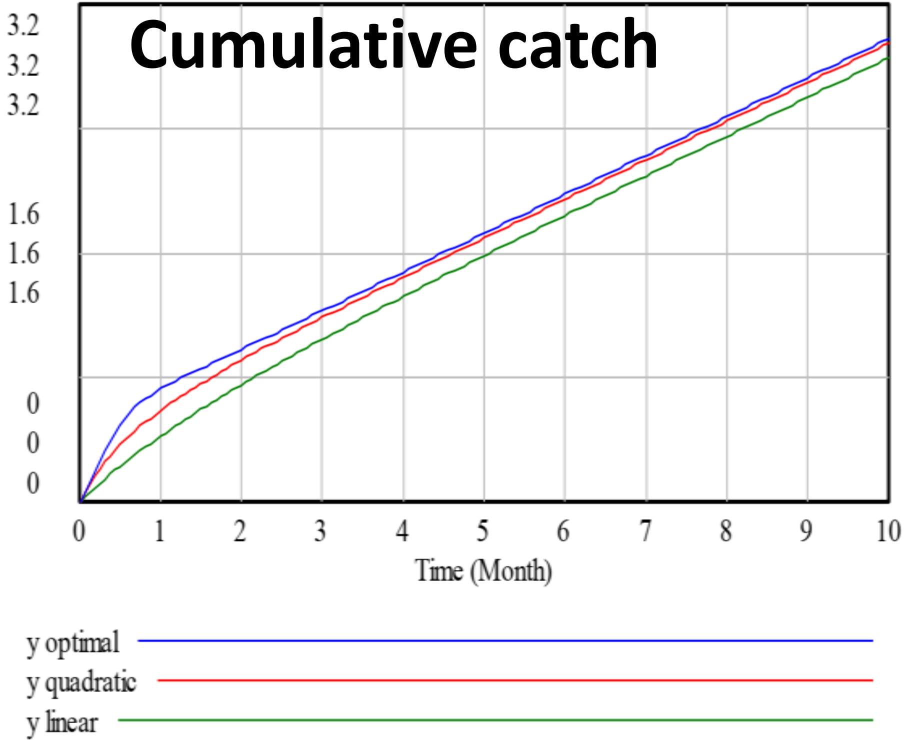
For $x_0 = 1$

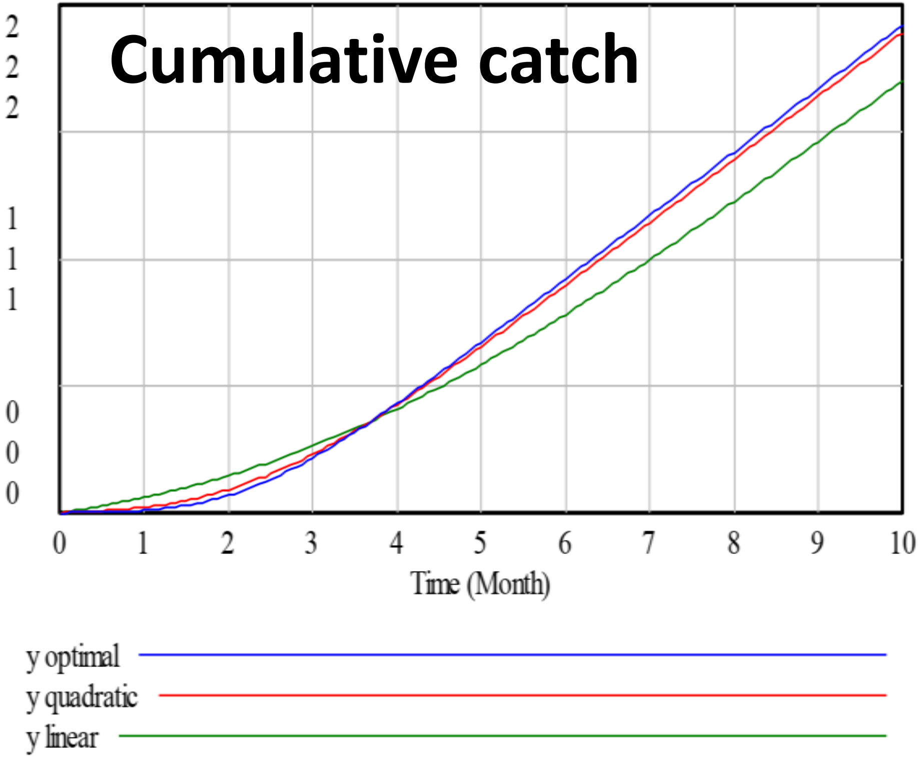
HCR	Stock x	Catch y	Net change $\dot{x}$
Linear	0.57	0.285	–0.051
Quadratic	0.54	0.294	–0.053
Optimal	0.52	0.297	–0.053

For $x_0 = 0.1$

HCR	Stock x	Catch y	Net change $\dot{x}$
Linear	0.341	0.171	0.039
Quadratic	0.419	0.189	0.040
Optimal	0.433	0.192	0.040

Harvesting in M-2 (k = 0.5), M-3 (m = 1) and S-1 (policy optimization) for x_0 = 1 (l.) and x_0 = 0.1 (r.), years 0–10





Conclusion

- **The aggravation regime arises from dominance of the positive feedback connecting biomass *x* and its growth rate in Arnold M-4 (open loop HCR with catch *c* = const).**
- **For reversing decline in fish stock *x*, catch *y* has to become lower than natural net increment $x - x^2 > 0$. Such a reduction promotes ability of depleted stocks to recover from otherwise a dangerously low level.**
- **The policies of improving biomass catch and renewal are elaborated in M-3 (quadratic HCR) and predator-prey S-1 (enhanced HCR); quadratic HCR is more cautious than linear.**
- **Cumulative sustainable catch is raised in S-1 in relation to M-2 (linear HCR) and M-3 (quadratic HCR) due to policy optimization over 10 years. This policy is even more cautious.**

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Salvador Dali: surrealism - realism feedback (sunlit reproduction)

Halldor Laxness (1957)
The Fish Can Sing

The Fish Can Sing is one of Islandic Nobel Prize winner Halldór Laxness's "most beloved novels, a poignant coming-of-age tale marked with his peculiar blend of light irony and dark humor."