Shedding Light on the Harvesting Control Rules in Abstract Bioeconomic Models

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Abstract

The theory of bifurcations and catastrophes is applied to the development of modifications of the Schaefer fishery model. The key variables are the stock of the bioresource, its natural net change, as well as the Man harvesting activity. The global and local analysis reveals quantitative and qualitative characteristics of open or closed loop control. Especially dangerous are the aggravation regimes arising from the dominance of the positive feedback connecting the biomass and the rate of its net change. The equations for excessive or sparing harvesting are derived. The time frames for collapses have been determined. This paper facilitates creation of more complex and realistic bioeconomic models and enhances Harvesting Control Rules.

Key words: renewable resource, depletion, maximum sustainable yield, harvesting control rule, aggravation mode, saddle-node bifurcation, catastrophe theory

Introduction

As well established by system dynamics research over decades, reserves of fish and other resources of flora and fauna, due to their natural reproductive capacity, can grow, contributing to the preservation and increase of natural capital [1, 2]. However, according to the World Bank experts [3, 4], a decrease in the biomass of global fish stocks, as a result of their excessive catch, created a threat to sustainable fishing.

"Global marine fisheries are in crisis. The proportion of fisheries that are fully fished, overfished, depleted, or recovering from overfishing increased from just over 60 percent in the mid-1970s to about 75 percent in 2005 and to almost 90 percent in 2013." [4: 1]. These conclusions are shared by OECD analysts [5] who believe that the current rate of resource utilization is far in excess of what is sustainable in the long run.[1]

[1] Unsustainable management of renewable resources can lead to their permanent depletion in much the same way as the finite extraction of nonrenewable resources. Stagnant or declining (even slightly) catches can accompany a long-term decline in fish stock. If left unchecked, harvesting could destroy the fisheries that would become biologically or commercially extinct over time.

There is a need for a transition to fundamentally more favourable natural-anthropogenic regimes. This transition should be based on in-depth studies of contrasting regimes of ecological and economic interaction based on system-dynamic models, starting with engaging ones such as Fish Banks Game developed by D. Meadows and his colleagues [2]. A great constructive role in clarification of such models and in their further development belongs to the mathematical control theory [6] with strong footage in mathematical analysis and theory of differential equations.

According to the control theory, open-loop control is completely determined at the initial instant t_0; here, the integration of the equation (or equations) of motion for fixed initial conditions defines the phase trajectory $x(t)$ of the states of the system [7]. Closed-loop control (with feedback) assumes the definition of control as a function of phase coordinates and time (ibid.). These concepts have wide theoretical and applied significance for economic theory and economic practice.

To simplify exposition of economics of renewable resources we will keep in mind their rich diversity and consider non-farming fish as their representative. Peculiarities of specific types of these resources are not considered on this stage of investigation.

Then according to existing conventions, biomass is total amount of fish resources, biomass net change is due to natural processes and harvesting by Man. Hereby harvest equals yearly catch. Table 1 lists model variables and their units of measurement. It may be a prompt on variables of differential equations below.

Table 1. The main variables of simplified biomass models

Variable	Notation	Measurement unit
Catch	y, c	fish/year
Fish stock (biomass)	x	fish
Carrying capacity	$1/\alpha$	fish
Birth rate	βx	fish/year
Death rate	$-\beta\alpha x^2$	fish/year
Net change of fish stock	$\dot{x}$	fish/year
The growth rate of fish stock	$\hat{x}$	1/year
The growth rate of catch	$\hat{y}$	1/year

The reader sees that global marine fish stock is considered as a scalar. This permits application of single equation technique akin to methods developed in the research on mineral resources and proved stocks (see references and critique in [12]).

1. Simplified Verhulst – Schaefer – Arnold models

1.1. Verhulst's textbook model M-1

The logistic equation, also known as the Verhulst equation (named for the first time formulated by a Belgian mathematician), originally appeared when considering the model of wild population growth. Denoting by x the population size, by $t \geq 0$ time, the model can be represented by a non-linear autonomous differential equation

$$\dot{x} = \phi(x) = \beta x(1-\alpha x). \tag{1}$$

where parameter β characterizes the potential rate of growth (multiplication) in the absence of intraspecific competition, and α – the reciprocal of the supporting capacity of the environment (that is, the inverse of the maximum possible population size).

Fish hatch (give birth), grow to maturity, lay eggs and die. Fish death rate is the number of fish per year that die from causes other than fish harvesting. Factors of fish population simple growth are depicted on Figure 1.

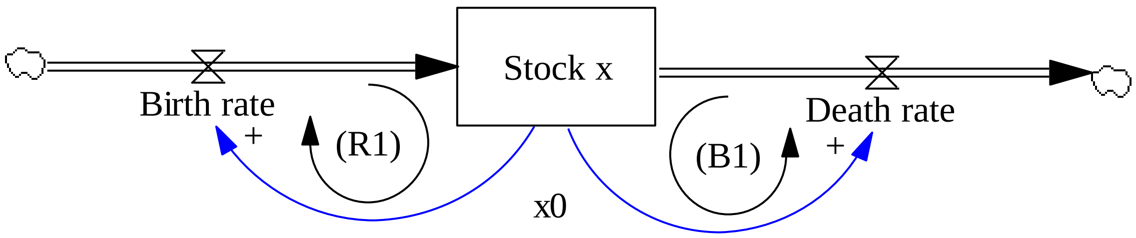


Figure 1 – The Vensim diagram of the Verhulst logistic model M-1

The initial assumptions for the derivation of the equation when considering population dynamics are as follows: the rate of reproduction of the population is proportional to its current level; the second term of the equation reflects intraspecific competition for resources, which limits the growth of the population, or, in plain words, the death rate increases as crowding increases.

The derivative of the natural net change is defined as

$$\phi_x' = \beta - 2\alpha\beta x. \tag{2}$$

When $\phi_x' = 0$, net increment $\phi(x)$ is maximal for $x_s = 1/(2\alpha)$.

The stationary states are found from the condition that the right-hand side of (1) is equal to zero. They differ qualitatively and quantitatively.

On the one hand, $x_1 = 1/\alpha$ is an asymptotically stable node, since $\phi_x'(x_1) = -\beta < 0$, on the other hand, $x_2 = 0$ – unstable node, as $\phi_x'(x_2) = \beta > 0$.

The population growth is S-shaped. Neither open nor closed loop control of the wild population by Man is active. The size of the population tends to dynamic equilibrium at the maximum number that can sustain most of random external shocks except huge calamities. M-1 is structurally stable.

1.2. Simplified Schaefer – Arnold model M-2

The model [8] supplements the assumptions of the logistic growth of biomass by the assumption that human fishing activities reduce the increase in the fish population by the catch amount *y*, the amount of which linearly depends on the available biomass without delay:

$$\dot{x} = f(x) = \beta x(1-\alpha x) - y, \qquad (3)$$

where $y = kx$, $1 > k = \text{const} \geq 0$ (Figure 2).

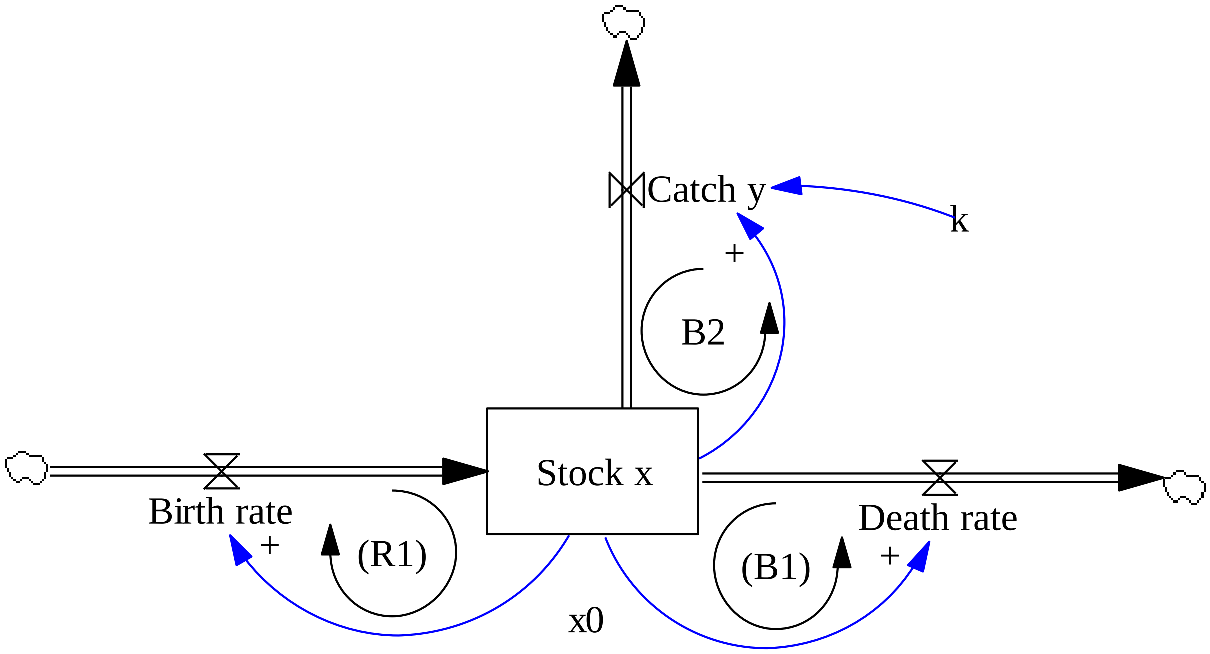


Figure 2 – A causal structure of M-2 without information delay in catch *y*

Table 2. Three feedback loops in M-2

Loops descendant from M-1	New loop
R1 of length 1 Stock $x \rightarrow$ Birth rate	B2 of length 1 Stock $x \rightarrow$ Catch y
B1 of length 1 Stock $x \rightarrow$ Death rate	

Without loss of generality, let $\alpha = 1$ and $\beta = 1$ [9: 98–99]. Then

$$\dot{x} = (1 - k - x)x, \qquad (4)$$

where $x_0 > 0$ for $t_0 = 0$, $f_x{}'(x) = 1 - k - 2x$.

For the linear harvesting control rule (HCR) there is a negative linear dependence of the growth rate of the stock on its magnitude like in M-1 (for $k = 0$):

$$\hat{x} = 1 - k - x. \tag{5}$$

The rate of change in catch and stock is, contrary to M-1, the same:

$$\hat{y} = \hat{x} = 1 - k - y/k. \tag{6}$$

Let us consider the properties of stationary states more closely. The first of these is a stable node $x_1 = 1 - k > 0$ as $f_x'(x_1) = k - 1 < 0$. The value of x_1 smoothly depends on control parameter k, the latter's changes in the specified boundaries do not affect the established mode qualitatively. The necessary and sufficient condition for dominance of the negative feedback $x \xrightarrow{-} \hat{x} \to \dot{x}$ is fulfilled in M-2 $\frac{d\hat{x}}{dx} = -1 < 0$. This property gives the equilibrium global asymptotic stability.

In addition, there is a second steady state $x_2 = 0$. It is unstable node since $f_x'(x_2) = 1 - k > 0$.

If $x_0 > 0$, $x_1 = 1 - k$, a solution to (6) is

$$x = \frac{x_1 x_0}{x_0 - e^{-(1-k)t}(x_0 - x_1)}. \tag{7}$$

This formula generalizes its particular case (1) for $k = 0$, $y = 0$ in M-1. Similarly to the former, M-2 is structurally stable for $0 \le k < 1$.

Next equation determines catch

$$y = \frac{y_1 y_0}{y_0 - e^{-(1-k)t}(y_0 - y_1)}. \tag{8}$$

Proposition 1. For $t \to \infty$ $y \to y_1 = k(1-k)$. Maximum sustainable catch MSY $y_s = c_s = 0.25$ is achieved at $k = k_s = 0.5$, when the biomass volume is determined by the conditions of a stable node $x_1 = x_s = 1 - k_s = 0.5$.

The general biological overexploitation of fish stocks beyond the biomass level corresponding to MSY inevitably leads to subsequent reduction in catches. Timely reduced fishing efforts can allow depressed fish stocks to recover.

As pointed out in [10: 6], “from a biological point of view the concept of MSY is simply not sufficient. Nevertheless, it should be stressed that it provides a valuable rough index of production potential. As a first rough cut at management policy for major commercial species, MSY is probably acceptable. But ones the level of MSY is attained, it should be expected that it may not be sustained.”

The simplified Schaefer – Arnold models represent social production in very (if not extremely) abstract form. They are enhancement for thought experiments on the rocky and hard way from abstract to concrete. Still teaching experience demonstrates that these models strongly stimulate interest of students and newcomers to the system dynamics field discovering how their mathematical knowledge can be applied for better understanding of acute – local and global – sustainability issues.

In more complex models, this deficiency is overcome by explicitly taking into account the goals of capitalist production and the methods for achieving them [1, 2–5, 11–13]. For economic reasons, economic entities that maximize profits and / or rents tend to choose $k \neq k_s$. Technological capabilities, property relations, as well as features of competition, narrow the boundaries of the choice of k and y.

According to [4], "stocks are defined as fully or overfished if their biomass is at or below the level that supports maximum sustainable yield (MSY). Maximum economic yield (MEY), which maximizes the sustainable net benefits flowing from the stocks, occurs at a stock size that is larger than that at MSY level."

Harvesting control rule in M-2 is not satisfactory from the control theory standpoint. It can be dangerous for low stock x to harvest it with rate kx if random fluctuations are taken into consideration. It violates to an extent – dangerous under some typical circumstances – the precautionary principle in the renewable resources management.

1.3. Modified Schaefer – Arnold model M-3

Guided by the precautionary principle the author transforms M-2 into M-3, preserving the logistic natural increase in the bioresource (1), but replacing the linear dependence of the catch on stock by a quadratic one:

$$y = mx^2, \tag{9}$$

where $m \geq 0$.

The net increase in biomass is now defined as

$$\dot{x} = f(x) = x[1 - (1 + m)x], \tag{10}$$

where $x_0 > 0$ for $t_0 = 0$, $f_x'(x) = 1 - 2(1 + m)x$.

Similarity to M-1 and M-2, for quadratic HCR the growth rate of the stock has a negative linear dependence on its positive magnitude as well

$$\hat{x} = 1 - (1+m)x. \tag{11}$$

However, now the product $(1 + m)x$ has replaced the algebraic sum $k + x$ in (5). Figure 3 presents causal-loop structure of M-3 that is very similar to that of M-2 (Figure 2).

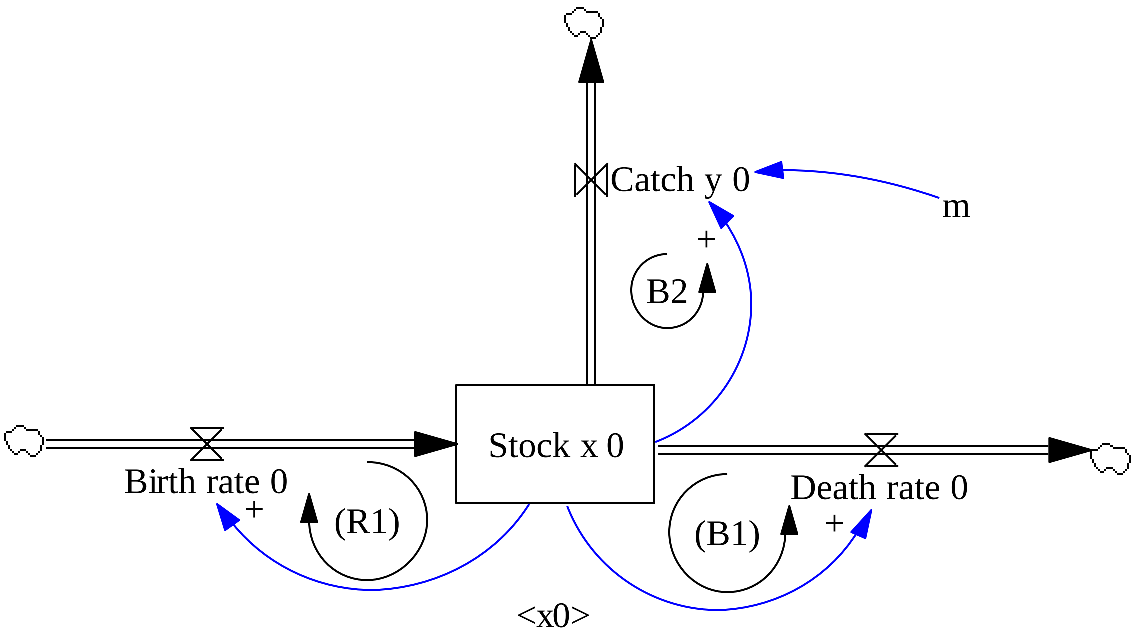


Figure 3 – A causal structure of M-3 without information delay in catch *y* 0

Still unlike M-2, the rate of increase in catch is now twice the rate of increase in stock. This brings about a non-linear negative dependence of this rate on the catch magnitude:

$$\hat{y} = 2\hat{x} \;=\; 2 - 2(1+m)\sqrt{\frac{y}{m}}\,. \tag{12}$$

The negative derivative $\dfrac{d\hat{x}}{dx} = -\,(1 + m) < 0$ guarantees the dominance of the stabilizing negative feedback $x \xrightarrow{-} \hat{x} \to \dot{x}$. The dominant negative feedback is deeper in M-3 than that in M-2. This deepening accelerates restoration of equilibrium after disturbance. Thanks to this deepening the threat of overshooting which is present for $k \geq 1$ in M-2 has disappeared in M-3 that is more structurally stable than M-2.
Let us consider stationary states for $m \geq 0$ explicitly.

Proposition 2. The system has two equilibrium states. One of them is a stable node $x_1 = 1/(1 + m) > 0$, because $f_x'(x_1) = -1 < 0$. The other is unstable node $x_2 = 0$, because $f_x'(x_2) = 1 > 0$.

Corollary. Stock *x* depends smoothly on control parameter *m*, the latter's changes do not affect the steady state qualitatively.

The solution to (9) is the logistic function

$$x = \frac{x_1 x_0}{x_0 + e^{-t}(x_1 - x_0)}\,. \tag{13}$$

This formula is valid for the Verhulst model (1) above, as a special case, in which $m = 0$.

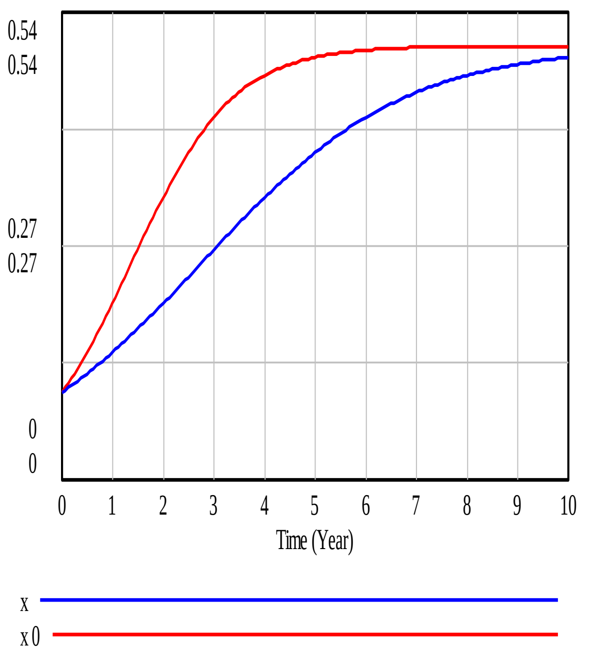
The catch is given by

$$y = \frac{y_1}{[1 + e^{-t}\left(\sqrt{\frac{y_1}{y_0}} - 1\right)]^2}. \tag{14}$$

Proposition 3. Catch $y \to y_1 = \frac{m}{(1+m)^2}$ for $t \to \infty$. Maximum sustainable yield (catch) $y_s = \frac{m_s}{(1+m_s)^2} = 0.25$ is achieved at $m_s = 1$ and $x_1 = x_s = 1/(1 + m_s) = 0.5$.

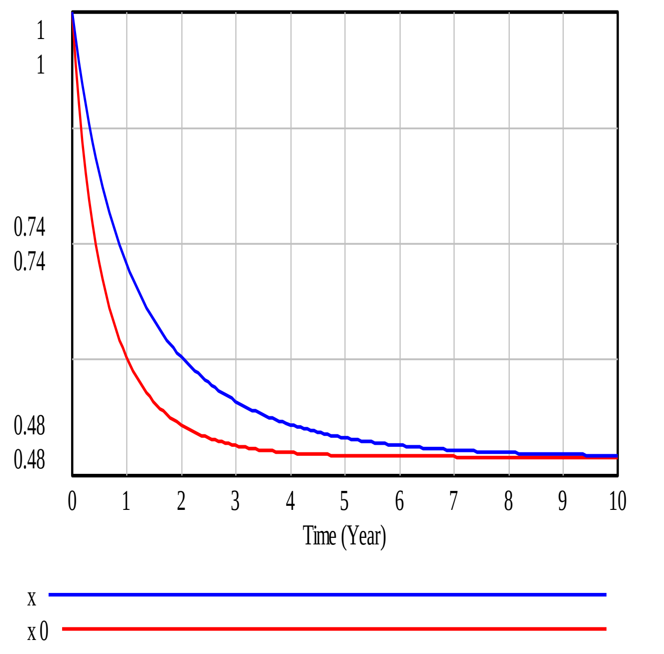
For low x_0, the integral (cumulative) catch in M-3 is lower than that in M-2 over short segments (few years) and higher when integrating over longer segments of 5–10 years (Figure 4).

Initial stock x_0 = 0.1

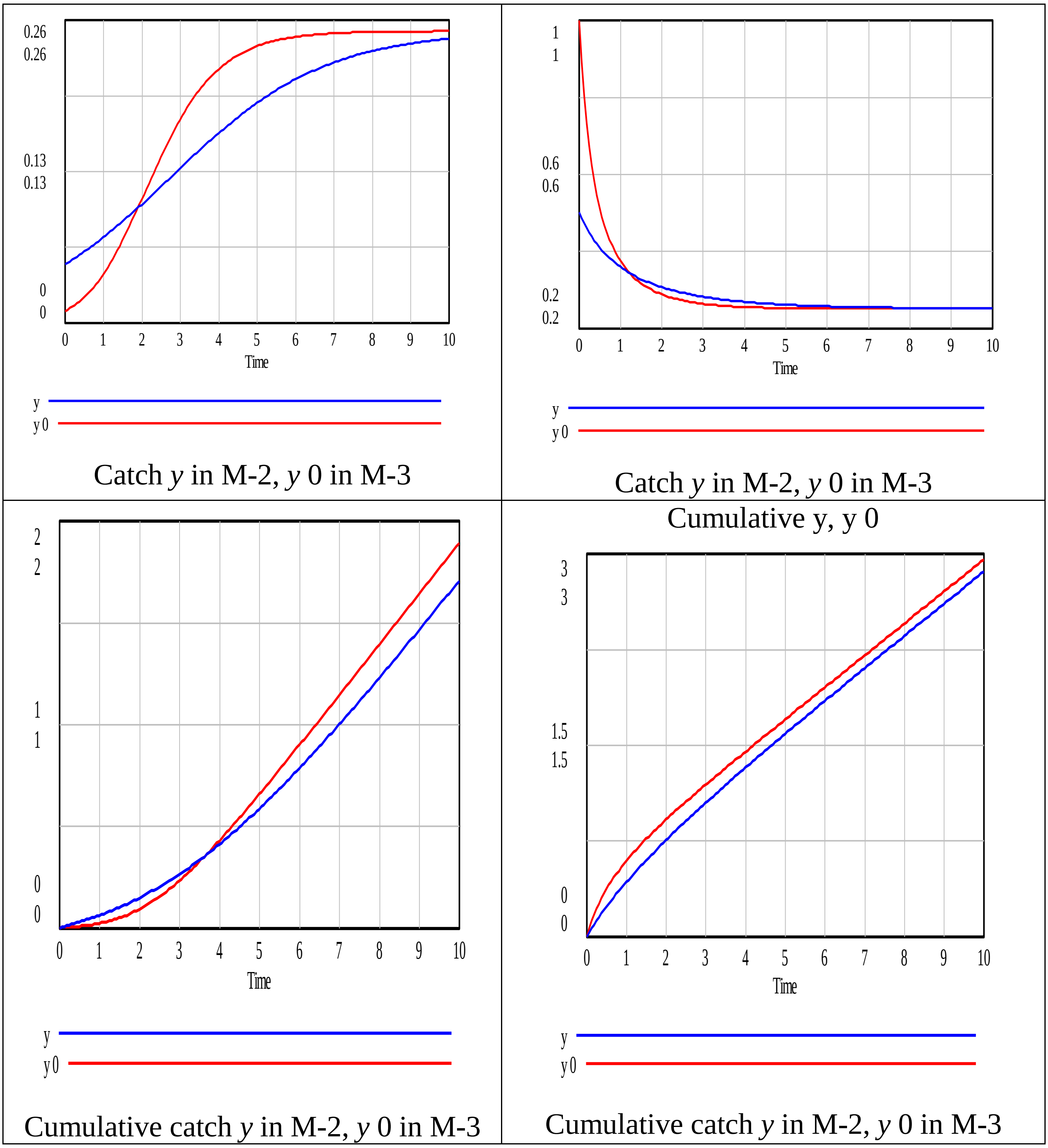


Stock *x* in M-2, *x* 0 in M-3

Initial stock x_0 = 1



Stock *x* in M-2, *x* 0 in M-3



Catch y in M-2, y 0 in M-3

Catch y in M-2, y 0 in M-3

Cumulative catch y in M-2, y 0 in M-3

Cumulative catch y in M-2, y 0 in M-3

Figure 4 – Stock, catch and integral (cumulative) catch under two HCRs (k = 0.5, m = 1, respectively) over 0–10 years for x_0 = 1 on the left and for x_0 = 0.1 on the right, blue curves – for linear HCR, red curves – for quadratic HCR

Table 3. Average stock, catch and net change of stock under three harvesting control rules for x_0 = 1 over 0–10 years

HCR	Stock x	Catch y	Net change $\dot{x}$
Linear	0.570	0.285	–0.051
Quadratic	0.536	0.294	–0.053
Enhanced in S-1	0.516	0.297	–0.053

Table 4. Average stock, catch and net change of stock under three harvesting control rules for $x_0 = 0.1$ over 0–10 years

HCR	Stock x	Catch y	Net change $\dot{x}$
Linear	0.341	0.171	0.039
Quadratic	0.419	0.189	0.040
Enhanced in S-1	0.433	0.192	0.040

Transition from linear to quadratic HCR strengthens precaution in nature management, due to its adherence to longer-term efficiency, which helps overcome "quarterly capitalism", aimed at immediate profit.

2. Aggravation modes and catastrophes in Arnold's model M-4

The author uses the concept of aggravation modes, investigated in different contexts, in particular, in [4, 11–12, 14, 15].

Let the natural increase in the biological resource be determined as before, whereas the catch y is redefined as constant c [9: 98]. Then open loop control determines HCR in M-4, whereas the net increase in stock is given as

$$\dot{x} = f(x) = x - x^2 - y = \phi(x) - c, \tag{15}$$

where $c > 0$, $x_0 > 0$, $f_x'(x) = 1 - 2x$.

The analysis reveals non-linear dependence of the rate of growth of the stock on itself $\hat{x} = \dfrac{\dot{x}}{x} = 1 - x - \dfrac{c}{x}$, where the last hyperbolic element is potent of an aggravation mode. Indeed, $\hat{x} \to -\infty$ for $x \to 0$.

A birth of the aggravation mode results from the transition from dominant negative feedback $x \xrightarrow{-} \hat{x} \to \dot{x}$ to dominant positive feedback $x \to \hat{x} \to \dot{x}$ at a tipping point, when the sign of $\dfrac{d\hat{x}}{dx} = -1 + \dfrac{c}{x^2} < 0$ turns into its opposite. Quite dramatically $\dfrac{d\hat{x}}{dx} \to \infty$ for $x \to 0$.

The conditions for emergence of an aggravation (exacerbation) regime and its unfolding are described in detail below. This aggravation mode arises when mismanagement in M-4 destroys structural stability present in M-1, M-2 and M-3.

For brevity, the author defines auxiliary parameter

$$a = \sqrt{|c_s - c|}\,. \tag{16}$$

Proposition 4. The stationary state for $c = c_s = 0.25$ and $a = 0$ is $x_s = 0.5$. The stationary states for $c < c_s$ are defined as

$$0 < x_{1,2} = 0.5 \pm a. \tag{17}$$

A lower stationary state $x_2 = \frac{1}{2} - a$ is an unstable node, since $f_x'(x_2) = 2a > 0$, while the higher stationary state $x_1 = \frac{1}{2} + a$ is a stable node, since $f_x'(x_1) = -2a < 0$.

Maximum catch $y_s = c_s = 0.25$ requires stock x_s. The quantity c_s is critical, or bifurcational: jump-like changes in dynamic regimes are generated by infinitesimal changes in this parameter's magnitude in M-4.

Proposition 5. Let $0 < c < c_s$ and $x_0 < x_2 < x_s$. There is a monotonous decrease in biomass down to complete exhaustion; a solution to (15) is

$$x = \frac{x_2 - x_1 \frac{x_2 - x_0}{x_1 - x_0} e^{2at}}{1 - \frac{x_2 - x_0}{x_1 - x_0} e^{2at}}. \tag{18}$$

Exhaustion $x = 0$ occurs at the moment

$$T_1 = \frac{1}{2a} \ln \left[\left(\frac{x_1 - x_0}{x_1} \right) \left(\frac{x_2}{x_2 - x_0} \right) \right]. \tag{19}$$

For example, if $c = 0.2499$, $a = 0.01$, $T_1 = 23.54$ for $x_0 = 0.46 < x_2 = 0.49 < x_1 = 0.51$.

Proposition 6. Let $c > c_s$. There is no stationary state. There is a monotonous decrease in available biomass up to its elimination; a solution to (15) is

$$x = x_s + \frac{a^2 tg(-at) + a(x_0 - x_s)}{a - tg(-at)(x_0 - x_s)}. \tag{20}$$

Exhaustion $x = 0$ occurs at the moment

$$T_2 = \frac{1}{a} \left[arctg\left(\frac{x_0 - x_s}{a} \right) + arctg\left(\frac{x_s}{a} \right) \right]. \tag{21}$$

For example, for $x_0 = 0.52$ and $c = 0.2501$, $T_2 = 265.81$.

It is easy to see that both stationary states merge into one x_s if $a = 0$. There is a catastrophic change in the system's regime in response to a smooth change of this control parameter.

Proposition 7. For $a = 0$, a saddle-node bifurcation takes place. This saddle-node state is unstable for $x < x_s$ and is stable for $x > x_s$.

Proof. The necessary and sufficient conditions for the saddle-node bifurcation are fulfilled [16: 84–84]: the fusion of the nodes with the conversion into the saddle is confirmed by the inversion of the derivative at the critical point to zero $f_x'(x_s,c_c)=1-2x_s=0$ in the absence of degeneracy in it, $f_x''(x_s,c_s) = -2 \neq 0$, and it is additionally supported by transversality condition $f_c'(x_s,c_s) = -1 \neq 0$ satisfied.

For the lower (unstable) branch of solutions $x < x_s$ the derivative $f_x'(x,c_c)=1-2x>0$, whereas for the upper (stable) branch of solutions $x > x_s$ the derivative $f_x'(x,c_c)=1-2x<0$. In other words, x_s is an attractor for $x > x_s$ and a repeller for $x < x_s$.

If $a = 0$ and $x_0 < x_s$, depletion of the resource occurs on the hyperbolic curve; a solution to (15) is shaped as

$$x = x_s + \frac{1}{t + \frac{1}{x_0 - x_s}}. \tag{22}$$

Complete extermination of the bioresource occurs at the moment

$$T_3 = 2x_0 \frac{1}{0.5 - x_0}. \tag{23}$$

For example, for $x_0 = 0.2$ and $a = 0$ $T_3 = 1.33$.

Proposition 8. A steady steady-state regime with attractor x_1 dies, colliding with an unstable regime with a repeller x_2, and at the moment of collision, the convergence rate is infinite.

Proof. For $c \to c_s$ there are $\frac{\partial x_1}{\partial c} = -\frac{1}{2a} \to -\infty$ and $\frac{\partial x_2}{\partial c} = \frac{1}{2a} \to \infty$.

The author would like to turn the reader attention to the notion of characteristic return time T_r near a threshold [17, 18]. This notion rests, in my view, on the concept of the first order delay in the system dynamics literature.

Proposition 9. For $x > x_2$ and values of the parameter c, increasingly close to the critical value c_s, the delay time T_r, which characterizes the asymptotic approximation of x to a so far stable node, increases unlimitedly.

The proof for the present case follows a more general proof [17, 18: 244]. Near x_1 stock adjustment process of the first order takes place

$$\dot{x} = \lambda(x_1 - x), \tag{24}$$

where $\lambda = \frac{1}{T_r} = 2a$.

As a result of integration (24), I obtain

$$x = x_1 - (x_1 - x_0)e^{-\lambda t}. \tag{25}$$

For $c \to c_S$, $\lambda \to 0$ holds, therefore unbounded growth of the delay in overcoming the initial deviation of x from the attractor x_1 happens: $T_r \to \infty$. Thus, the characteristic return time T_r stretches to infinity near threshold c_S of the parameter c.

Let us explain this important aspect. The excessively growing time interval required to eliminate imbalances can be a precursor for a catastrophe.[2] Catastrophic regimes are possible even with a catch locally close to MSY c_S with a minor excess.

The collapse will happen relatively faster if the author assumes that the catch is more or less steady growing in M-5. On the contrary, timely and accurate reduction of catch allows avoiding collapse. Table 5 and Figure 5 illustrate this principal difference in the evolutionary patterns in M-4 and in M-5. Notice that M-5 has the same equations and parameters as M-4 except those that directly affect catch y.

Table 5. Effects of HCR on fishery time frame for the same x_0 = 1 and c = 0.2501 in M-4 and M-5

Model	Catch y	Catch growth rate γ	Time left until full depletion of stock x
M-4	$y = c = const$	0	310.2
M-5	Growing $y = ce^{\gamma t}$	0.02	12.65
M-5	Declining $y = ce^{\gamma t}$	–0.02	Infinity

[2] In other bioeconomic models, saddle-node bifurcations and hysteresis occur with the initial presence of three rather than two, as in our case, stationary states. Examples are [19–21]. They have properties similar to those presented in the eighth and ninth Propositions.

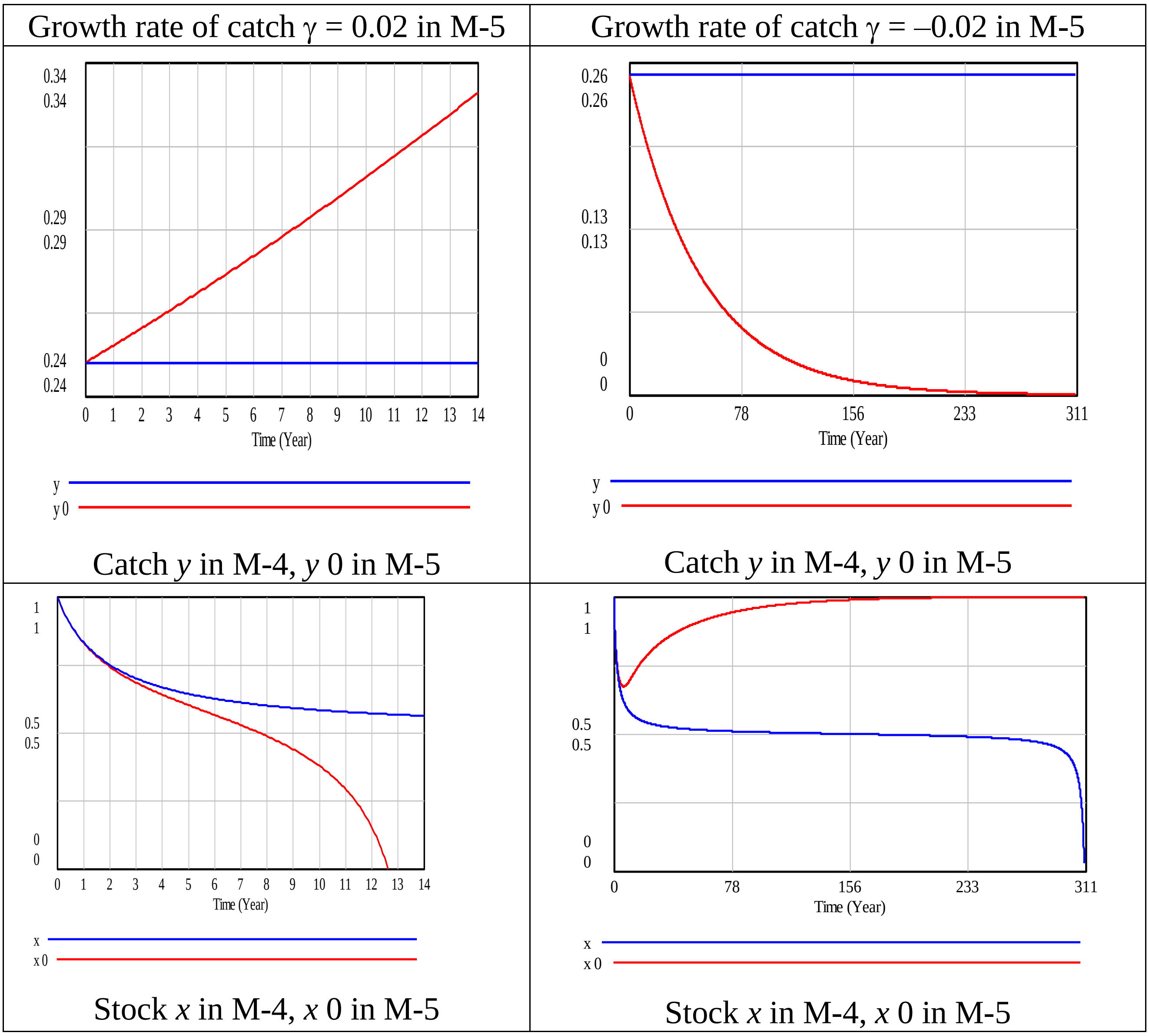


Figure 5 – Catch y and stock x under HCRs for $x_0 = 1$: blue curves for constant harvest in M-4 with collapse at $t = 311$, red curves for exponentially increasing catch on the left with stock depletion at $t = 12.65$ and exponentially declining catch with stock recovery on the right

Being severely or moderately harvested, fish stock x is completely depleted at very different moments separated by roughly three hundred years (in the year 12.65 in M-5 or in the year 311 in M-4). Fish stock is allowed to recover after the initial plunge to a high sustainable level if catch is reduced exponentially in M-5.

The increased overfishing makes collapse in fisheries closer and faster. To reverse the decline in fish stocks, catch y has to become lower than the natural net increment $x - x^2 > 0$. Such a reduction promotes the ability of depleted stocks to recover from otherwise a dangerously low level.

The rigorous mathematical control theory maintains these rather clear sustainability rules and assists in their further development. The author proceeds to a quite more elaborated closed loop control than developed by my predecessors in M-2 and even than newly proposed for M-3 in this paper earlier. HCRs will be upgraded as well.

3. MSY-centred stabilization in predator-prey model S-1

An appropriate stabilization policy for proved mineral reserves for avoiding their depletion has been proposed in the system dynamics literature [12]. This section elaborates a stabilization policy for a renewable resource that improves economic efficiency and maintains bioresource sustainability in the middle- and long-term.

The recent World Bank and FAO studies have identified lack of prudent control as one of the main factors detrimental for the global renewable resources [3, 4]: "…the state of governance worldwide varies greatly and, despite some encouraging successful approaches, is in dire need of improvement [4: 17]."

3.1. Enhancing harvesting control rule

Transforming the previous models into a predator-prey model is the necessary step for designing more reliable and efficient HCR than considered above. Catch *c* becomes the new phase variable in addition to stock *x*.

The urgent question arises: how do we turn the hyperbolic element *c*/*x* as a foe of sustainability under open loop control as in M-4 or M-5 in its champion? In real life similar transformation of certain quality into the opposite are ubiquitous: for instance, an explosive gas that destroys homes accidentally faster than a blink of eye can be thoughtfully applied for home construction purposes instead.

The new phase variable *c* has become a subject of proportional and derivative control as Table 6 and Figure 6 demonstrate. Thereby catch *c* is targeted at MSY c_s, and stock *x* is targeted at corresponding optimal level x_s.

Table 6. Four feedback loops in S-1

Loops descendant from M-3	New loops
A1 of length 1 $x \xrightarrow{A} \dot{x}$	B1 of length 2 $c \xrightarrow{N} \hat{x} \rightarrow \dot{c}$
A2 of length 2 $x \xrightarrow{A} \hat{x} \rightarrow \dot{x}$	B2 of length 4 $x \rightarrow \dot{c} \rightarrow c \xrightarrow{N} \hat{x} \rightarrow \dot{x}$

Note. Only a negative partial derivative and partial derivative with alternating algebraic sign are explicitly shown with *N* and *A*, respectively, immediately above arrows. All other first partial derivatives are positive.

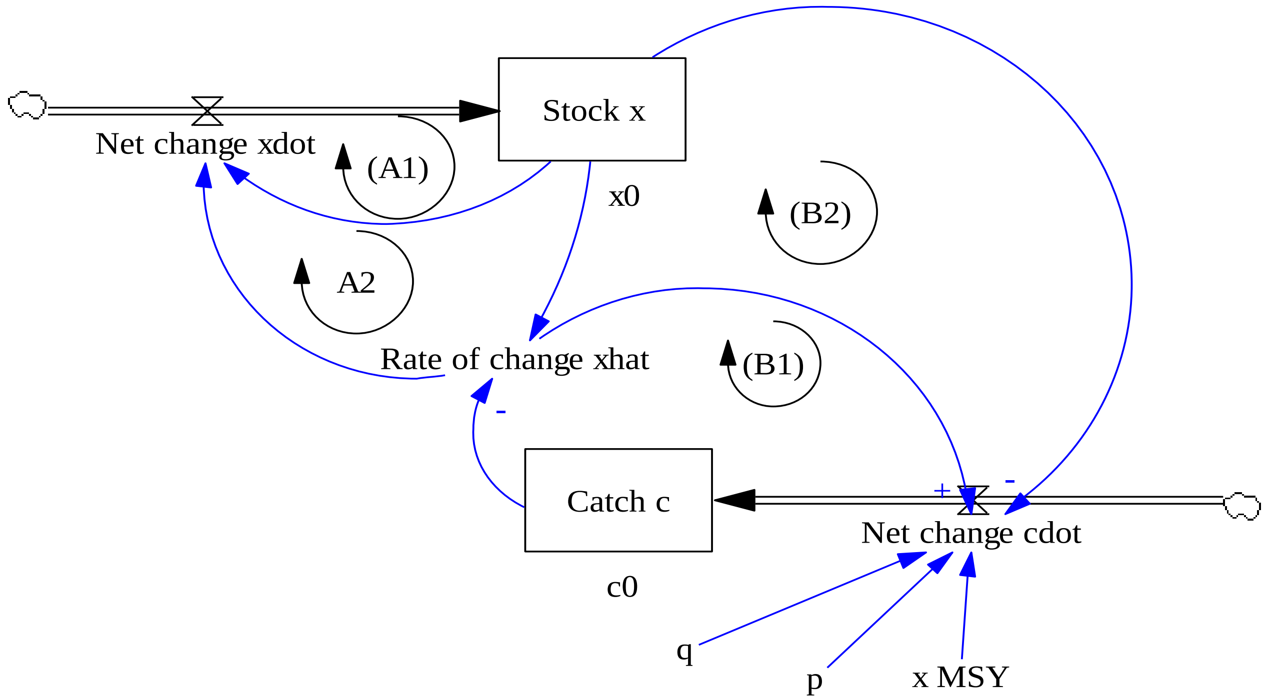


Figure 6 – A condensed causal loop structure of S-1; total number of feedback loops – 4, among them: 1st order – 3 (1 – negative, 2 – alternative), 2nd order – 1 (negative)

The positive feedback $x \rightarrow \hat{x} \rightarrow \dot{x}$ is transformed into one with alternating polarity A2 of length 2 (Table 6) that is much safer indeed. A mathematical analysis that follows maintains this expectation.

The above causal loop diagram is the basis for the 2nd order system of ODEs

$$\dot{x} = f(x) = x - x^2 - c \tag{26}$$

$$\dot{c} = p(x_s - x) + q\,\hat{x} = p(x_s - x) + q(1 - x - \frac{c}{x}), \tag{27}$$

where $p \leq 0$ and $q \geq 0$.

For this system the Jacobi matrix is defined as

$$J_{S\text{-}1} = \begin{array}{|c|c|} \hline 1 - 2x & -1 \\ \hline -(p+q) + q\dfrac{c}{x^2} & -q\dfrac{1}{x} < 0 \\ \hline \end{array}\ . \tag{28}$$

The reader sees S-1 can belong to predator (c) – prey (x) models whenever $\frac{\partial \dot{c}}{\partial x} > 0$ as $\frac{\partial \dot{x}}{\partial c} < 0$ is always satisfied. There is predator intra-specific competition as $\frac{\partial \dot{c}}{\partial c} < 0$. Preys co-operate with each other if $x < 0.5$ and $\frac{\partial \dot{x}}{\partial x} > 0$ or compete with each other if $x > 0.5$ and $\frac{\partial \dot{x}}{\partial x} < 0$, a neutral case is for $x = x_s = 0.5$.

The above system has the non-trivial stationary state:

$$E_s = (x_s, c_s) \tag{29}$$

with corresponding Jacoby matrix

$J_s =$

0	–1
$-p > 0$	$-2q < 0$

. (30)

Proposition 10 (a). The stationary state E_s (29) is locally asymptotically stable.

Proposition 10 (b). If $0 > p \geq -q^2$ the stationary state E_s (29) is stable node; if $p < -q^2 < 0$ it is stable focus. In both cases, it is hyperbolic.

Proof (applying the Routh–Hurwitz stability criterion).
The necessary and sufficient conditions for asymptotically local stability of (29) are satisfied:

$$|J_s| = -p > 0 \qquad (31)$$

as $p < 0$ and

$$Trace(J_s) = -2q < 0 \qquad (32)$$

as $q > 0$.

For gaining additional information consider a characteristic equation that is written as

$$\lambda^2 + 2q\lambda - p = 0. \qquad (33)$$

It has one real root or two roots

$$\lambda_{1,2} = -q \pm \sqrt{q^2 + p}\,. \qquad (34)$$

For having one negative real root $\lambda_{1,2} = -q$ there must be

$$p = -q^2. \qquad (35)$$

Roots (34) are real and negative if $p > -q^2$. They are complex-conjugate with a negative real part if $p < -q^2$.

A stable focus arises for $p < -q^2$ with

$$\lambda_{1,2} = -q \pm i\sqrt{-(q^2 + p)}\,. \qquad (36)$$

The period of converging fluctuations is then

$$T_c \approx \frac{2\pi}{\sqrt{-\left(q^2 + p\right)}}\,. \qquad (37)$$

The author has proved that at $q = 0$ sufficient requirements for Andronov – Hopf bifurcation are satisfied. Yet this case in not immensely relevant for selecting appropriate HCL and it is skipped therefore.

Explicit solutions to the linearized at the stationary state system are different for two distinct negative $\lambda_{1,2}$, on the one hand, and for negative $\lambda_1 = \lambda_2 = -q$ for $p = -q^2$, on the other hand.

3.2. Policy optimization

The author has carried out two parameters policy optimization for c_0 and q for one stable node with $p = -q^2$. The optimization criterion is mostly grasped as cumulative catch over 0–T years. Besides this, Penalty for negative c is added in Pay-off:

$$\text{Penalty} = \int_0^T \delta dt \tag{38}$$

where $\delta = 0$, if $c \geq 0$, $\delta = -100$, if $c < 0$.

Formally, this policy optimization in Vensim is based on a restricted dynamic optimization problem:

$$\text{Max}\left(\int_0^T cdt + \int_0^T \delta dt\right), \tag{39}$$

subject to (26) and (27)

with $z_0 = (x_0, c_0)$,

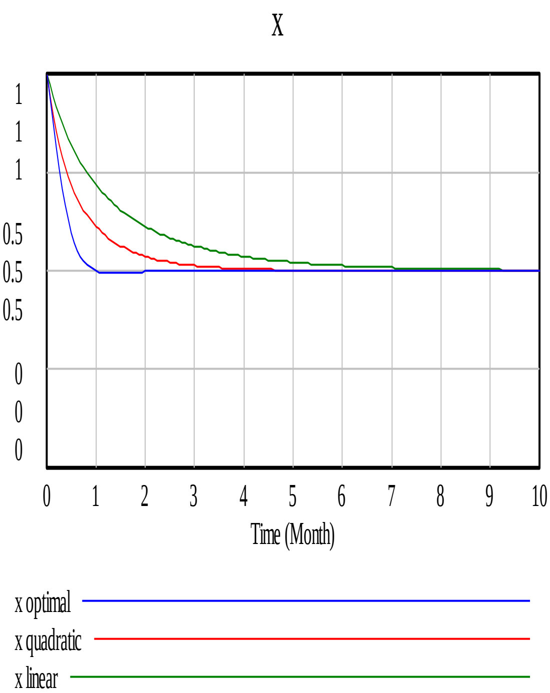
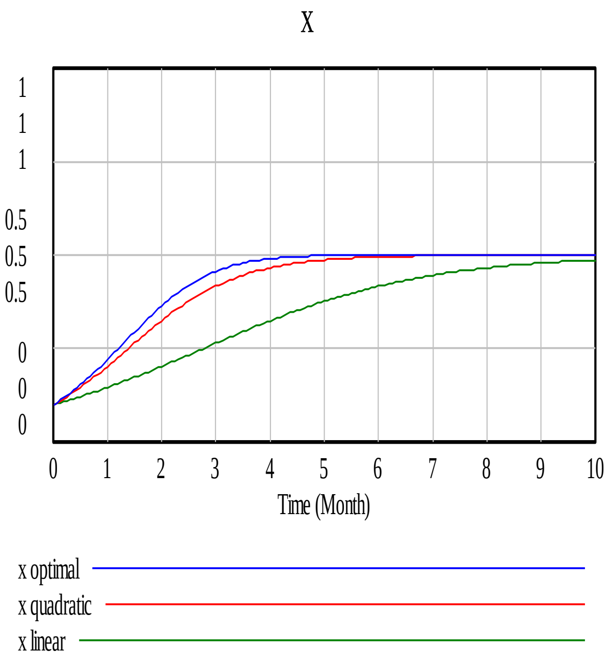
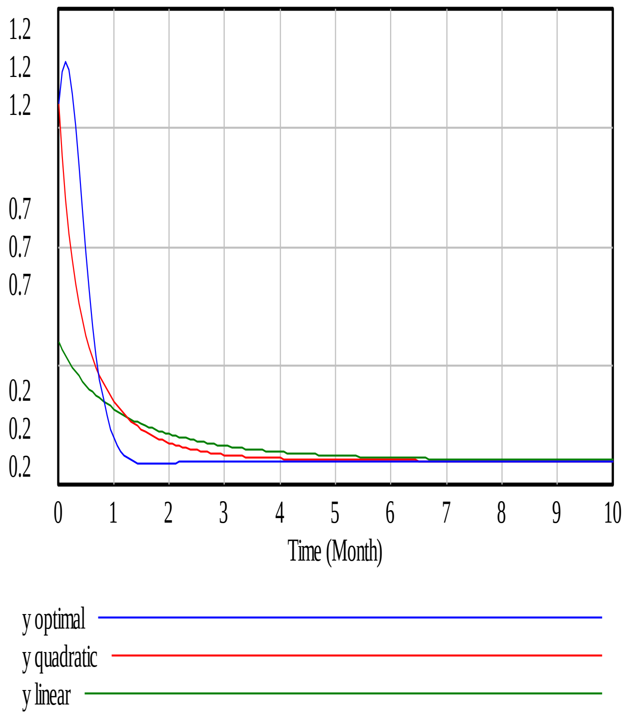
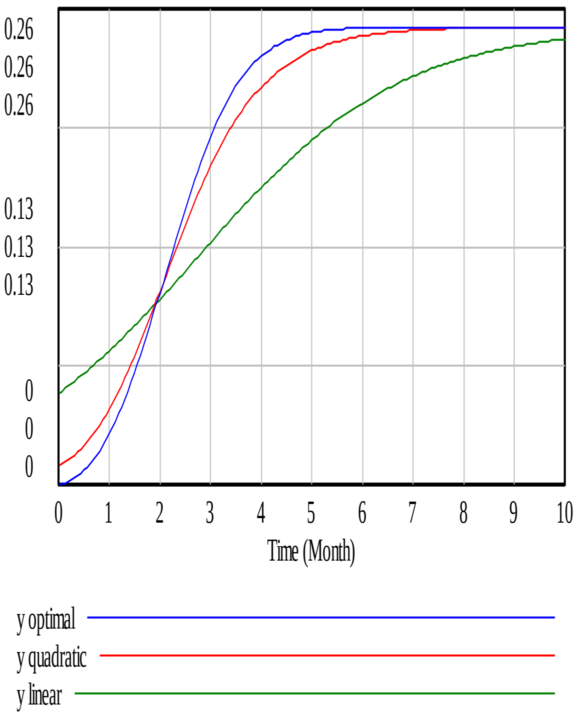
initially: $0 \leq c_0 = 0 \leq 1$ and $1 \leq q = 1 \leq 3$.

A solution for the stable node depends on x_0 (Table 7).

Table 7. Policy optimization results depending on x_0 in S-1

Run's No.	x_0	c_0	q	p
2	0.1	0	2.261	–5.112
1	1	1	3	–9

The convergence of biomass x and catch c to its distant attractor E_s (29) is almost monotonous in the both runs. Results for this HCL judged by average catch c and integral catch $\left(\int_0^T cdt\right)$ over years 0–10 are better now than for aforementioned M-2 and M-3 with linear and quadratic HCL correspondingly when all other conditions are the same (Tables 3 and 4, Figure 7).

Stock x (green in M-2, red in M-3, blue in S-1) for $x_0 = 1$	Stock x (green in M-2, red in M-3, blue in S-1) for $x_0 = 0.1$
	
Catch $y = c$ (green in M-2, red in M-3, blue in S-1) for $x_0 = 1$	Catch $y = c$ (green in M-2, red in M-3, blue in S-1) for $x_0 = 0.1$
	

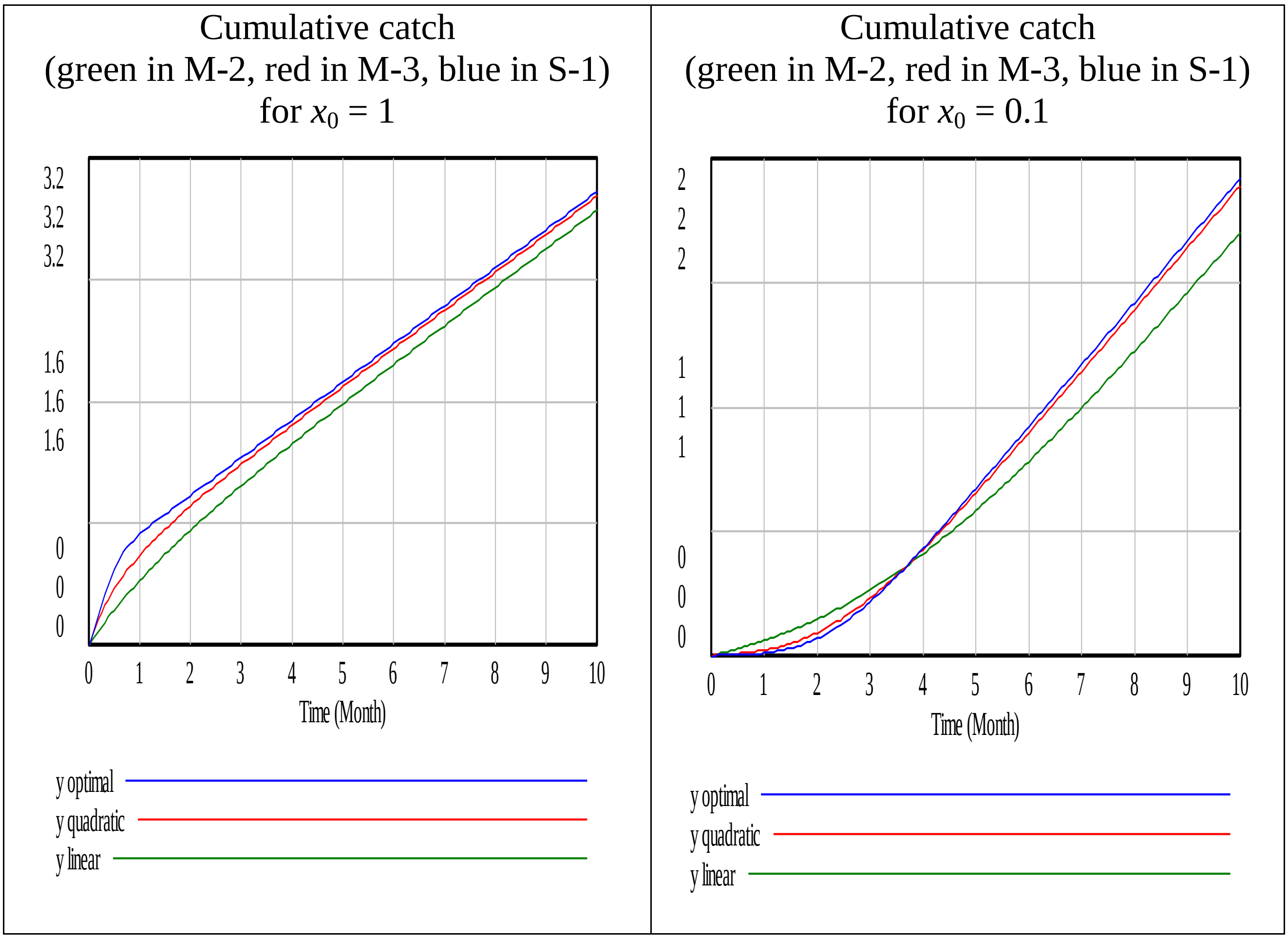


Figure 7 – Comparison of HCL results in M-2, M-3 and S-1 for years 0–10 for $x_0 = 1$ on the left and for $x_0 = 0.1$ on the right

The upgraded closed loop control in S-1 gives a bit more time for the bioresource to grow from low x_0 by choosing $c_0 = 0$ initially. Quite contrary this control sets $c_0 = 1$ initially for high x_0.

Conclusion

Typical modes of renewable resource management are considered for open-loop or closed-loop control. Using the theory of bifurcations and catastrophes, the policies of improving bioresource catch and renewal, with raised long-term effectiveness in relation to the policy proposed in the simplified Schaefer – Arnold bioeconomic model M-2, are elaborated in M-3 and S-1.

The obtained results related to the compared modes of nature-use are not only local, as is often the case in the applications of catastrophe theory, but also global in nature (particularly, in M-4 with open loop control). For all the considered regimes in one-dimensional models (except M-5), the original formulas of integral curves for stocks and catches are derived. Still the analytical results for the proposed two-dimensional predator-prey model S-1 are mostly local, they are extended to broader areas thanks to Vensim simulations.

A more concrete presentation of the ecological and economic reproduction and its current global crisis is expected to be carried out in further studies with detailed elaboration of technological and institutional aspects.

The transition from the above simplified analysis of sustainability to the study of the evolutionary ecological stability of interacting bioresources is promising [22]. The research should also enhance the probabilistic approach to bioeconomic modelling.

However, there is no doubt that depletion of bioresources is Damocles sword for the world economy. Management of reproduction based on scientific foresight becomes more and more pressing necessity. Only this path, traditionally favoured by advanced system dynamics, opens the rich opportunities for overcoming the global crisis of nature management.

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