Rethinking of Continuous Models with Approximate Discrete Variable Calculations in Simulation Software

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1. Introduction

Most of system dynamics software implement a common calculation process based on Euler method of numerical integration. This calculation is continuous models' discrete approximation and has an assumption that the definition of derivatives has no change during one time unit. Therefore, when a modeler sets time unit too long which reflects plural decision makings for a flow variable during one time unit, its simulation result differs with the analytical result of same numerical models. In addition, a typical issue appears when a modeler uses impulse functions whose implementations are different among system dynamics software; the same meaning definitions can make different results. This paper expresses a term "time unit" as time horizon's basic unit, and DT as a computation interval usually set equal to or less than one.

2. Flow variables' change during one time unit

When a modeler sets too a long time unit, its simulation result differs with the analytical result of same numerical models. For example, if one set a time unit as one year and or she gives a quarterly decision at each a quarter time, the setting that the time unit is one too long. In this case, the time unit should correspond to less than quarter. Otherwise, its simulation result is inaccurate. The discretization of the calculation process brings about this difference.

Following case accompanies with a numerical simulation comparison: Given a simple stock flow structure (figure 1) with the definition that a variable "flow" changes as figure 2 and the initial value of a variable "stock" is zero, one would be able to guess the behavior of the variable "stock."

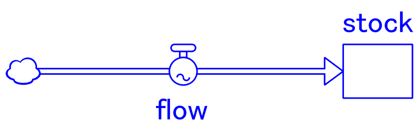

Figure 1. Stock flow diagram

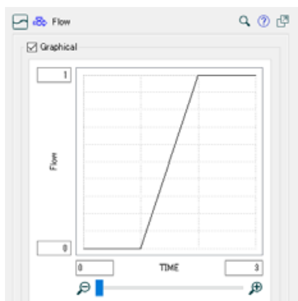

Figure 2. The variable "flow" in the figure 1.

The final value (at time 3) of the variable "stock" should be 1.5 which is the area below the graph line in figure 2.

Of course, one can run a simulation of this model. With the setting of simulation start at time 0, finishes at time 3, and DT is 0.25, one has the simulation result shown in the Table 1.

The final value of the variable "stock" is 1.375. This is not the same as calculated in advance. The reason of the difference between the simulation result is discretization in the simulation process. Of course, given sufficiently small DT with Euler method, the calculation results would be correct. However, the setting DT=0.0001 is still insufficient to obtain a correct answer for the simulation result of model above. If a modeler needs to implement quarterly decision making, a time unit must correspond to one quarter (three months) or shorter term. Modelers should avoid referring a value at a time between precise time points to calculate flow variables.

Table 1. Simulation result of the variable “stock” in the figure 1.

Time	0.00-1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
Stock	0.0000	0.0000	0.0625	0.1875	0.3750	0.6250	0.8750	1.1250	1.3750
flow	0.0000	0.2500	0.5000	0.7500	1.0000	1.0000	1.0000	1.0000	1.0000

3. Discrete built-in function differences

Even if functions’ meanings or nuances are similar, usages of the functions in each software are not always the same. Apparent differences appear in impulse functions, whose names are PULSE in many products of software, although modelers frequently use them.

The same structure models (figure 3) generate different results (figure 4 and 5), with the setting of DT=0.25. “Pulse input” ignites at time one, and the volume is one. “Flow” equal this. “Stock” has an initial value zero.

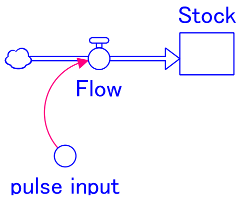

Figure 3. Stock Flow Diagram

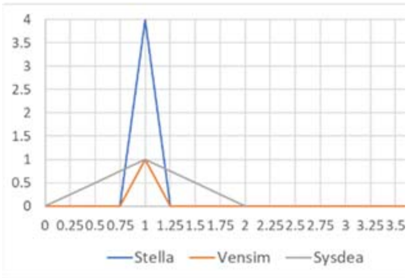

Figure 4. Flow Transitions

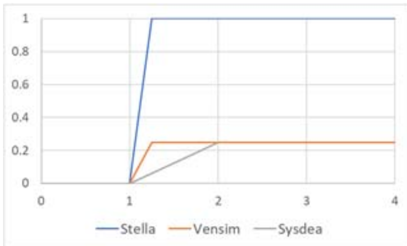

Figure 5. Stock Transitions

Stella generates a value of specified volume divided by DT for “inflow.” The flow equation values are multiplied by DT through integration. Then, the net flow to “stock” is specified in the PULSE function. Sysdea generates the specified value in the PULSE function for “inflow,” then net flow is smaller (specified value in the function multiplied by DT). Sysdea reports only values at precise times; therefore, the graph looks different from Vensim’s output. However, Sysdea’s calculated values are the same as Vensim’s ones.

4. Conclusion

System dynamics models are continuous models; however, the calculation processes are discrete approximations. Modelers need to consider appropriate time settings for their models and simulations. Besides, modelers must not give certain meanings to time points during a time-unit. At the same time, discrete approximation does not mean that discrete functions’ behaviors are intuitive. Definitions of functions related to DT require significant awareness. Of course, neither of the problems shown above imply a weakness of system dynamics nor of simulation software. One should simply attribute these problems to modelers. Modelers need to be careful not only in verification and validation of models but also in simulation settings.

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