CERC-BEE Impact Model

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1 Framework and Summary

1.1 Motivation

Currently there exist few building energy consumption models to test the national impact of building energy efficiency technology. The number of such models that are publicly available models is even fewer. In order to predict correctly the effectiveness of policy concepts, this is a need for realistic models of technology adoption and regulation.

This document describes a mathematical framework to accomplish this, and as an example estimates the impact of the portfolio of Building Energy Efficiency technologies advanced by the first five program years of US-China Clean Energy Research Center (CERC-BEE 1.0).[1]

The system dynamics simulation [1] framework here calculates national building energy consumption. While the framework could be applied to any country (e.g. US or China), this paper describes a calibrated to the Reinventing Fire China baseline scenario.[2, 3] The resolution of the framework extends to Climate Zone, Building Type, End Use, and End Use Technology Types. The framework includes sub-models that realistically describe technology adoption, new building construction and building retrofit, gross domestic product and population. The framework naturally incorporates the impact of regulation and commercialization impact on technology availability. Finally, the framework simulation computer code is human-readable and open-source.

The framework can give a calibrated understanding of: adoption rates of building efficiency technology based on total cost of ownership with respect to incumbent (competitor) technologies. Building on the framework described herein, and incorporating mathematical representations of policy concepts, the parameter space can be explored to maximize anticpated policy impact. Possible extensions include: understanding building retrofit rates based on decision factors; and code adoption rates; and recommended procedures to update learning (actual vs projected) and adjust policy to maximize adoption rates.

1.2 Framework

Figure 1 shows an graphical overview of the mathematical framework. Starting from the end and working backwards, impact is determined by the difference in national building energy consumption between two scenarios: a baseline scenario and the test scenario. A trajectory of national building energy consumption is determined by the installed base of end use technology with its respective intensity, as modifed by the building energy efficiency technologies. The turnover of end use technology installed base depends on rate of new technology installation which accompanies new buidling construction and replacement of installed technology due to failures or to existing building retrofit. Individual technology sales market share are determined by the availability, benefits, and total cost of ownership, of that technology relative to all alternatives relevant in that market. Technology installed base is tracked by building type area, whose turnover is also modeled. The driving force of

[1] `https://cercbee.lbl.gov/`

new construction is increases in per-capita GDP, which is determined by separate models for gross domestic product and population. Wherever possible, model parameters are adjusted to obtain agreement with historical data or projections from publicly available sources. While impact estimates for a specific geographic region depend on the model calibration for that region, the mathematical framework itself can be applied to any region.

1.3 Summary

Figure 2 shows the model estimate of building energy consumption in China for the Reinventing Fire China baseline scenario (red curve), starting at around 6×10^{15}Wh/year in 2010 and increasing to about 14×10^{15}Wh/year in 2050. The scenario with free-market adoption of the CERC-BEE 1.0 portfolio of advanced building energy efficiency technology, the energy consumption is reduced from the baseline scenario. The magnitude of the reduction of energy consumption, the impact, is estimated to be 8% by 2030 and 11% by 2050.

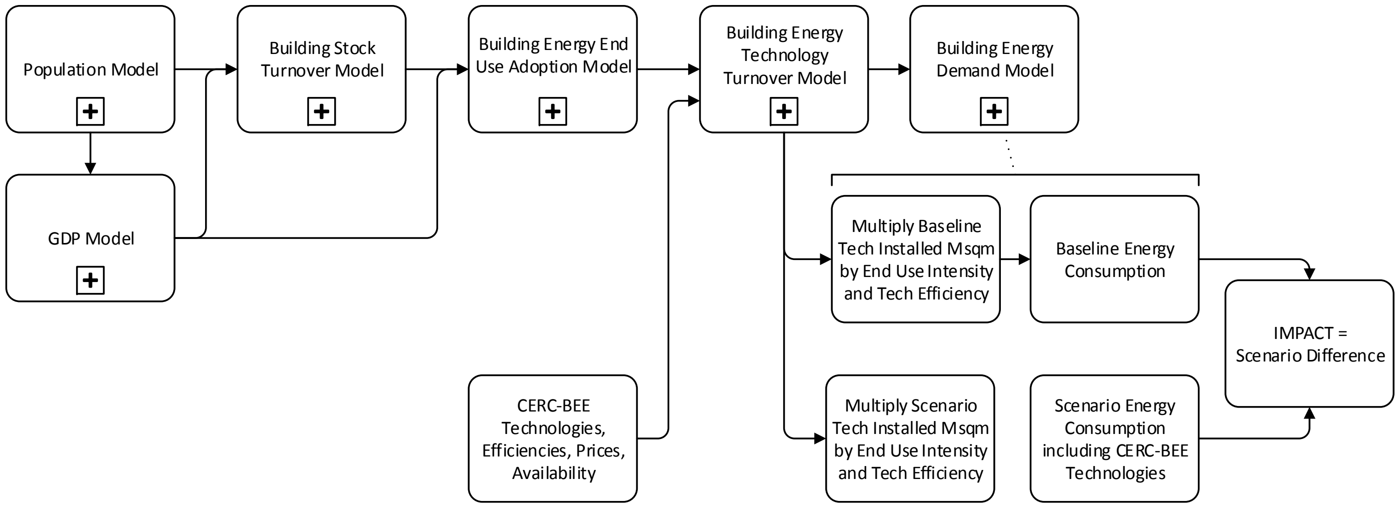


Figure 1: Framework for determining impact of CERC-BEE 1.0 Advanced Technology portfolio.

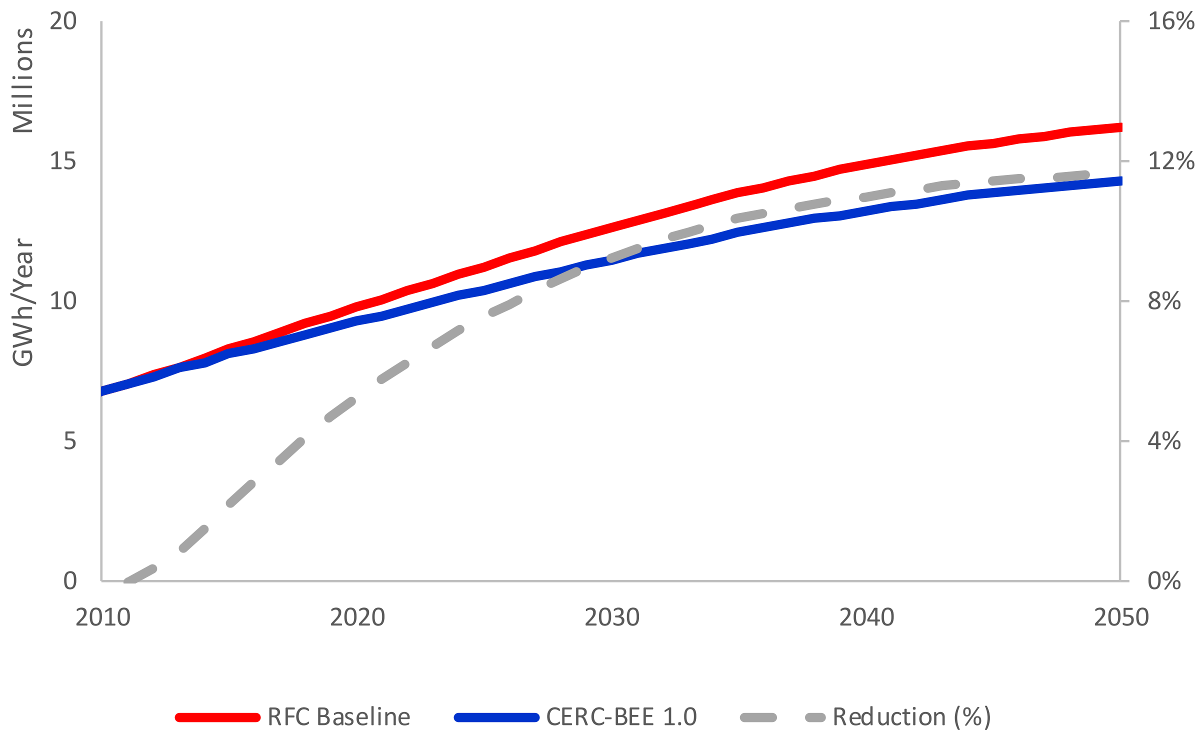


Figure 2: Summary of CERC-BEE 1.0 impact, compared with Reinventing Fire China baseline scenario.

2 Population and Demographic Shifts

2.1 Population Stock and Flow

Figure 3 shows the stock-and-flow structure for population. As can be seen from the basic equations in Figure 4, the structure is repeated for each country (`ref WDI`), each climate zone (`ClimateZone`), rural or urban areas (`RurUrb`). Generically, the population in one time period is equal to the population in the previous time period, but increased by the integral of the birth rate and net immigration rate over the intervening time, and decreased by the integral of the death rate over the intervening time.

In a more detailed fashion, the structure repeated for each age cohor (`Age Group`), through use of the Vensim external function `COHORT CONTROL`: population is tracked by age as well, wherein the number of people within a certain age group is increased by the number of people who "graduate in" from the younger age group since the previous time, and decreased by the number of people who "graduate out" to the older age group. See Eberline [4, 5] for more details.

Tracking population by age is required to give an accurate estimate of employable labor. Figure 5 shows the equations for births, deaths, and immigration that change population (aside from internal shifts amongst age groups). Before 2013, the simulation uses the historical value of the crude birth rate, and afterwards uses an extrapolation.

2.2 Crude Birth Rates

Figure 7 shows a simple model showing behavior of crude birth rate, along with the simulation equations. In the model, crude birth rate is assumed to approach an asymptotic value (`CBR goal w policy`) with a rate that is portional to the difference between the existing crude birth rate and the asymptotic value. The asymptotic value and the decay rate are adjusted to give an approximate fit to historical data, as well as reasonable projections.

2.3 Population Age Fractions

The population fraction functions and simulation equations of the model for population age fractions are shown in Figure 8. Because we track population by age cohort, determining the population age fractions is a simple matter of summing over the appropriate age cohorts.

2.4 Gompertz-Makeham Mortality Model

To reproduce historical measures of population, we require a realistic mortality hazard function. We adopt the following notation[2]:

$F(t)$	lifetime distribution function,
$S(t)$	survival function,

[2] `http://en.wikipedia.org/wiki/Survival_analysis`

$f(t)$	density function of the lifetime distribution,
$h(t)$	hazard function,
$\Lambda(t)$	cumulative hazard function

The following basic relations apply:

$$
\begin{aligned}
f(t) &= \frac{dF(t)}{dt}, \\
h(t) &= \frac{f(t)}{S(t)}, \\
\Lambda(t) &= \int_0^t h(u)\,du, \qquad (1) \\
S(t) &= 1 - F(t), \\
&= \int_t^\infty f(u)\,du, \\
&= \exp\left[-\Lambda(t)\right], \qquad (2) \\
f(t) &= h(t)\,S(t),
\end{aligned}
$$

The expected life given survival to age x is given by

$$L(x) = \frac{1}{S(x)} \int_x^\infty S(u)\,du.$$

We will approximate the hazard function by the average over the interval $t_1 \leq t \leq t_2$, given by

$$\langle h \rangle \;=\; \frac{\int_{t_1}^{t_2} f(t)\,dt}{\int_{t_1}^{t_2} S(t)\,dt} = \frac{\int_{t_1}^{t_2} h(t)\,S(t),dt}{\int_{t_1}^{t_2} S(t)\,dt} = \frac{S(t_1) - S(t_2)}{\int_{t_1}^{t_2} S(t)\,dt}. \qquad (3)$$

For $t_2 \to \infty$, $\langle h \rangle \to 1/L(t_1)$.

From Gage et al [6, 7, 8], we employ a five-parameter model for the hazard function $h(x)$, building on the Gompertz-Makeham hazard function[3], and insert into the equations for the cumulative hazard function and survival fraction, Eqs.(1)–(2):

$$
\begin{aligned}
h(x) &= \alpha \exp(\beta x) + \lambda + \epsilon \exp(-\gamma x), \\
\Lambda(t) &= \lambda t + \frac{\alpha}{\beta}\left[\exp(\beta t) - 1\right] - \frac{\epsilon}{\gamma}\left[\exp(-\gamma t) - 1\right], \\
S(t) &= \exp\left[-\left(\lambda t + \frac{\alpha}{\beta}\left[\exp(\beta t) - 1\right] - \frac{\epsilon}{\gamma}\left[\exp(-\gamma t) - 1\right]\right)\right]
\end{aligned}
$$

Given age-specific probabilities q_i, find the estimates of parameters $\theta = \{\alpha, \beta, \lambda\,\epsilon, \gamma\}$ which minimize

$$\text{SSE} = \sum_i \left[\ln\left(\langle h_i(\theta) \rangle / q_i\right)\right]^2$$

[3] http://en.wikipedia.org/wiki/Gompertz%E2%80%93Makeham_law_of_mortality

Figures 9 and 10 show a sketch and of the hazard function, using the Gompertz-Makeham formula, and the simulation equations, respectively.

Figure 11 shows a simple model and equations for the time dependence of the coefficients θ in the Gompertz-Makeham formula. Each of the coefficient has an initial and an asymptotic value, and the rate of change is proportional to the difference between the instantaneous value and the asymptotic value. The parameters are adjusted such that quantities derived from the hazard function match with reference data. The values used in the calibration for China are given in Table 1.

2.5 Rural and Urban Populations and Migration

Figure 13 shows a simple model of rural-to-urban migration, along with the equations. In the model, the fraction of population that lives in rural area is assumed to approach an asymptotic value (`aym Rural Pop Pct`) with a rate that is proportional to the difference between the existing fraction and the asymptotic value. The initial value, asymptotic value, and decay rate are adjusted to give an approximate fit to historical data, as well as reasonalble projections.

2.6 Calibration

Figures 14–18 show the calibration of the population model. The model is calibrated against a weighted combination of the crude birth rate, mortality rates of infants (less than one year old), children (less than 5 years old), and adults, crude death rate, adult survival, life expectancy at birth, population age fractions (between zero and fourteen years, between fifteen and sixty-four years, and sixty five years and older), and urban and rural populations. In the calibration, the model output is compared with values taken World Development Indicators.[9] Table 2 shows the indicator codes of the time series used for calibration.

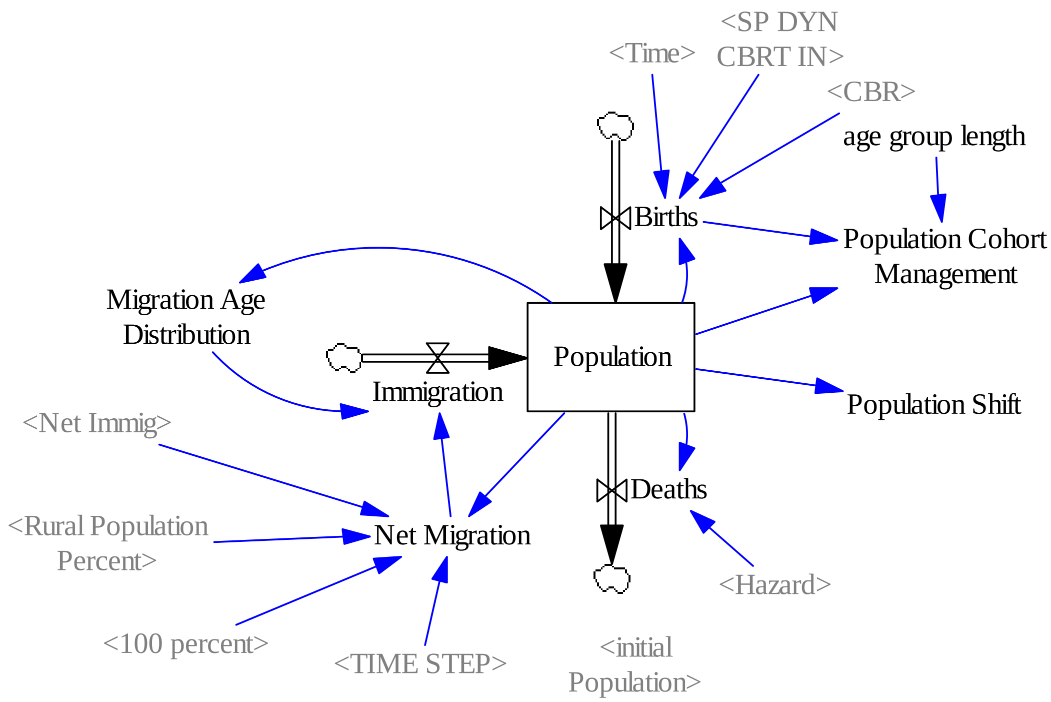


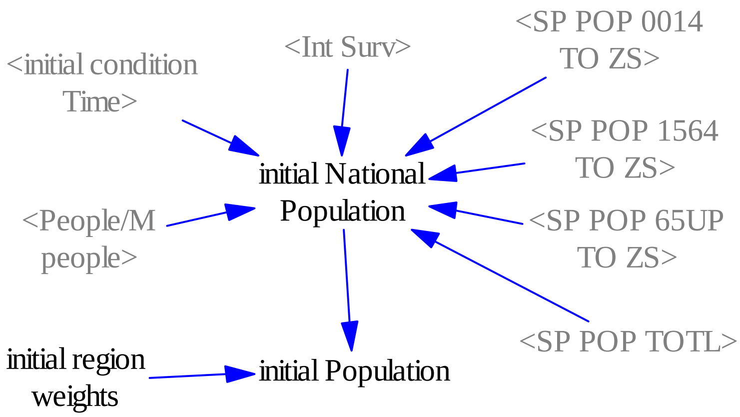
Figure 3: Population stock and flow model, with cohort aging and age-dependent Gompertz-Makeham hazard function.

```
Population[region WDI,ClimateZone,RurUrb,a0000]= INTEG (
Births[region WDI,ClimateZone,RurUrb]-Deaths[region WDI,ClimateZone,RurUrb,a0000],
initial Population[region WDI,ClimateZone,RurUrb,a0000]) ~~|
Population[region WDI,ClimateZone,RurUrb,ag01UP]= INTEG (
Immigration[region WDI,ClimateZone,RurUrb,ag01UP]-
Deaths[region WDI,ClimateZone,RurUrb,ag01UP],
initial Population[region WDI,ClimateZone,RurUrb,ag01UP])
~ Mpeople
~ |

Population Cohort Management[region WDI,ClimateZone,RurUrb,Age Group]=
COHORT CONTROL(Population[region WDI,ClimateZone,RurUrb,a0000],
age group length[Age Group] ,
Births[region WDI,ClimateZone,RurUrb] , ELMCOUNT(Age Group) , 1 )
~ Mpeople
~ |

Population Shift[region WDI,ClimateZone,RurUrb]=
COHORT SHIFT( Population[region WDI,ClimateZone,RurUrb,a0000])
~ Mpeople
~ |
```

Figure 4: Equations for Population stock and flow model.

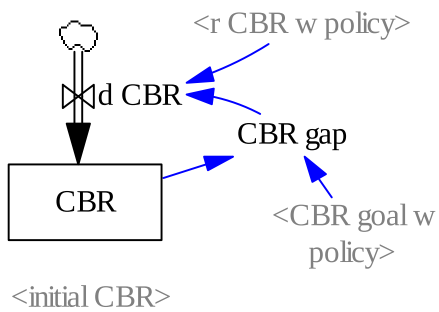
```
Births[region WDI,ClimateZone,RurUrb]=
SUM(Population[region WDI,ClimateZone,RurUrb,Age Group!])*
IF THEN ELSE(Time < 2013, SP DYN CBRT IN[region WDI], CBR[region WDI])/1000
~ Mpeople/Year
~ |

Deaths[region WDI,ClimateZone,RurUrb,Age Group]=
Population[region WDI,ClimateZone,RurUrb,Age Group]*
Hazard[region WDI,Age Group]/1000
~ Mpeople/Year
~ |

Immigration[region WDI,ClimateZone,RurUrb,Age Group]=
Net Migration[region WDI,ClimateZone,RurUrb]*
Migration Age Distribution[region WDI,ClimateZone,RurUrb,Age Group]
~ Mpeople/Year
~ |
```

Figure 5: Equations for rate of change of Population.



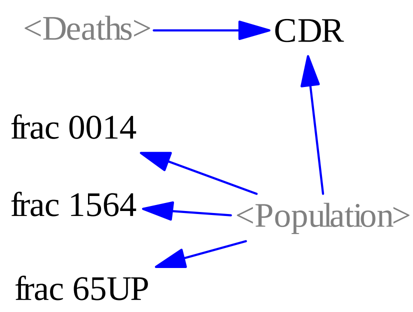

```
initial National Population[region WDI,ag0014]=
GET DATA AT TIME(SP POP 0014 TO ZS[region WDI], initial condition Time)*
GET DATA AT TIME(SP POP TOTL[region WDI],initial condition Time)/
"People/Mpeople"/100*
Int Surv[region WDI,ag0014]/SUM(Int Surv[region WDI,ag0014!]) ~~|

initial National Population[region WDI,ag1564]=
GET DATA AT TIME(SP POP 1564 TO ZS[region WDI], initial condition Time)*
GET DATA AT TIME(SP POP TOTL[region WDI],initial condition Time)/
"People/Mpeople"/100*
Int Surv[region WDI,ag1564]/SUM(Int Surv[region WDI,ag1564!]) ~~|

initial National Population[region WDI,ag65UP]=
GET DATA AT TIME(SP POP 65UP TO ZS[region WDI], initial condition Time)*
GET DATA AT TIME(SP POP TOTL[region WDI],initial condition Time)/
"People/Mpeople"/100*
Int Surv[region WDI,ag65UP]/SUM(Int Surv[region WDI,ag65UP!])
~ Mpeople
~ |

initial Population[region WDI,ClimateZone,RurUrb,Age Group]=
initial National Population[region WDI,Age Group]*
initial region weights[region WDI,ClimateZone,RurUrb]/
SUM(initial region weights[region WDI,ClimateZone!,RurUrb!])
~ Mpeople
~ |
```

Figure 6: Structure and equations for initial population.




```
CBR[region WDI]= INTEG (
d CBR[region WDI],
initial CBR[region WDI])
~ 1/Year
~ |

d CBR[region WDI]=
CBR gap[region WDI]*r CBR[region WDI]
~ 1/Year/Year
~ |

CBR gap[region WDI]=
asym CBR[region WDI]-CBR[region WDI]
~ 1/Year
~ |

initial CBR[region WDI]=
GET DATA AT TIME(SP DYN CBRT IN[region WDI], initial condition Time)
~ 1/Year [0,60,0.5]
~ |
```

Figure 7: Causal loop diagram and equations for simple model of Crude Birth Rate.




```
frac 0014[region WDI]=
SUM(Population[region WDI,ClimateZone!,RurUrb!,ag0014!])/
SUM(Population[region WDI,ClimateZone!,RurUrb!,Age Group!])*100
~ Dmnl
~ |

frac 1564[region WDI]=
SUM(Population[region WDI,ClimateZone!,RurUrb!,ag1564!])/
SUM(Population[region WDI,ClimateZone!,RurUrb!,Age Group!])*100
~ Dmnl
~ |

frac 65UP[region WDI]=
SUM(Population[region WDI,ClimateZone!,RurUrb!,ag65UP!])/
SUM(Population[region WDI,ClimateZone!,RurUrb!,Age Group!])*100
~ Dmnl
~ |

CDR[region WDI]=
SUM(Deaths[region WDI,ClimateZone!,RurUrb!,Age Group!])/
SUM(Population[region WDI,ClimateZone!,RurUrb!,Age Group!])*1000
~ 1/Year
~ |
```

Figure 8: Causal loop diagram and equations for population aging fractions.

Variable \ region	CHN	IND	OED	USA	WLD
asym CBR (births/year/1000 people)	11.43	7.00	7.13	14.42	7.12
asym Rural Pop Pct (%)	-	54.24	16.82	0.07	31.64
asym Theta[alpha] (1/year)	0.000225	0.000054	0.000006	0.000002	0.000079
asym Theta[beta] (1/year)	0.081	-	0.077	0.064	0.118
asym Theta[epsilon] (1/year)	-	0.167	0.002	0.116	0.887
asym Theta[gamma] (1/year)	35.00	35.00	35.00	35.00	35.00
asym Theta[lambda] (1/year)	-	-	0.000043	0.000043	-
init Rural Pop Pct (%)	80.21	79.67	34.22	29.32	65.11
init Theta[alpha] (1/year)	0.000598	0.000120	0.000089	0.000089	0.000133
init Theta[beta] (1/year)	0.069	0.098	0.089	0.088	0.088
init Theta[epsilon] (1/year)	2.672	4.774	1.582	0.798	5.134
init Theta[gamma] (1/year)	35.00	35.00	35.00	35.00	35.00
init Theta[lambda] (1/year)	-	-	0.000429	0.000780	-
initial Net Immigration (Mpeople/year)	0.17	(0.14)	0.35	0.30	-
initial Urban Pop Pct (%)	17.96	17.13	51.69	28.13	24.45
r CBR (1/year)	0.06	0.02	0.02	0.10	0.02
r Rural Pop Pct (1/year)	0.09	0.16	0.12	0.02	0.08
r Theta[alpha] (1/year)	0.80	0.08	0.02	0.01	0.09
r Theta[beta] (1/year)	-	0.00	0.00	0.00	0.00
r Theta[epsilon] (1/year)	0.04	0.01	0.03	0.04	0.04
r Theta[gamma] (1/year)	-	-	-	-	-
r Theta[lambda] (1/year)	0.10	0.10	1.00	1.00	0.09

Table 1: Values of parameters used to determine time dependent coefficients of Gompertz-Makeham function.

Indicator Code	**Description**
SH.DTH.IMRT	Number of infant deaths
SH.DTH.MORT	Number of under-five deaths
SH.DYN.MORT	Mortality rate, under-5 (per 1,000 live births)
SH.DYN.MORT.FE	Mortality rate, under-5, female (per 1,000 live births)
SH.DYN.MORT.MA	Mortality rate, under-5, male (per 1,000 live births)
SM.POP.NETM	Net migration
SP.DYN.AMRT.FE	Mortality rate, adult, female (per 1,000 female adults)
SP.DYN.AMRT.MA	Mortality rate, adult, male (per 1,000 male adults)
SP.DYN.CBRT.IN	Birth rate, crude (per 1,000 people)
SP.DYN.CDRT.IN	Death rate, crude (per 1,000 people)
SP.DYN.IMRT.FE.IN	Mortality rate, infant, female (per 1,000 live births)
SP.DYN.IMRT.IN	Mortality rate, infant (per 1,000 live births)
SP.DYN.IMRT.MA.IN	Mortality rate, infant, male (per 1,000 live births)
SP.DYN.LE00.FE.IN	Life expectancy at birth, female (years)
SP.DYN.LE00.IN	Life expectancy at birth, total (years)
SP.DYN.LE00.MA.IN	Life expectancy at birth, male (years)
SP.DYN.TO65.FE.ZS	Survival to age 65, female (% of cohort)
SP.DYN.TO65.MA.ZS	Survival to age 65, male (% of cohort)
SP.POP.0014.TO.ZS	Population ages 0-14 (% of total)
SP.POP.1564.TO.ZS	Population ages 15-64 (% of total)
SP.POP.65UP.TO.ZS	Population ages 65 and above (% of total)
SP.POP.TOTL	Population, total
SP.RUR.TOTL	Rural population
SP.RUR.TOTL.ZS	Rural population (% of total population)
SP.URB.TOTL	Urban population

Table 2: List of WDI time series use for calibration of the population sub-model. Reference: World Development Indicators, [9]

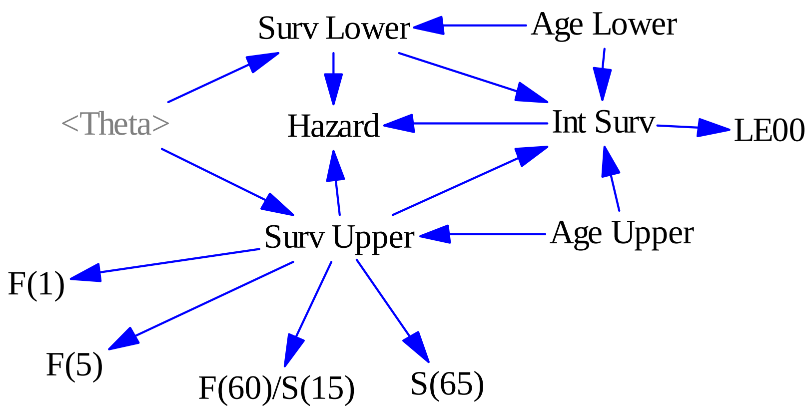
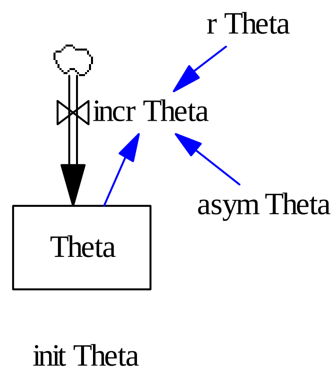


Figure 9: Diagram for Gompertz-Makeham age-dependent mortality (hazard) function.

```
Hazard[region WDI,Age Group]=
(Surv Lower[region WDI,Age Group]-Surv Upper[region WDI, Age Group])/
Int Surv[region WDI,Age Group]*1000
~ 1/Year
~ |

Surv Lower[region WDI,Age Group]=
exp(-(
Theta[lambda,region WDI]*Age Lower[Age Group]
+ Theta[alpha,region WDI]/Theta[beta,region WDI]*
(exp(Theta[beta,region WDI]*Age Lower[Age Group])-1)
- Theta[epsilon,region WDI]/Theta[gamma,region WDI]*
(exp(-Theta[gamma,region WDI]*Age Lower[Age Group])-1)))
~ Dmnl
~ |

Surv Upper[region WDI, Age Group]=
exp(-(
Theta[lambda,region WDI]*Age Upper[Age Group]
+ Theta[alpha,region WDI]/Theta[beta,region WDI]*
(exp(Theta[beta,region WDI]*Age Upper[Age Group])-1)
- Theta[epsilon,region WDI]/Theta[gamma,region WDI]*
(exp(-Theta[gamma,region WDI]*Age Upper[Age Group])-1)))
~ Dmnl
~ |

Int Surv[region WDI,Age Group]=
(Surv Upper[region WDI, Age Group]+Surv Lower[region WDI, Age Group])/2*
(Age Upper[Age Group]-Age Lower[Age Group])
~ Year
~ |
```

Figure 10: Equations for Gompertz-Makeham age-dependent mortality (hazard) function.




```
Theta[itheta,region WDI]= INTEG (
incr Theta[itheta,region WDI],
init Theta[itheta, region WDI])
~ 1/Year
~ |

incr Theta[itheta,region WDI]=
(asym Theta[itheta,region WDI]-Theta[itheta,region WDI])*r Theta[itheta,region WDI]
~ 1/(Year*Year)
~ |
```

Figure 11: Time dependent coefficients of Gompertz-Makeham function.

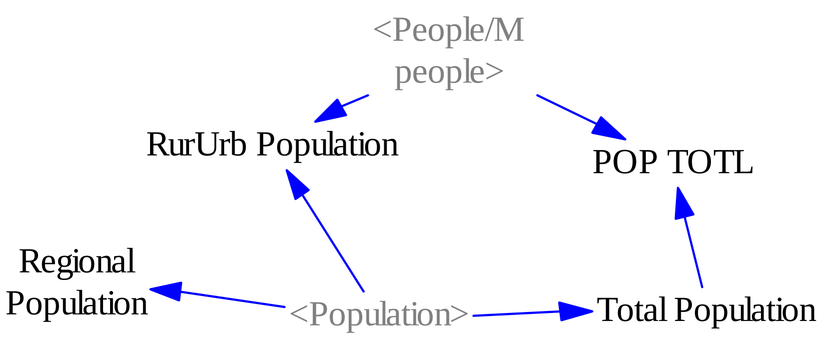
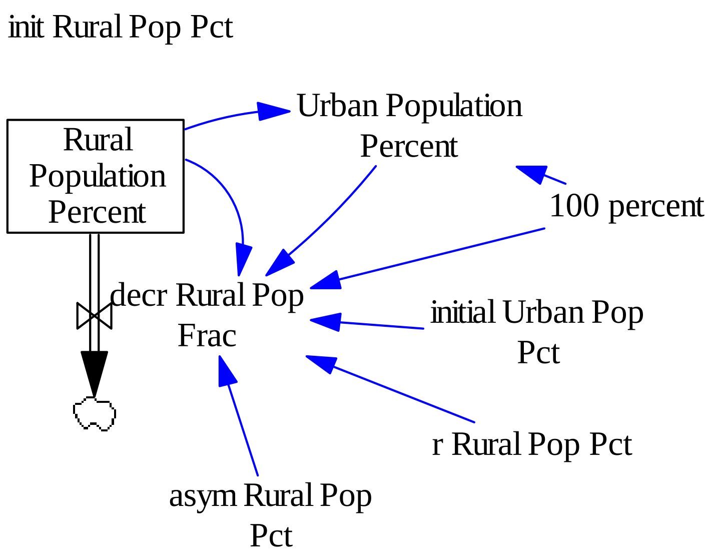


Figure 12: Relationship of variables to calculate Rural Population.




```
Rural Population Percent[region WDI]= INTEG (
-decr Rural Pop Frac[region WDI],
init Rural Pop Pct[region WDI])
~ Percent
~ |

decr Rural Pop Frac[region WDI]=
(Rural Population Percent[region WDI]-asym Rural Pop Pct[region WDI])*
(Urban Population Percent[region WDI]-initial Urban Pop Pct[region WDI])/
"100 percent"*r Rural Pop Pct[region WDI]
~ Percent/Year
~ |

Urban Population Percent[region WDI]=
"100 percent"-Rural Population Percent[region WDI]
~ Percent
~ |
```

Figure 13: Simple model of and equations for Rural to Urban migration.

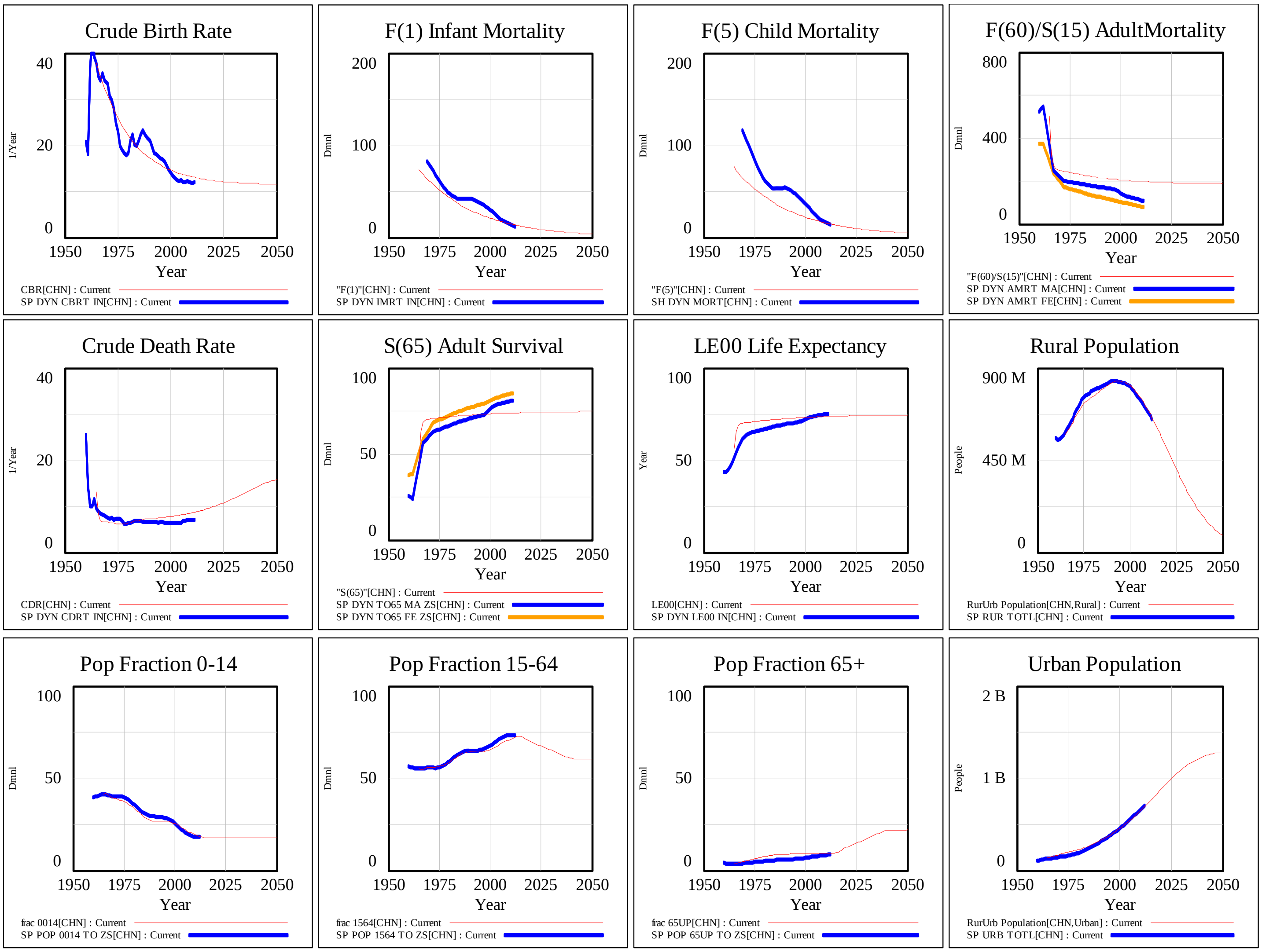


Figure 14: Calibration of population parameters. Thick blue lines represent values for China (CHN). Reference: World Development Indicators, [9]

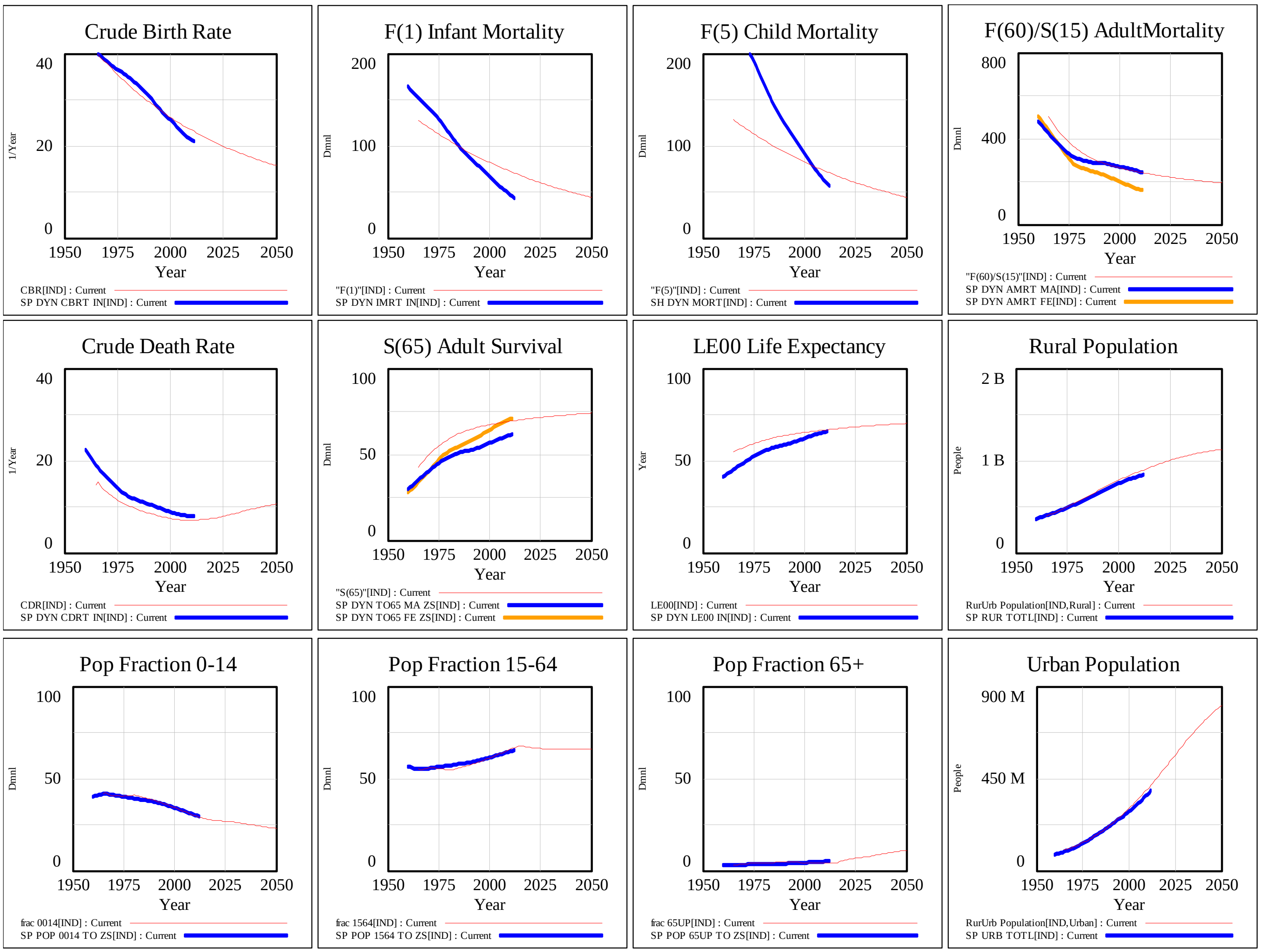


Figure 15: Calibration of population parameters. Thick blue lines represent values for India (IND). Reference: World Development Indicators, [9]

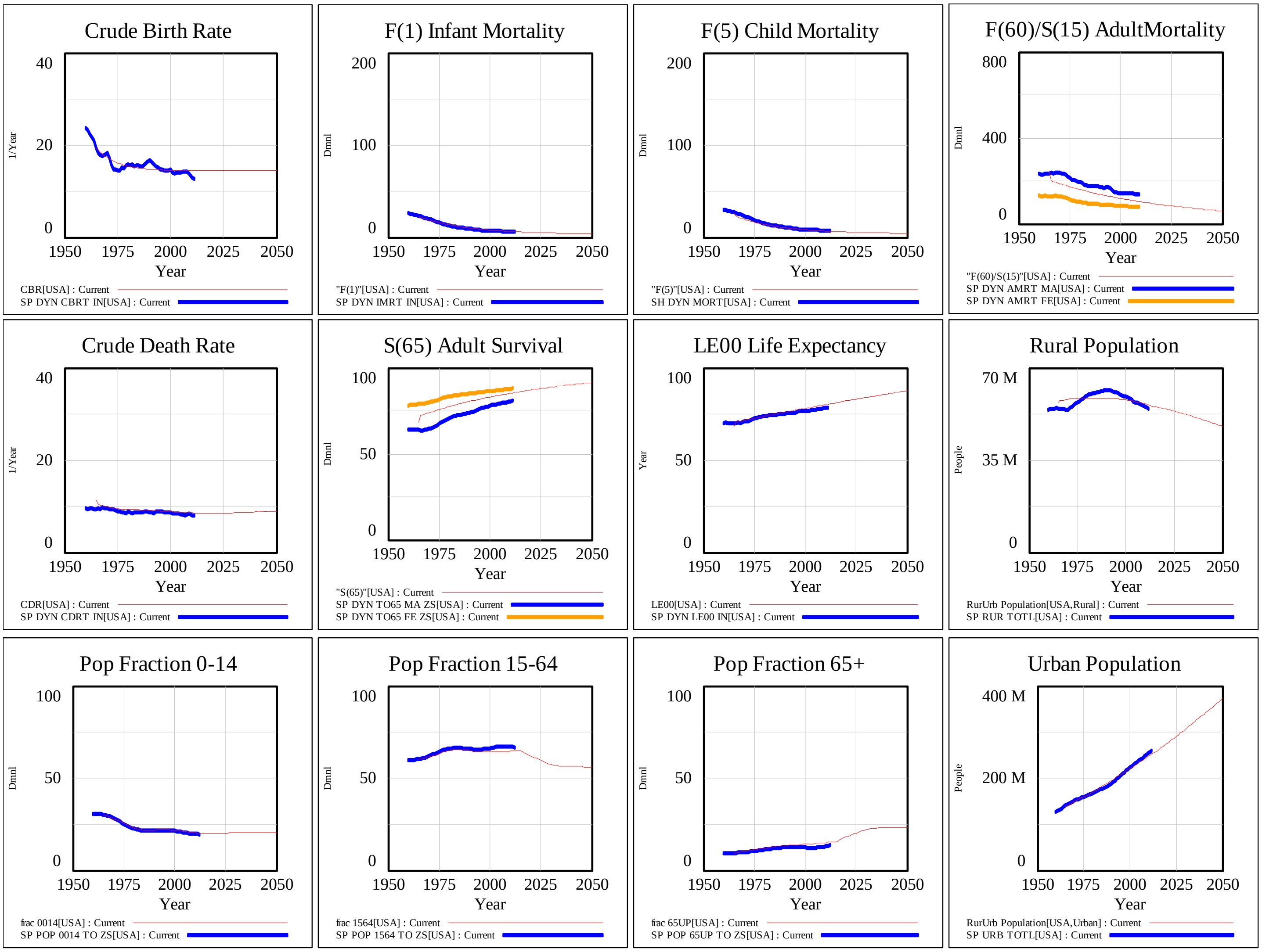


Figure 16: Calibration of population parameters. Thick blue lines represent values for the United States (USA). Reference: World Development Indicators, [9]

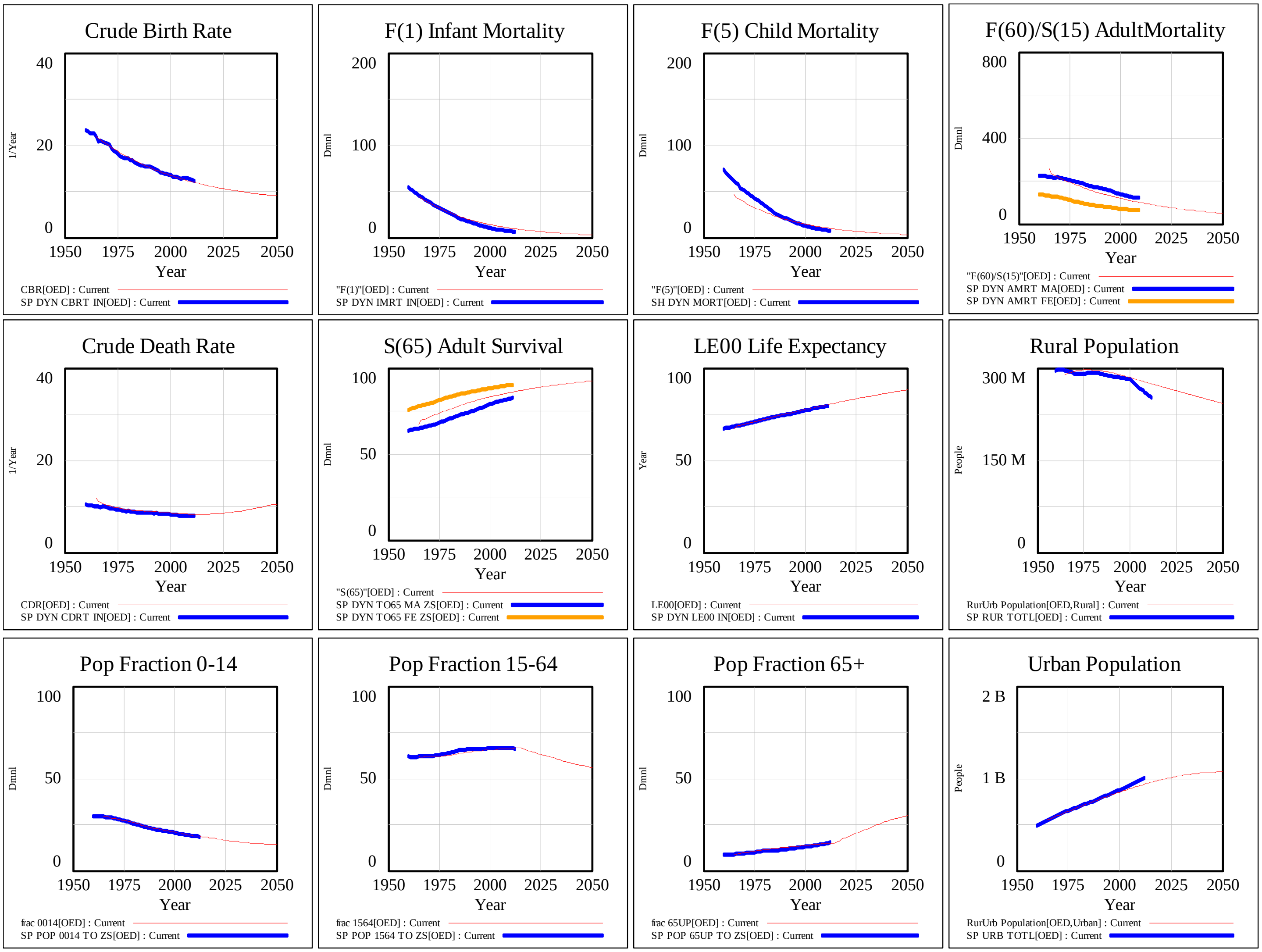


Figure 17: Calibration of population parameters. Thick blue lines represent values for countries in the Organization for Economic Cooperation and Development (OED). Reference: World Development Indicators, [9]

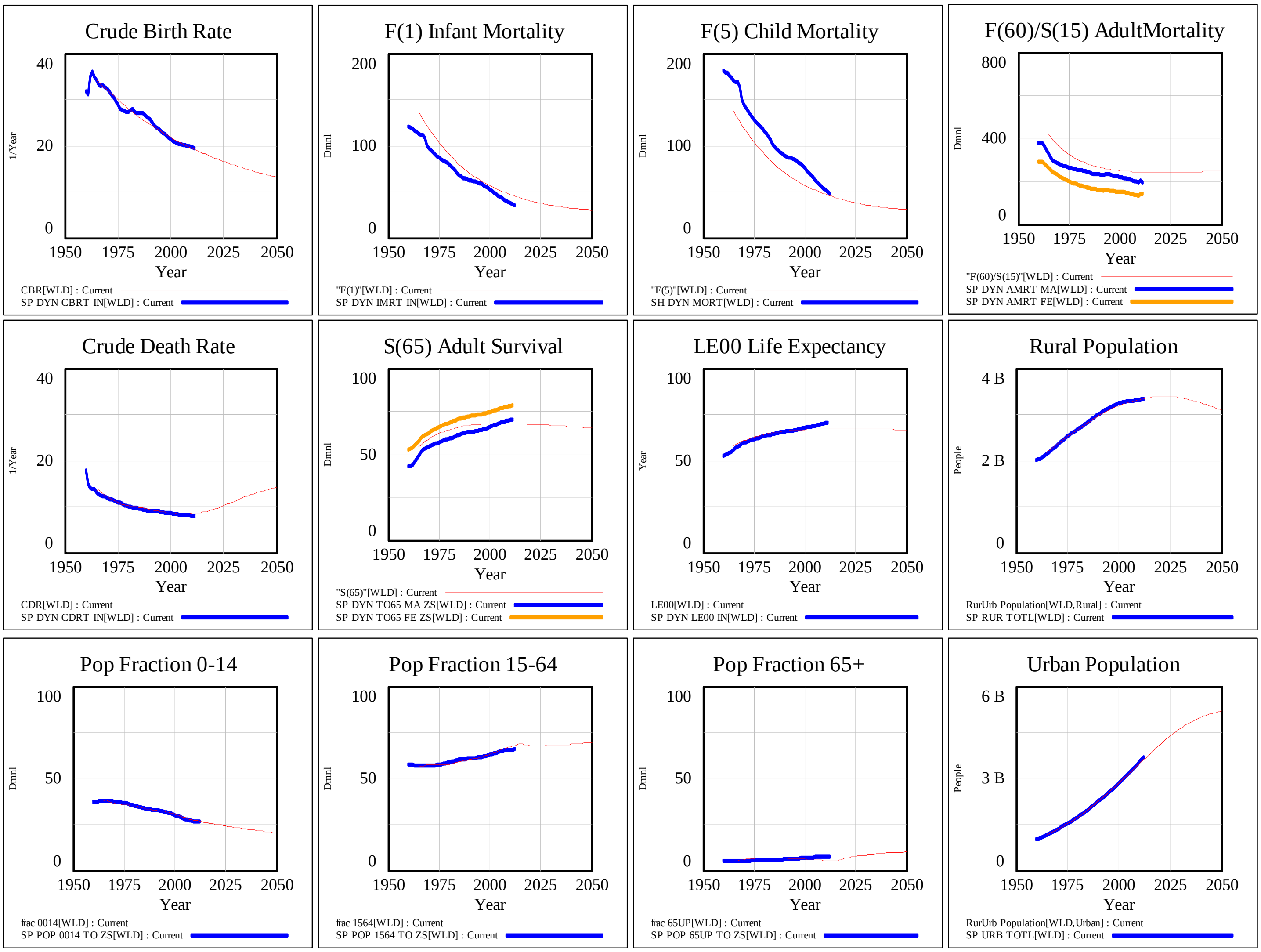


Figure 18: Calibration of population parameters. Thick blue lines represent values for all countries (WLD). Reference: World Development Indicators, [9]

3 Gross Domestic Product

3.1 Solow Growth Model

Figure 19 shows the complete Solow Growth Model for calculating national output (Gross Domestic Product). See original papers by Solow [10, 11]. While more sophisticated economic models may apply, the Solow model is sufficient to reproduce the essential elements as shown by the calibration time series, and yet simple enough to provide reasonable extrapolations, and to provide hooks for policy and scenario analysis. Subsequent sections will detail the equations for output, available labor, capital, investment in capital, and total factor productivity.

3.2 Output

Figure 20 shows the factors and equations determining output. Output is given by a Cobb-Douglas function for capital and labor, multiplied by a productivity factor.

3.3 Available Labor

Figure 21 shows the factors and equations determining employed labor. Labor is approximated by a constant fraction (e.g. 85%) of the population between fifteen and sixty-four years of age. Output increases with labor, scaled by `Labor norm`, and with an exponent equal to the complement of the Cobb-Douglass coefficient.

3.4 Capital

Figure 22 shows the stock and flow structure and equations that determine capital. Output increases with capital, scaled by `Capital norm`, and with an exponent equal to the Cobb-Douglass coefficient. Capital increases by investment: allocating a fraction of the output to make new capital. Capital decreases by depreciation with a characteristic lifetime.

3.5 Investment Fraction of GDP

Figure 23 shows the simple model and equations for the fraction of GDP allocated to investment in new capital. For simplicity, the savings fraction is assumed to approach an asymptotic goal, `Savings Fraction goal w policy`. As the name implies, the asymptotic value could be determined, for example, by monetary policy.

3.6 Total Factor Productivity

Figure 24 shows the simple model and equations fortotal factor productivity. In this model, `Productivity` approaches an asymptotic value with a rate proportional to the difference between the asymptotic and current values. The initial and asymptotic values and decay rate are chosen to give a good agreement with historical values as well as reasonable projections.

3.7 Calibration

Figures 25–29 show the calibrated results of the GDP model. For the case of China, the model is calibrated to reproduce twenty years of historical values of gross domestic product, employed labor, GDP per capita, capital, capital investment, and total factor productivity. The calibration sources and coefficients are given in Tables 3 and 4, respectively.

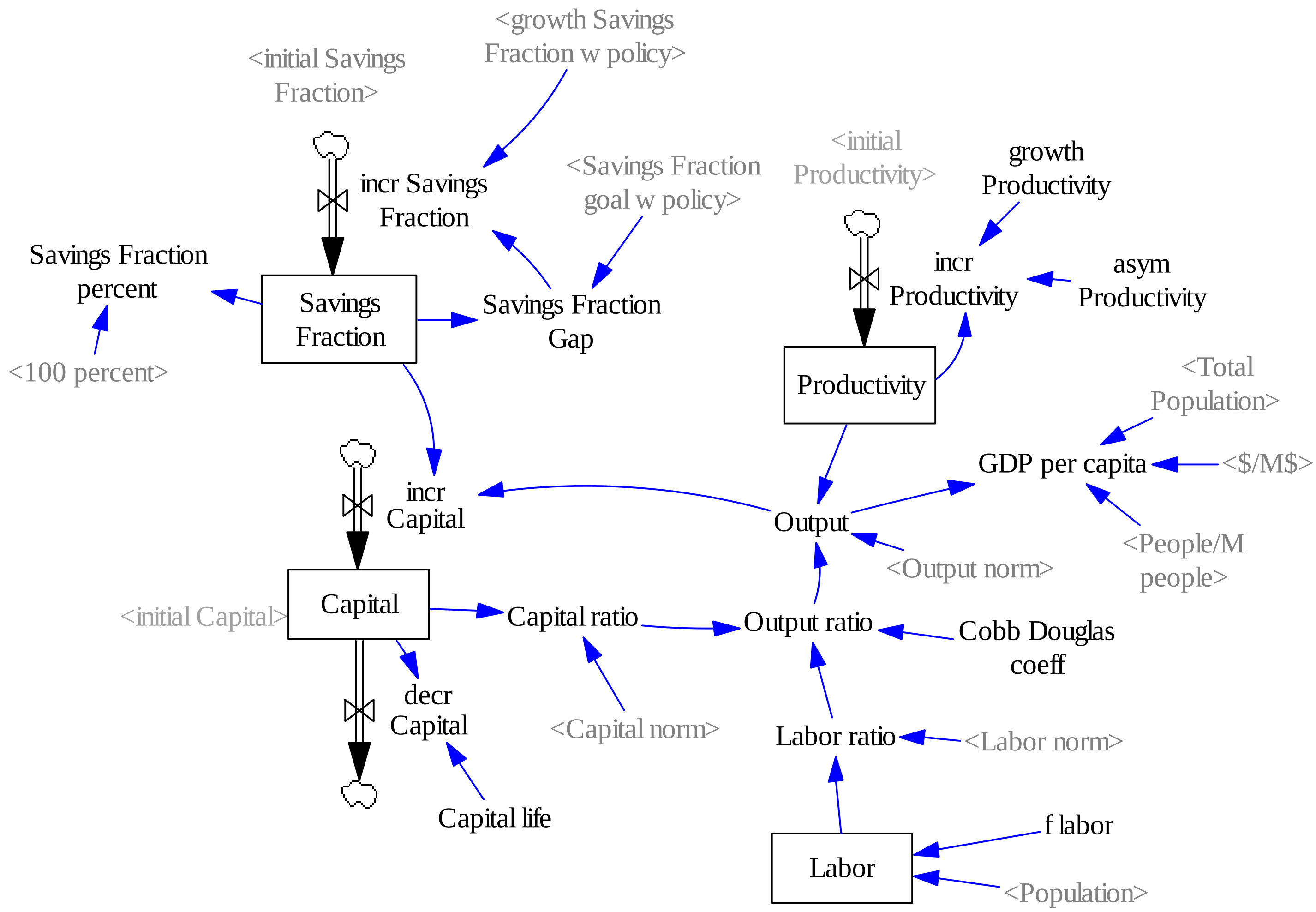
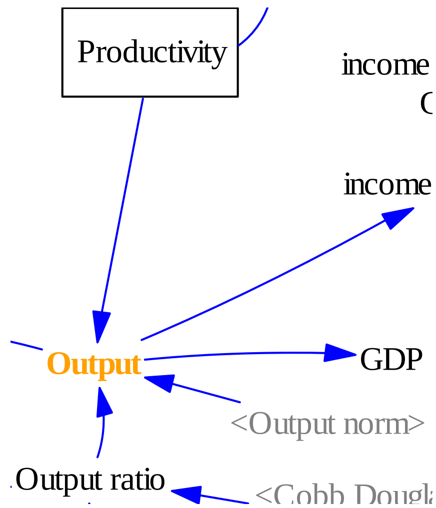
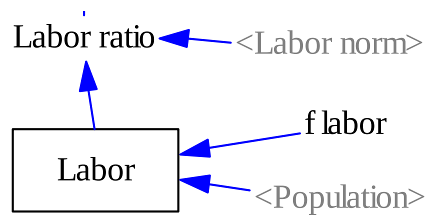


Figure 19: Relationship amongst all variables in Solow Growth Model for Gross Domestic Product.



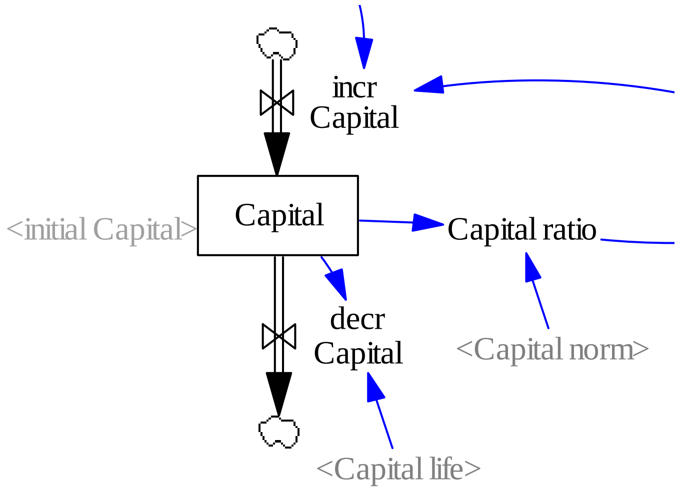

```
Output[region PWT80]=
Productivity[region PWT80]*Output norm[region PWT80]*Output ratio[region PWT80]
~ M$/Year
~ Rate of production, Y = F(K,L).  Using Cobb-Douglas function with constant \
returns to scale.
|

Output norm[region PWT80]=
GET DATA AT TIME(RGDPNA[region PWT80], norm time)
~ M$/Year [1e+006,5e+007,100000]
~ |

Output ratio[region PWT80]=
Capital ratio[region PWT80]^Cobb Douglas coeff[region PWT80]*
Labor ratio[region PWT80]^(1-Cobb Douglas coeff[region PWT80])
~ Dmnl
~ |
```

Figure 20: Relationship amongst variables and equations for Output (GDP) in Solow Growth Model.



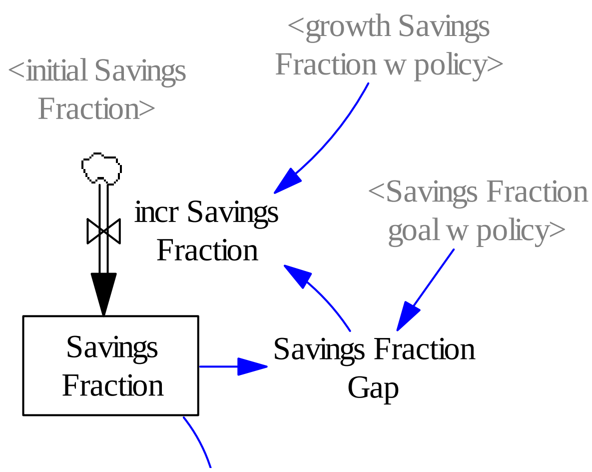

```
Labor ratio[region PWT80]=
Labor[region PWT80]/Labor norm[region PWT80]
~ Dmnl
~ |

Labor[region WDI]=
SUM(Population[region WDI,ClimateZone!,RurUrb!,ag1564!])*f labor[region WDI] ~~|

Labor[OED']=
Labor[OED]-SUM(Labor[in OED!]) ~~|

Labor[ROW]=
Labor[WLD]-SUM(Labor[in OED!])-SUM(Labor[non OED!])
~ Mpeople
~ labor
|

Labor norm[region PWT80]=
GET DATA AT TIME(EMP[region PWT80], norm time)
~ Mpeople
~ |
```

Figure 21: Relationship amongst variables and equations for Labor ratio.



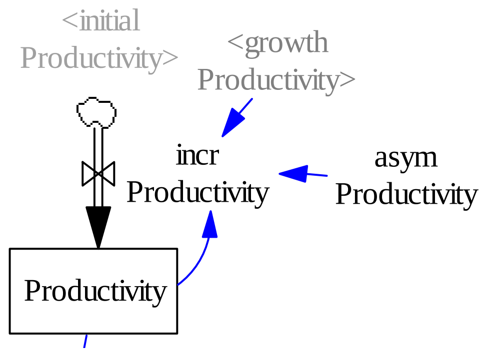

```
Capital ratio[region PWT80]=
Capital[region PWT80]/Capital norm[region PWT80]
~ Dmnl
~ |

Capital[region PWT80]= INTEG (
incr Capital[region PWT80]-decr Capital[region PWT80],
initial Capital[region PWT80])
~ M$
~ capital
|

Capital norm[region PWT80]=
GET DATA AT TIME( RKNA[region PWT80], norm time)
~ M$ [1e+006,5e+007,1e+006]
~ |

incr Capital[region PWT80]=
Output[region PWT80]*Savings Fraction[region PWT80]
~ M$/Year
~ net increase of capital stock
|

decr Capital[region PWT80]=
Capital[region PWT80]/Capital life[region PWT80]
~ M$/Year
~ |
```

Figure 22: Relationship amongst variables and equations determining Capital ratio, and rate of change of Capital.




```
Savings Fraction[region PWT80]= INTEG (
incr Savings Fraction[region PWT80],
initial Savings Fraction[region PWT80])
~ Dmnl [0,0.5,0.001]
~ Savings fraction of output
|

Savings Fraction Gap[region PWT80]=
Savings Fraction goal w policy[region PWT80]-Savings Fraction[region PWT80]
~ Dmnl
~ |

incr Savings Fraction[region PWT80]=
Savings Fraction Gap[region PWT80]*growth Savings Fraction w policy[region PWT80]
~ 1/Year
~ |
```

Figure 23: Relationship amongst variables and equations determining Savings (investment in Capital).




```
Productivity[region PWT80]= INTEG (
incr Productivity[region PWT80],
initial Productivity[region PWT80])
~ Dmnl
~ |

incr Productivity[region PWT80]=
Productivity[region PWT80]*growth Productivity[region PWT80]*
(1-Productivity[region PWT80]/asym Productivity[region PWT80])
~ 1/Year
~ |
```

Figure 24: Relationship amongst variables and equations determining Total Factor Productivity.

Source	Variable name	Variable definition
PWT	EMP	Number of persons engaged (in millions)
PWT	RGDPNA	Real GDP at constant 2005 national prices (in mil. 2005US$)
PWT	RKNA	Capital stock at constant 2005 national prices (in mil. 2005US$)
PWT	RTFPNA	TFP at constant national prices (2005=1)
WDI	NY.GDP.PCAP.PP.KD	GDP per capita, PPP (constant 2005 international $)
WDI	NE.GDI.FTOT.ZS	Gross fixed capital formation (% of GDP)

Table 3: GDP model calibration time series from Penn World Table [12] and World Development Indicators [9].

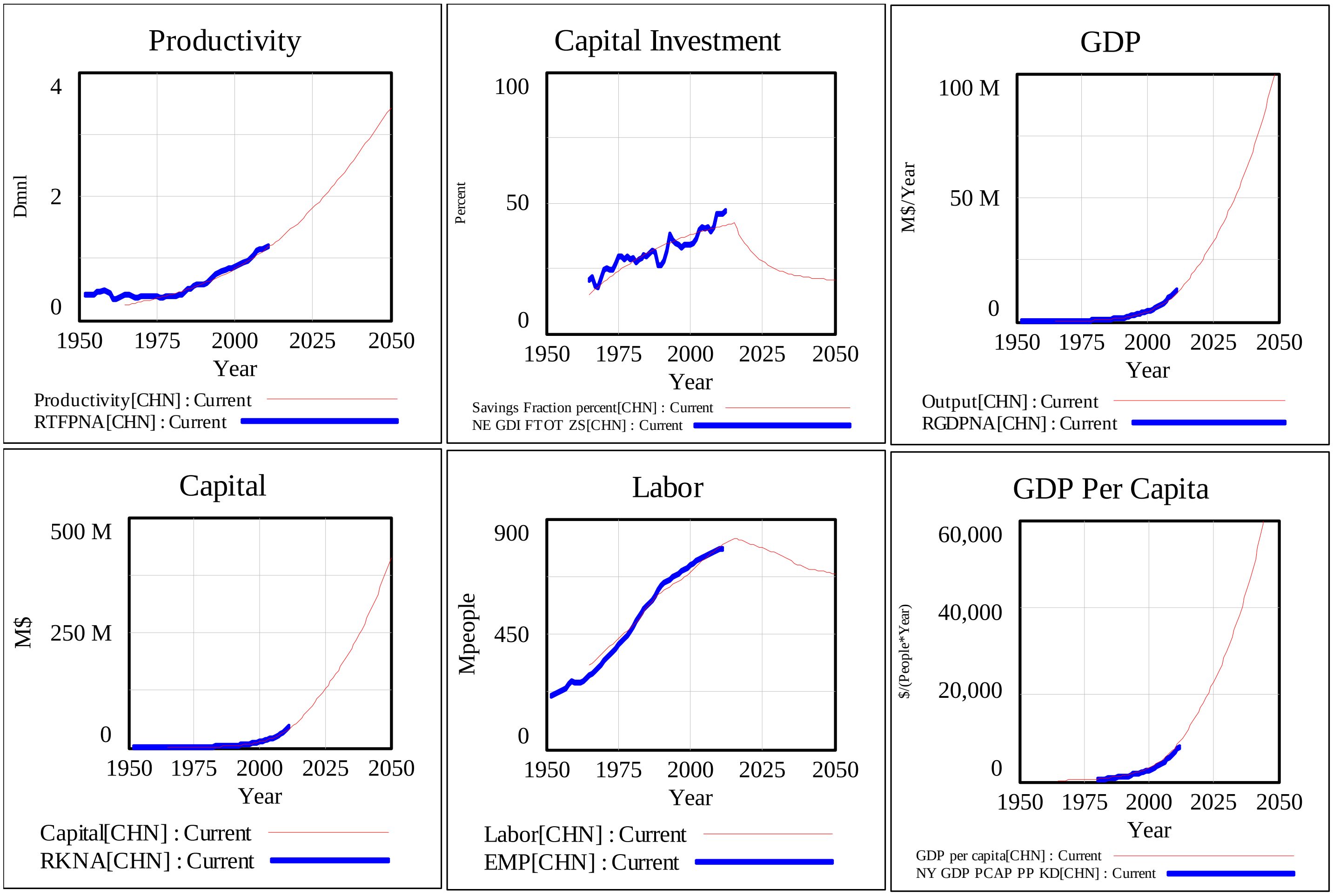


Figure 25: GDP Model Calibration, China. All currency values given in 2005 USD.

Variable \ Region	CHN	IND	USA	OED	WLD
asym Productivity (Dmnl)	8.46	1.20	1.42	1.27	7.42
Capital Life (Year)	100	29	26	24	28
Cobb Douglas coeff (Dmnl)	0.54	0.87	0.44	0.38	0.12
f labor (Dmnl)	0.83	0.59	0.73	0.68	0.66
growth Productivity (1/Year)	0.037	0.028	0.020	0.023	0.013
growth Savings Fraction (1/Year)	0.028	0.012	0.100	0.100	0.100
initial Productivity (Dmnl)	0.25	0.62	0.69	0.73	0.62
Savings Fraction goal (Dmnl)	0.51	0.43	0.22	0.23	0.24

Table 4: GDP model calibration coefficients.

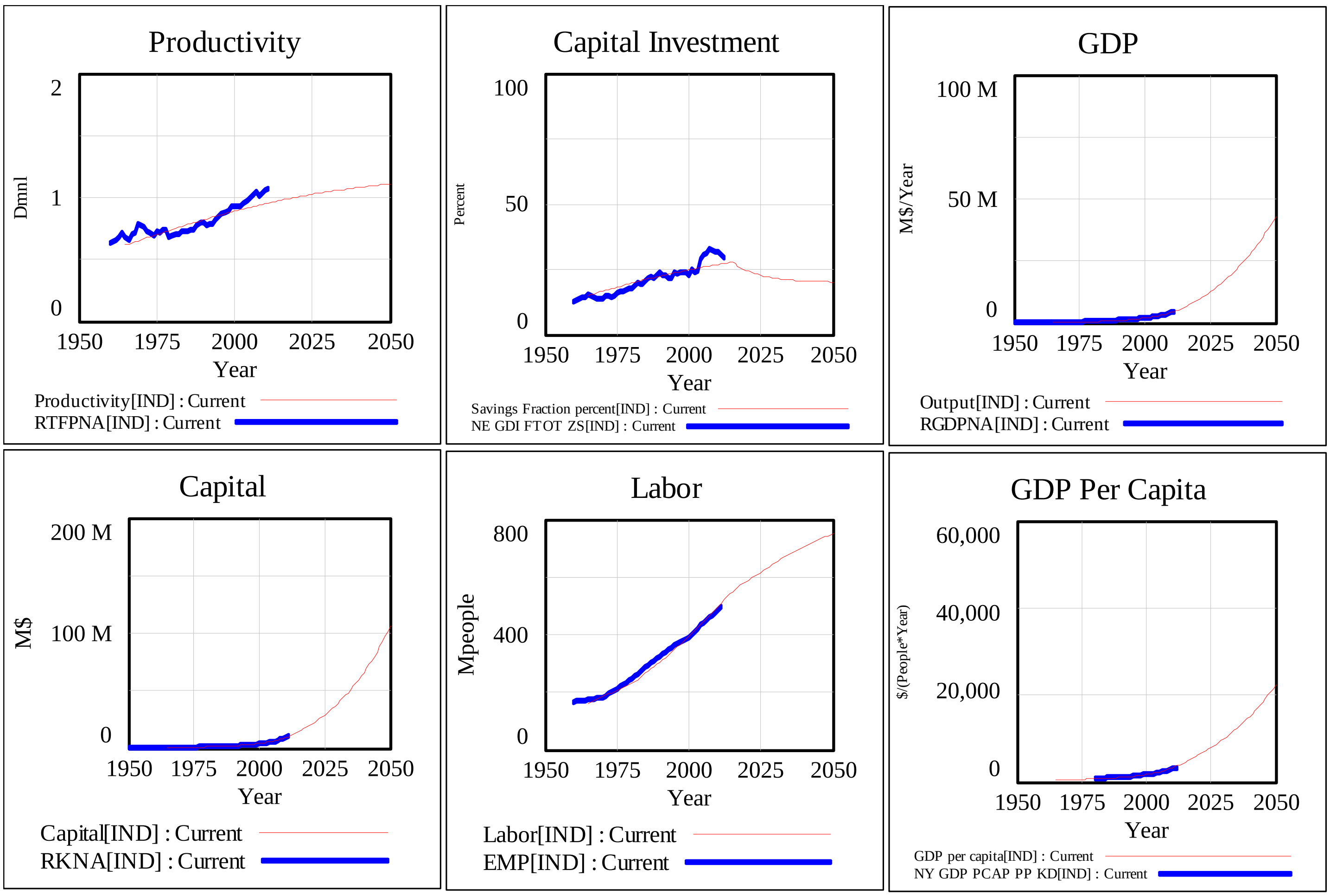


Figure 26: GDP Model Calibration, India. All currency values given in 2005 USD.

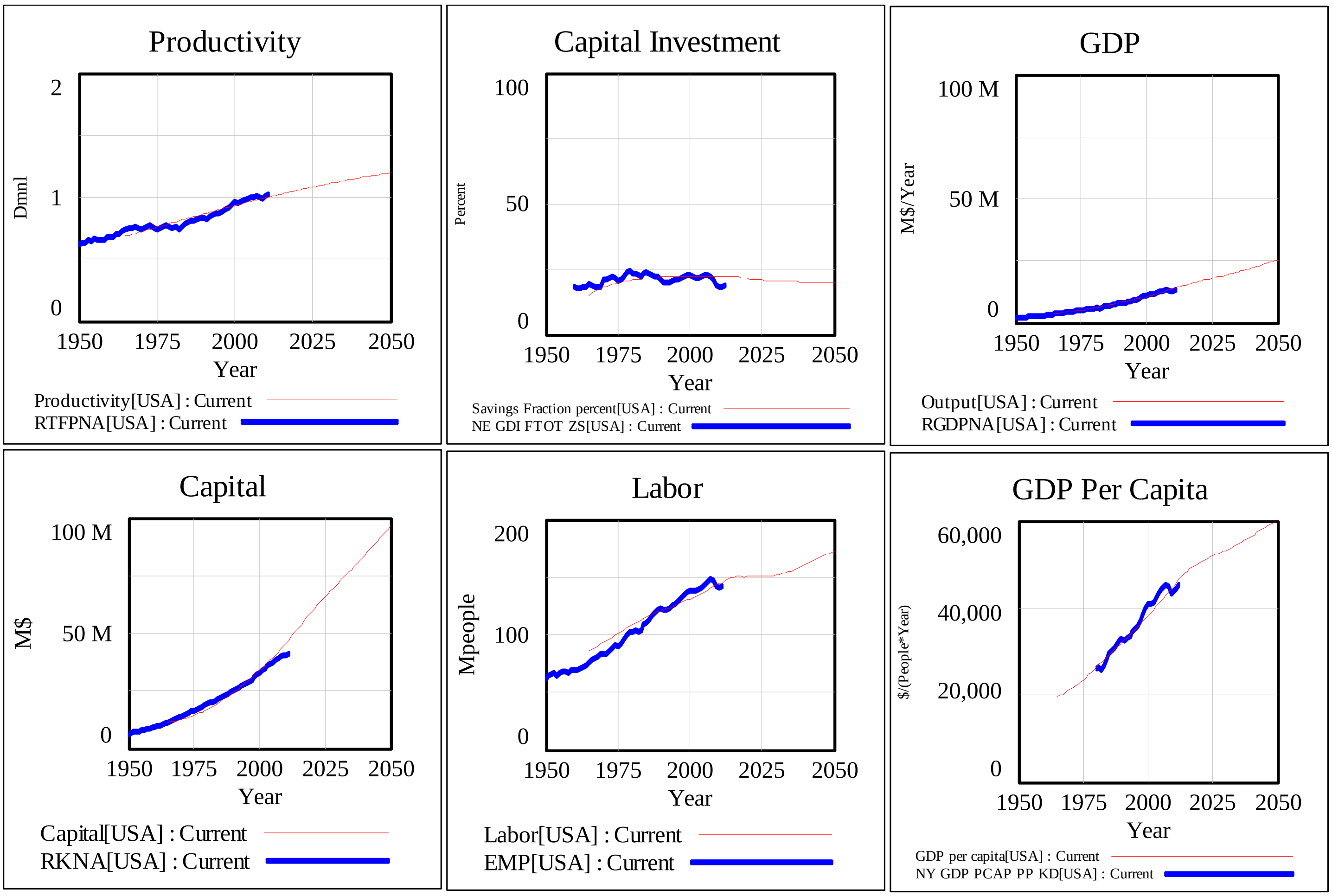


Figure 27: GDP Model Calibration, United States. All currency values given in 2005 USD.

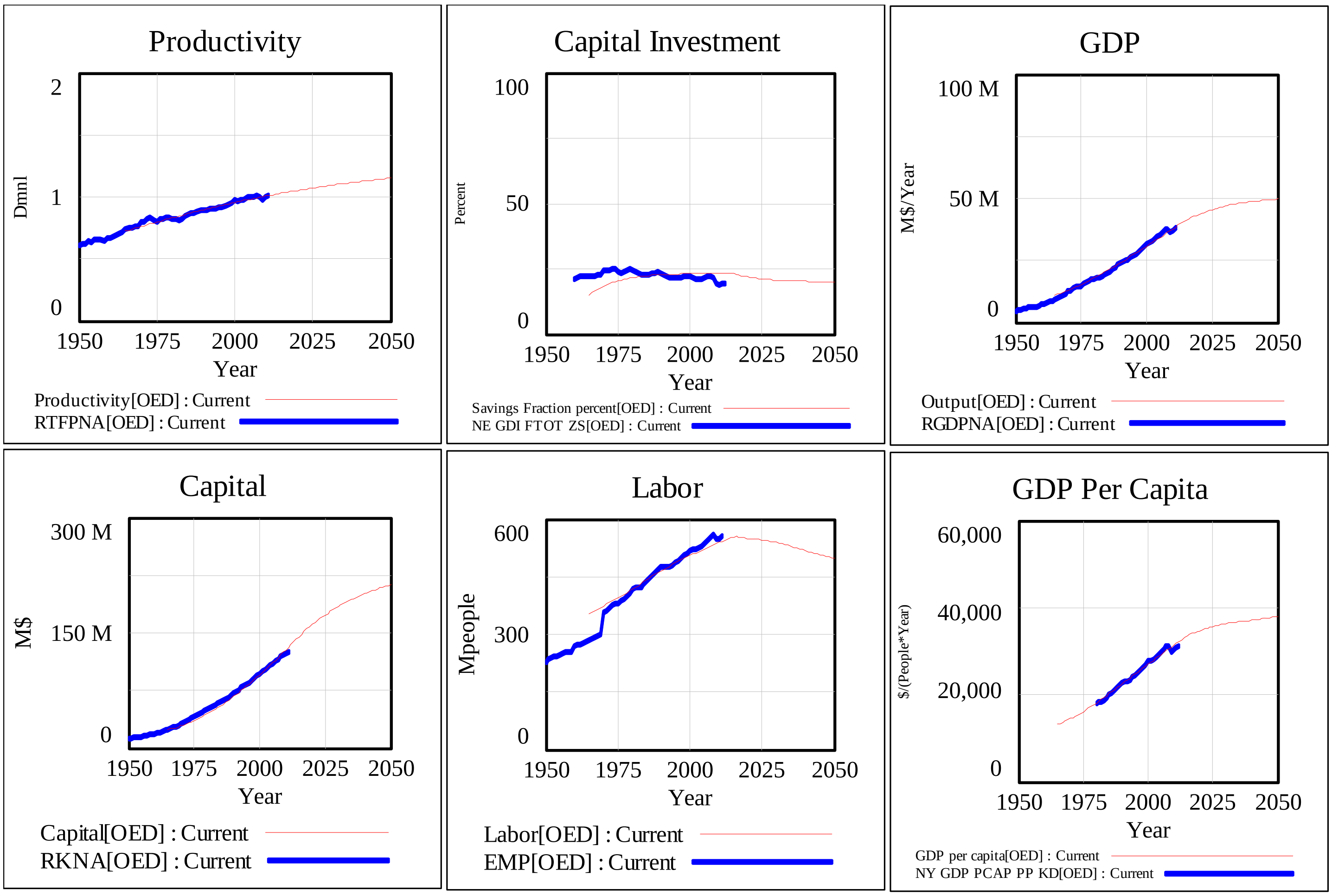


Figure 28: GDP Model Calibration, OECD. All currency values given in 2005 USD.

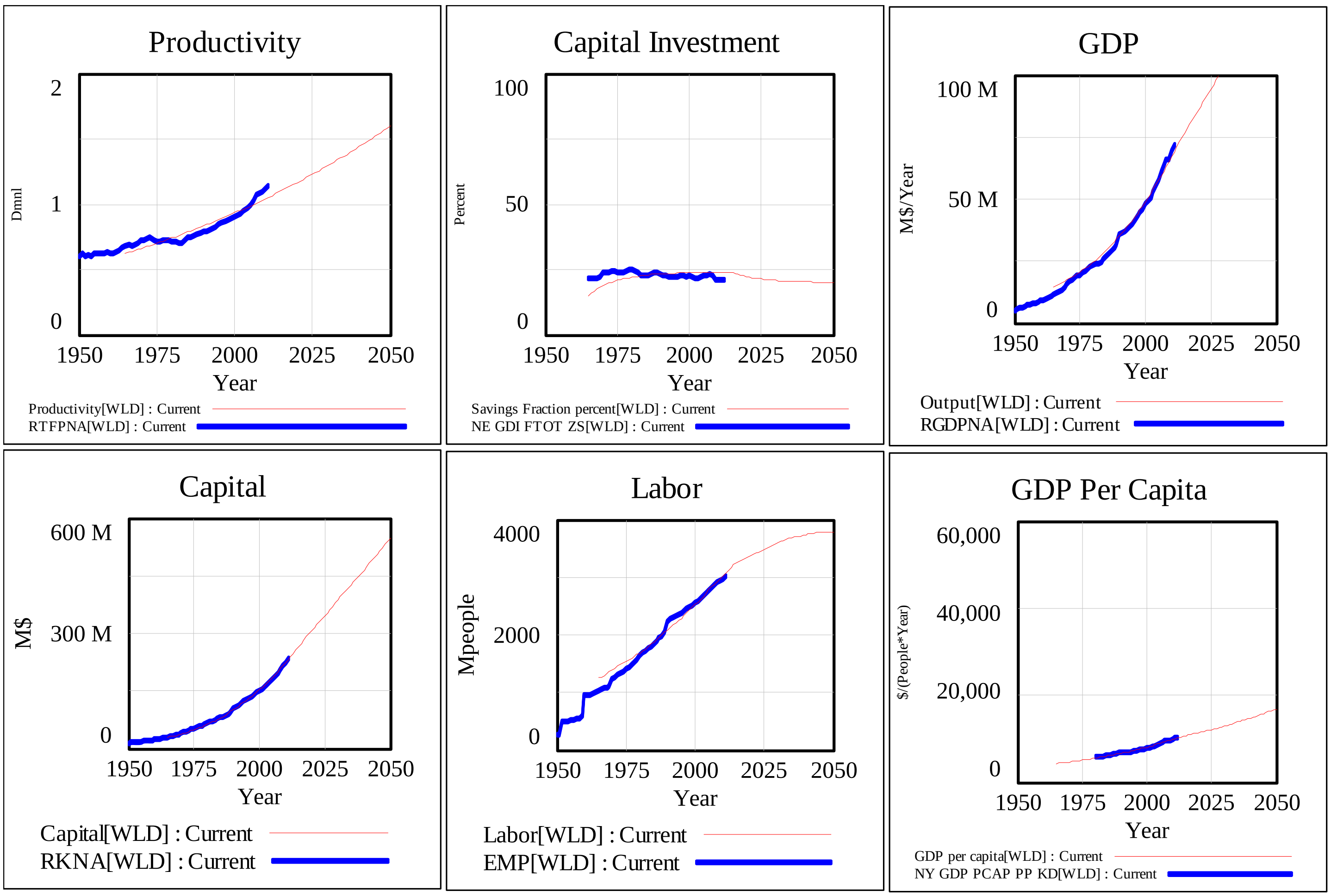


Figure 29: GDP Model Calibration, World. All currency values given in 2005 USD.

4 Income Distribution

4.1 Lognormal Distribution

The frequency distributions for income is well-described by a log-normal distribution.[13] The log-normal cumulative distribution function for location parameter μ and scale parameter σ is given by[4]

$$\int_0^x \textsc{pdf}(u)\,du = \textsc{cdf}(x) = \frac{1}{2} + \frac{1}{2}\text{erf}\left[\frac{\ln x - \mu}{\sqrt{2}\sigma}\right] = \Phi\left(\frac{\ln x - \mu}{\sigma}\right), \quad (4)$$

$$\int_0^x \textsc{pdf}(u)\,u\,du = \exp\left(\mu + \frac{\sigma^2}{2}\right)\Phi\left(\frac{\ln x - \mu - \sigma^2}{\sigma}\right) \quad (5)$$

The log-normal distriution has mean $\exp(\mu + \sigma^2/2)$ and median $\exp(\mu)$, which allows for quick estimation of the location and scale parameters. The argument of the exponent must be dimensionless, so μ and σ must both be dimensionless. The argument of the natural logarithm must be dimensionless, so x must be dimensionless. Therefore the integral makes sense only if we integrate over *dimensionless* income.

We are often given fraction of total income held by population percentiles. As an example, Table 6 shows the fractions of income earned by different percentiles of the population in China in 2009, reported by World Development Indices [9]. Figure 30 shows the same data, plotted as cumulative distributions of population vs cumulative distribution of income. We will fit a log-normal distribution to this data.

Let x_{10} represent the (dimensionless) income at the upper range of the lowest decile, etc. Then

$$\textsc{cdf}(x_{10}) = 0.1, \quad (6)$$

$$\dots$$

$$\textsc{cdf}(x_{90}) = 0.9. \quad (7)$$

In this example, $\textsc{cdf}x$ represents the fraction of population with (dimensionless) income less than x. We could multiply by the total population $\textsc{pop}$ to get the *number* of people with (dimensionless) income less than x.

Similarly the total (dimensionless) income is given by

$$\textsc{pop}\int_0^\infty u\,\textsc{pdf}(u)\,du = \textsc{tot inc}.$$

Therefore the (dimensionless) percapita income is given by

$$\int_0^\infty u\,\textsc{pdf}(u)\,du = \frac{\textsc{tot inc}}{\textsc{pop}}, \\ = \exp\left(\mu + \frac{\sigma^2}{2}\right).$$

[4] http://en.wikipedia.org/wiki/Log-normal_distribution

where the last equation follows from Eq.(5).

Therefore the fraction of total income held by the fraction of population with (dimensionless) income less than x is given by

$$G(x) = \frac{\int_0^x u\,\mathrm{PDF}(u)\,du}{\int_0^\infty u\,\mathrm{PDF}(u)\,du} = \Phi\left(\frac{\ln x - \mu - \sigma^2}{\sigma}\right).$$

$$G(x_{10}) = \textsc{si.dst.frst}.10, \tag{8}$$
$$G(x_{20}) = \textsc{si.dst.frst}.20, \tag{9}$$
$$G(x_{40}) - G(x_{20}) = \textsc{si.dst.02nd}.20, \tag{10}$$
$$G(x_{60}) - G(x_{40}) = \textsc{si.dst.03rd}.20, \tag{11}$$
$$G(x_{80}) - G(x_{60}) = \textsc{si.dst.04th}.20, \tag{12}$$
$$1 - G(x_{80}) = \textsc{si.dst.05th}.20, \tag{13}$$
$$1 - G(x_{90}) = \textsc{si.dst.10th}.10. \tag{14}$$

Experimentally we determine that the values of x can be scaled by a factor $k > 0$, leaving the value of σ unchanged and scaling μ by a factor of $1 + \ln k$:

$$x \to k\,x, \quad \mu \to (1 + \ln k)\mu.$$

If we wish to scale all values by k (say, the *number* of 2013 dollars corresponding to the percapita income), the value of μ is scaled as described above. Without loss of generality, we scale income by the percapita income (leaving it dimensionless), which implies $\exp(\mu + \sigma^2/2) = 1$, i.e.

$$\begin{aligned}
\mu &= -\frac{\sigma^2}{2}, \\
\mathrm{CDF}(x) &= \Phi\left(\frac{\ln x + \sigma^2/2}{\sigma}\right), \\
G(x) &= \Phi\left(\frac{\ln x - \sigma^2/2}{\sigma}\right), \\
\ln x_{10} &= \sigma\,\Phi^{-1}(0.1) - \frac{\sigma^2}{2}, \\
&\dots \\
\ln x_{90} &= \sigma\,\Phi^{-1}(0.9) - \frac{\sigma^2}{2}, \\
G(x_{10}) &= \Phi\left(\Phi^{-1}(0.1) - \sigma\right), \\
&\dots \\
G(x_{90}) &= \Phi\left(\Phi^{-1}(0.9) - \sigma\right).
\end{aligned}$$

Using Excel to evaluate the fraction of total income held by the fraction of population with income less than x_k, we would have

$$G(x_k) = \textsc{normsdist}(\textsc{normsinv}(k) - \sigma)$$

By minimizing least squares, we can find values for σ that produce values of $x \in \{x_{10}, x_{20}, \ldots\}$ which satisfiy Eqs.(8)–(14). Figure 31 shows the least-squares fit to historical distribution data for the USA and China.

Figure 32 shows a simple model and equations for the evolution of the single parameter, `income sigma`, describing the log-normal distribution of incomes. The parameter approaches an asymptotic value at a rate that is proportional to the difference between the asymptotic value and the instantaneous value. The initial and asymptotic values, as well as the decay rate, are chosen to give an approximate fit with historical values, as well as give reasonable projections.

4.2 Fraction of Population that Affords

As an application, let us consider a product whose price p (measured in \$/year) can be afforded by households who have a fraction d of income above the price. Let income x be measured in multiples of the percapita income I. Then all households with

$$d\,x\,I \geq p$$

can afford the price. The fraction M of households that can afford price p is given by

$$M(p) = 1 - \text{CDF}(p/I/d).$$

Given average p of a product, household fraction with ownership, average household income and income distribution parameters, we can estimate the fraction d of income at the income threshold.

4.3 Calibration

Figure 33 shows the calibration of a simple model for income distribution to historical data from China. This section describes the simple model for evolution of a single-parameter model for the distribution of income. The next section describes the fitting of historical data of income distribution to a single-parameter log-normal function. The detailed income distribtion data used for calibration are the WDI time series described in Table 5.

Variable Name	Description
SI.DST.FRST.10	Income share held by lowest 10%
SI.DST.FRST.20	Income share held by lowest 20%
SI.DST.02ND.20	Income share held by second 20%
SI.DST.03RD.20	Income share held by third 20%
SI.DST.04TH.20	Income share held by fourth 20%
SI.DST.05TH.20	Income share held by highest 20%
SI.DST.10TH.10	Income share held by highest 10%

Table 5: World Development Indicators income distribution measure time series.[9]

Variable	2009		G(x)	function	fit	SSE
					sigma	0.77
SI DST FRST 10 [CHN]	1.69	G(0.1)	2.03	G(0.1)	2.03	0.12
SI DST FRST 20 [CHN]	4.67	G(0.2)	5.40	G(0.2)	5.40	0.54
SI DST 02ND 20 [CHN]	9.74	G(0.4)	15.42	G(0.4)-G(0.2)	10.01	0.08
SI DST 03RD 20 [CHN]	15.31	G(0.6)	30.43	G(0.6)-G(0.4)	15.02	0.09
SI DST 04TH 20 [CHN]	23.19	G(0.8)	53.04	G(0.8)-G(0.6)	22.61	0.34
SI DST 05TH 20 [CHN]	47.09	G(0.9)	69.72	1-G(0.8)	46.96	0.02
SI DST 10TH 10 [CHN]	29.98			1-G(0.9)	30.28	0.09
						1.15

Table 6: Example of income distribution in China (2009), with least squares log-normal fit $\sigma \approx 0.77$. Source: WDI.[9]

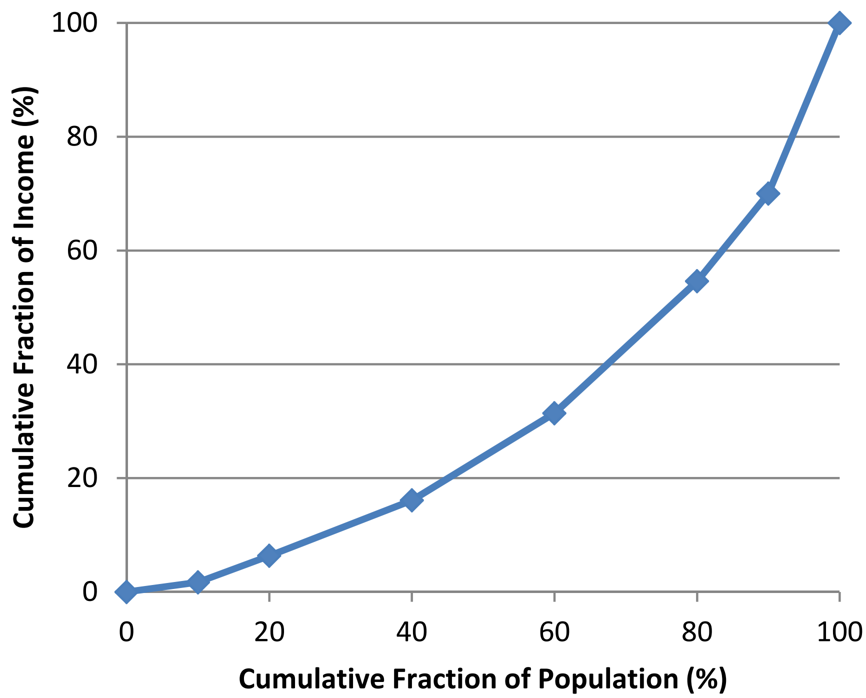


Figure 30: Example of cumulative distribution of income distribution in China (2009). Source: WDI.[9]

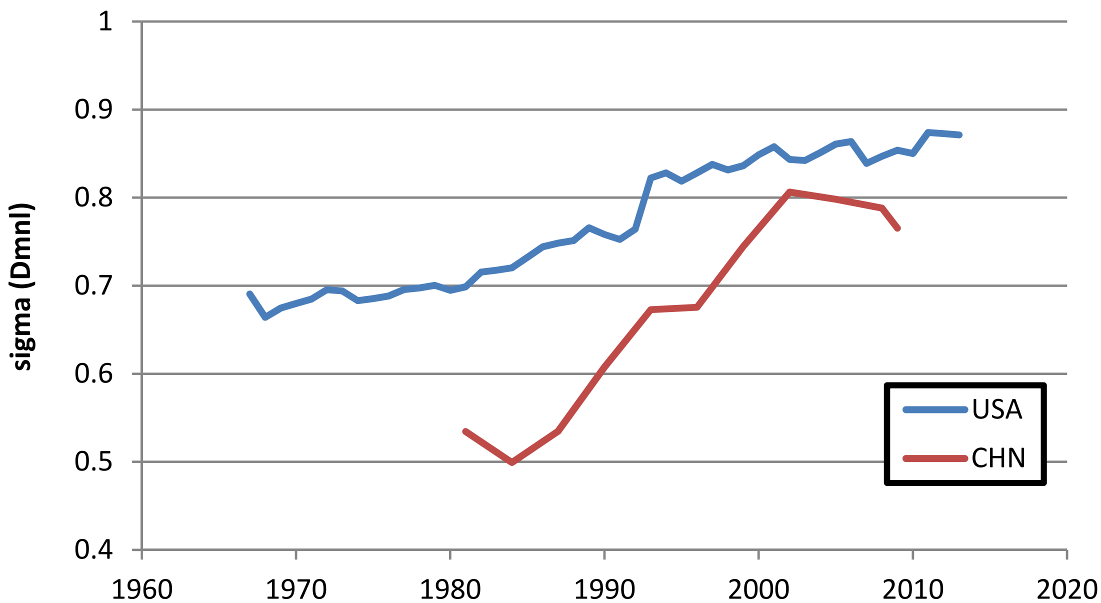
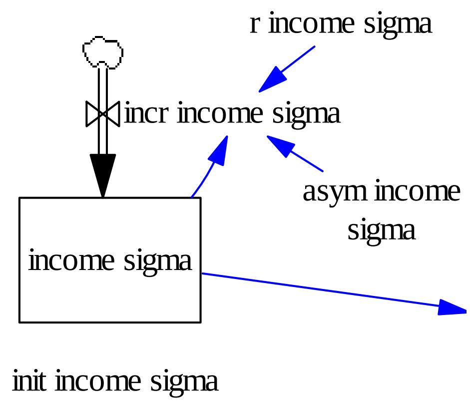


Figure 31: Scale parameter σ of log-normal distribution of income for USA and China, from least squares fit to data from WDI.[9] Lower values of σ correspond to more uniformly distributed income.




```
income sigma= INTEG (
incr income sigma,
init income sigma)
~ Dmnl
~ |

incr income sigma=
r income sigma*(asym income sigma-income sigma)
~ 1/Year
~ |

init income sigma=
0.5
~ Dmnl
~ |

asym income sigma=
0.94
~ Dmnl [0,1.5,0.01]
~ |

r income sigma=
0.05
~ 1/Year [0,0.1,0.01]
~ |
```

Figure 32: Relationship amongst variables and equations to model income distribution.

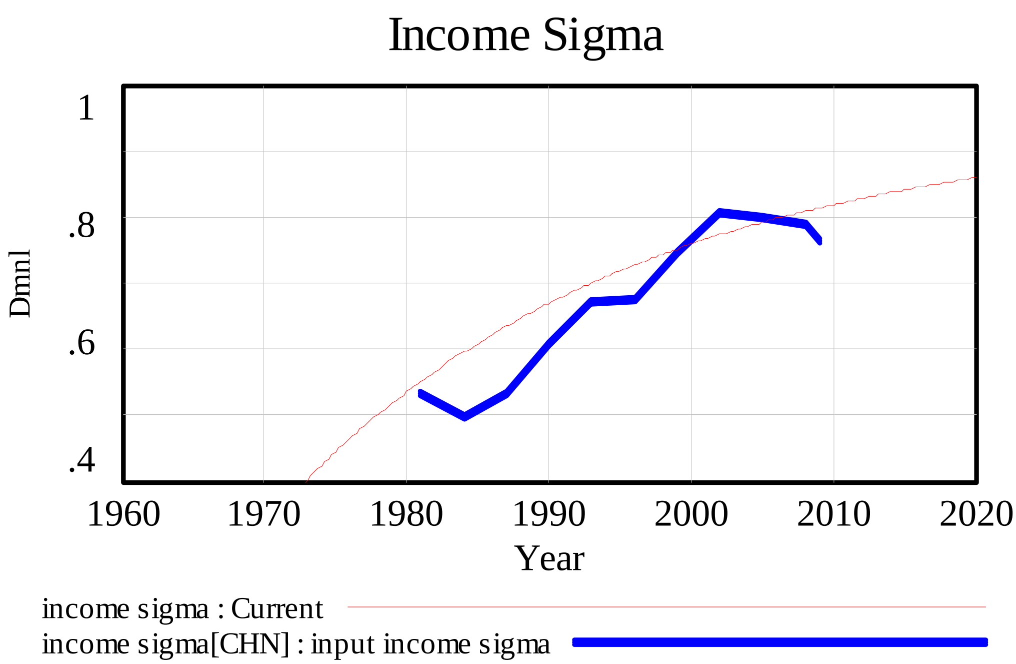


Figure 33: Income distribution model Calibration, China.

5 Buildings Turnover Model

5.1 Building Stock

The building types considered in this model are taken from RFC [2, 3]. Figure 34 lists the building types as shows in the simulation subscript control panel.

The complete stock and flow structure for buildings is shown in Figure 35. Using the subscript feature of Vensim, the building stock and flow model is repeated for each of the building types shown in Figure 34.

Figure 36 shows the building existing stock submodel and associated equations. As with the population model, buildings are tracked by age since construction, since the probability of demolition depends on the age. `Building Area` increases with `Construction Completions` and decreases with `Demolitions`, and at any time the `Building Area all Vintage` is the sum of `Building Area` of all ages. Roughly speaking, construction is determined by comparing `Building Area all Vintage` with `Indicated Area`.

5.2 Indicated Area and Indicated Area per Capita

Figure 37 shows the causal loop diagram for `Indicated Area` and the associated equations. `Indicated Area` is given by the product of population (summed over all ages), a normalizing value of building area per capita, and a dimensionless factor adjusting for `GDP per capita`. The dependence `Area Income Factor` on `GDP per capita` is given by a concave function ($f''(x) < 0$), in this case $f \sim 1 - \exp(-x)$.

5.3 Indicated Construction

Figure 38 shows the causal loop diagram and equations for `Indicated Construction`, which is driven primarily by the gap between the by `Indicated Area` and the actual building area. The equations also compensate for the pipeline of `Building Area in Construction` as well as `Demolition`. See Sterman [1, Ch.17] for more details.

5.4 Buildings in Construction

Figure 39 shows the stock and flow structure and equations for `Building Area in Construction`, which increases with `Construction Starts` and decreases with `Construction Completions`. `Construction Starts` is given naturally by the positive portion of `Indicated Construction`. `Construction Completions` is given by a third-order delay of `Construction Starts`.

5.5 Building Aging and Hazard Function

The rate of building demolition is assumed to increase with the age of the building. We employ an Gompertz-Makeham hazard function just as was done for population. Figure 40 shows the causal loop diagrram and equations for the building hazard function. Figure 41 shows the simple model for the time dependence of the Gompertz-Makeham coefficients, as well as the initial and asymptotic values of the coefficients themselves.

5.6 Retrofit

Figure 42 shows the stock and flow structure of building retrofit. With periodic retrofit, buildings can easily be in good service for a hundred years. The model structure tracks buildings by the number of retrofit cycles they have experienced. The average interval between retrofits is given, but instantaneous rate of retrofits depends on the continuous aging structure of the building stock. Since the retrofit cycle structure largely repeats, Figure 42 gives the equations only for the first retrofit cycle. Figure 43 shows the causal loop diagram and the equations for the total retrofit rate.

5.7 Total Building Area and Total Construction

Figure 44 shows the causal loop diagram and equations for calculating the total building area by the number of retrofit cycles. Using Chinese commercial office area in the transition climate zone as an example, Figure 45 shows total building area by retrofit generation. For the same example, Figure 46 shows the new construction and total retrofit construction rates.

5.8 Calibration

Figure 47 shows projections for commercial and residential building area by building type and Chinese climate zone: the thick blue lines are from from Reinventing Fire China (RFC) studies[2, 3], and the thin red lines are reproductions of the simulation.

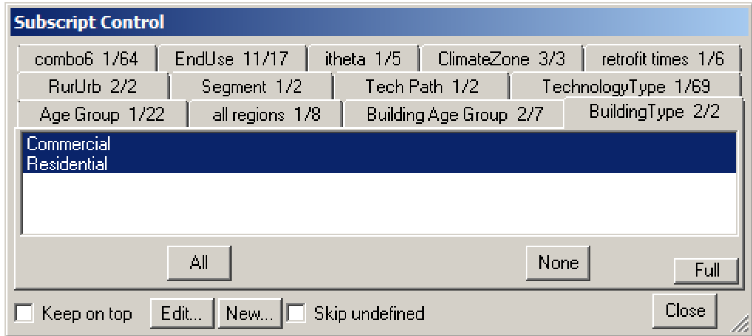


Figure 34: Subscript control showing the building types considered for China model.

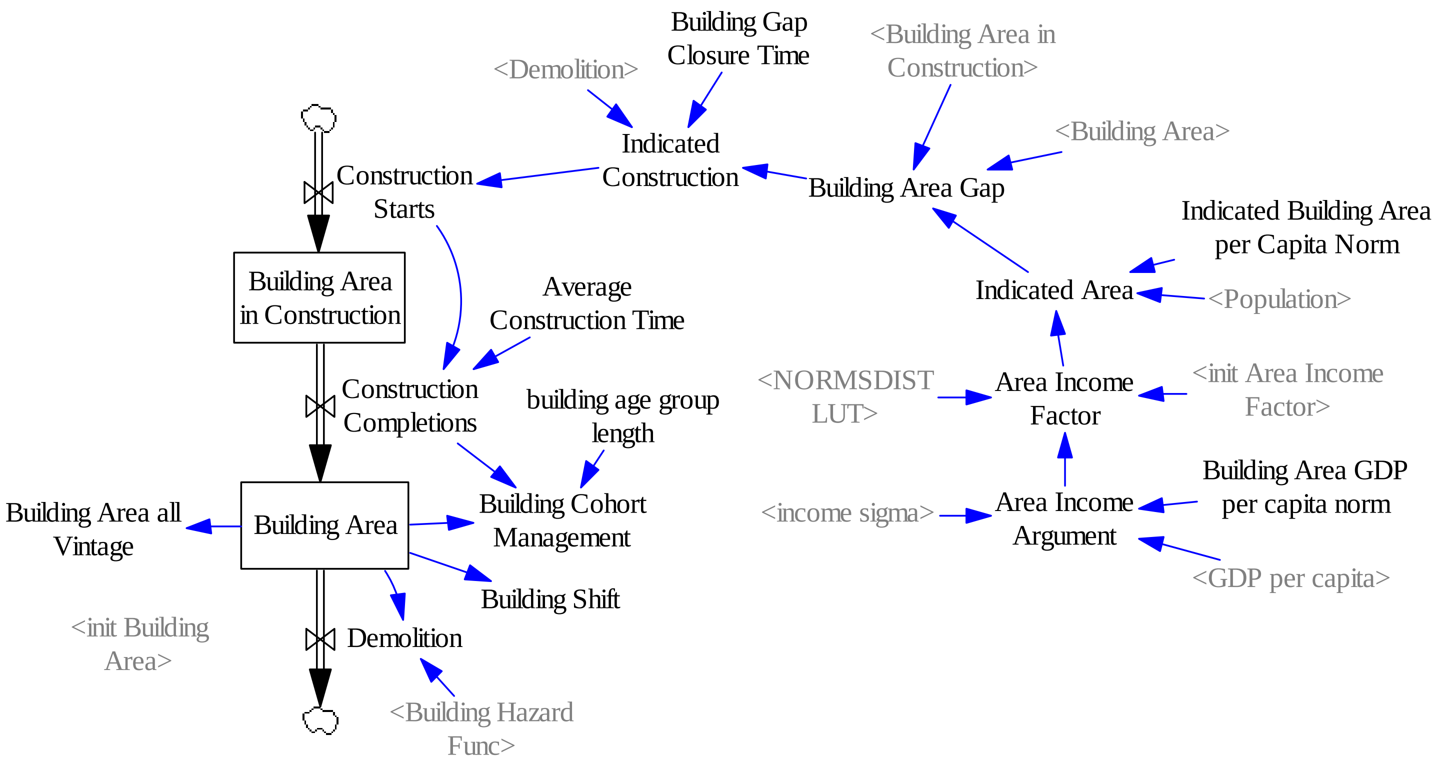
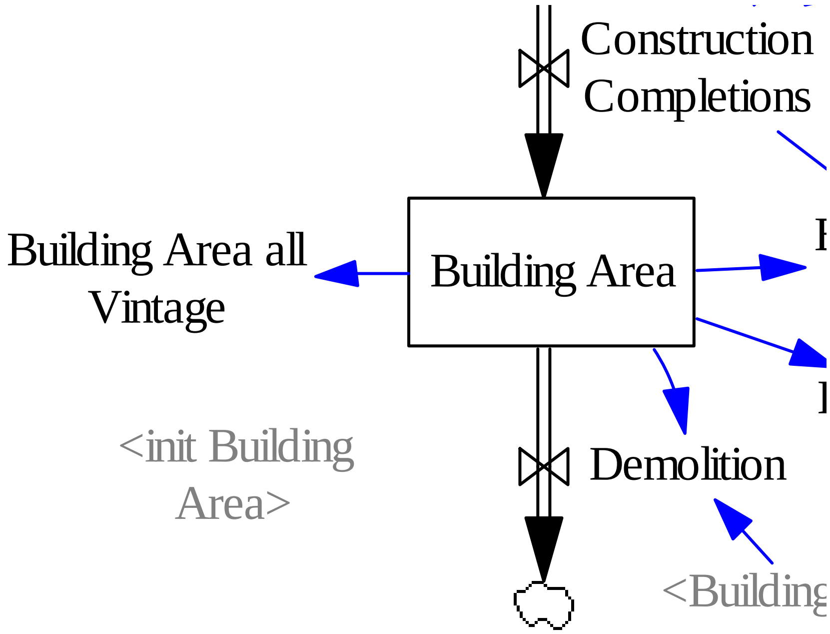
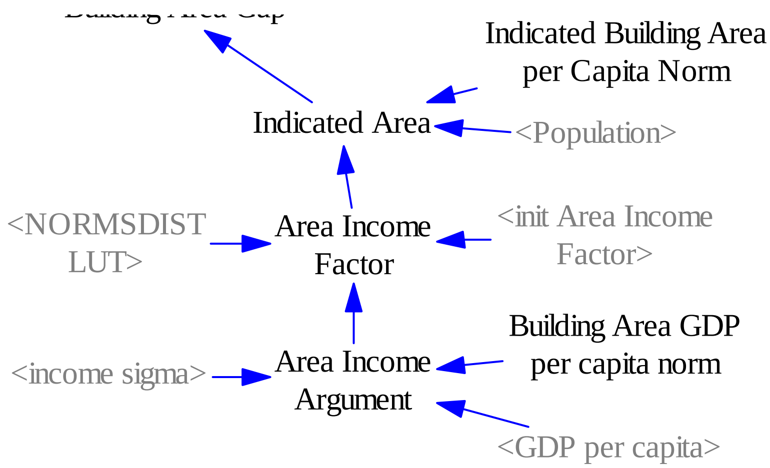


Figure 35: Building turnover model stock and flow structure.



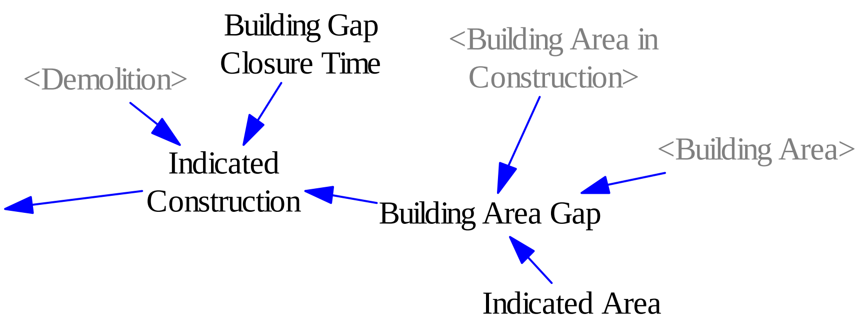

```
Building Area[region,ClimateZone,RurUrb,BuildingType,b0000]= INTEG (
Construction Completions[region,ClimateZone,RurUrb,BuildingType]-
Demolition[region,ClimateZone,RurUrb,BuildingType,b0000],
init Building Area[region,ClimateZone,RurUrb,BuildingType,b0000]) ~~|

Building Area[region,ClimateZone,RurUrb,BuildingType,b01UP]= INTEG (
-Demolition[region,ClimateZone,RurUrb,BuildingType,b01UP],
init Building Area[region,ClimateZone,RurUrb,BuildingType,b01UP])
~ Msqm
~ |

Building Area all Vintage[region,ClimateZone,RurUrb,BuildingType]=
SUM(Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group!])
~ Msqm
~ |
```

Figure 36: Building existing stock submodel.



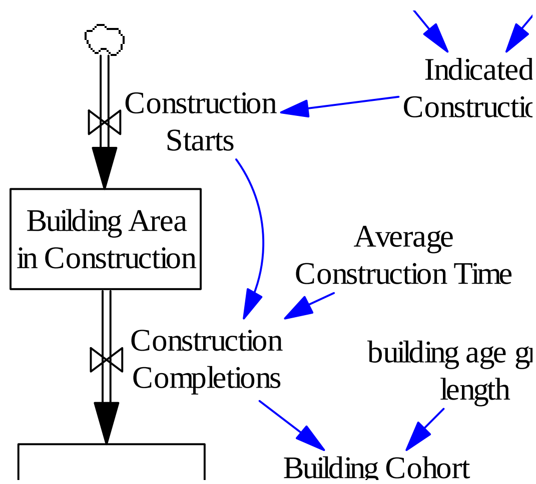

```
Indicated Area[region,ClimateZone,RurUrb,BuildingType]=
SUM(Population[region,ClimateZone,RurUrb,Age Group!])*
Indicated Building Area per Capita Norm[region,ClimateZone,RurUrb,BuildingType]*
Area Income Factor[region,RurUrb,BuildingType]
~ Msqm
~ |

Area Income Factor[region,RurUrb,BuildingType]=
1-NORMSDIST LUT(Area Income Argument[region,RurUrb,BuildingType])+
init Area Income Factor
~ Dmnl
~ |
```

Figure 37: Building indicated area and indicated dependence on income.



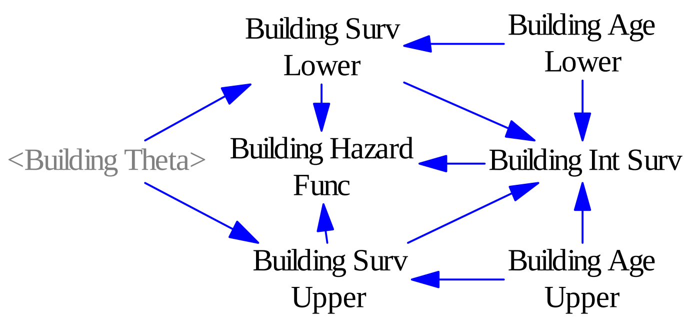

```
Indicated Construction[region,ClimateZone,RurUrb,BuildingType]=
Building Area Gap[region,ClimateZone,RurUrb,BuildingType]/Building Gap Closure Time+
SUM(Demolition[region,ClimateZone,RurUrb,BuildingType,Building Age Group!])
~ Msqm/Year
~ |

Building Area Gap[region,ClimateZone,RurUrb,BuildingType]=
Indicated Area[region,ClimateZone,RurUrb,BuildingType] -
SUM(Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group!])-
Building Area in Construction[region,ClimateZone,RurUrb,BuildingType]
~ Msqm
~ |
```

Figure 38: Building turnover model structure for indicated construction.



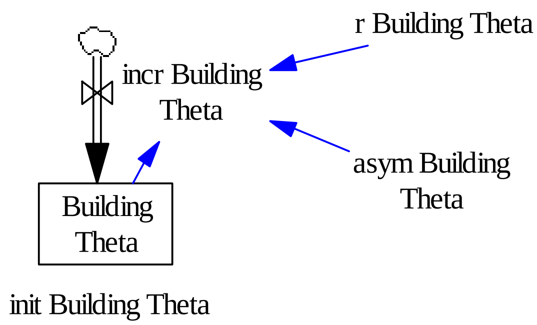

```
Building Area in Construction[region,ClimateZone,RurUrb,BuildingType]= INTEG (
Construction Starts[region,ClimateZone,RurUrb,BuildingType]-
Construction Completions[region,ClimateZone,RurUrb,BuildingType],
init Building Area in Construction[region,ClimateZone,RurUrb,BuildingType])
~ Msqm
~ |

Construction Completions[region,ClimateZone,RurUrb,BuildingType]=
DELAY3(Construction Starts[region,ClimateZone,RurUrb,BuildingType],
Average Construction Time)
~ Msqm/Year
~ |

Construction Starts[region,ClimateZone,RurUrb,BuildingType]=
max(0,Indicated Construction[region,ClimateZone,RurUrb,BuildingType])
~ Msqm/Year
~ |
```

Figure 39: Building construction submodel.




```
Building Hazard Func[Building Age Group]=
(Building Surv Lower[Building Age Group]-Building Surv Upper[Building Age Group])/
Building Int Surv[Building Age Group]*1000
~ 1/Year
~ |

Building Int Surv[Building Age Group]=
(Building Surv Upper[Building Age Group]+Building Surv Lower[Building Age Group])/2*
(Building Age Upper[Building Age Group]-Building Age Lower[Building Age Group])
~ Year
~ |

Building Surv Lower[Building Age Group]=
exp(-(
Building Theta[lambda]*Building Age Lower[Building Age Group]
+ Building Theta[alpha]/Building Theta[beta]*
(exp(Building Theta[beta]*Building Age Lower[Building Age Group])-1)
- Building Theta[epsilon]/Building Theta[gamma]*
(exp(-Building Theta[gamma]*Building Age Lower[Building Age Group])-1)))
~ Dmnl
~ |

Building Surv Upper[Building Age Group]=
exp(-(
Building Theta[lambda]*Building Age Upper[Building Age Group]
+ Building Theta[alpha]/Building Theta[beta]*
(exp(Building Theta[beta]*Building Age Upper[Building Age Group])-1)
- Building Theta[epsilon]/Building Theta[gamma]*
(exp(-Building Theta[gamma]*Building Age Upper[Building Age Group])-1)))
~ Dmnl
~ |
```

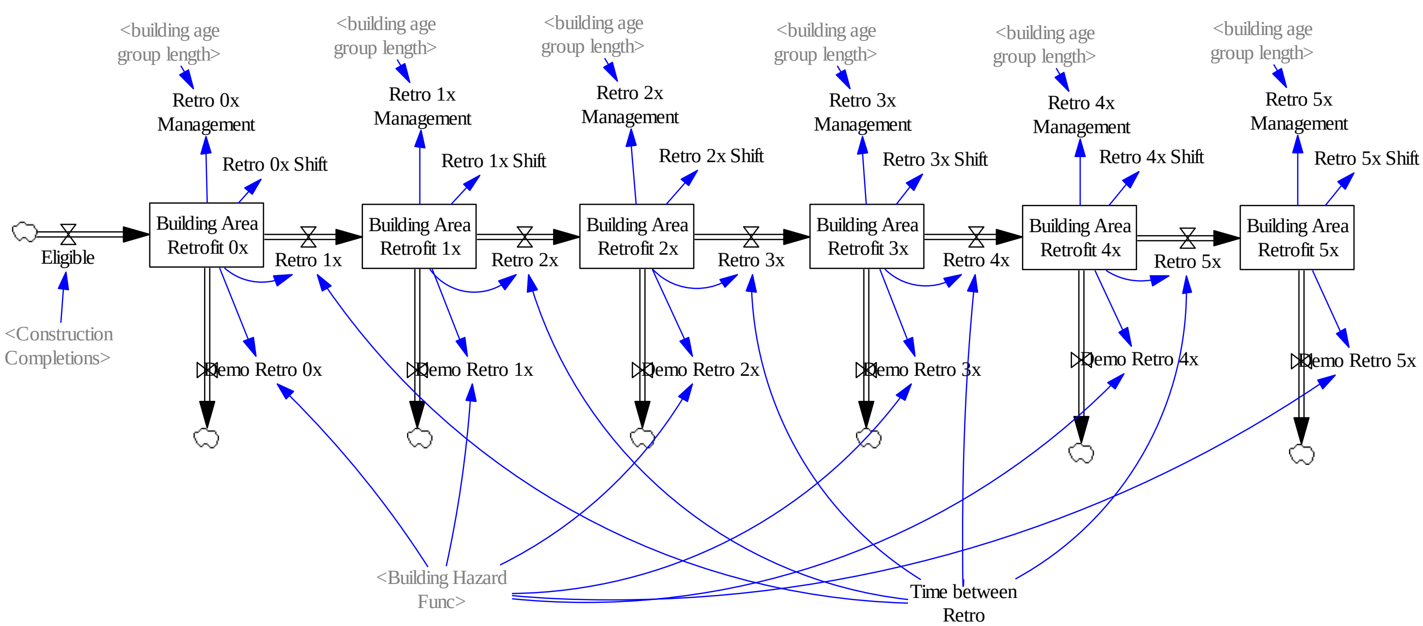
Figure 40: Building “hazard” (lifetime) sub model.




```
Building Theta[itheta]= INTEG (
incr Building Theta[itheta],
init Building Theta[itheta])
~ 1/Year
~ |
```

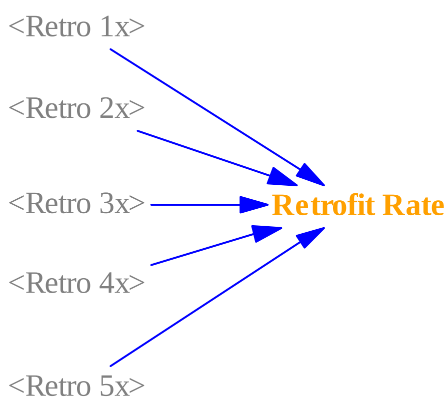
itheta	init Building Theta	asym Building Theta	r Building Theta
alpha	0.0039	0.0013	0.05
beta	0.0913	0.0680	0.05
lambda	0	0	0
epsilon	0	0	0
gamma	1	1	0

Figure 41: Evolution model for building hazard function coefficients.



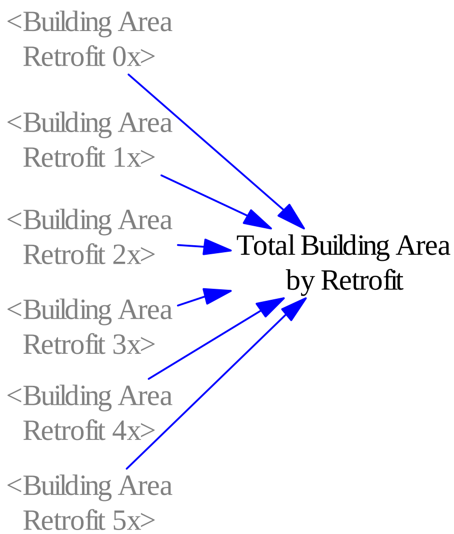

```
Building Area Retrofit 0x[region,ClimateZone,RurUrb,BuildingType,b0000]= INTEG (
Eligible[region,ClimateZone,RurUrb,BuildingType]-
Demo Retro 0x[region,ClimateZone,RurUrb,BuildingType,b0000]-
Retro 1x[region,ClimateZone,RurUrb,BuildingType,b0000],
init Building Retro Area[region,ClimateZone,RurUrb,BuildingType,b0000,x0]) ~~|

Building Area Retrofit 0x[region,ClimateZone,RurUrb,BuildingType,b01UP]= INTEG (
-Demo Retro 0x[region,ClimateZone,RurUrb,BuildingType,b01UP]-
Retro 1x[region,ClimateZone,RurUrb,BuildingType,b01UP],
init Building Retro Area[region,ClimateZone,RurUrb,BuildingType,b01UP,x0])
~ Msqm
~ |

Demo Retro 0x[region,ClimateZone,RurUrb,BuildingType,Building Age Group]=
Building Area Retrofit 0x[region,ClimateZone,RurUrb,BuildingType,Building Age Group]\
*Building Hazard Func[Building Age Group]/1000
~ Msqm/Year
~ |

Retro 1x[region,ClimateZone,RurUrb,BuildingType,Building Age Group]=
Building Area Retrofit 0x[region,ClimateZone,RurUrb,BuildingType,Building Age Group]\
/Time between Retro
~ Msqm/Year
~ |
```

Figure 42: Building turnover model structure for retrofit cycles.




```
Retrofit Rate[region,ClimateZone,RurUrb,BuildingType]=
SUM(
Retro 1x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]+
Retro 2x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]+
Retro 3x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]+
Retro 4x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]+
Retro 5x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]
)
~ Msqm/Year
~ |
```

Figure 43: Causal Loop Diagram and equations for total retrofit rate.




```
Total Building Area by Retrofit[region,ClimateZone,RurUrb,BuildingType,x0]=
SUM(Building Area Retrofit 0x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]) ~~|

Total Building Area by Retrofit[region,ClimateZone,RurUrb,BuildingType,x1]=
SUM(Building Area Retrofit 1x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]) ~~|

Total Building Area by Retrofit[region,ClimateZone,RurUrb,BuildingType,x2]=
SUM(Building Area Retrofit 2x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]) ~~|

Total Building Area by Retrofit[region,ClimateZone,RurUrb,BuildingType,x3]=
SUM(Building Area Retrofit 3x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]) ~~|

Total Building Area by Retrofit[region,ClimateZone,RurUrb,BuildingType,x4]=
SUM(Building Area Retrofit 4x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!]) ~~|

Total Building Area by Retrofit[region,ClimateZone,RurUrb,BuildingType,x5]=
SUM(Building Area Retrofit 5x[region,ClimateZone,RurUrb,BuildingType,Building Age Group!])
~ Msqm
~ |
```

Figure 44: Causal Loop Diagram and equations for Total Building Area by number of Retrofit cycles.

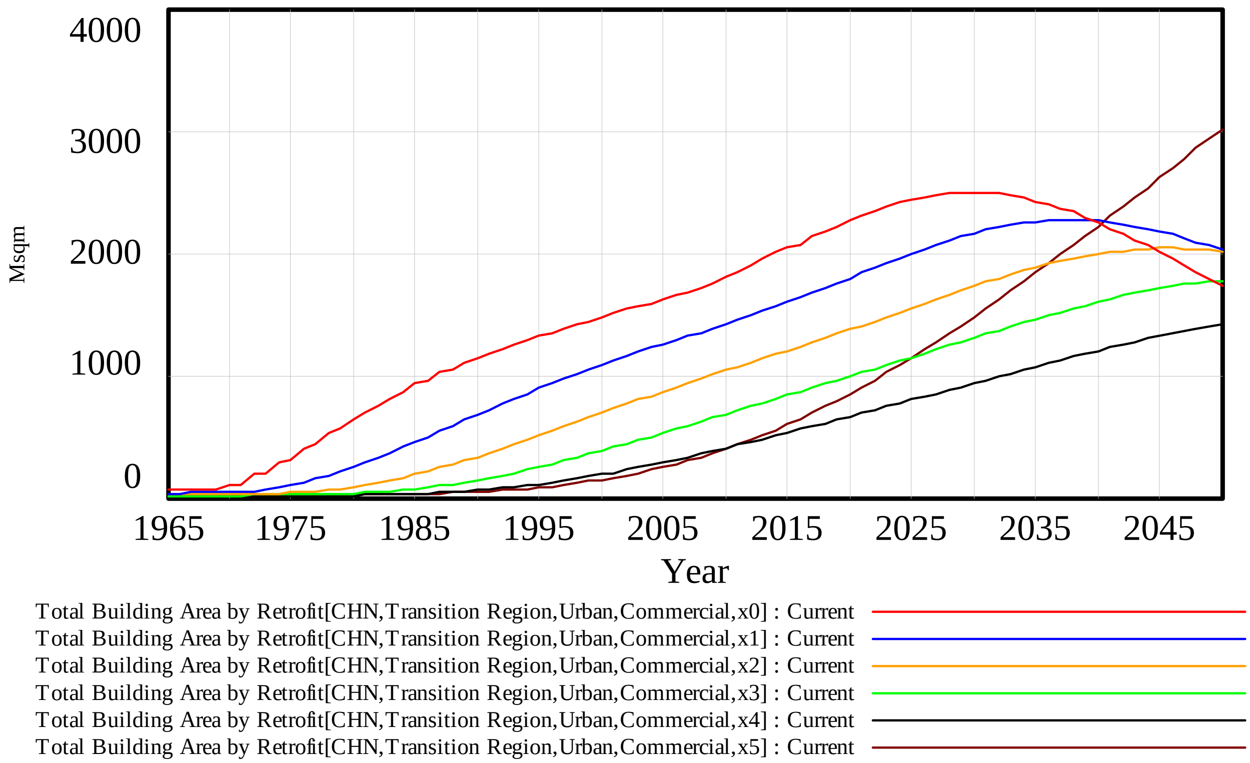


Figure 45: Example of total building area by number of retrofit cycles: Chinese commercial area in transition zone.

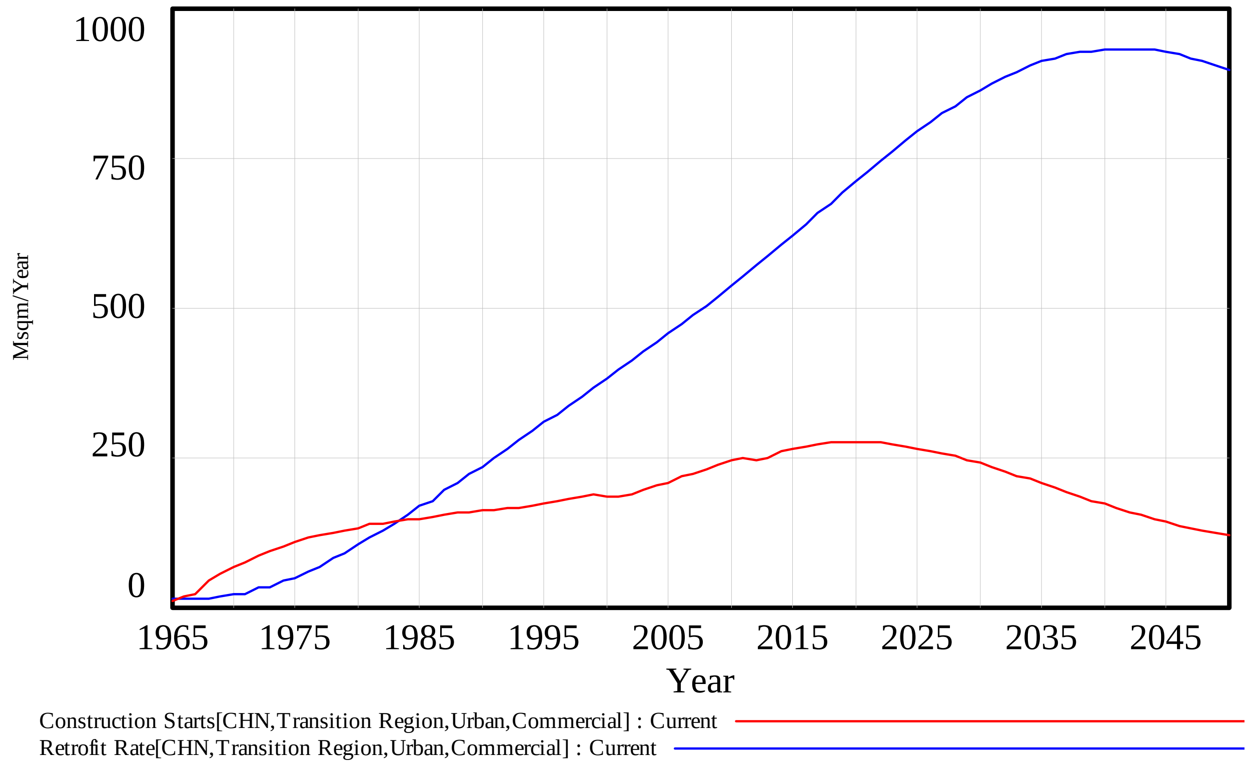


Figure 46: Example of new construcdtion and total retrofit construction: Chinese commercial area in transition zone.

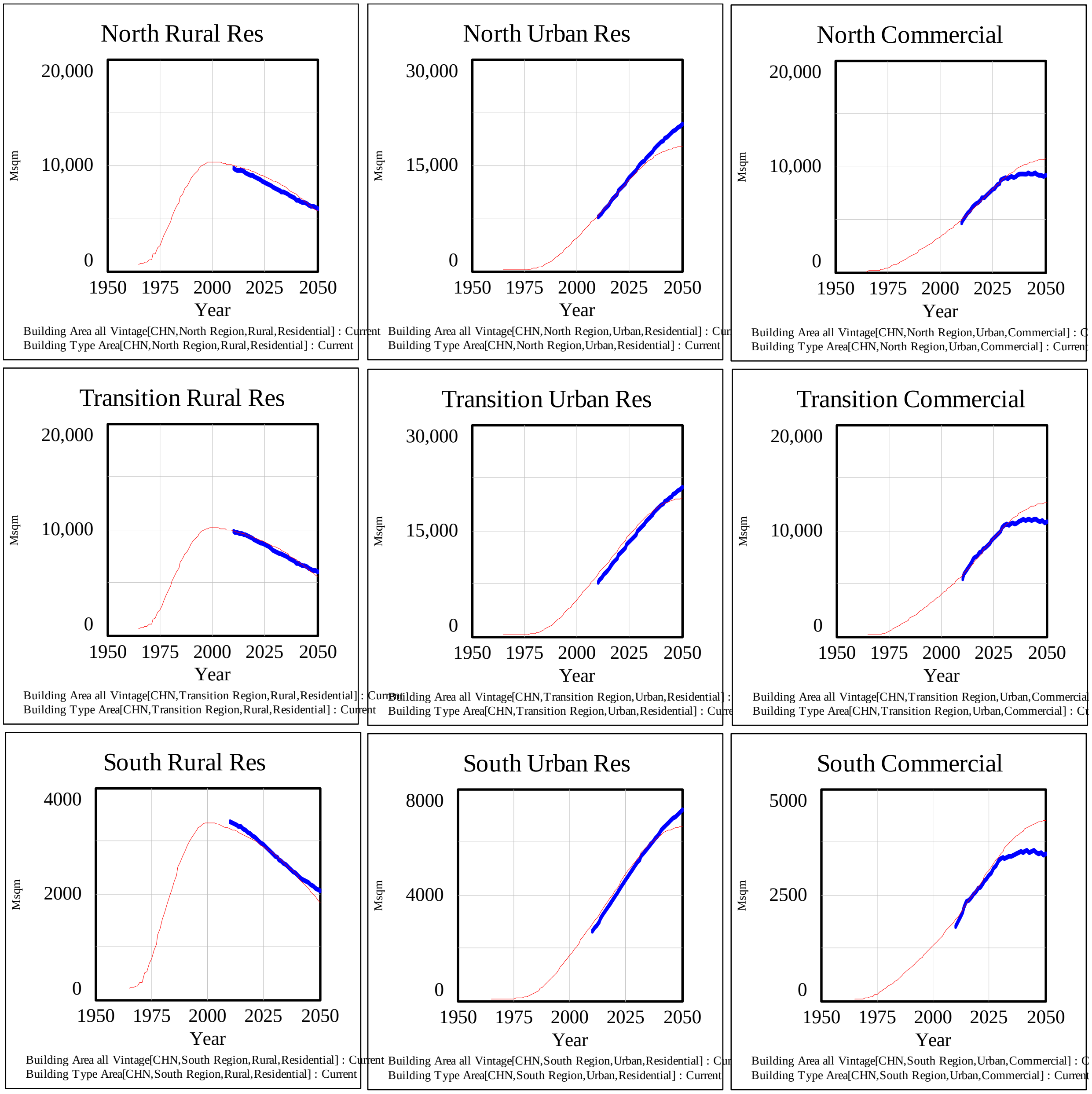


Figure 47: Residential and commercial building indicated area and by rural/urban and climate zone. Units are in millions of square meters (Note different scales for vertical axes.) Source: RFC.[2, 3]

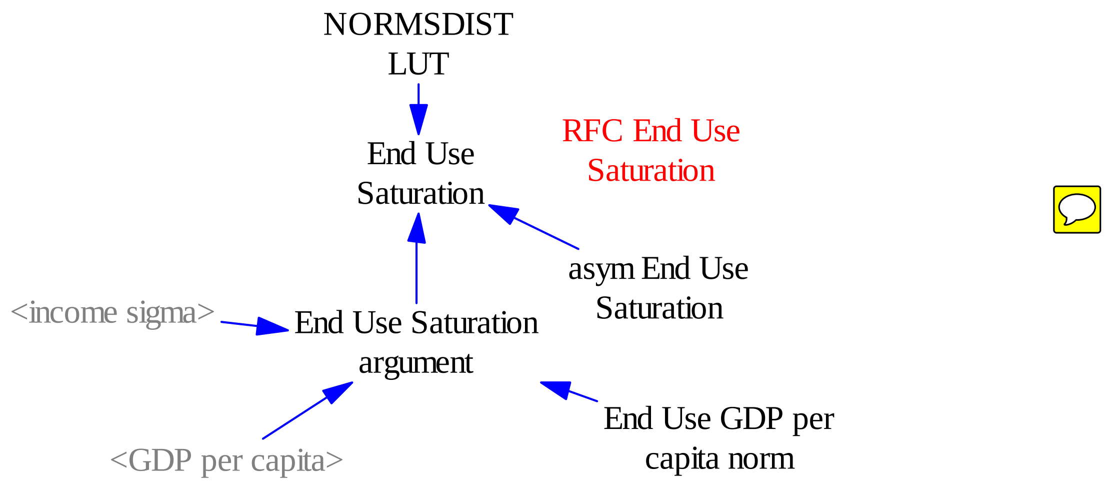
6 End Use Saturation

6.1 Model

Figure 48 shows the relationship amongst variables and equations that describe saturation of End Use, determined by the fraction of a population that can afford the End Use. Each end use has its own asymptotic saturation value as well as its own normal value of GDP per capita.

6.2 Calibration

Figures 49 – 51 show the calibration of the model to projections from the Reinventing Fire China studies.[2, 3]. The values of asymptotic saturation value and nominal value of GDP per capita for each end use, climate zone, and buiilding type are found the the accompanying Excel spreadsheets.




```
End Use Saturation[region,ClimateZone,RurUrb,BuildingType,EndUse]=
asym End Use Saturation[region,ClimateZone,RurUrb,BuildingType,EndUse]*
(1-NORMSDIST LUT(
End Use Saturation argument[region,ClimateZone,RurUrb,BuildingType,EndUse]))
~ Percent
~ |

End Use Saturation argument[region,ClimateZone,RurUrb,BuildingType,EndUse]=
(LN(End Use GDP per capita norm[region,ClimateZone,RurUrb,BuildingType,EndUse]/
GDP per capita[region])+income sigma*income sigma/2)/income sigma
~ Dmnl
~ |
```

Figure 48: Relationships amongst variables determining End Use Saturation, including income distribution.

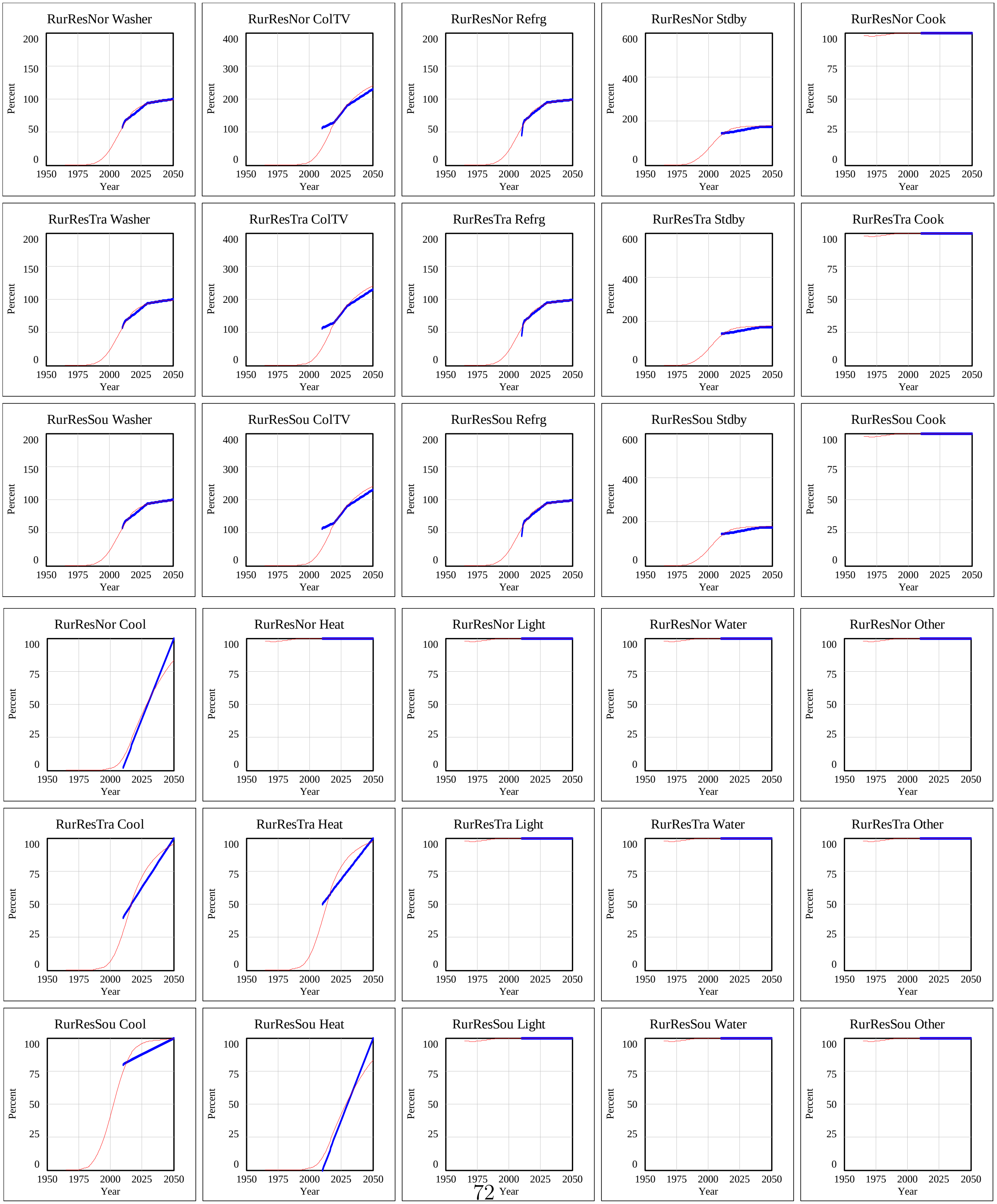


Figure 49: End Use Saturation, Rural Residential.

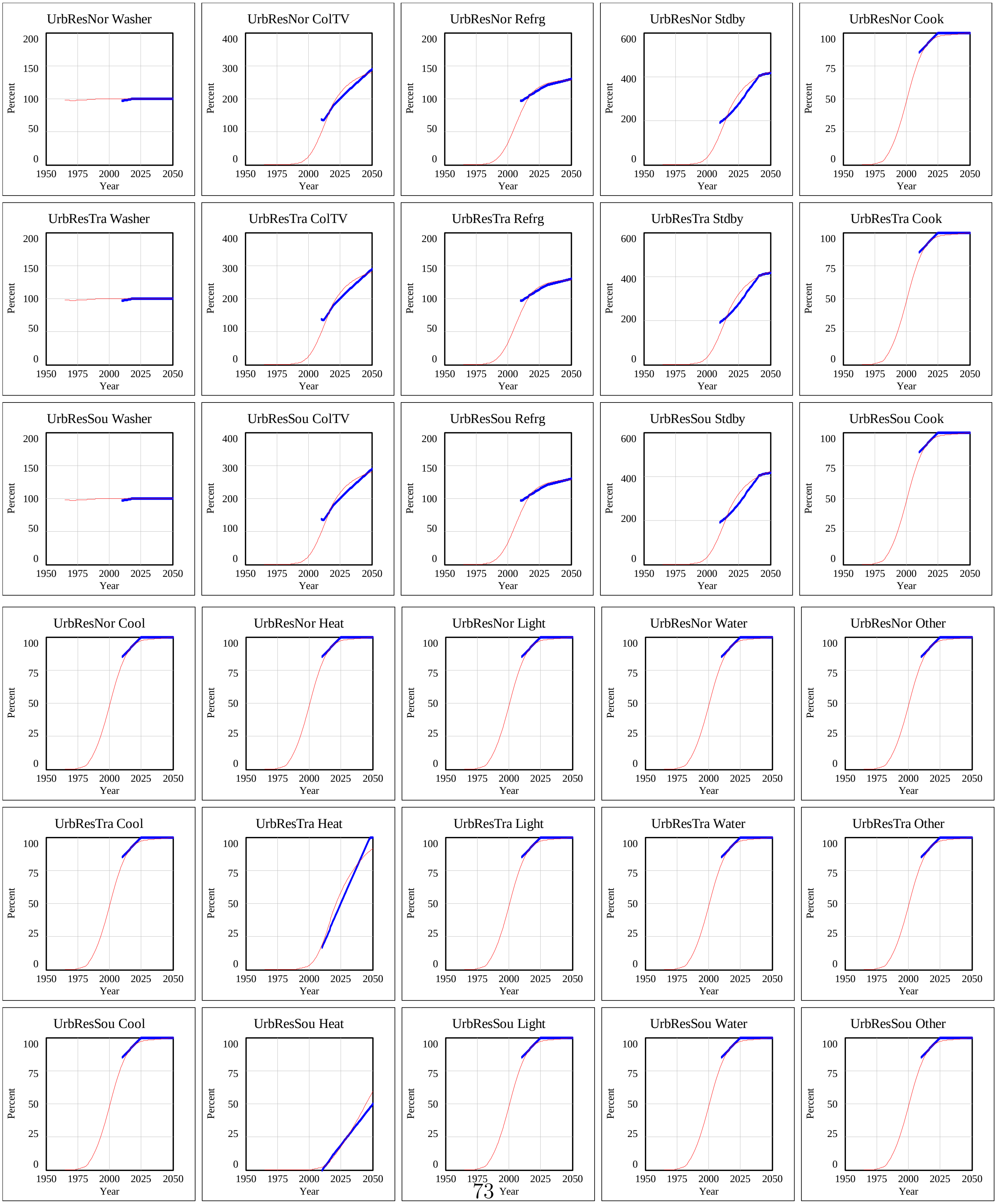


Figure 50: End Use Saturation, Urban Residential.

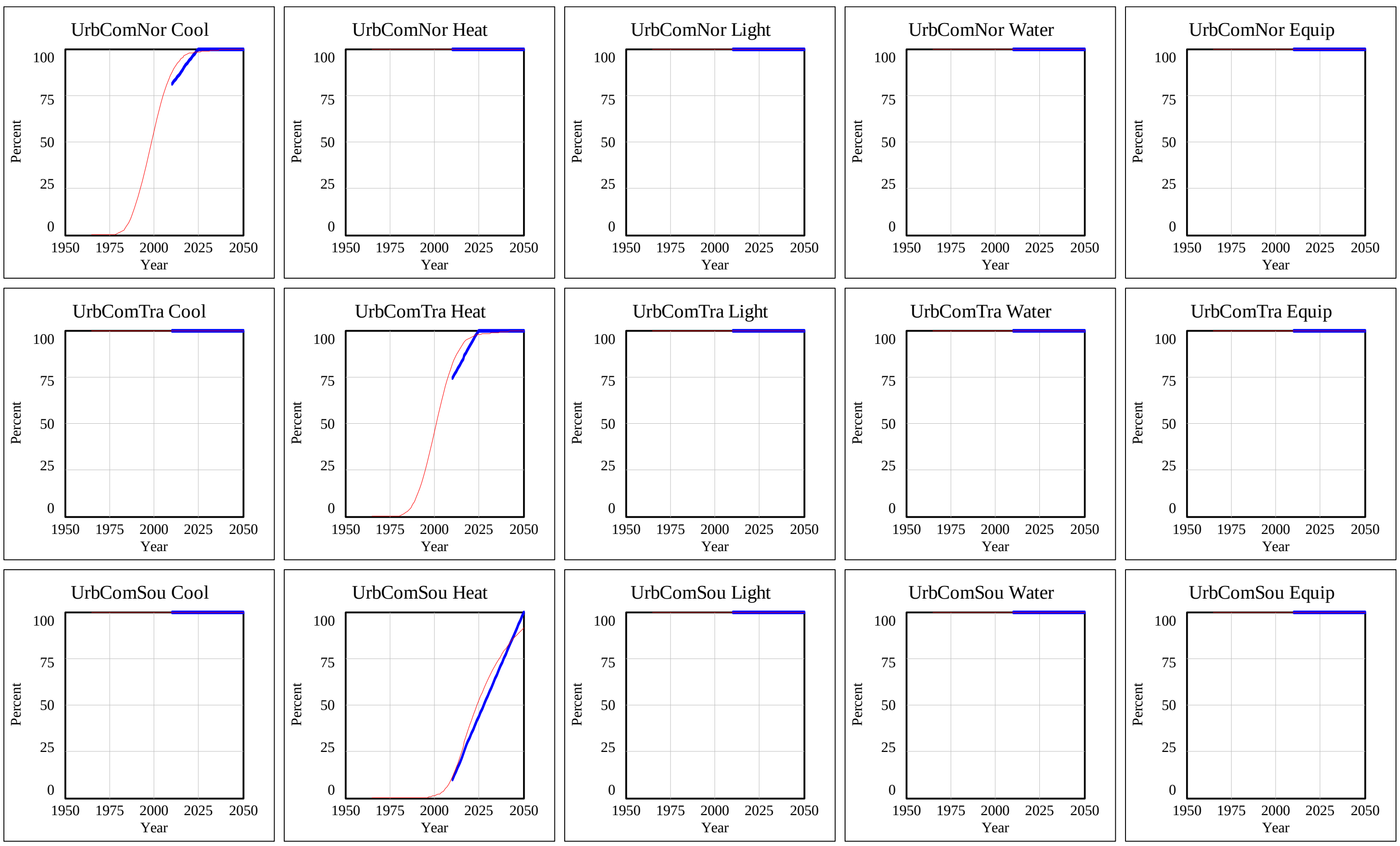


Figure 51: End Use Saturation, Urban Commercial.

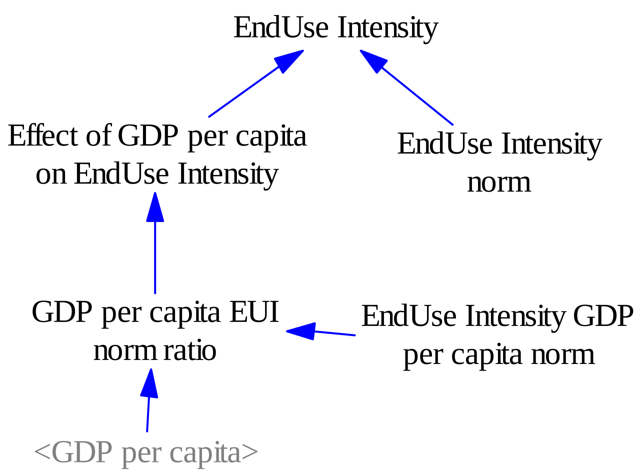
7 End Use Energy Intensity

7.1 Model

Figure 52 shows the causal loop diagram and equations determining `EndUse Intensity`, assumed to be a simple function of GDP per capita.

7.2 Calibration

Figures 53 – 55 show the calibration of the model to projections of end use energy intensity from the Reinventing Fire China studies.[2, 3]. The values of end use intensity norm and end use intensity GDP per capita norm are found the the accompanying Excel spreadsheets.




```
EndUse Intensity[region,ClimateZone,RurUrb,BuildingType,EndUse]=
EndUse Intensity norm[region,ClimateZone,RurUrb,BuildingType,EndUse]*
Effect of GDP per capita on EndUse Intensity[region,ClimateZone,RurUrb,BuildingType,EndUse]
~ GWh/(Year*Msqm)
~ |

Effect of GDP per capita on EndUse Intensity[region,ClimateZone,RurUrb,BuildingType,EndUse]=
LN(1+GDP per capita EUI norm ratio[region,ClimateZone,RurUrb,BuildingType,EndUse])/
LN(2)
~ Dmnl
~ LN(1+GDP per capita EUI norm ratio[ref \
WDI,ClimateZone,RurUrb,BuildingType,EndUse])/LN(2)
|

GDP per capita EUI norm ratio[region,ClimateZone,RurUrb,BuildingType,EndUse]=
GDP per capita[region]/EndUse Intensity GDP per capita norm[region,ClimateZone,RurUrb\
,BuildingType,EndUse]
~ Dmnl
~ |
```

Figure 52: Causal Loop Diagram showing and equations for end use intensity.

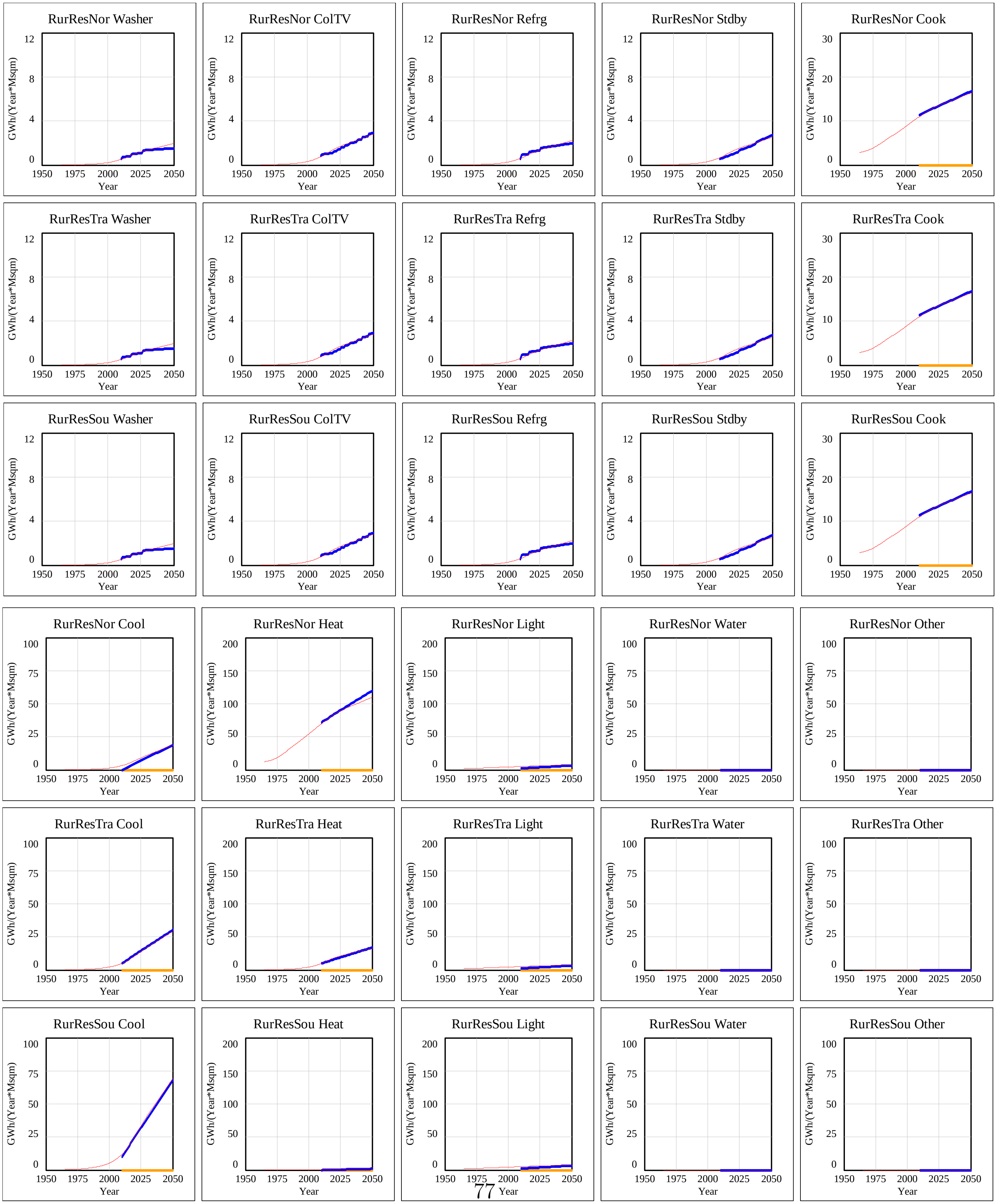


Figure 53: End Use Energy Intensity, Rural Residential.

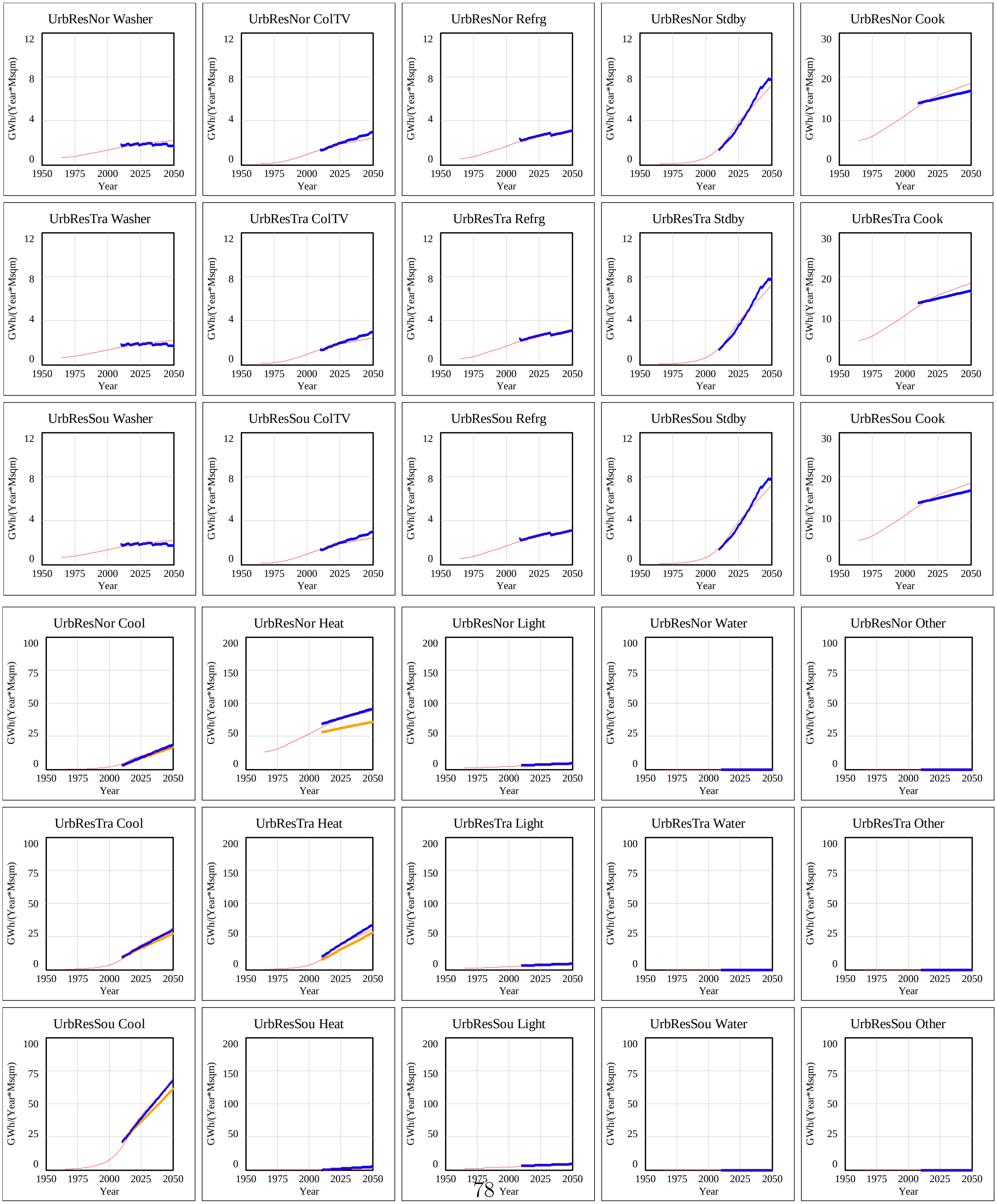


Figure 54: End Use Energy Intensity, Urban Residential.

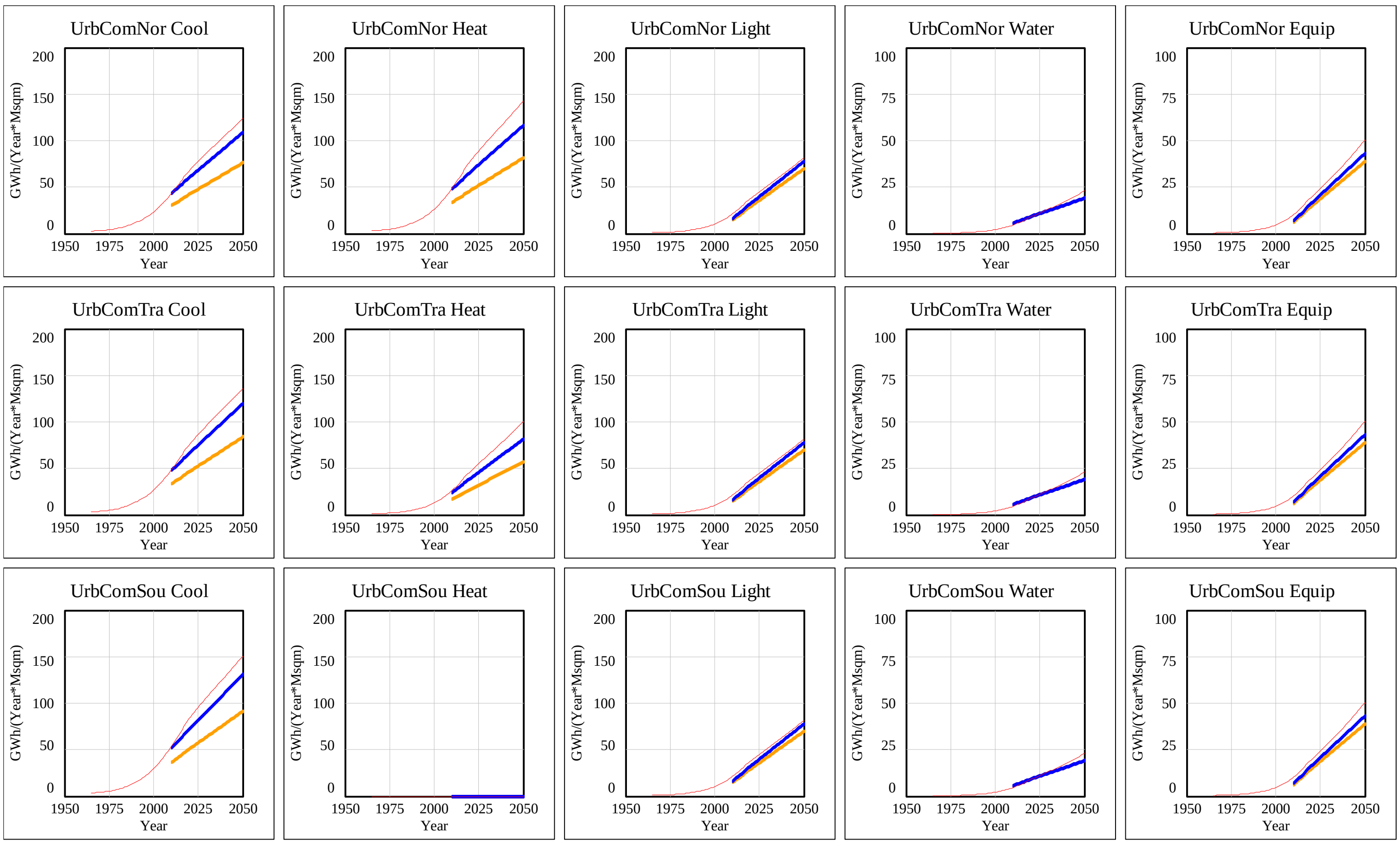
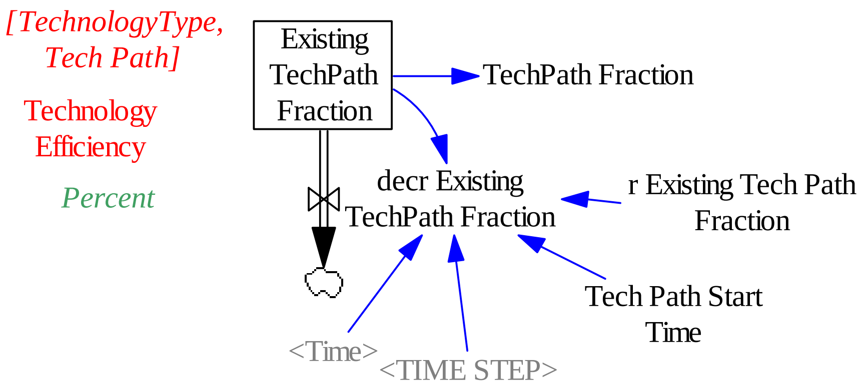


Figure 55: End Use Energy Intensity, Urban Commercial.

8 Technology Efficiency

Incumbent technology is assumed to improve more or less linearly as a function of time. Figure 7 shows the simple model and equations. Table 8 shows the technology efficiency assumed by the Reinventing Fire China studies.[2, 3] in 2015 (Existing) and 2030 (Efficient).




```
TechPath Fraction[Existing]=
Existing TechPath Fraction ~~|

TechPath Fraction[Efficient]=
1 -Existing TechPath Fraction
~ Dmnl
~ |

Existing TechPath Fraction= INTEG (
-decr Existing TechPath Fraction,1)
~ Dmnl
~ |

decr Existing TechPath Fraction=
IF THEN ELSE(Time < Tech Path Start Time,0,
MIN(Existing TechPath Fraction/TIME STEP,r Existing Tech Path Fraction))
~ 1/Year
~ |
```

Table 7: Simple model for technology efficiency migration path.

Technology Type	Existing	Efficient
Appliances Clothes Washer	100	160
Appliances Color TV	100	170
Appliances Refrigerator	100	127
Appliances Stand By	100	197
Cooking Biogas	55	70
Cooking Biomass	16.6	46.6
Cooking Coal Gas Stove	55	70
Cooking Coal Stove	40	65
Cooking Electric Cooker	81	90
Cooking LPG Cooker	55	70
Cooking NG Cooker	55	70
Cooking Other Cooker	40	65
Cooling AC	257	364
Cooling Centralized AC	284	410
Cooling Geothermal HP	300	420
Cooling NG Centralized AC	80	260
Cooling Room AC	257	364
Equipment Commercial Equip Tech	100	150
Heating Air Source Heat Pump	257	364
Heating Biomass	16.6	46.6
Heating Coal Boiler	63	89
Heating Coal Boiler Rural	31.5	41.4
Heating Coal District	85	91
Heating Coal Stove	40	65
Heating Distributed Coal Boiler	63	89
Heating Distributed Gas Boiler	78	94
Heating Electric	95	98
Heating Gas Boiler	78	94
Heating Gas District	85	91
Heating Gas Stove	90	94
Heating Ground Source Heat Pump	300	420
Heating Oil Boiler	78	94
Heating Small Cogen	76	91
Heating Solar Thermal	100	100

Technology Type	Existing	Efficient
Lighting CFL	100	100
Lighting Commercial Lighting Tech	100	300
Lighting Fluorescent	100	100
Lighting Incandescent	100	100
Lighting LED	100	100
Other Uses Resi Other Tech	100	100
Water Heating Air Source Heat Pump	257	364
Water Heating Biogas	55	70
Water Heating Biomass	16.6	46.6
Water Heating Coal Boiler	40	70
Water Heating Coal Gas	60	96
Water Heating Coal Stove	40	65
Water Heating Electric	89	93
Water Heating Gas Boiler	78	94
Water Heating Ground Source Heat Pump	300	420
Water Heating LPG	60	96
Water Heating LPG Cooker	60	96
Water Heating Natural Gas	60	96
Water Heating Oil Boiler	78	94
Water Heating Other	40	85
Water Heating Small Cogen	76	91
Water Heating Solar	100	100

Table 8: Technology Efficiency.

9 Technology Adoption

9.1 Adoption Model

We will assume that sales market share follows a multi-nomial logit function (see Ben-Akiva [14, Ch.5] for details), modified to allow for availability that increases from zero (say, due to commercialization of a new technology) or decreases to zero (say, due to a regulatory ban):

$$\begin{aligned}
S_j & \quad \text{sales of model } j \text{ (units/year)}, \\
A_j & \quad \text{availability of model } j \text{ (dimensionless)}, \\
U_j & \quad \text{utility of model } j \text{ (dimensionless)}, \\
W_j & \quad \text{willingess to pay for model } j \text{ (\$/unit)}, \\
P_j & \quad \text{price of model } j \text{ (\$/unit)}, \\
\beta & \quad \text{price coefficient (unit/\$)}, \\
S & \quad \text{market size (units/year)}, \\
S_j/S &= A_j \exp(U_j) / \sum_j A_j \exp(U_j), \\
U_j &= \beta(W_j - P_j).
\end{aligned}$$

Figure 56 shows an overview of the model for sales market share of individual end use technologies, depending broadly on the availability and the economic utility relative to competitive technologies.

Figure57 shows local dependence and equations of the model for sales market share. `Market Share` is assumed to follow a multi-nomial logit function[14, Ch.5]. Where technologies are fully available, the market share is proportional to the exponent of the economic `Utility`. The normalization function `Z factor` serves to ensure that no market share exceeds unity, and the `tiny` factor assures that the `Z factor` is finite even if all technology availabilities are zero. The exogenous binary matrix `EndUse Tech Map` indicates which technologies are appropriate for which end use, and therefore determines the technologies that "compete" with each other.

Figure 58 shows local dependence and equations of the model for technology availability. If a technology is fully commercialized and available for selection in an end use, then `Availability` assumes a value of unity. At the other extreme, if a technology has not been invented or commercialized yet, or has been banned by regulation, or is otherwise unavailable in a particular region, then `Availability` assumes a value of zero. Partial commercialization is represented by a value of `Availability` between zero and one. The availability of every technology is assume to take a linear trajectory between value `A1` at time `T1` to a value `A2` at time `T2`. By appropriate selection of the values of `A1`, `A2`, `T1` and `T2` for each technology, for each climate and building type, the many assumptions of the Reinventing Fire China studies [2, 3] can be reproduced. In particular, the dramatic shifts in technology installed base are determined primarily by reductions in availability of once-market-leading technologies.

Figure 59 shows the local dependence and equations for the economic `Utility` of each technology, which together with the `Availability` determine the `Tech Sales Factor`. As one would expect, `Utility` decreases with `Dimensionless Tech Cost`, so that decreasing the technology cost results in an exponential increase in the market share. All other attractive and unattracdtive characteristics of the technology are lumped into a single constant parameter called willingness-to-pay, `Tech WTP`. Each technology has its own value of `Tech WTP`, and values are selected to reproduce historical values of sales market share. In the multinomial logit model [14], any two available technologies with the same value of `Tech WTP` are distinguished only on the basis of price, i.e. may be considered as commodities.

9.2 Turnover Model

Figure 60 shows an overview of the model for the installed base of technologies, *which in many ways is the heart of the building energy consumption model.* The turnover of building area equipped with particular technolgies depends on the various rates of removal of technologies, the market need for new end use technologies, and the particular market shares enjoyed by individual technologies.

Figure 61 shows the stock and flow structure and equations for the `Installed Tech Area`. The `Installed Tech Area` basically increases with the `Installation Rate`, and decreases with the rate of removal for a number of reasons.

Figure 62 shows the local dependence of the `Installation Rate` on the overall market size, `EndUse Area without Tech`, and the `Market Share` of individual technologies.

Figure 63 shows the stock and flow structure and equations for the overall market size, `EndUse Area without Tech`. The overall market can increase with construction, both new and retrofit, and is satisfied (reduced) when technologies are installed.

Figure 64 shows the local dependencies and equations for the various rates of technology removal.

9.3 Calibration

Figure 65 shows how `Tech Installed Base Fraction` is calculated from the `Tech Installed Area` and `EndUse Tech Map`. Figures 66–82 compare calibrated model values of `Tech Installed Base Fraction` to projections `RFC Technology Share` from the Reinventing Fire China studies.[2, 3]

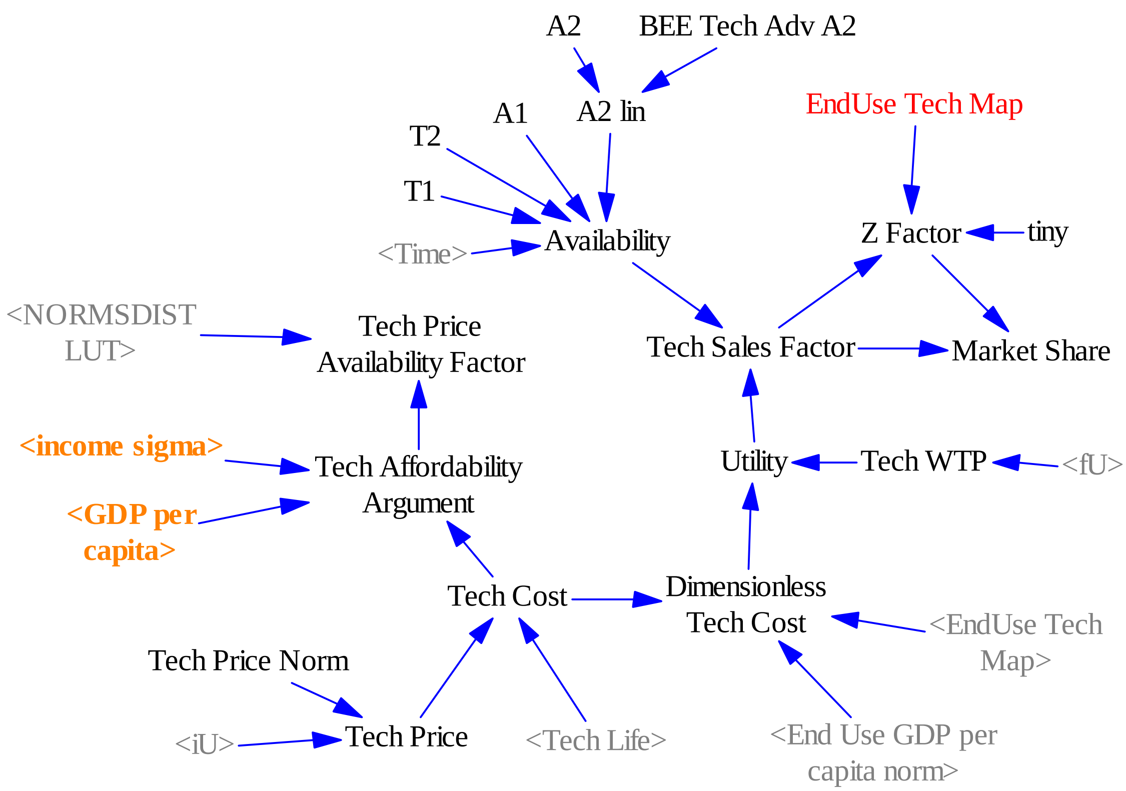
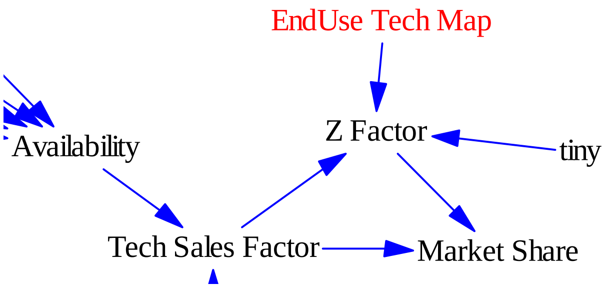
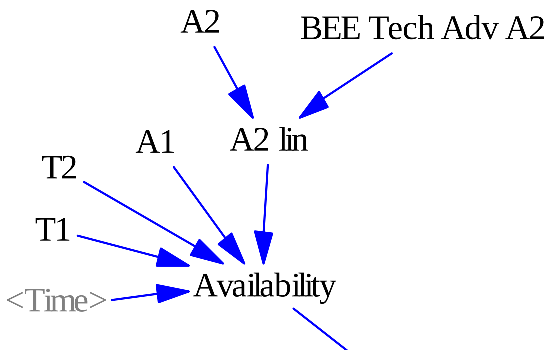


Figure 56: Overview of model determining sales market share.



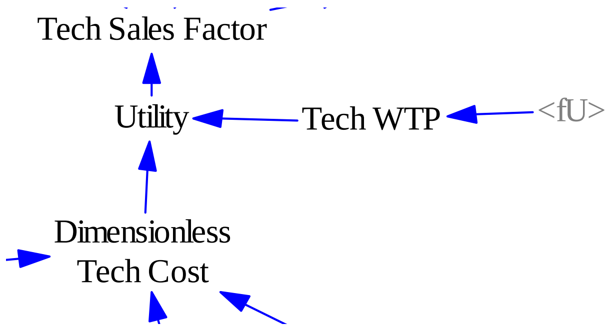

```
Market Share[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
Tech Sales Factor[region,ClimateZone,RurUrb,BuildingType,TechnologyType]/
Z Factor[region,ClimateZone,RurUrb,BuildingType,TechnologyType]
~ Dmnl
~ Technology sales market share (not installed base share)
|

Tech Sales Factor[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
Availability[region,ClimateZone,RurUrb,BuildingType,TechnologyType]*
exp(Utility[region,ClimateZone,RurUrb,BuildingType,TechnologyType])
~ Dmnl
~ product of availability and exponent of utility, proportional to the sales \
market share.
|

Z Factor[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
SUM(EndUse Tech Map[TechnologyType,EndUse!]*
Tech Sales Factor[region,ClimateZone,RurUrb,BuildingType,TechnologyType])+tiny
~ Dmnl
~ Normalization factor, equal to sum of sales market share factors for a \
given market segment
|

tiny=
1
~ Dmnl
~ |
```

Figure 57: Immediate factors determining Sales Market Share.




```
Availability[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
max(0,MIN(1,
A1[ClimateZone,RurUrb,BuildingType,TechnologyType]+
(Time-T1[ClimateZone,RurUrb,BuildingType,TechnologyType])*
(A2 lin[ClimateZone,RurUrb,BuildingType,TechnologyType]-
A1[ClimateZone,RurUrb,BuildingType,TechnologyType])/
(T2[ClimateZone,RurUrb,BuildingType,TechnologyType]-
T1[ClimateZone,RurUrb,BuildingType,TechnologyType])
))
~ Dmnl [0,1]
~ Technology availability, including commercialization (increasing \
availability) and regulation (decreasing availability).  Approximated by a \
linear function, bounded by [0,1].
|

A2 lin[ClimateZone,RurUrb,BuildingType,TechnologyType] :EXCEPT:
[ClimateZone,RurUrb,BuildingType,BEE Tech Adv]=
A2[ClimateZone,RurUrb,BuildingType,TechnologyType] ~~|
A2 lin[ClimateZone,RurUrb,BuildingType,BEE Tech Adv]=
BEE Tech Adv A2
~ Dmnl [3.5593e-043,?]
~ |
```

Figure 58: Detail of Availability.




```
Utility[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
Tech WTP[ClimateZone,RurUrb,BuildingType,TechnologyType]-
Dimensionless Tech Cost[region,ClimateZone,RurUrb,BuildingType,TechnologyType]
~ Dmnl
~ economic utility of a end use technology, increases with willingness to \
pay (WTP) and decreases with cost
|

Tech WTP[ClimateZone,RurUrb,BuildingType,TechnologyType]=
fU[ClimateZone,RurUrb,BuildingType,TechnologyType]
~ Dmnl
~ |

Dimensionless Tech Cost[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
Tech Cost[ClimateZone,RurUrb,BuildingType,TechnologyType]/
SUM(End Use GDP per capita norm[region,ClimateZone,RurUrb,BuildingType,EndUse!]*
EndUse Tech Map[TechnologyType,EndUse!])
~ Dmnl
~ |
```

Figure 59: Factors determining economic Utility.

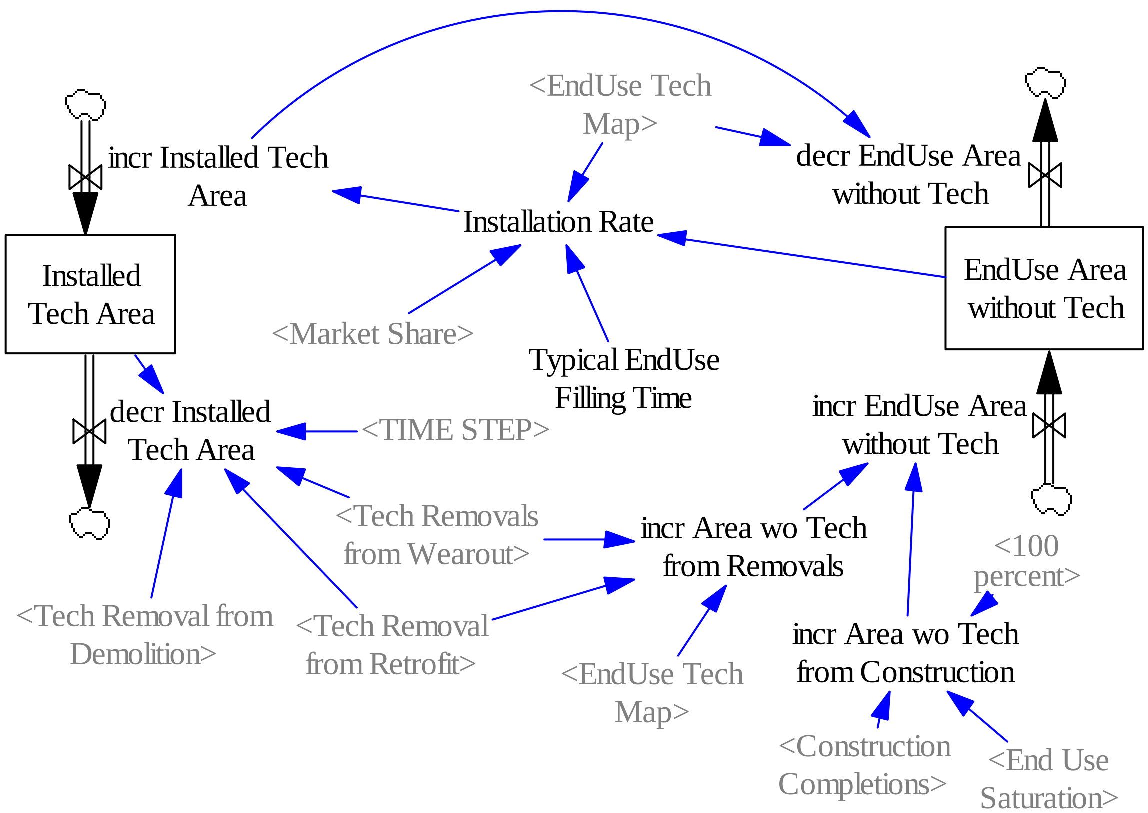
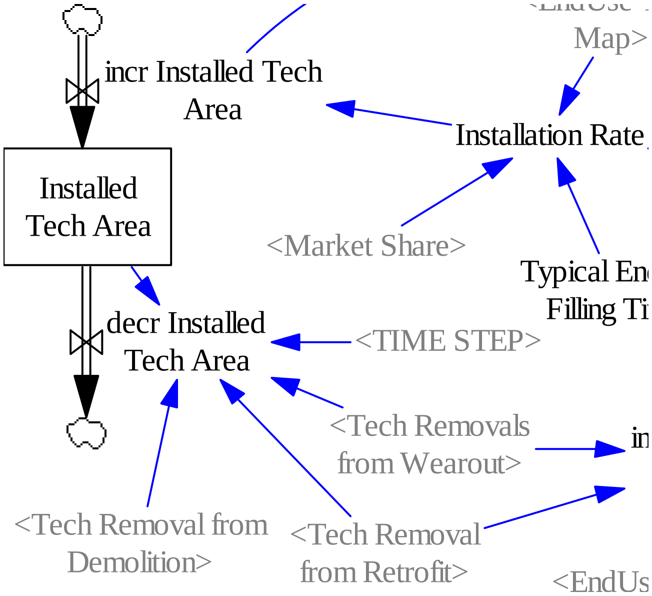
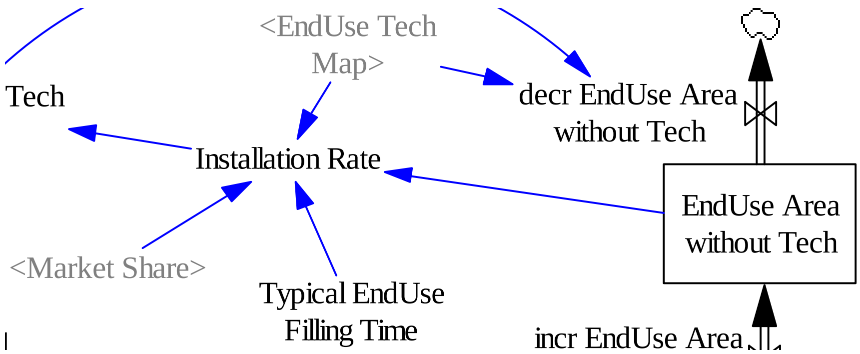


Figure 60: Overiew of model determining Installed Tech Area.



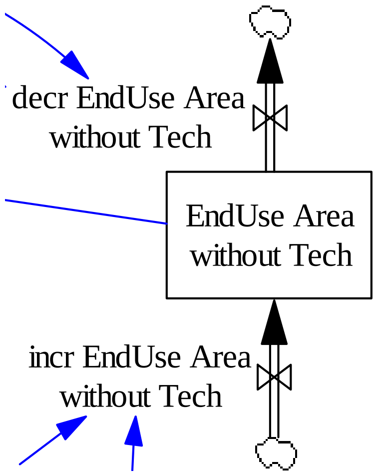

```
Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType]= INTEG (
incr Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType]-
decr Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType],
init Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType])
~ Msqm
~ Area equipped with particular end use technology
|

incr Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
Installation Rate[region,ClimateZone,RurUrb,BuildingType,TechnologyType]
~ Msqm/Year
~ rate of increase of area equipped with particular end use technology
|

decr Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
max(0,MIN(
Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType]/
(10*TIME STEP),
Tech Removal from Demolition[region,ClimateZone,RurUrb,BuildingType,TechnologyType]+
Tech Removal from Retrofit[region,ClimateZone,RurUrb,BuildingType,TechnologyType]+
Tech Removals from Wearout[region,ClimateZone,RurUrb,BuildingType,TechnologyType]))
~ Msqm/Year
~ rate of decrease of area equipped with particular end use technology
|
```

Figure 61: Detailed model and equations of installed technology area.



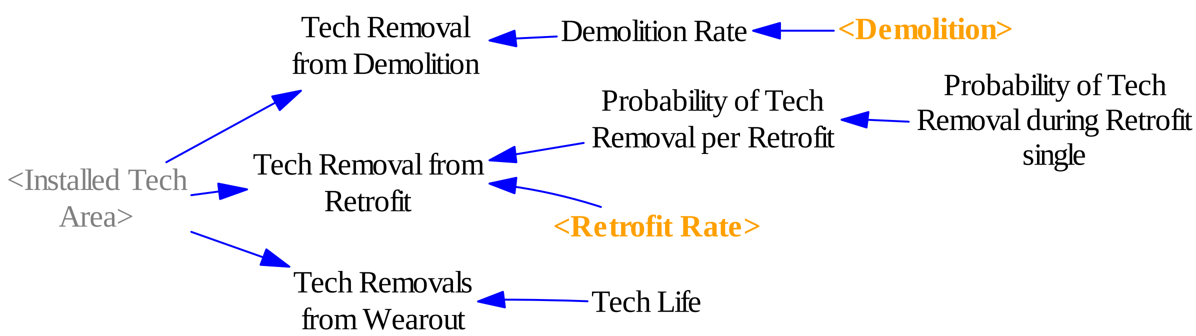

```
Installation Rate[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
SUM(
EndUse Area without Tech[region,ClimateZone,RurUrb,BuildingType,EndUse!]/
Typical EndUse Filling Time*
Market Share[region,ClimateZone,RurUrb,BuildingType,TechnologyType]*
EndUse Tech Map[TechnologyType,EndUse!])
~ Msqm/Year
~ rate of installation of particular end use technology
|
```

Figure 62: Detailed model and equations of technology installation rate.



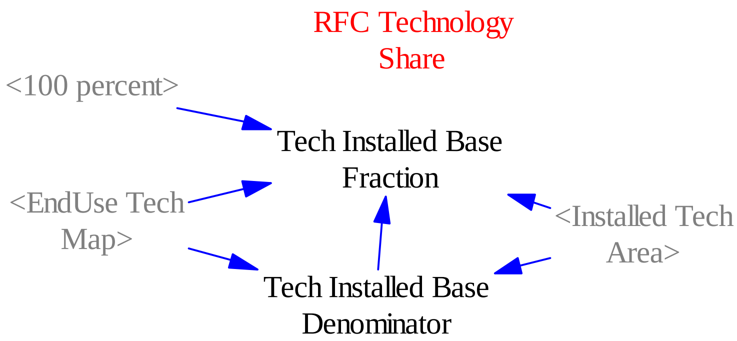

```
EndUse Area without Tech[region,ClimateZone,RurUrb,BuildingType,EndUse]= INTEG (
incr EndUse Area without Tech[region,ClimateZone,RurUrb,BuildingType,EndUse]-
decr EndUse Area without Tech[region,ClimateZone,RurUrb,BuildingType,EndUse],
init EndUse Area without Tech[region,ClimateZone,RurUrb,BuildingType,EndUse])
~ Msqm
~ total area demanding a particular end use
|

decr EndUse Area without Tech[region,ClimateZone,RurUrb,BuildingType,EndUse]=
SUM(incr Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType!]*
EndUse Tech Map[TechnologyType!,EndUse])
~ Msqm/Year
~ rate of decrease of area demanding a particular end use
|

incr EndUse Area without Tech[region,ClimateZone,RurUrb,BuildingType,EndUse]=
max(0,
incr Area wo Tech from Construction[region,ClimateZone,RurUrb,BuildingType,EndUse]+
incr Area wo Tech from Removals[region,ClimateZone,RurUrb,BuildingType,EndUse])
~ Msqm/Year
~ rate of increase in area demanding a particular end use
|
```

Figure 63: Detailed model and equations of end use area without installed technology.




```
Tech Removal from Demolition[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
Demolition Rate[region,ClimateZone,RurUrb,BuildingType]*
Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType]/
(SUM(Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType!]))
~ Msqm/Year
~ rate of removal of end use technology due to demolition of the area
|

Tech Removal from Retrofit[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
Retrofit Rate[region,ClimateZone,RurUrb,BuildingType]*
Probability of Tech Removal per Retrofit[TechnologyType]*
Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType]/
(SUM(Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType!]))
~ Msqm/Year
~ rate of removal of end use technology due to retrofit of the area
|

Tech Removals from Wearout[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType]/
Tech Life[TechnologyType]
~ Msqm/Year
~ rate of removal of end use technology due to wearout of the technology
|
```

Figure 64: Factors determining Technology Removals.




```
Tech Installed Base Fraction[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
"100 percent"*
ZIDZ(
Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType],
SUM(Tech Installed Base Denominator[region,ClimateZone,RurUrb,BuildingType,EndUse!]*
EndUse Tech Map[TechnologyType,EndUse!]))
~ Percent
~ End use technology share of installed base
|

Tech Installed Base Denominator[region,ClimateZone,RurUrb,BuildingType,EndUse]=
SUM(Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,TechnologyType!]*
EndUse Tech Map[TechnologyType!,EndUse])
~ Msqm
~ Denominator for End use technology share of installed base, i.e. total \
Msqm of end use
|
```

Figure 65: Factors determining Technology Installed Base Fraction.

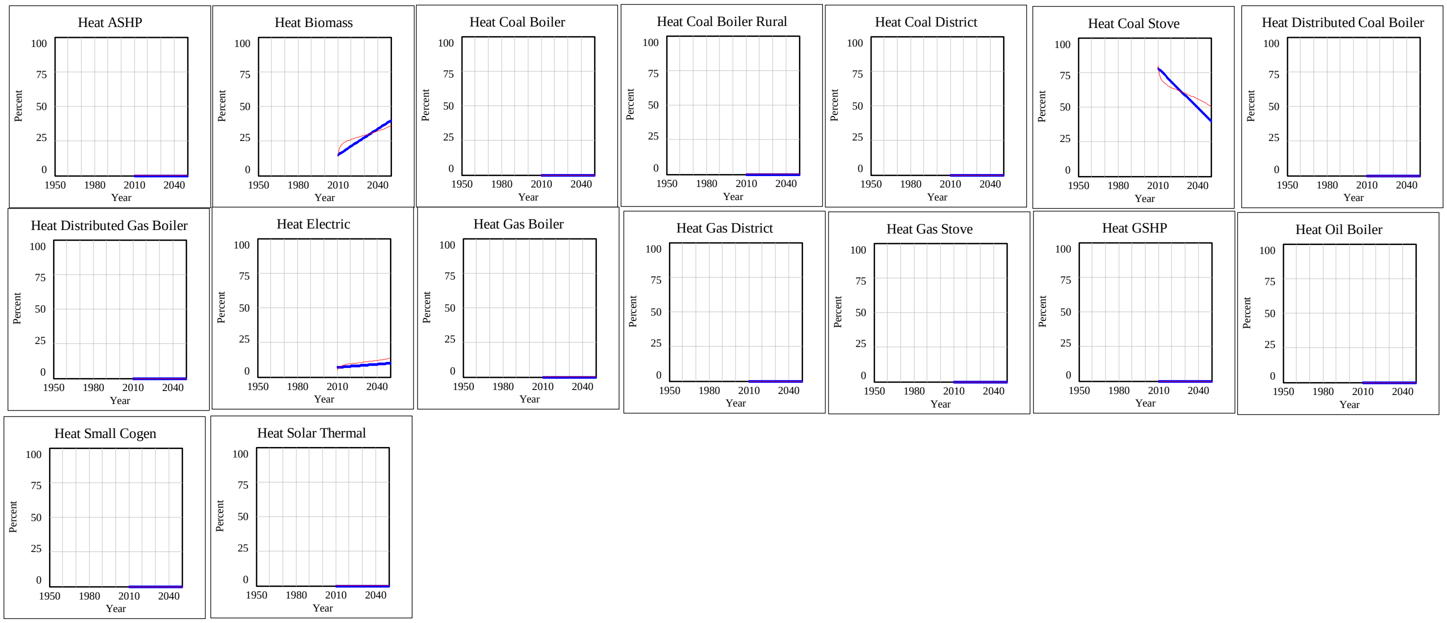


Figure 66: Calibration of Heating Technology Adoption, North Rural Residential.

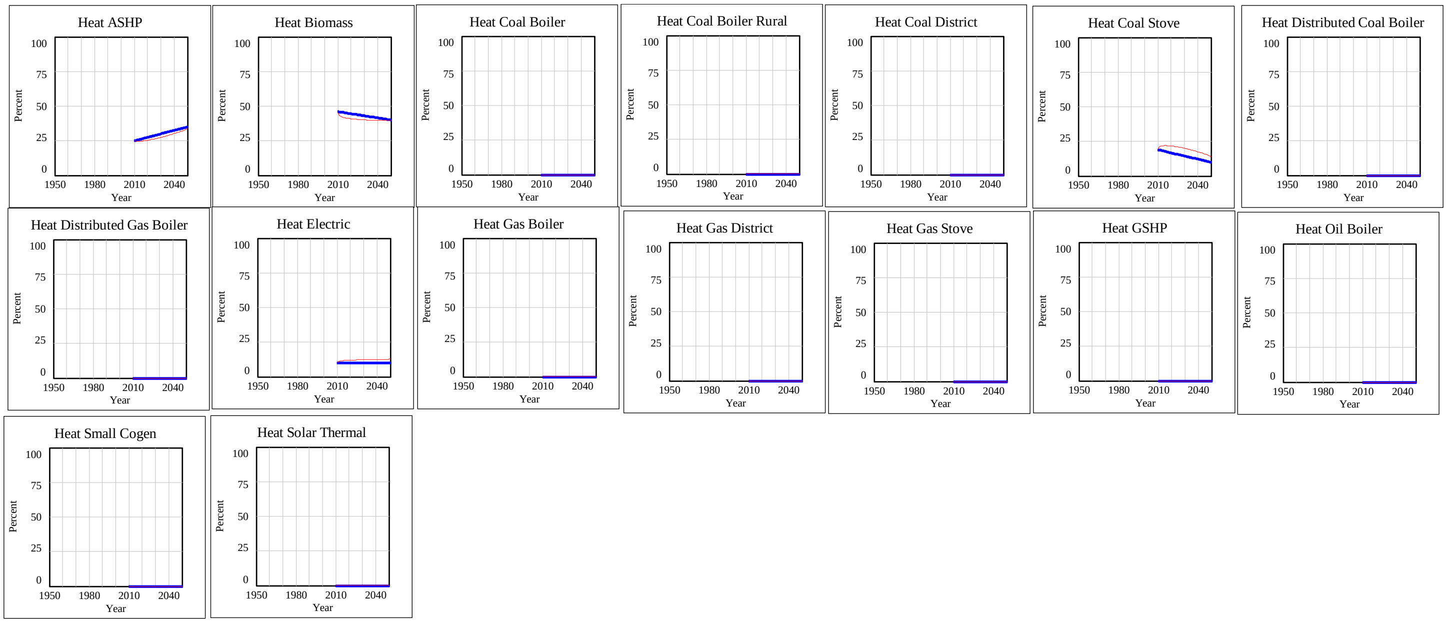


Figure 67: Calibration of Heating Technology Adoption, Transition Rural Residential.

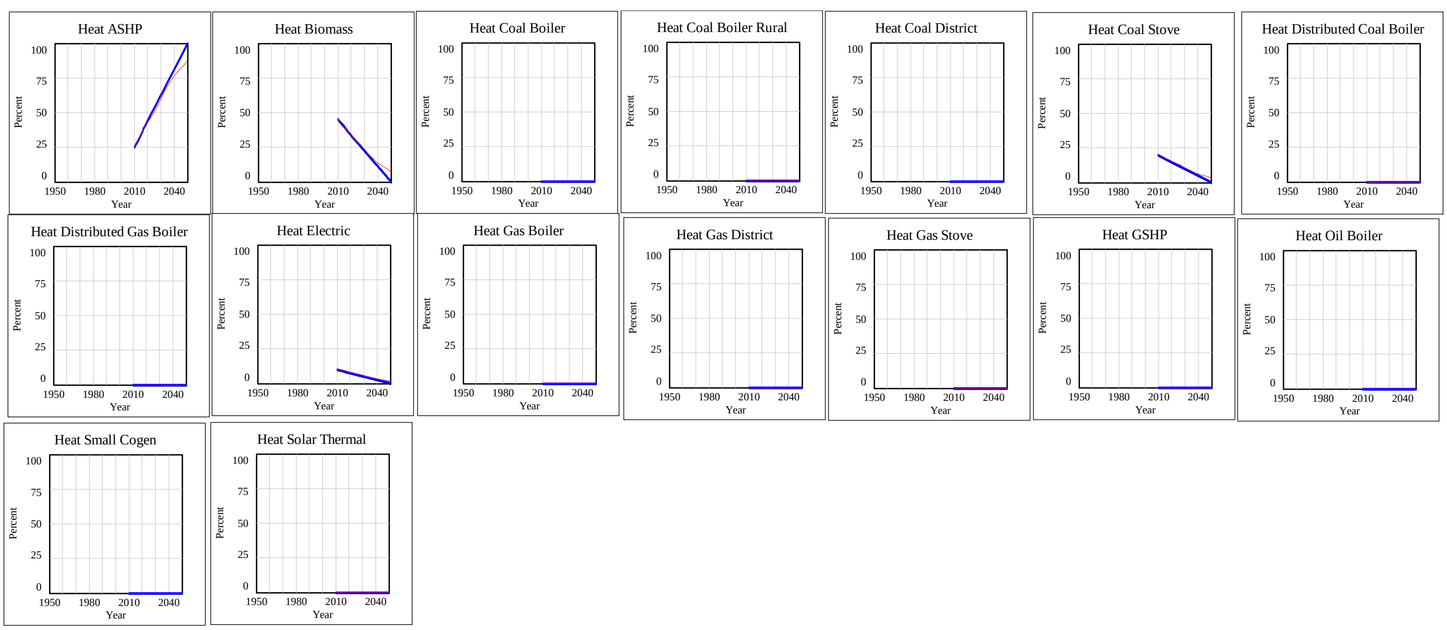


Figure 68: Calibration of Heating Technology Adoption, South Rural Residential.

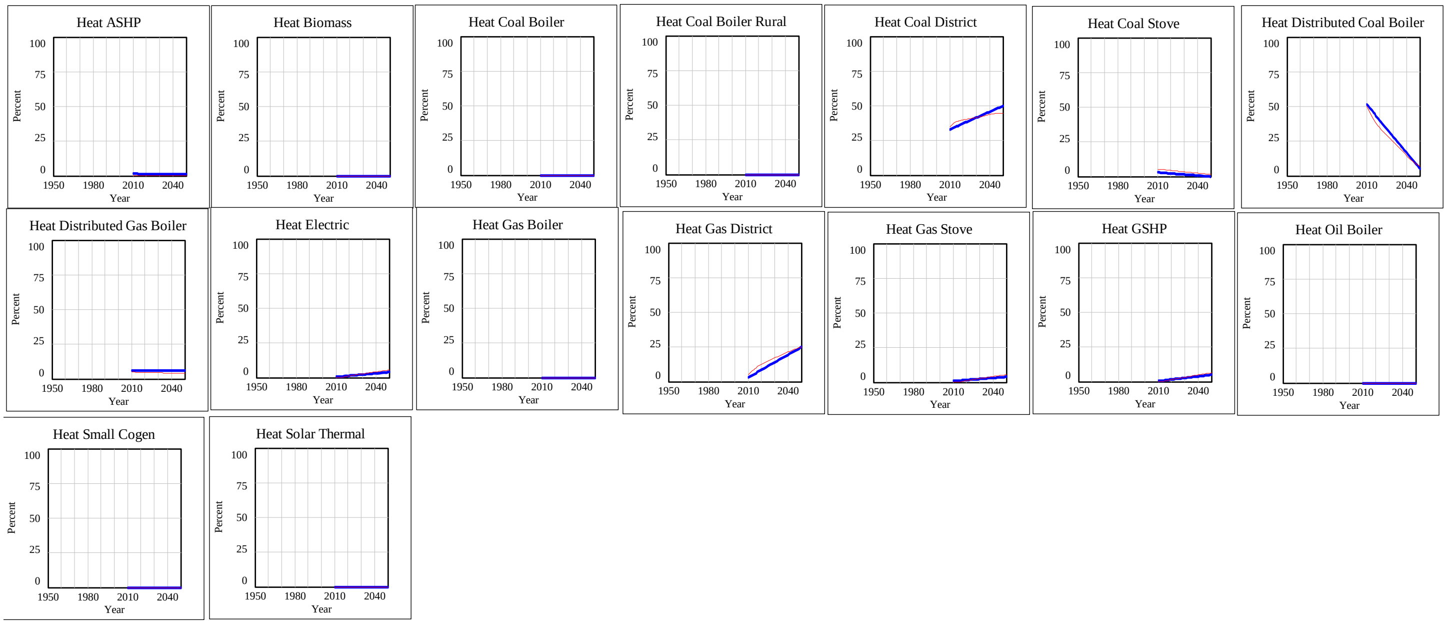


Figure 69: Calibration of Heating Technology Adoption, North Urban Residential.

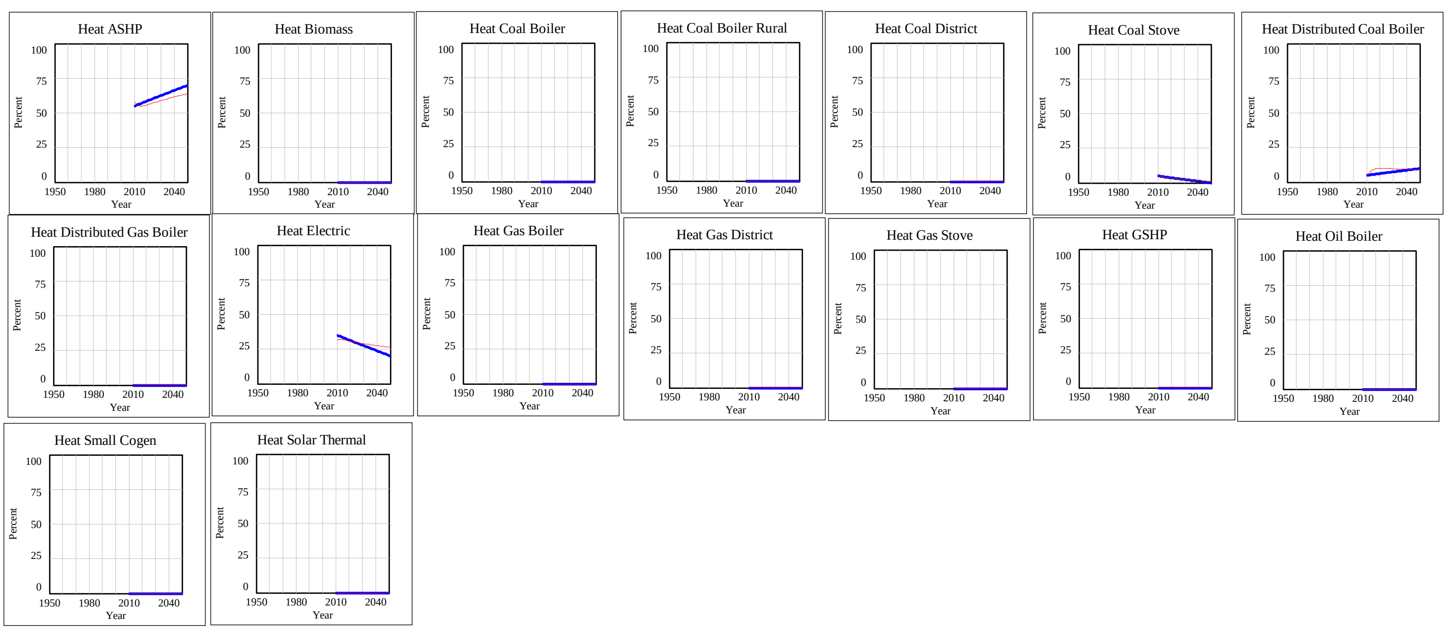


Figure 70: Calibration of Heating Technology Adoption, Transition Urban Residential.

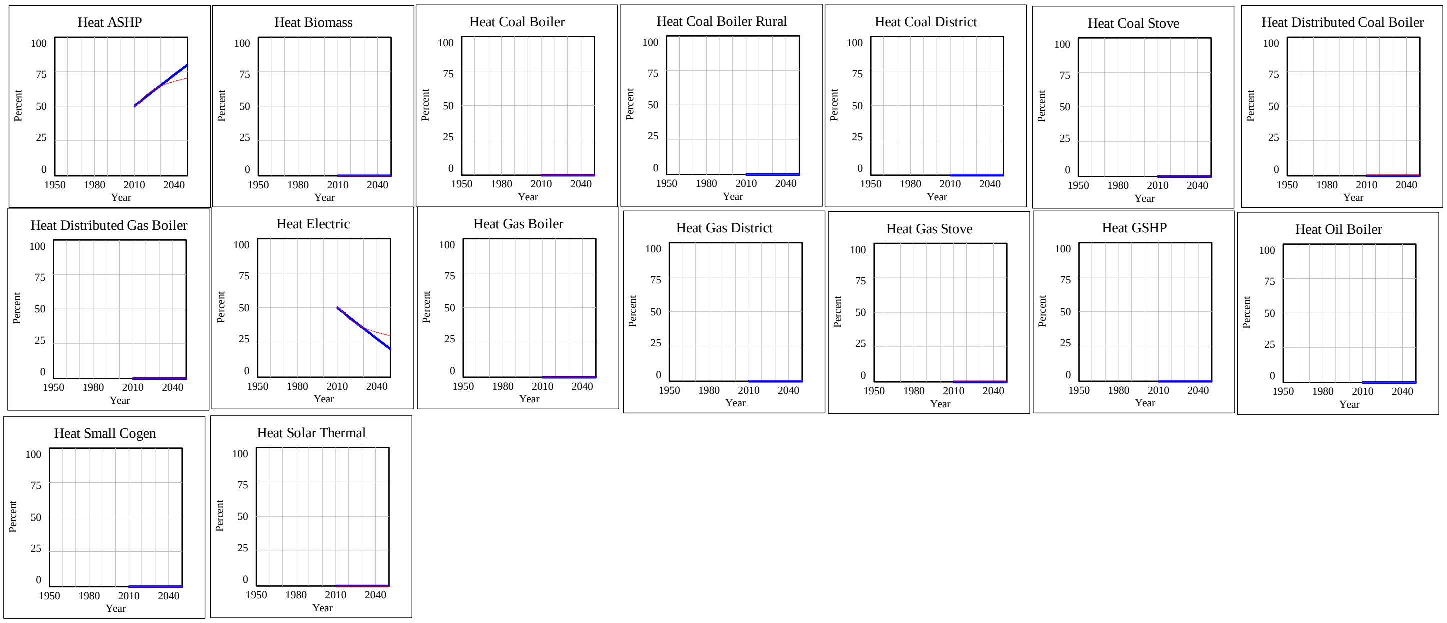


Figure 71: Calibration of Heating Technology Adoption, South Urban Residential.

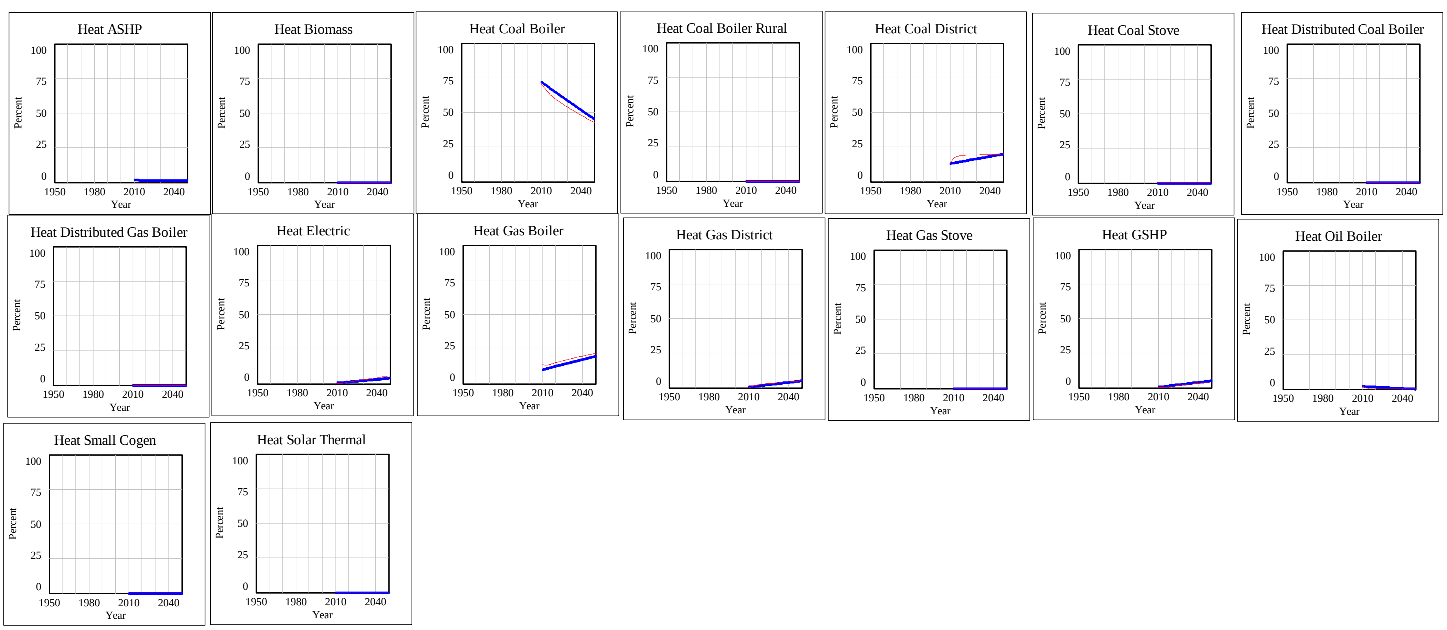


Figure 72: Calibration of Heating Technology Adoption, North Urban Commercial.

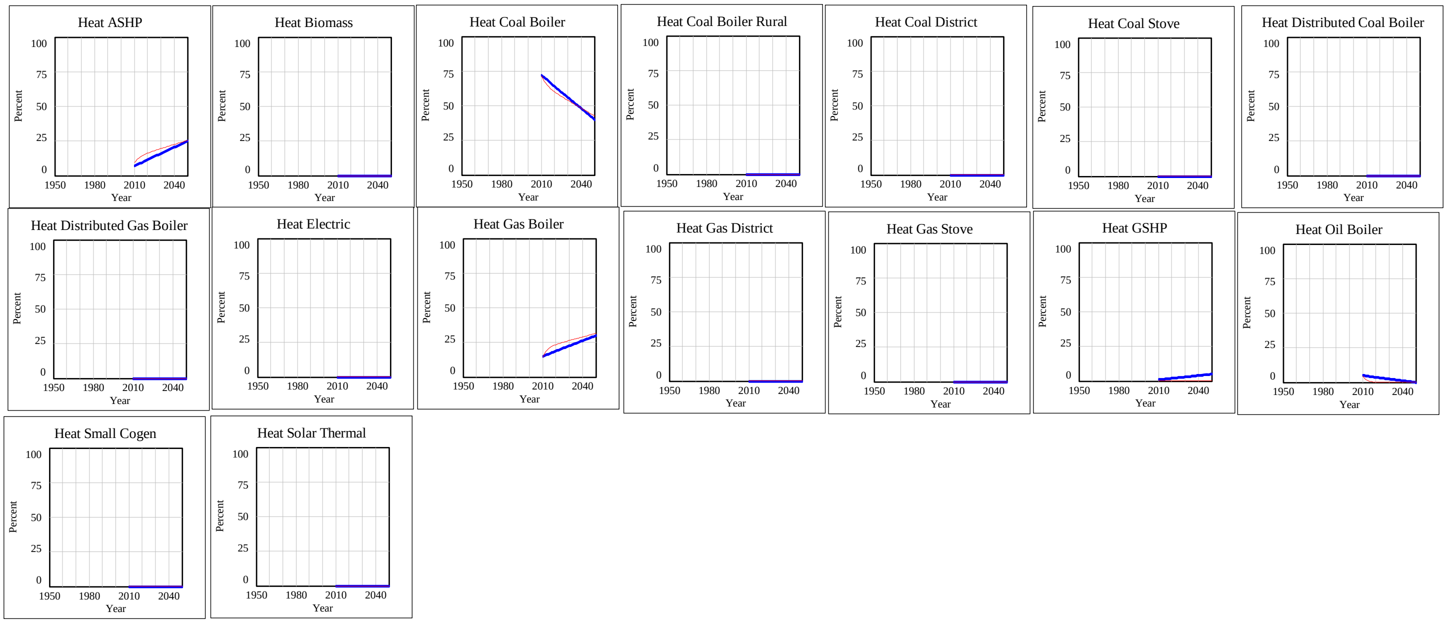


Figure 73: Calibration of Heating Technology Adoption, Transition Urban Commercial.

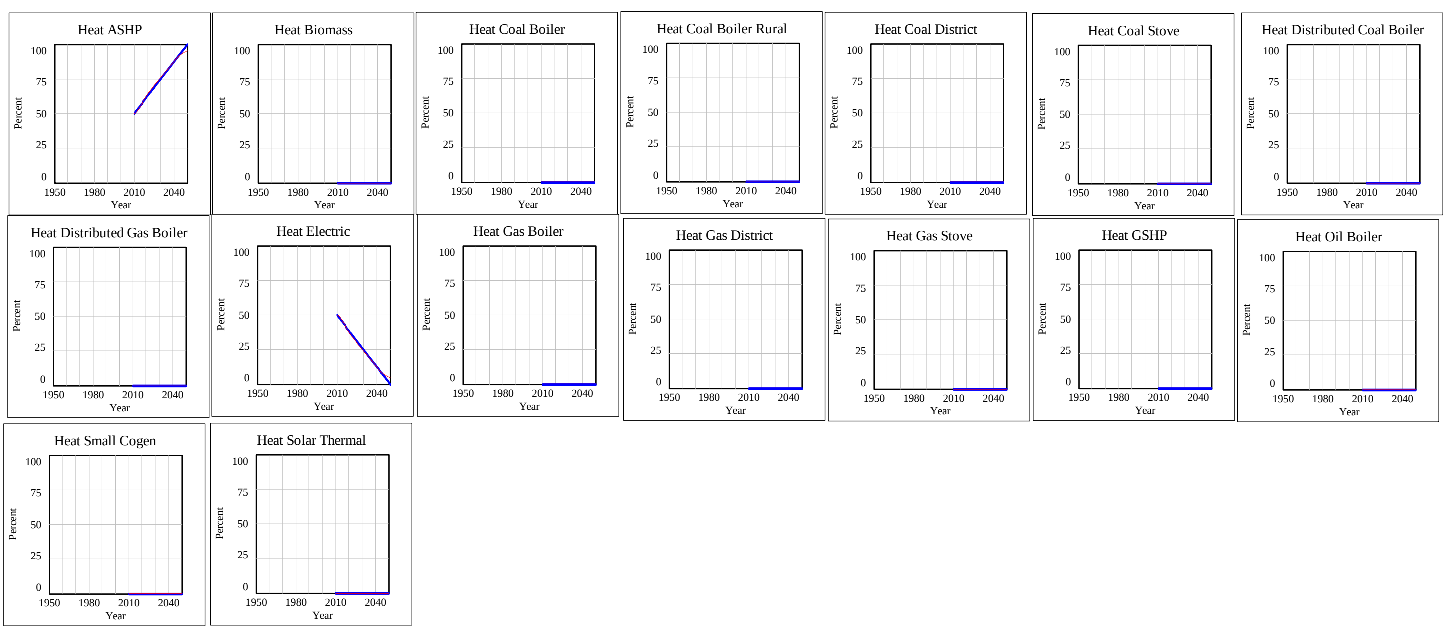


Figure 74: Calibration of Heating Technology Adoption, South Urban Commercial.

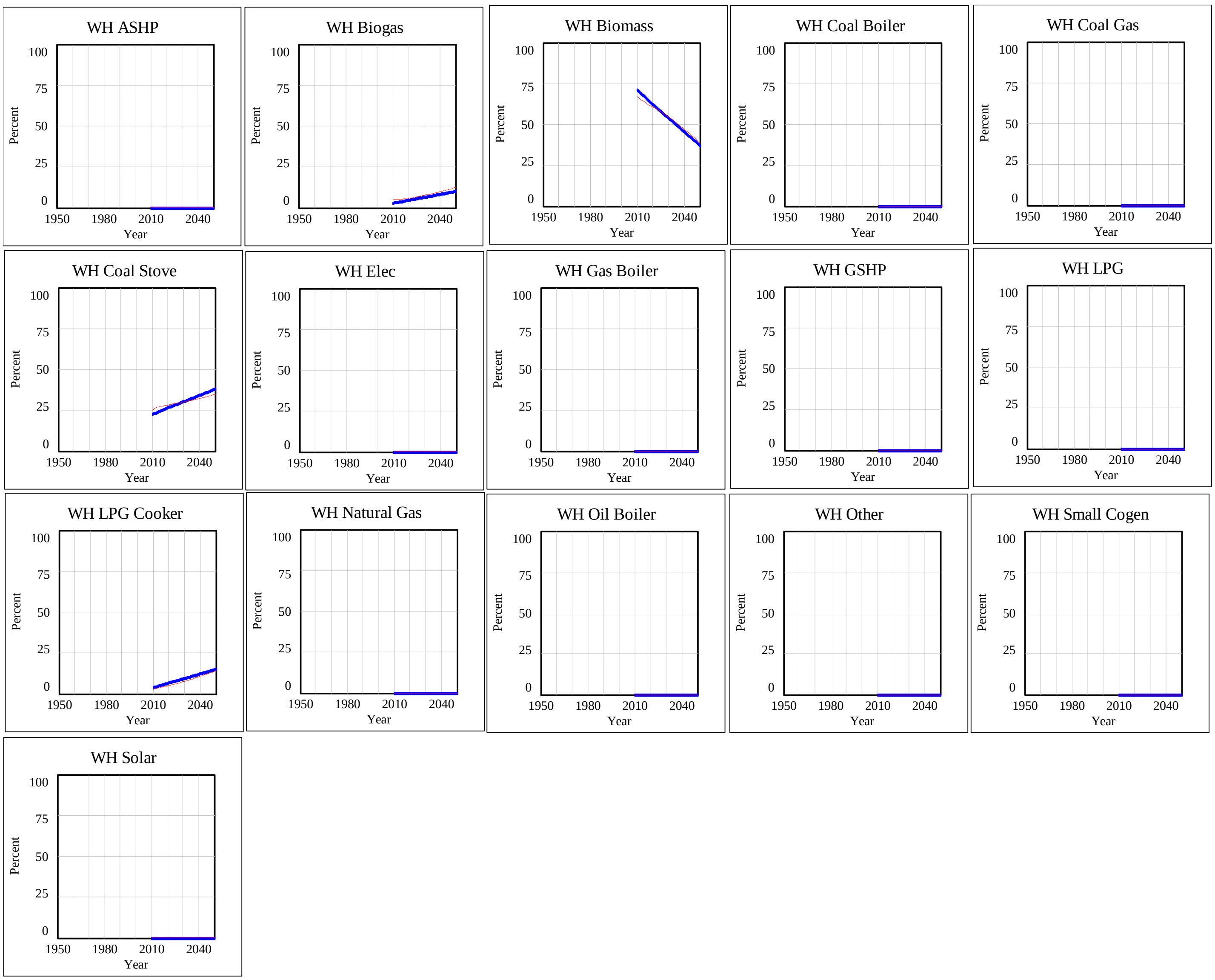


Figure 75: Calibration of Water Heating Technology Adoption, Rural Residential.

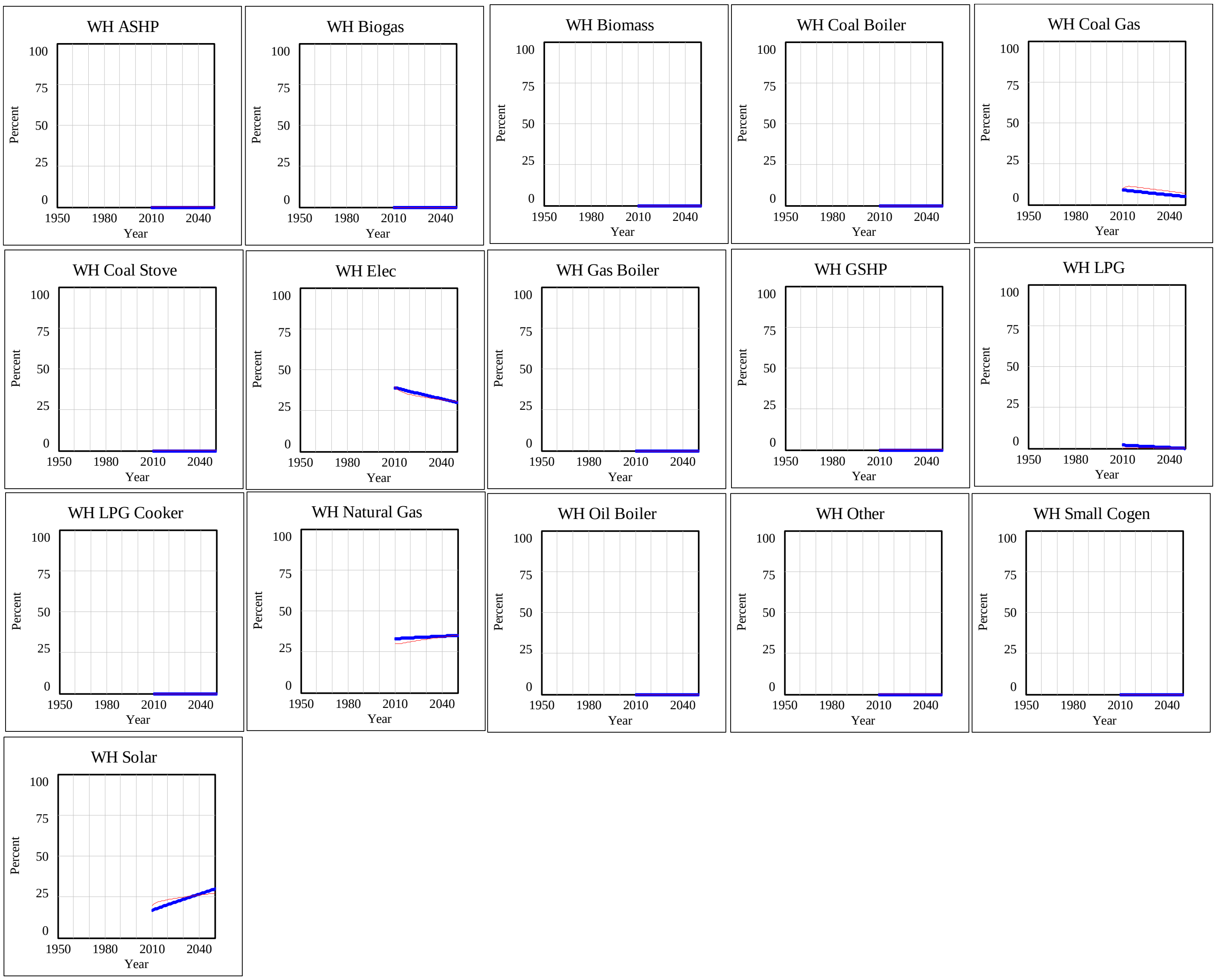


Figure 76: Calibration of Water Heating Technology Adoption, Urban Residential.

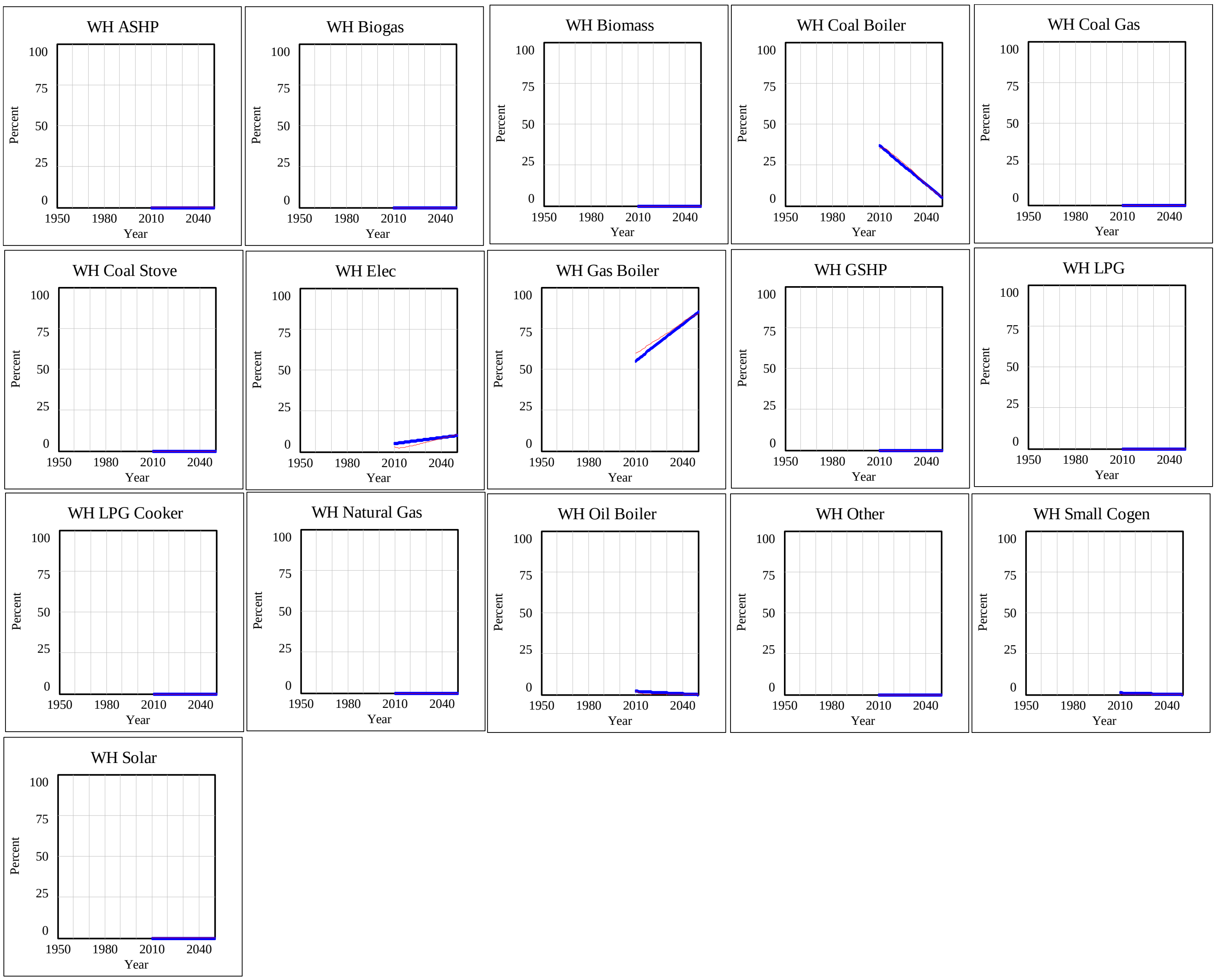


Figure 77: Calibration of Water Heating Technology Adoption, Urban Commercial.

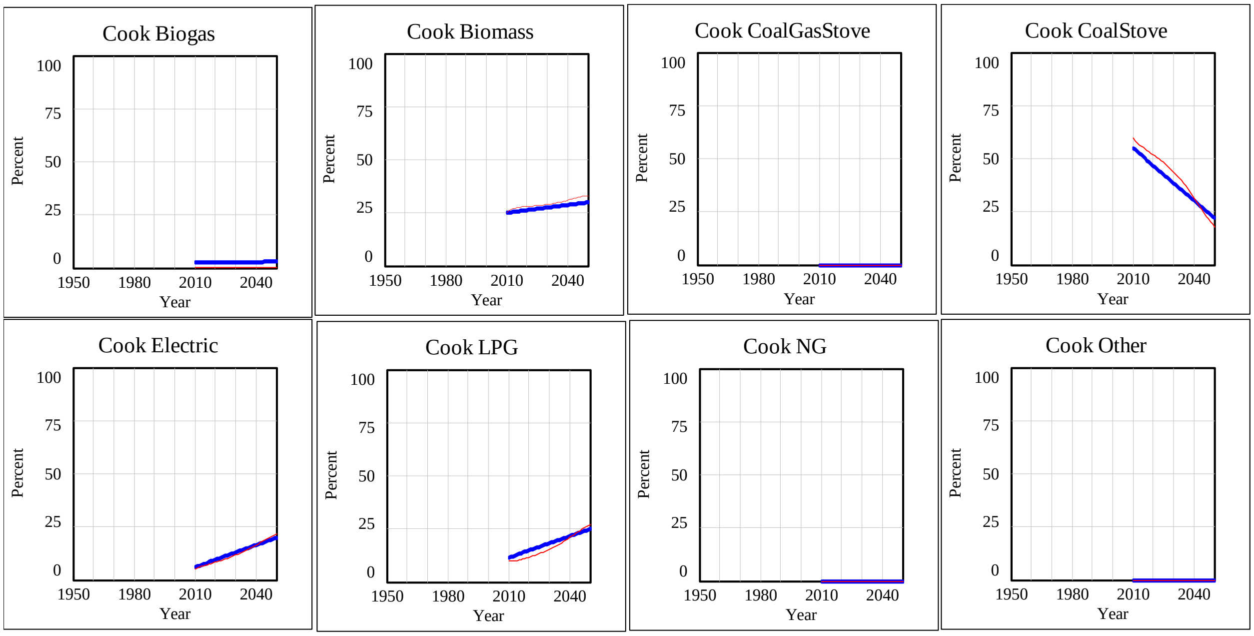


Figure 78: Calibration of Cooking Technology Adoption, Rural.

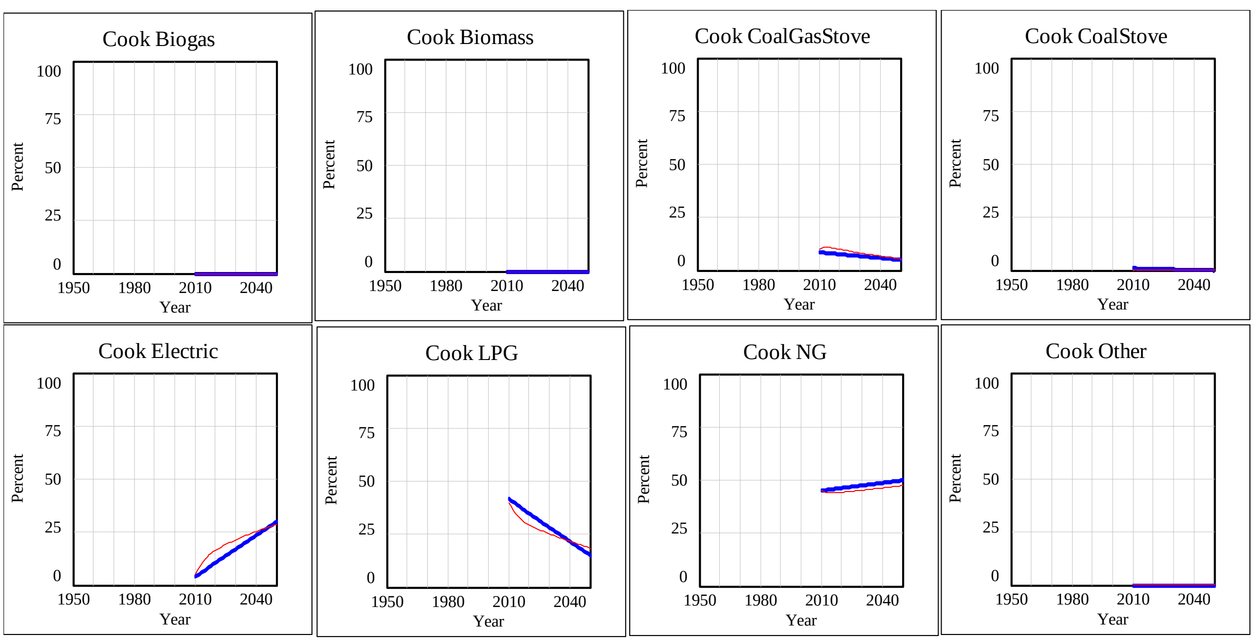


Figure 79: Calibration of Cooking Technology Adoption, Urban.

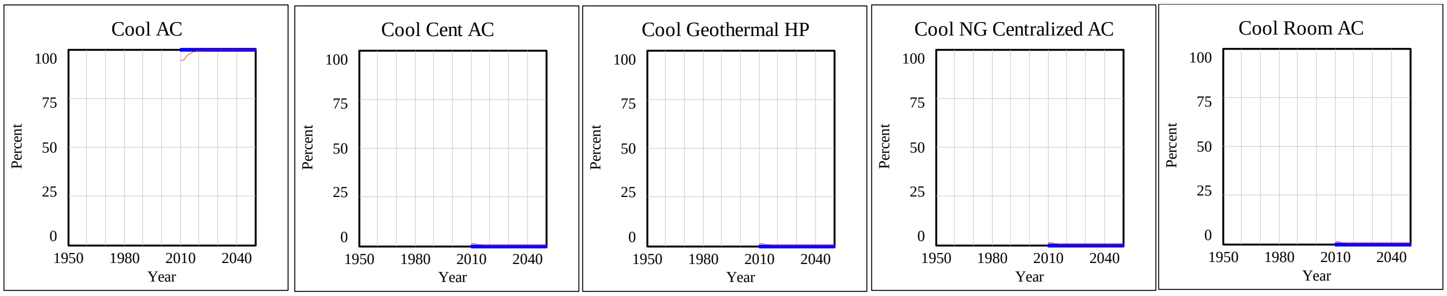


Figure 80: Calibration of Cooling Technology Adoption, Residential.

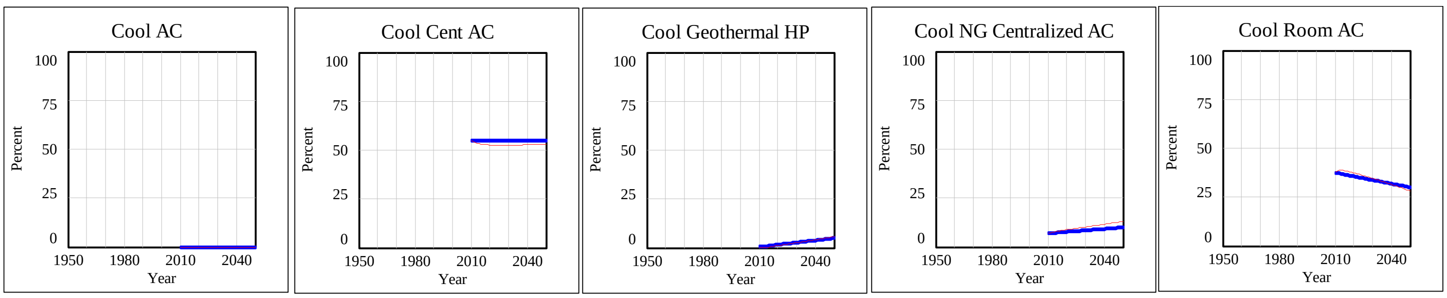


Figure 81: Calibration of Cooling Technology Adoption, Commercial.

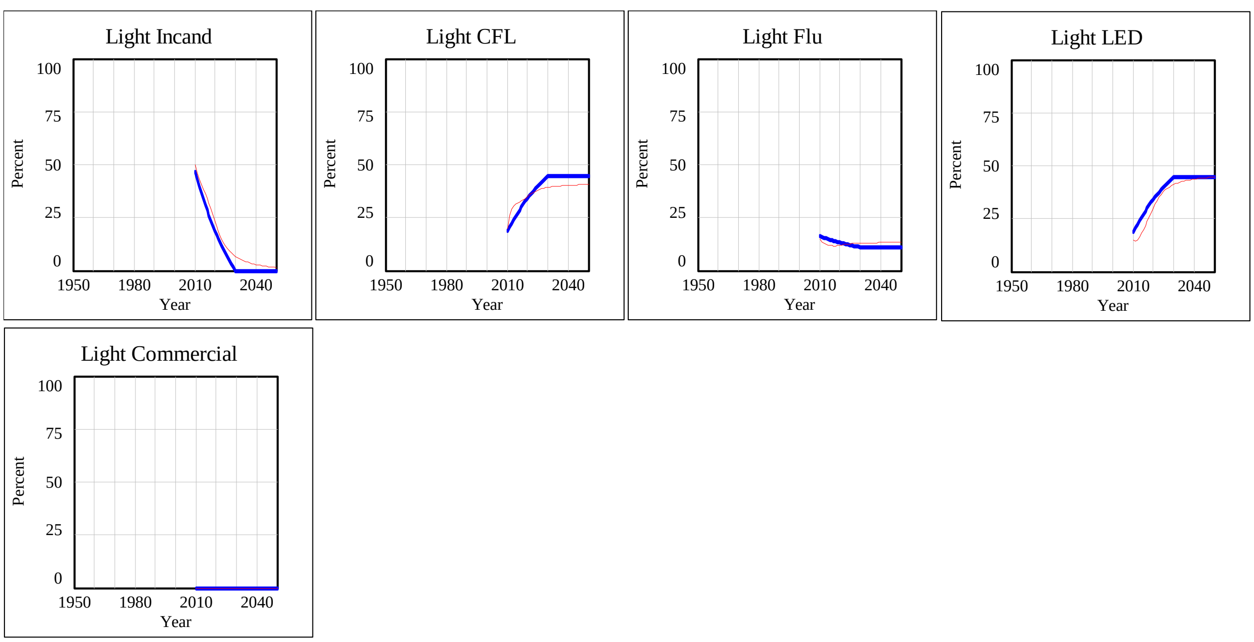


Figure 82: Calibration of Cooking Technology Lighting, Residential.

10 CERC-BEE Advanced Technologies

10.1 Portfolio Energy Savings Model

Let there be n advanced technologies which have the potential to save energy. Let s_i and f_i represent the savings potential and market share of technology $i = 1, \ldots, n$, respectively.

Let us assume that the technologies are *market independent* and *savings independent*. By "market independent" we mean that the market share of locations has multiple technologies is the product of the market shares of the individual technologies. By "savings independent" we mean that the usage factor obtained from multiple technologies is the product of the usage factors of the individual technologies.

Consider a binary choice between each advanced technology and the reference incumbent technology. Then for n advanced technologies, there are 2^n possible combinations of choices. Let $j = 1, \ldots, 2^n$ represent the choice combination index, and let δ_{ij} indicate whether the ith technology was chosen in the j combination. Then we can represent the average energy usage factor as follows:

$$\begin{aligned} f_i & \quad \text{market share of } i\text{th tech,} \\ s_i & \quad \text{savings potential of } i\text{th tech,} \\ \delta_{ij} & \quad i\text{th tech chosen in } j\text{th combination,} \\ 1 - S &= \sum_j \prod_i \left[\delta_{ij}\, f_i\, (1 - s_i) + (1 - \delta_{ij})(1 - f_i)\right] \end{aligned}$$

The consumption factor $E = 1 - S$ is multiplied by the energy usage resulting from the incumbent technologies. If the market shares of the advanced technologies are all zero, then $E = 1$, as expected.

We justify the assumptions of market and savings independence by noting:

- $\text{COR}(\text{Air Barrier}, \text{all others}) = 0$. No other technologies impact convection losses.
- $\text{COR}(\text{Cool Roof}, \text{Windows \& Shading}) = 0$. Roof and windows are independent.
- $\text{COR}(\text{Hybrid Ventilation}, \text{Ground Source Heat Pump}) = 0$. Building ventilation independent of type of heat exchange
- $\text{COR}(\text{Dew Point Fluid Cooling}, \text{Ground Source Heat Pump}) = 0$. Assume ground heat exchange, dry air evaporation are independent
- $\text{COR}(\text{Lighting}, \text{Windows \& Shading}) \sim 0$. Weakly dependent: windows and shading reduce solar heat gain, but also reduce natural light, and so increase need for artificial light

Figure 83 shows the causal connections and equations for `Average Consumption Factor` that implement the mathematics above.

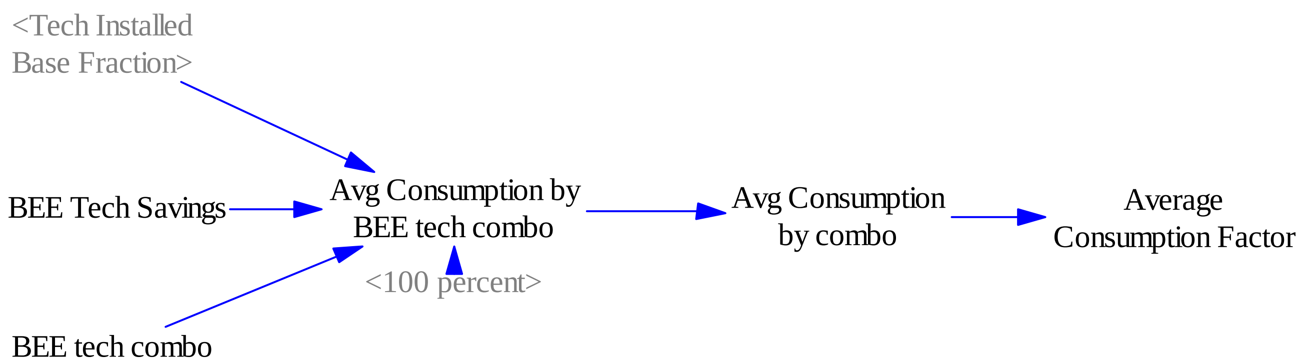
Table 9 lists the $n = 6$ advanced CERC-BEE technologies with the anticipated percentage energy savings for each end use category. Note that certain technologies have impact in multiple end use categories. Where the savings is negative, use of the CERC-BEE technology would increase end use consumption. For example, adoption of Advanced Controls would reduce the use of energy for lighting, but would increase the use of energy for heating. Table 10 shows the availability parameters and economic utilities for the CERC-BEE advanced technologies: full commercialization is assumed by 2030; economic utilities and price are assume to be identical with the incumbent technologies, i.e. the advanced technologies command half of the sales market share when fully available.

10.2 Portfolio Adoption

Given the availability and economic utility of the CERC-BEE advanced technologies, market adoption and installed base are given by the same equations as end use technologies. Figures 84–92 show the regional installed base fractions of the incumbent technologies and the CERC-BEE advanced technologies.

Row Labels	AC exchange Dewpoint	AC ventilation hybrid	Controls Advanced	Insulation AirBarrier	Roof Cool	Window Advanced
Commercial						
Cooling	10%	30%	7%	7%	10%	15%
Heating			-7%	35%	-2%	-1%
Lighting			20%			10%
Residential						
Cooling	10%	30%	7%	7%	10%	15%
Heating			-7%	35%	-2%	-1%
Lighting			20%			10%

Table 9: Estimated energy savings percentages by end use area for the CERC-BEE technologies.




```
Average Consumption Factor[region,ClimateZone,RurUrb,BuildingType,EndUse]=
SUM(Avg Consumption by combo[region,ClimateZone,RurUrb,BuildingType,EndUse,combo6!])
~ Dmnl
~ Factor of End Use Energy Consumption, determined by combination of \
building energy efficiency technologies
|

Avg Consumption by BEE tech combo[region,ClimateZone,RurUrb,BuildingType,EndUse,combo6\
,BEE Tech Adv]=
BEE tech combo[combo6,BEE Tech Adv]*
Tech Installed Base Fraction[region,ClimateZone,RurUrb,BuildingType,BEE Tech Adv]/
"100 percent"*
(1-BEE Tech Savings[BuildingType,EndUse,BEE Tech Adv])
+
(1-BEE tech combo[combo6,BEE Tech Adv])*
(1-Tech Installed Base Fraction[region,ClimateZone,RurUrb,BuildingType,BEE Tech Adv]/
"100 percent")
~ Dmnl
~ Inter-calculation step to determine Average Consumption Factor.  This step \
gives the consumption of a particular BEE tech combination and installed \
market shares.
|

Avg Consumption by combo[region,ClimateZone,RurUrb,BuildingType,EndUse,combo6]=
PROD(Avg Consumption by BEE tech combo[region,ClimateZone,RurUrb,BuildingType,EndUse\
,combo6,BEE Tech Adv!])
~ Dmnl
~ Inter-calculation step to determine Average Consumption Factor
|
```

Figure 83: Causal diagram for average consumption factor including CERC-BEE technologies.

ClimateZon	RurUrb	BuildingTyp	EndUse	Technology Type	T1	T2	A1	A2	xU = init installed	fU = WTP	iU = Price
ClimateZone	RurUrb	BuildingType	Insulation	Insulation Conventional	2010	2050	1	1	100	0	0
ClimateZone	RurUrb	BuildingType	Insulation	Insulation AirBarrier	2010	2030	0	1	0	0	0
ClimateZone	RurUrb	BuildingType	Roof	Roof Conventional	2010	2050	1	1	100	0	0
North Region	RurUrb	BuildingType	Roof	Roof Cool	1900	2100	0	0	0	-5	0
Transition Reg	RurUrb	BuildingType	Roof	Roof Cool	2010	2030	0	1	0	0	0
South Region	RurUrb	BuildingType	Roof	Roof Cool	2010	2030	0	1	0	0	0
ClimateZone	RurUrb	BuildingType	Windows	Window Conventional	2010	2050	1	1	100	0	0
North Region	RurUrb	BuildingType	Windows	Window Advanced	1900	2100	0	0	0	-5	0
Transition Reg	RurUrb	BuildingType	Windows	Window Advanced	2010	2030	0	1	0	0	0
South Region	RurUrb	BuildingType	Windows	Window Advanced	2010	2030	0	1	0	0	0
ClimateZone	RurUrb	BuildingType	AC exchange	AC exchange Conventional	2010	2050	1	1	100	0	0
North Region	RurUrb	BuildingType	AC exchange	AC exchange Dewpoint	2010	2030	0	0.1	0	0	0
Transition Reg	RurUrb	BuildingType	AC exchange	AC exchange Dewpoint	2010	2030	0	0	0	0	0
South Region	RurUrb	BuildingType	AC exchange	AC exchange Dewpoint	2010	2030	0	0	0	0	0
ClimateZone	RurUrb	BuildingType	AC ventilation	AC ventilation Conventional	2010	2050	1	1	100	0	0
North Region	RurUrb	BuildingType	AC ventilation	AC ventilation hybrid	2010	2030	0	1	0	0	0
Transition Reg	RurUrb	BuildingType	AC ventilation	AC ventilation hybrid	2010	2030	0	1	0	0	0
South Region	RurUrb	BuildingType	AC ventilation	AC ventilation hybrid	2010	2030	0	0	0	0	0
ClimateZone	RurUrb	BuildingType	Controls	Controls Conventional	2010	2050	1	1	100	0	0
ClimateZone	RurUrb	BuildingType	Controls	Controls Advanced	2010	2030	0	1	0	0	0

Table 10: Projected availability and marginal utility for the CERC-BEE technologies.

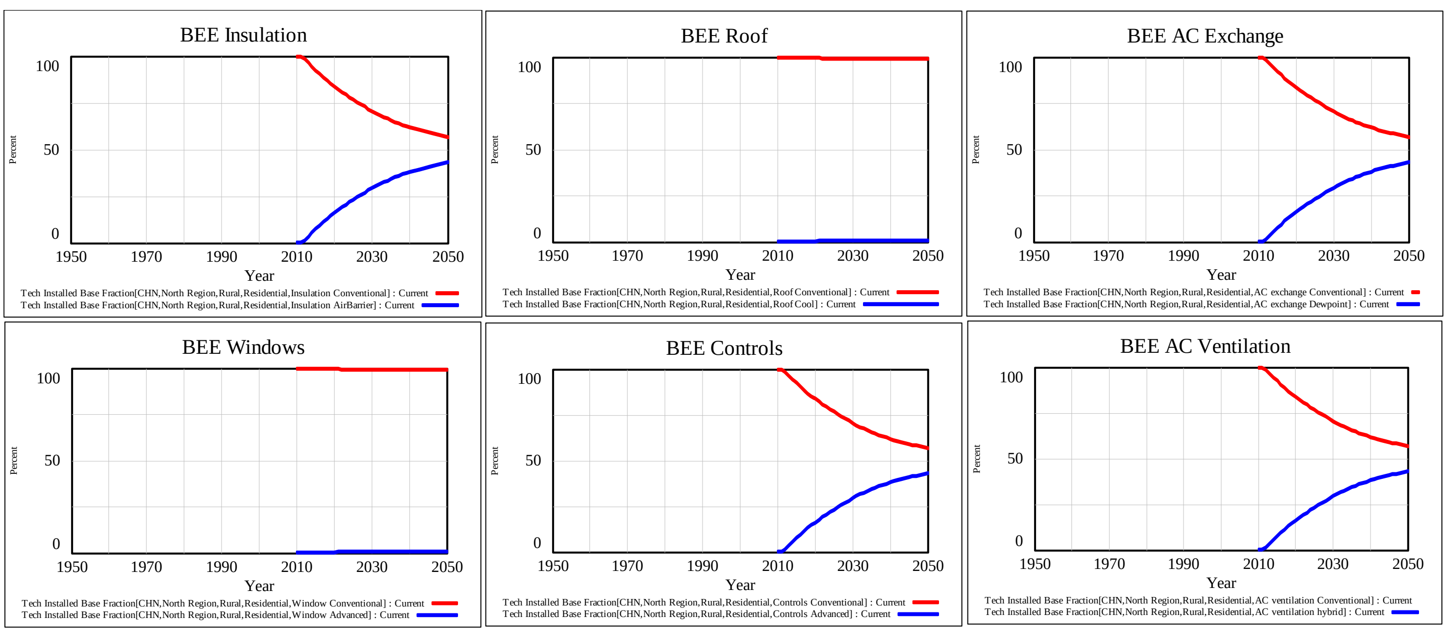


Figure 84: Installed Base Fraction of CERC-BEE Advanced Technologies, North Rural Residential.

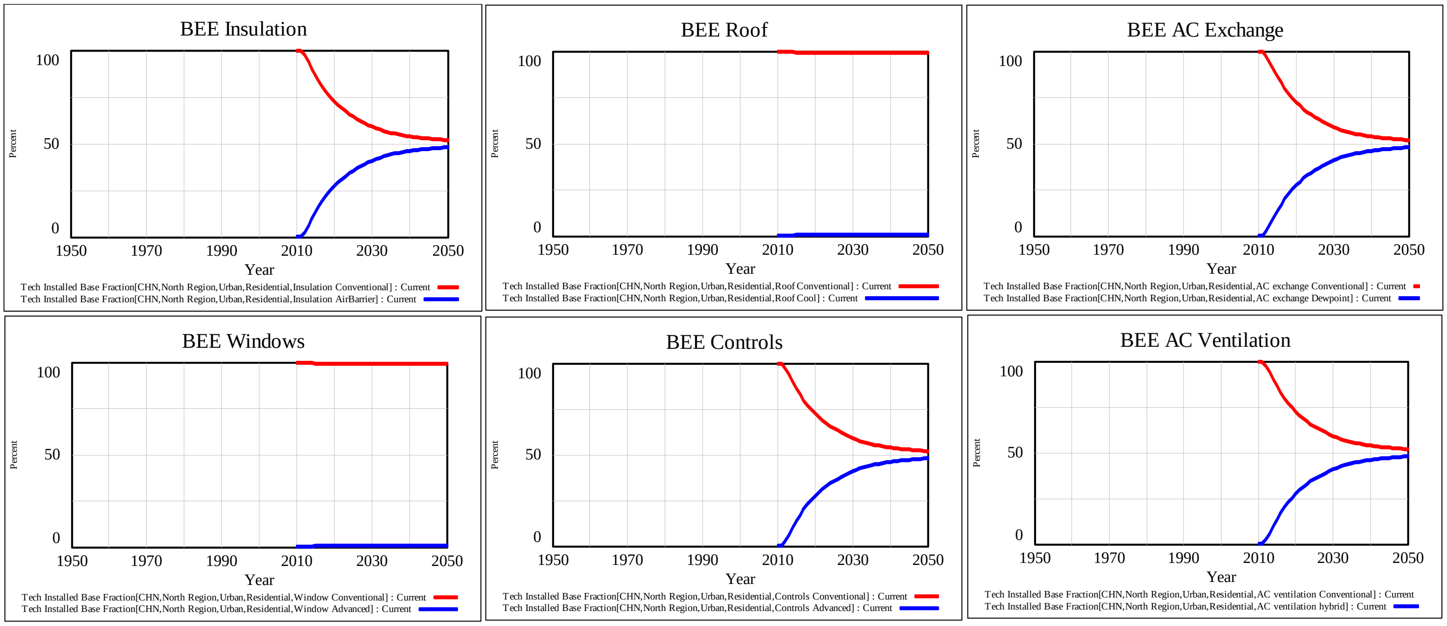


Figure 85: Installed Base Fraction of CERC-BEE Advanced Technologies, North Urban Residential.

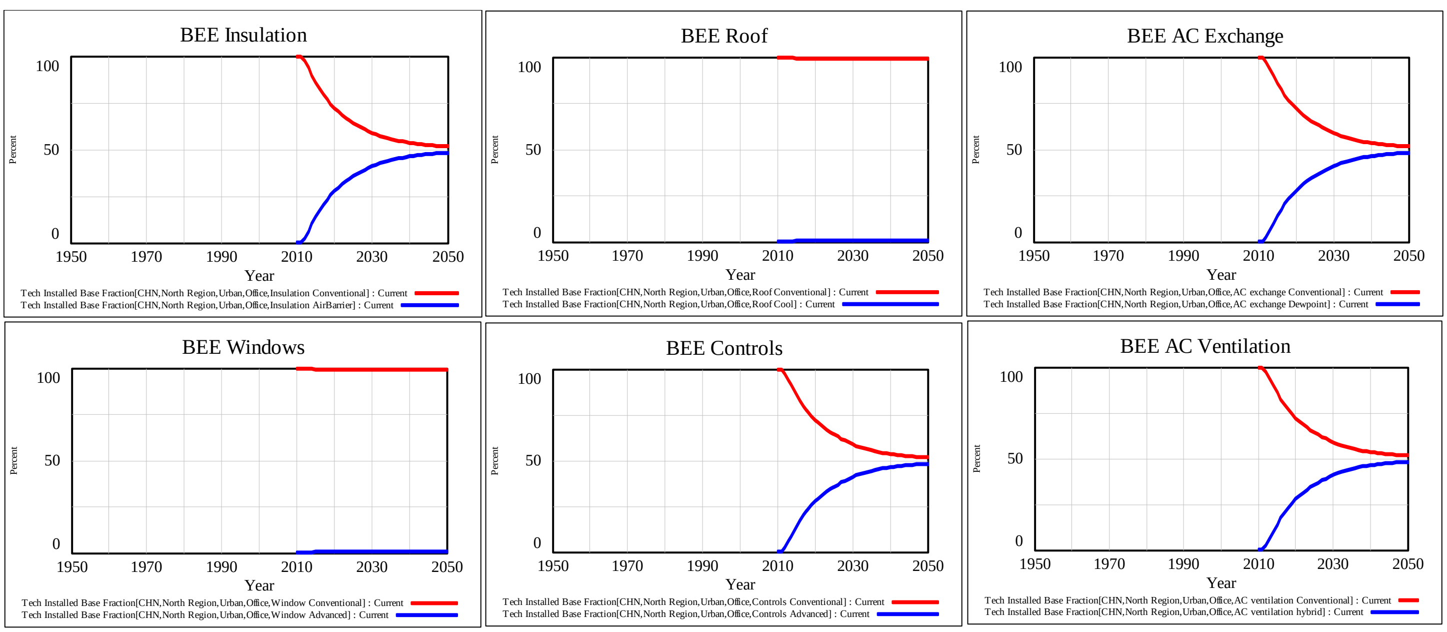


Figure 86: Installed Base Fraction of CERC-BEE Advanced Technologies, North Urban Commercial.

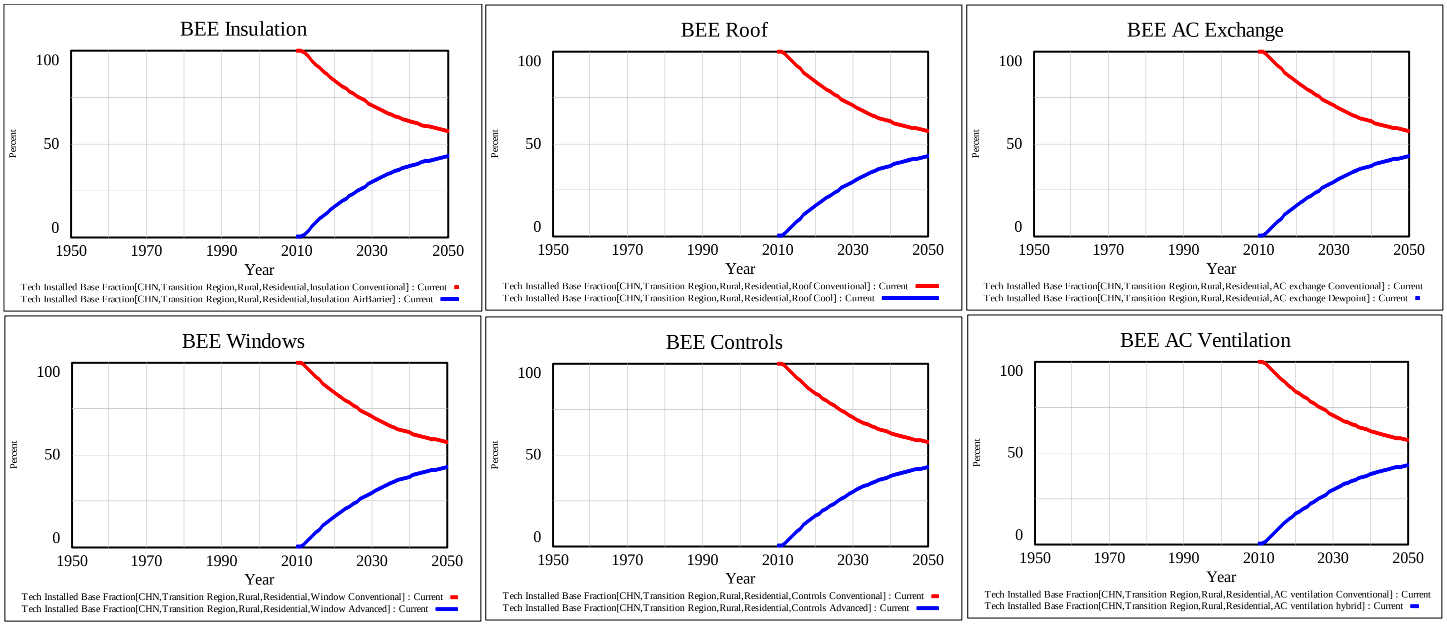


Figure 87: Installed Base Fraction of CERC-BEE Advanced Technologies, Transition Rural Residential.

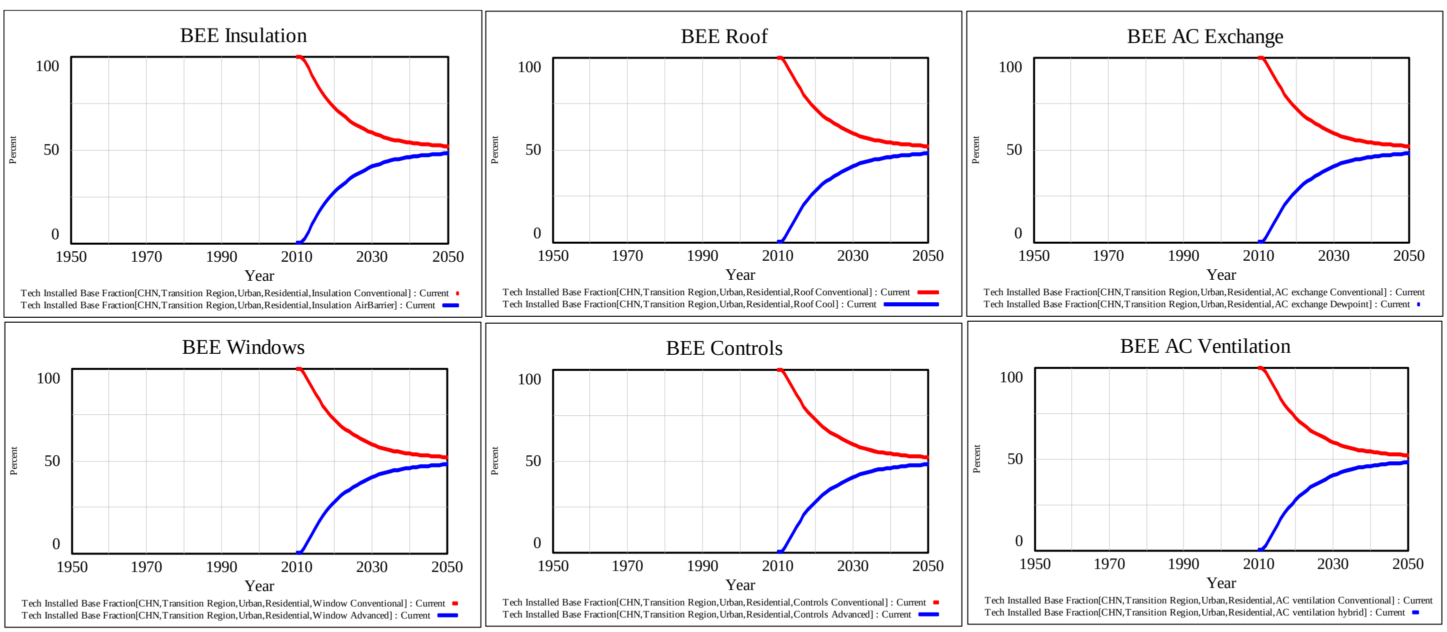


Figure 88: Installed Base Fraction of CERC-BEE Advanced Technologies, Transition Urban Residential.

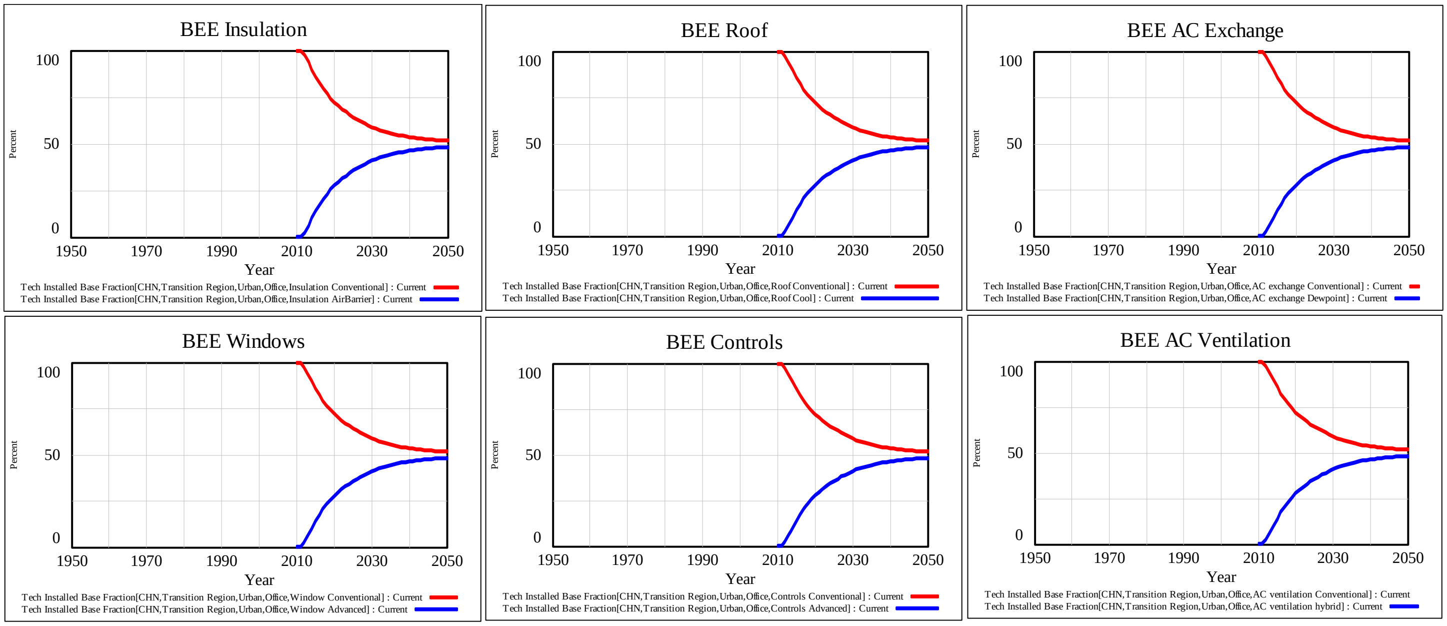


Figure 89: Installed Base Fraction of CERC-BEE Advanced Technologies, Transition Urban Commercial.

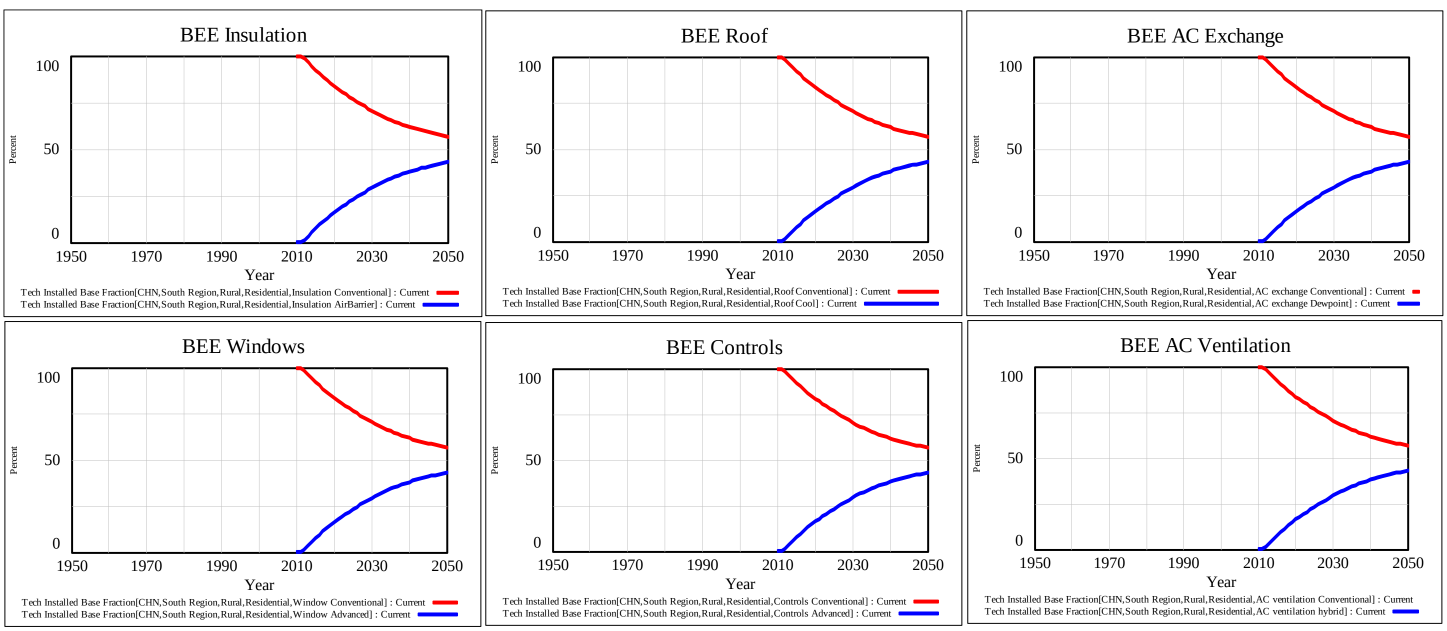


Figure 90: Installed Base Fraction of CERC-BEE Advanced Technologies, South Rural Residential.

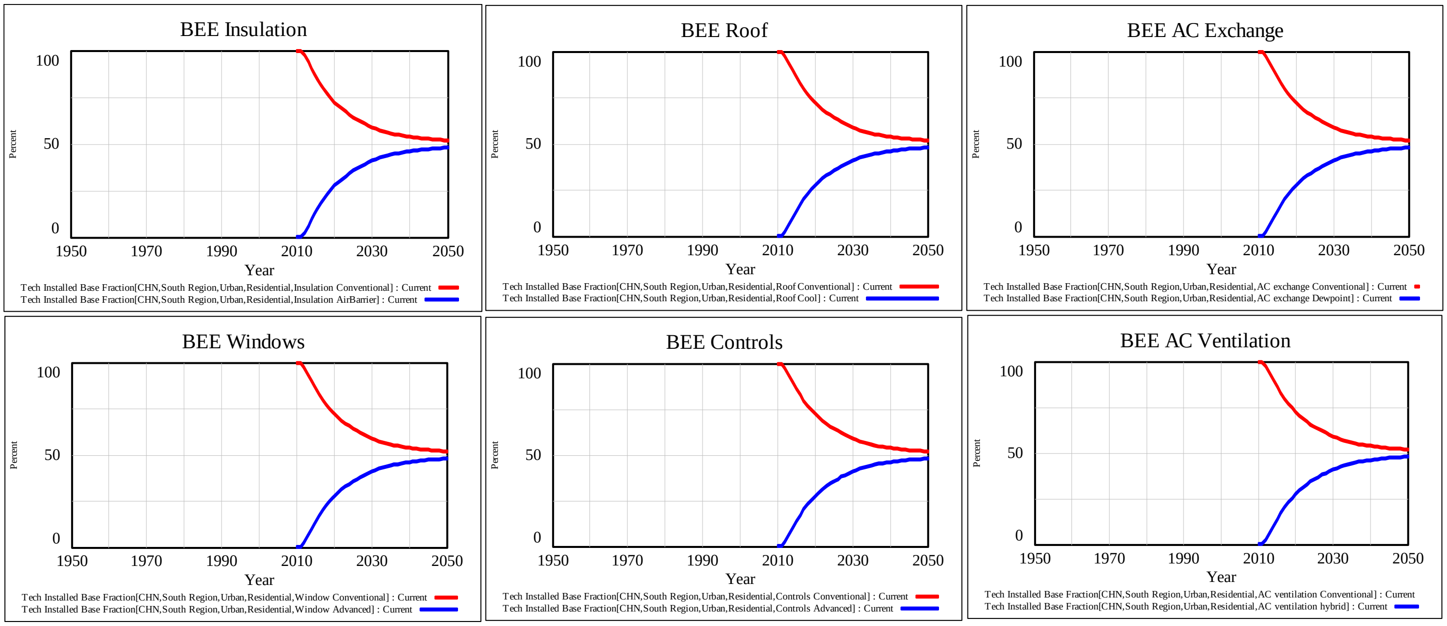


Figure 91: Installed Base Fraction of CERC-BEE Advanced Technologies, South Urban Residential.

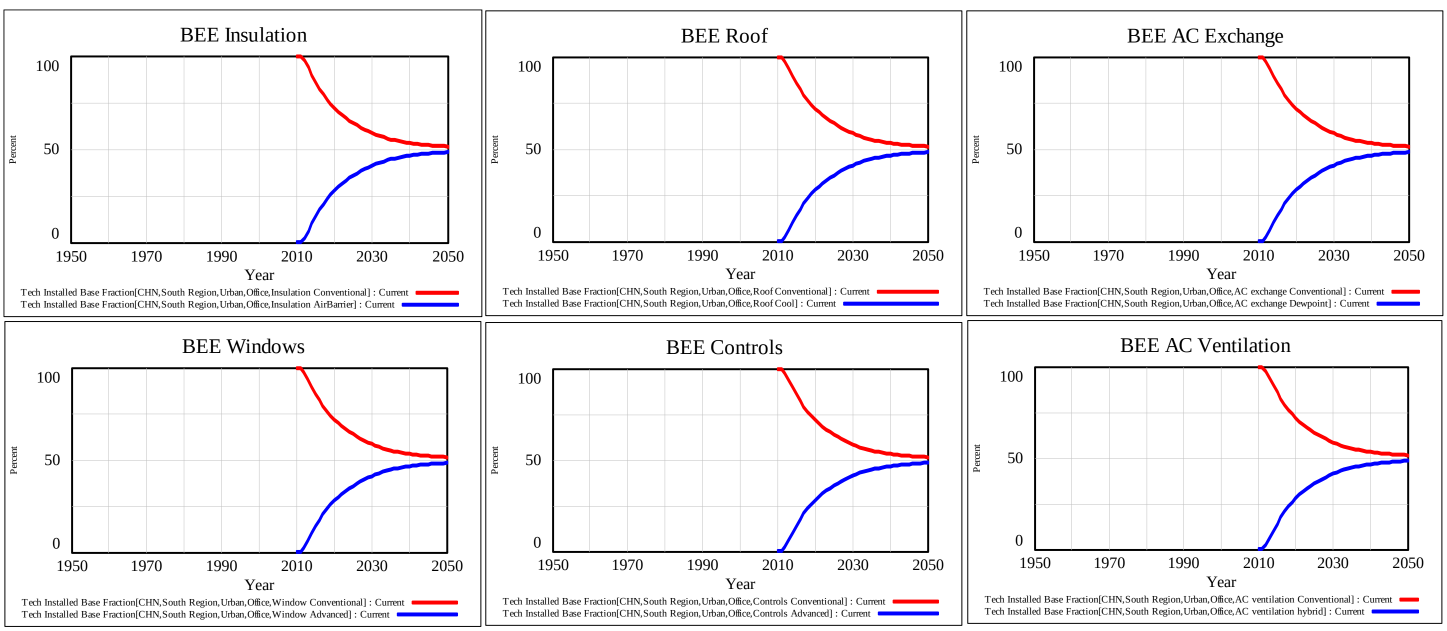


Figure 92: Installed Base Fraction of CERC-BEE Advanced Technologies, South Urban Commercial.

11 Building Energy Consumption and Impact

11.1 Building Energy Consumption

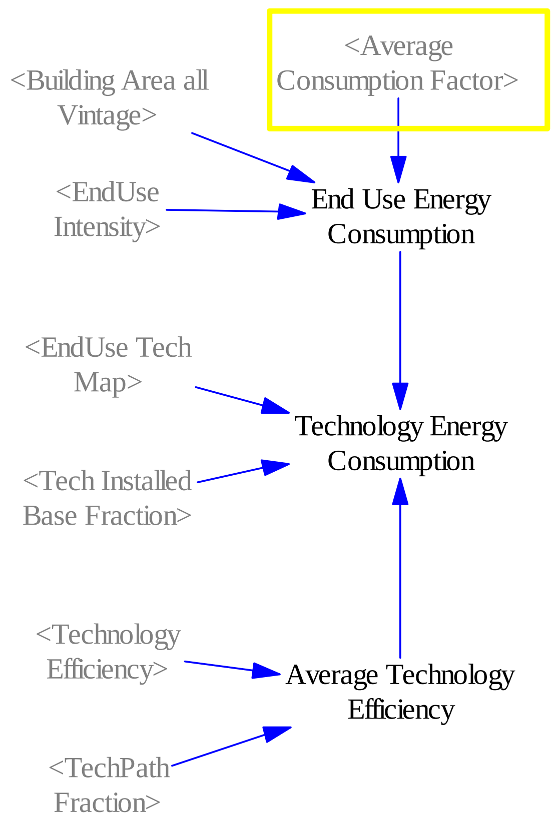
Figure 93 shows the dependence of and equations for `Technology Energy Consumption` on `Tech Installed Base Fraction`, `End Use Energy Consumption`, and `Average Technology Efficiency`.

Figures 94 and 95 show the dependencies and equations, respectively, for energy consumption totals derived from `Total Energy Consumption`.

Figure 96 and Figure 97 show stacked timeseries for the energy consumption totals for the Reinventing Fire China baseline scenario, without and with adoption of the CERC-BEE Advanced Technology portfolio, respectively.

11.2 Impact

Impact of the CERC-BEE Advanced Technologies portfolio is calculated as the difference in building energy consumption between two scenarios: with and without adoption of the CERC-BEE technologies, starting from the Reinventing Fire China baseline scenario. Figure2 (and also Figure 98) shows the building `Total Energy Consumption` time series for the Reinventing Fire China baseline with and without the CERC-BEE Advanced Technology portfolio, as well as the percentage differerence of the scenarios.




```
Technology Energy Consumption[region,ClimateZone,RurUrb,BuildingType,TechnologyType]=
SUM(End Use Energy Consumption[region,ClimateZone,RurUrb,BuildingType,EndUse!]*
EndUse Tech Map[TechnologyType,EndUse!])*
Tech Installed Base Fraction[region,ClimateZone,RurUrb,BuildingType,TechnologyType]/
Average Technology Efficiency[TechnologyType]
~ GWh/Year
~ Annual Energy Consumption of End Use Technology at finest level of \
resolution: Country, ClimateZone, Rural/Urban, Building Type.
|

End Use Energy Consumption[region,ClimateZone,RurUrb,BuildingType,EndUse]=
Building Area all Vintage[region,ClimateZone,RurUrb,BuildingType]*
EndUse Intensity[region,ClimateZone,RurUrb,BuildingType,EndUse]*
Average Consumption Factor[region,ClimateZone,RurUrb,BuildingType,EndUse]
~ GWh/Year
~ Annual End Use Energy Consumption by Country, ClimateZone, Rural/Urban, \
and Building Type
|

Average Technology Efficiency[TechnologyType]=
SUM(Technology Efficiency[TechnologyType,Tech Path!]*TechPath Fraction[Tech Path!])
~ Percent
~ End Use Technology Efficiency, in transition from initial to final \
efficiency values.
|
```

Figure 93: Factors determining Technology Energy Consumption.

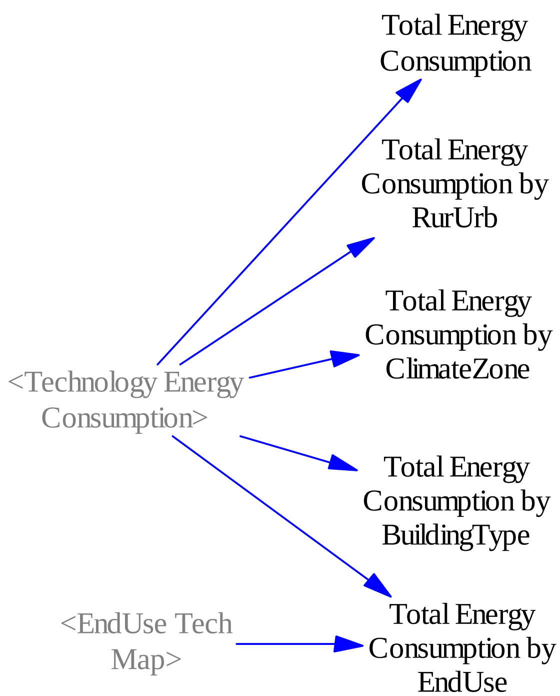


Figure 94: Dependence of energy totals on Technology Energy Consumption.

```
Total Energy Consumption[region]=
SUM(Technology Energy Consumption[region,ClimateZone!,RurUrb!,BuildingType!,TechnologyType!])
~ GWh/Year
~ Total (national building) energy consumption per year.
|

Total Energy Consumption by BuildingType[region,BuildingType]=
SUM(Technology Energy Consumption[region,ClimateZone!,RurUrb!,BuildingType,TechnologyType!])
~ GWh/Year
~ Total (building) energy consumption per year, by Building Type: \
Residential, Office, Hotel, ...
|

Total Energy Consumption by ClimateZone[region,ClimateZone]=
SUM(Technology Energy Consumption[region,ClimateZone,RurUrb!,BuildingType!,TechnologyType!])
~ GWh/Year
~ Total (building) energy consumption per year, by Region: North, \
Transition, South
|

Total Energy Consumption by EndUse[region,EndUse]=
SUM(Technology Energy Consumption[region,ClimateZone!,RurUrb!,BuildingType!,TechnologyType!]*
EndUse Tech Map[TechnologyType!,EndUse])
~ GWh/Year
~ Total (building) energy consumption per year, by EndUse: Heating, Water \
Heating, Cooling, ...
|

Total Energy Consumption by RurUrb[region,RurUrb]=
SUM(Technology Energy Consumption[region,ClimateZone!,RurUrb,BuildingType!,TechnologyType!])
~ GWh/Year
~ Total (building) energy consumption per year, by Rural/Urban
|
```

Figure 95: Equations for energy totals derived from Technology Energy Consumption.

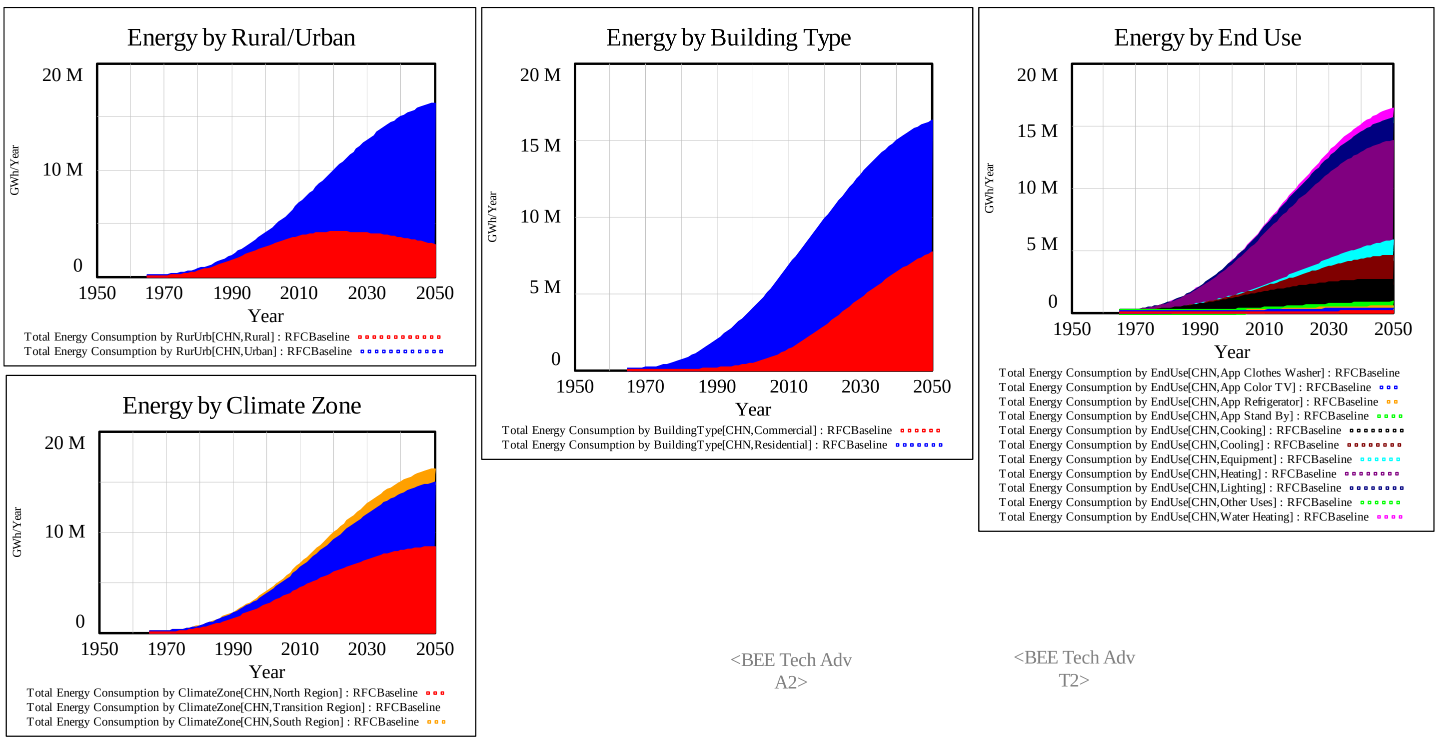


Figure 96: Total Energy Consumption under scenario of Reinventing Fire China Baseline. `TotalEnergyConsumptionBaseline`

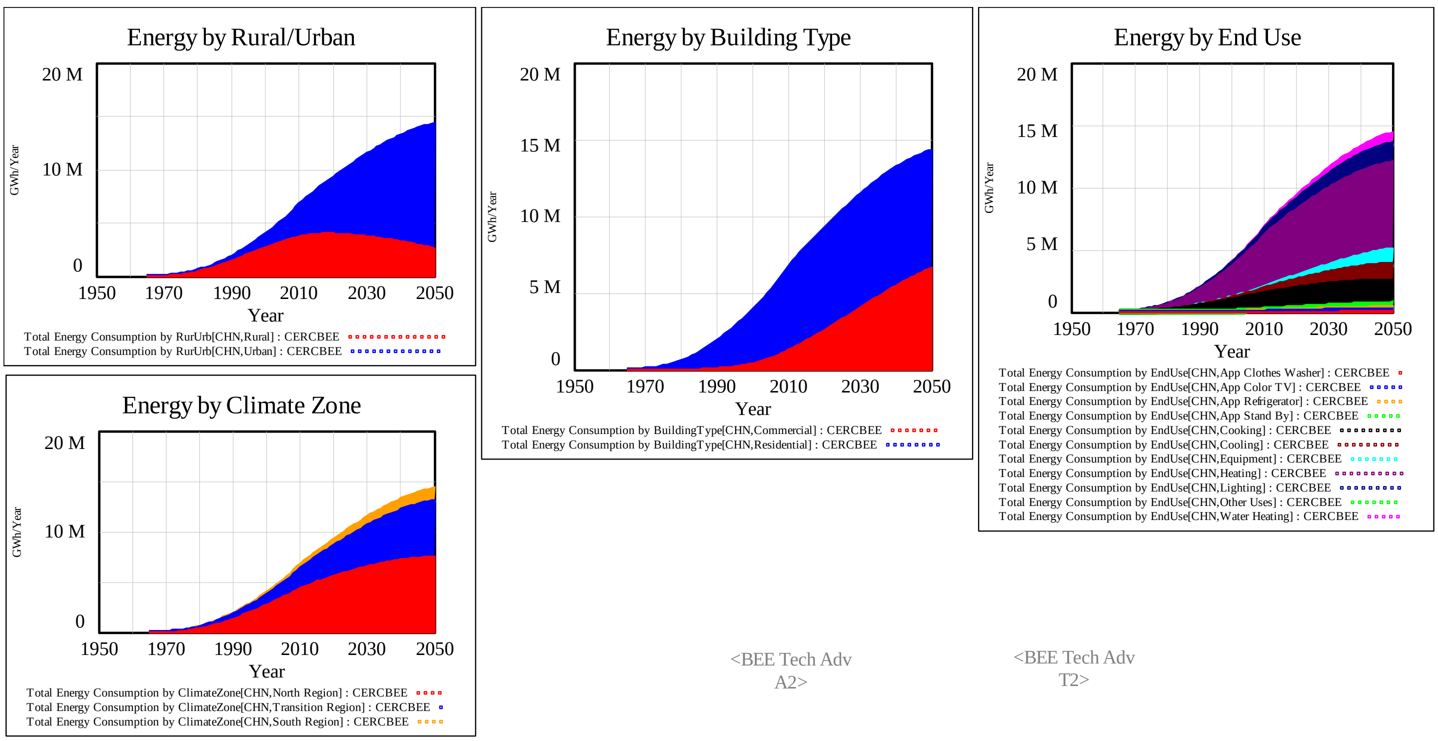


Figure 97: Total Energy Consumption under scenario of CERC-BEE Advanced Technology Adoption.

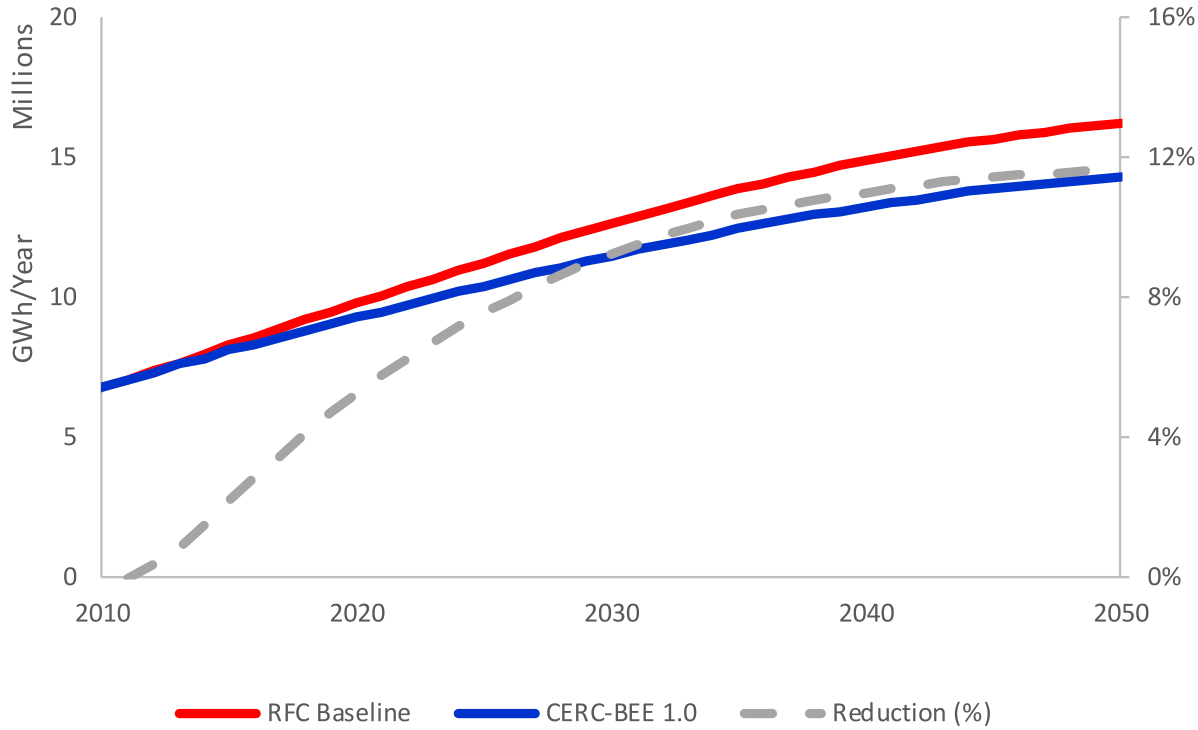


Figure 98: Summary of CERC-BEE 1.0 impact, compared with Reinventing Fire China baseline scenario (repeated).

12 Validation: Extreme Value Stress Tests

We can conceive of many numerical stresses to impose on the framework. While the conditions represented by the numerical stresses are not likely to occur in the real world, in all cases the framework should behave in accordance with physical constraints of the real world. For example, if we perform a numercial stress on the birth rate, under all conditions the resulting population shall never be negative.

The following subsections describe some stresses imposed and the resulting response of the framework. For each extreme value test, we describe the imposed change, the behavior we would expect, and the results of the test. There is no limit to the number of conceivable extreme value stress tests, and the authors would be grateful for suggestions of additional stress tests to perform.

12.1 Birth Rate Doubled

Figure 99 shows the equations for the stress test doubling the birth rate. Figure 100 shows the birth rate doubling in 2020, and Figure 101 shows the population dashboard reflecting the result. Specifically, the population between zero and fourteen years of age grows, increasing the fraction of population in that age range and depressing the population fraction between fifteen and sixty four years of age. All population subgroups (by region, urban or rural, and age group) remain positive, as expected.


```
"Birth Rate Doubled, Population Positive"[region,ClimateZone,RurUrb,Age Group]
:THE CONDITION:
Births[region,ClimateZone,RurUrb] =
RC STEP( Births[region,ClimateZone,RurUrb] , 2, RC START TIME)
:IMPLIES:
Population[region,ClimateZone,RurUrb,Age Group] >=
 RC STEP CHECK(0, Population[region,ClimateZone,RurUrb,Age Group] , 0)
~
~ |
```

Figure 99: Equations for stress test: if birth rate doubles, population for each age group should always remain positive.

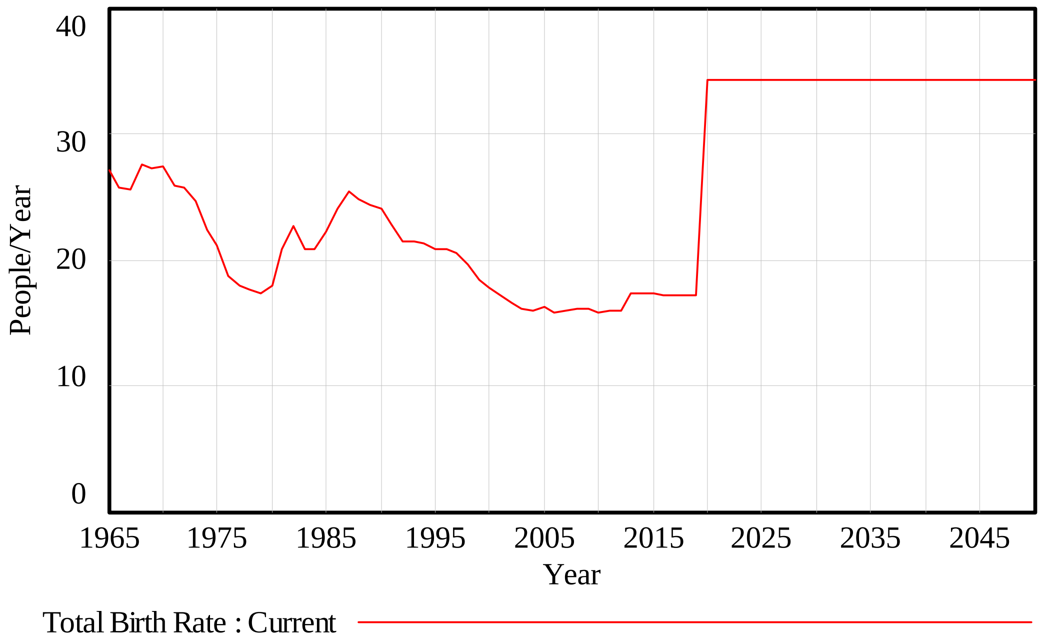


Figure 100: Test input for stress test, showing doubling of total birth rate in 2020.

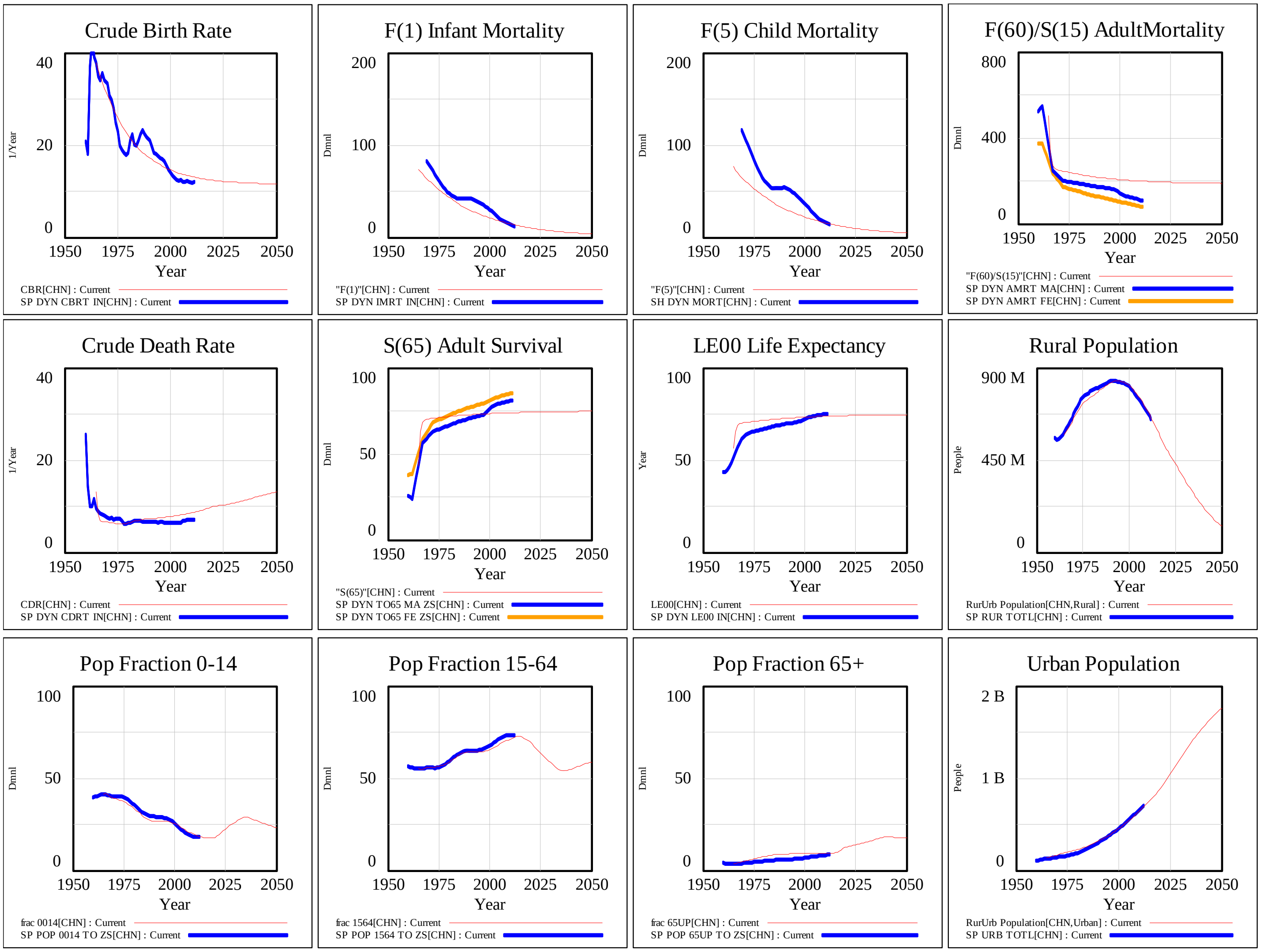


Figure 101: Population dashboard given doubling of total birth rate in 2020.

12.2 Birth Rate Zero

Figure 102 shows the equations for the stress test forcing the birth rate to zero. Figure 103 shows the birth rate vanishing in 2020, and Figure 104 shows the population dashboard reflecting the result. Specifically, the population between zero and fourteen years of age drops, decreasing the fraction of population in that age range and increasing the population fraction between fifteen and sixty four years of age. All population subgroups (by region, urban or rural, and age group) remain positive, as expected. Figure 105 shows the GDP dashboard, reflecting the impact evenutally declining labor force on GDP.


```
"Birth Rate Zero, Population Positive"[region,ClimateZone,RurUrb,Age Group]
:THE CONDITION:
Births[region,ClimateZone,RurUrb] =
RC STEP( Births[region,ClimateZone,RurUrb] , 0, RC START TIME)
:IMPLIES:
Population[region,ClimateZone,RurUrb,Age Group] >=
RC STEP CHECK(0, Population[region,ClimateZone,RurUrb,Age Group] , 0)
~
~ |
|
```

Figure 102: Equations for stress test: if birth rate goes to zero, population for each age group should always remain positive.

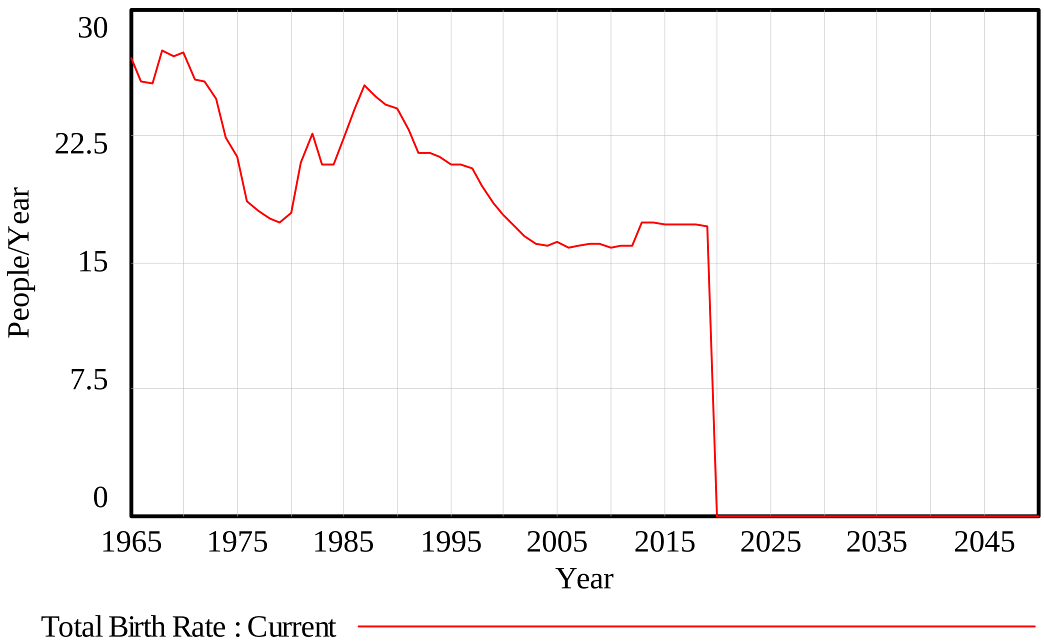


Figure 103: Test input for stress test, showing zero total birth rate in 2020.

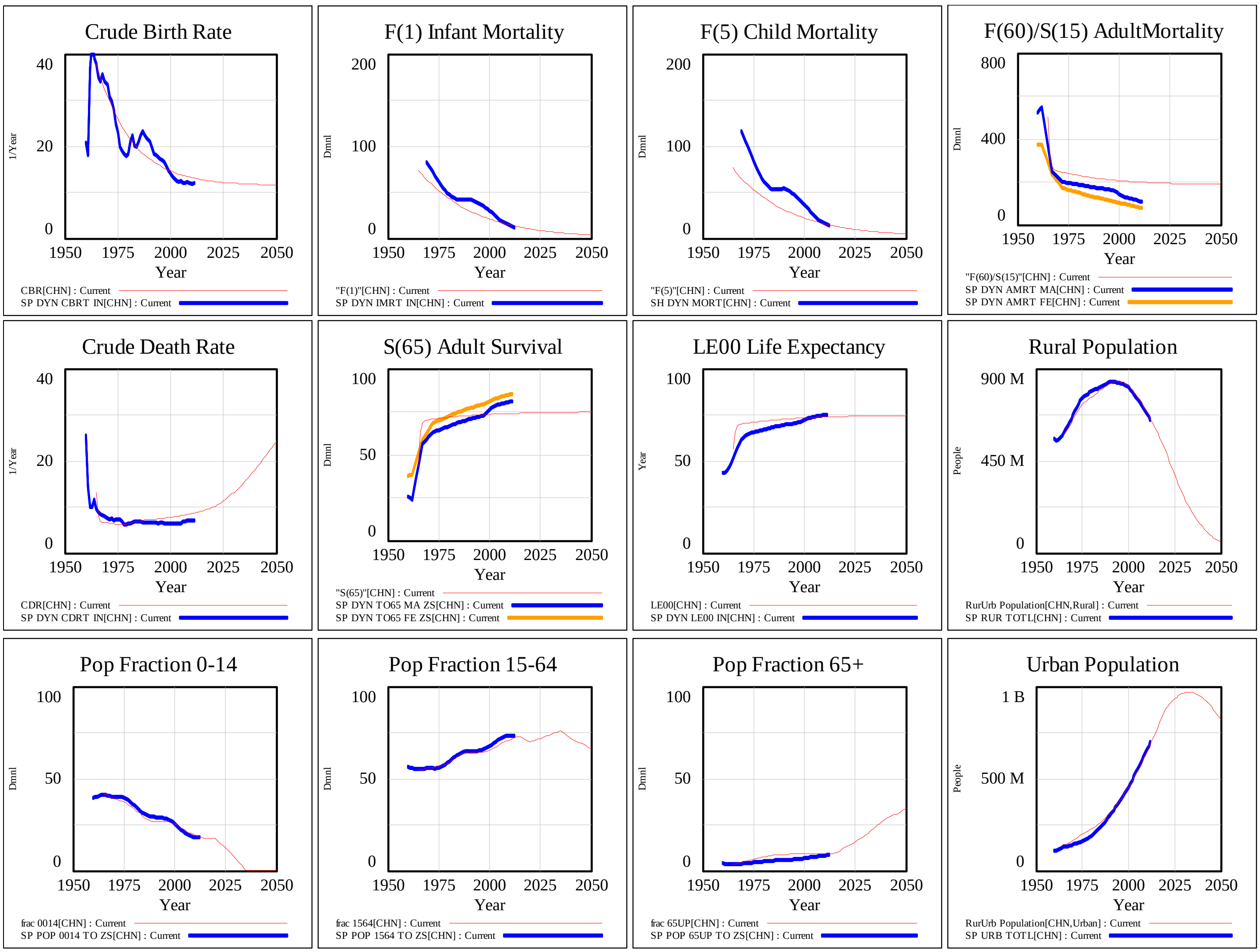


Figure 104: Population dashboard given zero total birth rate in 2020.

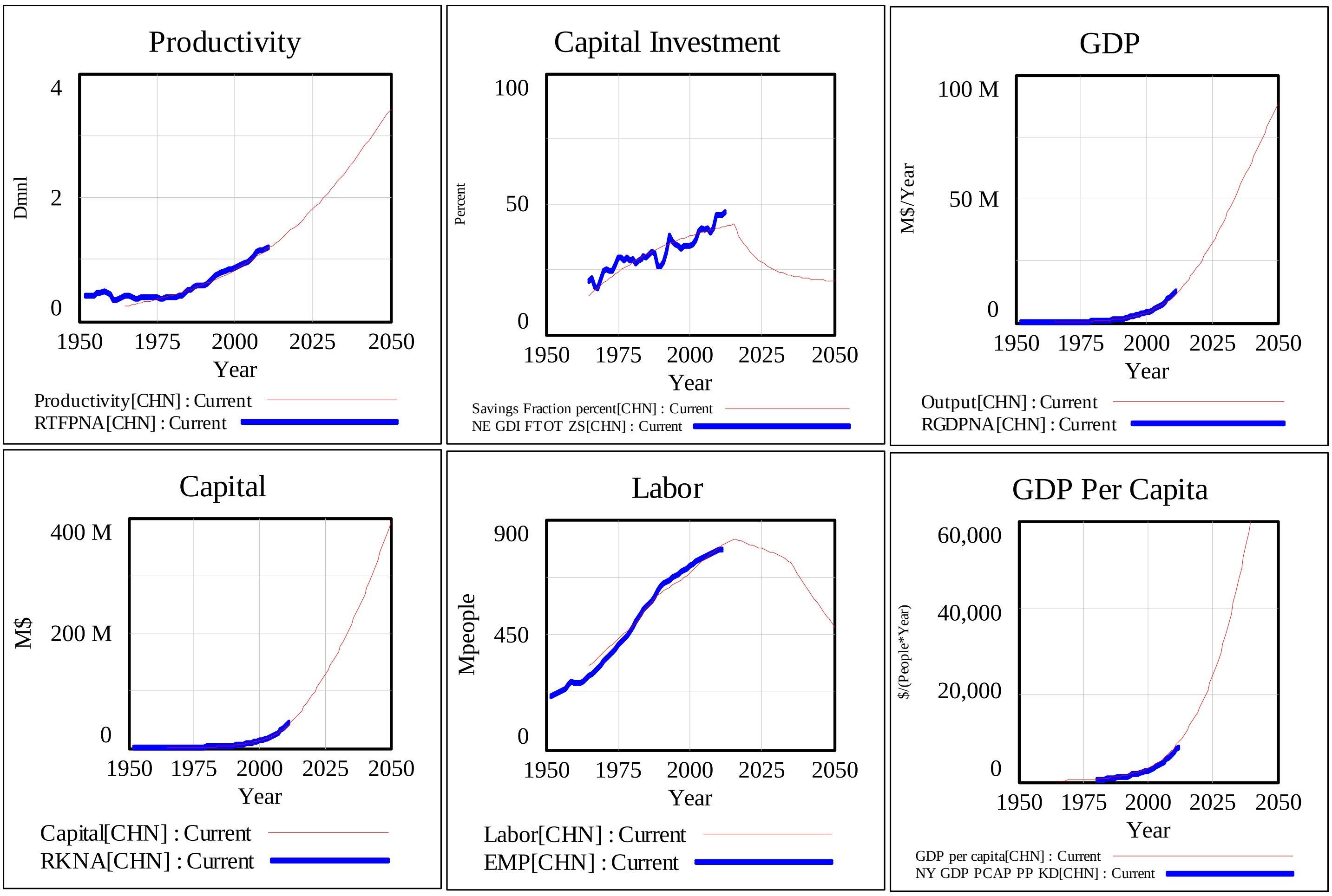
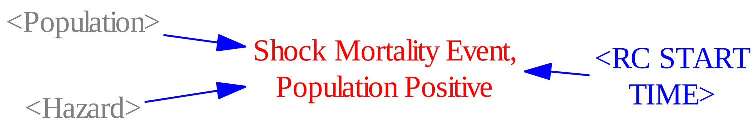


Figure 105: GDP dashboard given zero of total birth rate in 2020.

12.3 Shock Mortality Event

Figure 106 shows the equations for the stress test imposing a high-mortality event. Figure 107 shows the death rate spiking in 2020, and Figure 108 shows the population dashboard reflecting the result. Specifically, the crude death rate spikes in 2020, affecting all population age groups. All population subgroups (by region, urban or rural, and age group) remain positive, as expected. Figure 109 shows the GDP dashboard, reflecting the impact of a one-time reduction in labor force on GDP.




```
"Shock Mortality Event, Population Positive"[region,ClimateZone,RurUrb,Age Group]
:THE CONDITION:
Hazard[region,Age Group] =
RC STEP( Hazard[region,Age Group], 5, RC START TIME,2 )
:IMPLIES:
Population[region,ClimateZone,RurUrb,Age Group] >=
 RC STEP CHECK(0, Population[region,ClimateZone,RurUrb,Age Group], 0)
~
~ |
```

Figure 106: Equations for stress test: if the death rate increases for a period of time, population for each age group should always remain positive.

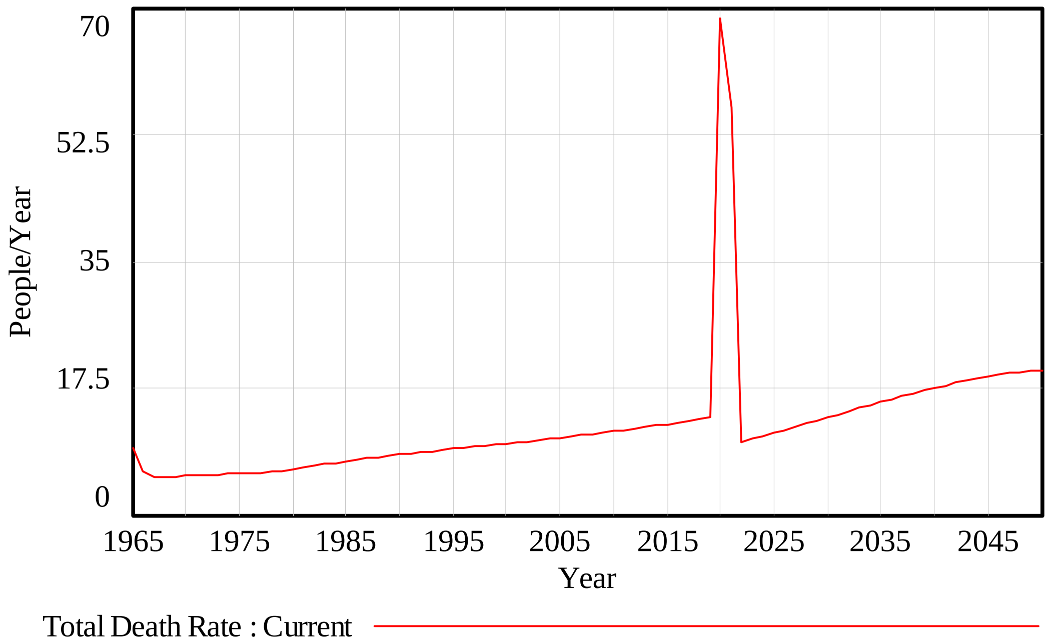


Figure 107: Test input for stress test, showing period of very high mortality rate starting in 2020.

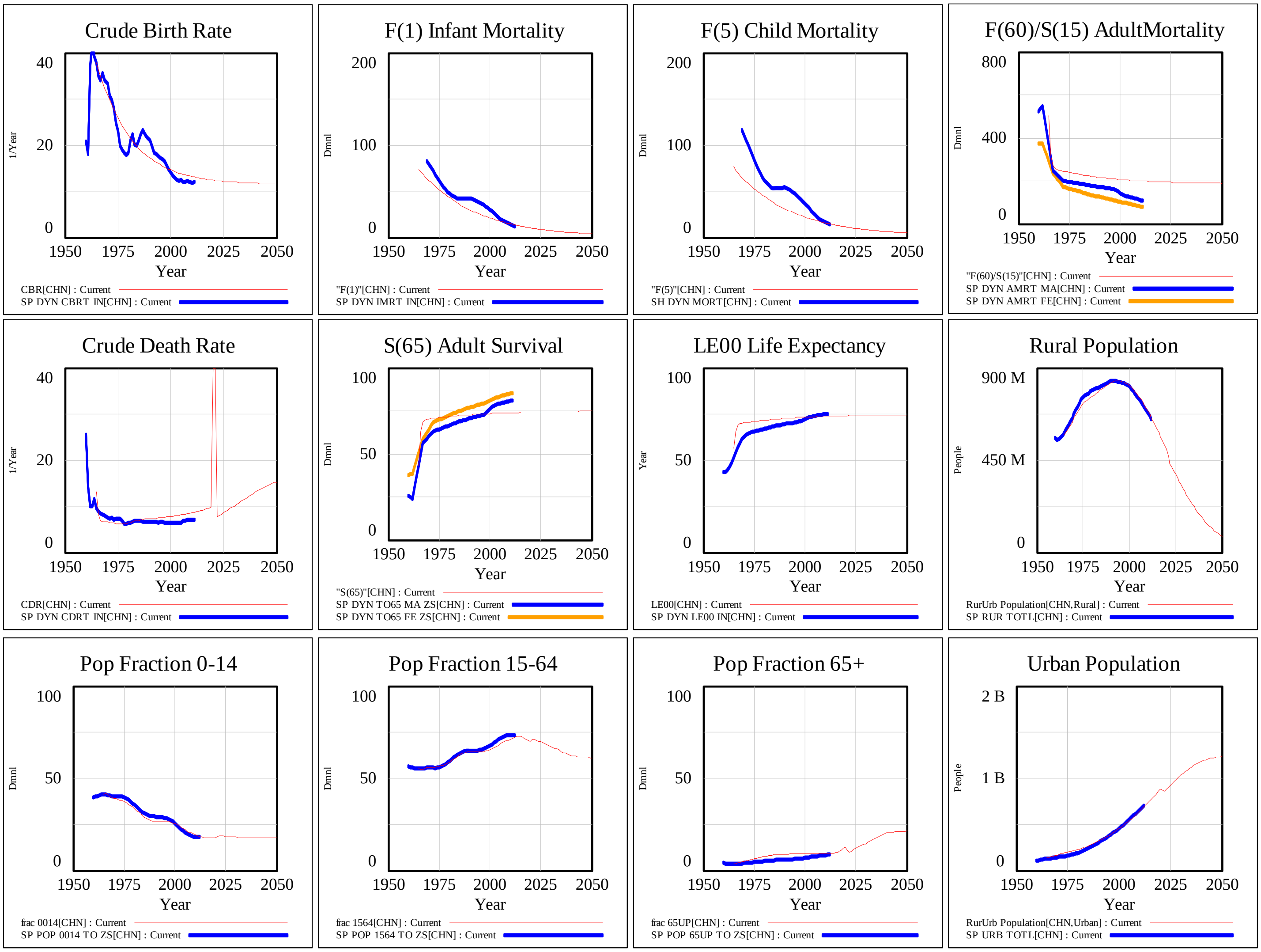


Figure 108: Population dashboard given period of high mortality starting in 2020.

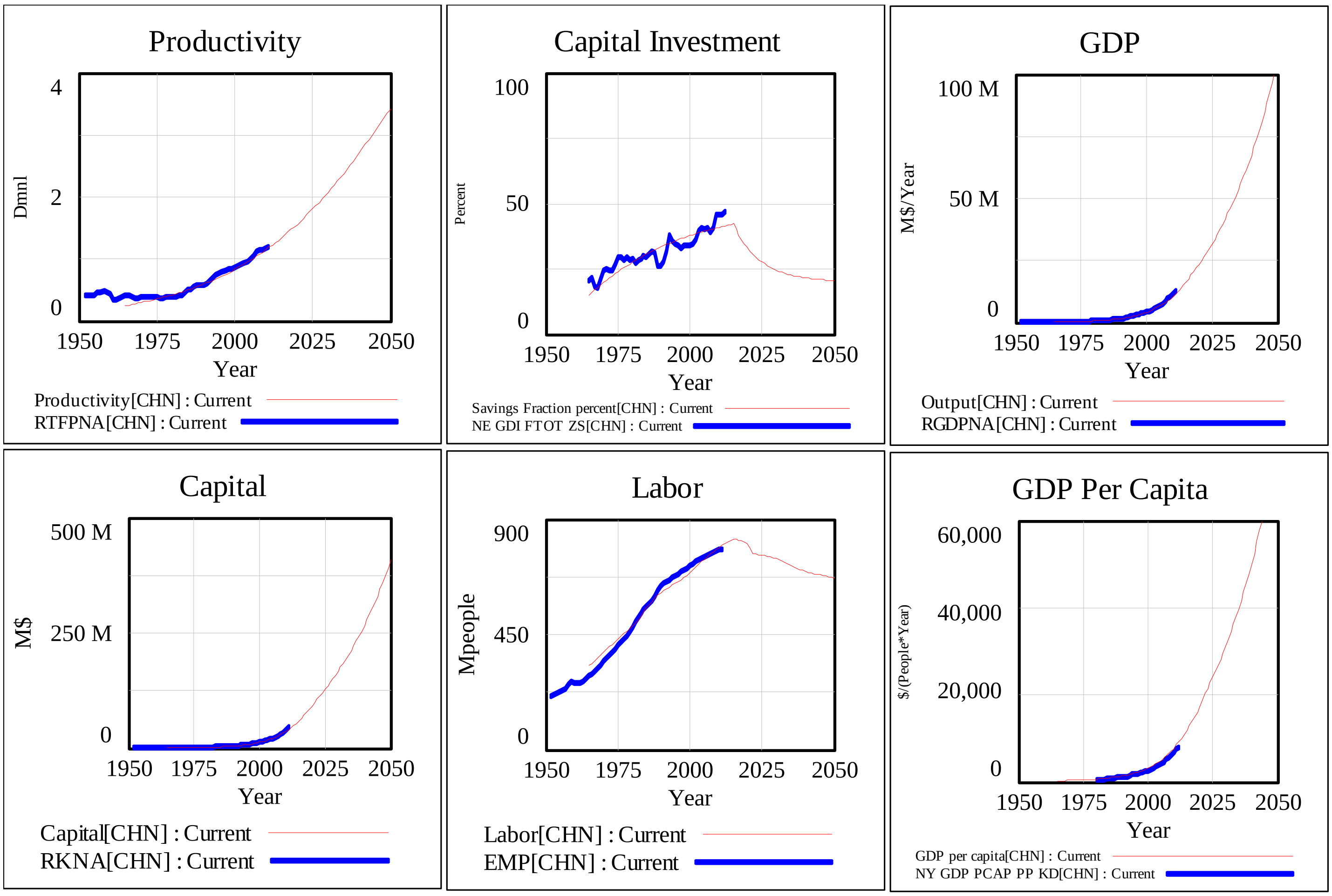


Figure 109: GDP dashboard given period of high mortality starting in 2020.

12.4 Instant Urban Migration

Figure 110 shows the equations for the stress test imposing instant migration of all rural population. Figure 111 shows the rural population fraction abruptly falling to zero in 2020, and Figure 116 shows the population dashboard reflecting the result. Specifically, the rural population abruptly falls to zero, and the urban population increases by the same amount. All population subgroups (by region, urban or rural, and age group) remain positive, as expected. Figure 113 shows the building dashboard, reflecting the increase of urban buildings to accomodate the increased population. Despite zero rural population, the rural building area continues to fall at the previous rate as the stress test leaves the building life-time (and thus demolition rate) unaffected.


```
"Mass Migration, Buildings Positive"[region,ClimateZone,RurUrb,BuildingType,Building Age Group]
:THE CONDITION:
Rural Population Percent[region] =
RC STEP( Rural Population Percent[region], 0,RC START TIME)
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 110: Equations for stress test: instant rural to urban migration.

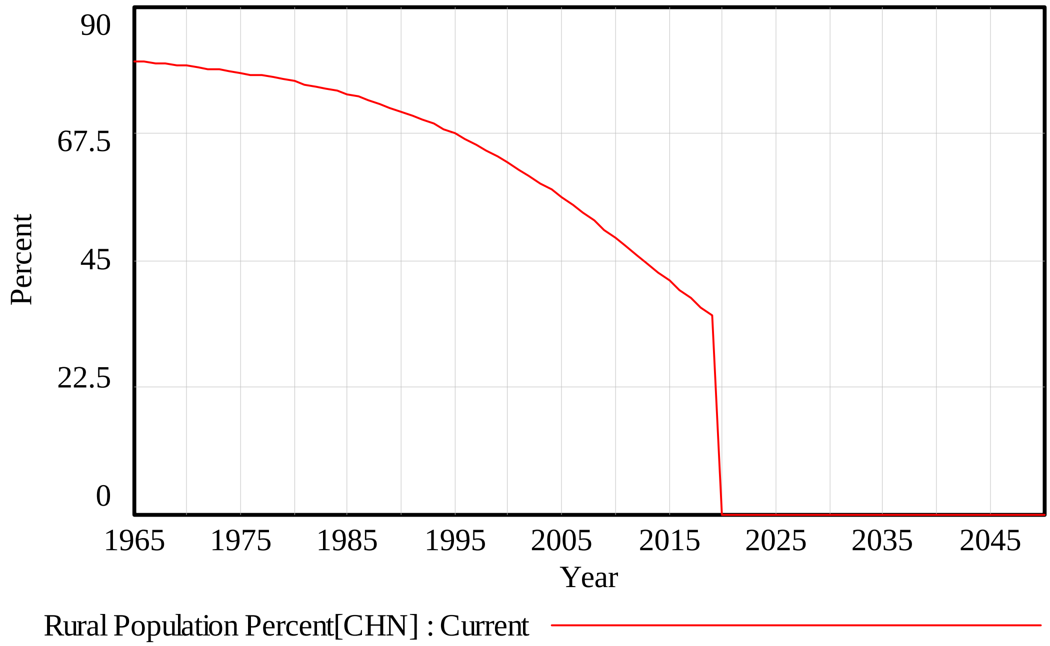


Figure 111: Test input for stress test, showing instant relocation of rural popoulation to urban area in 2020.

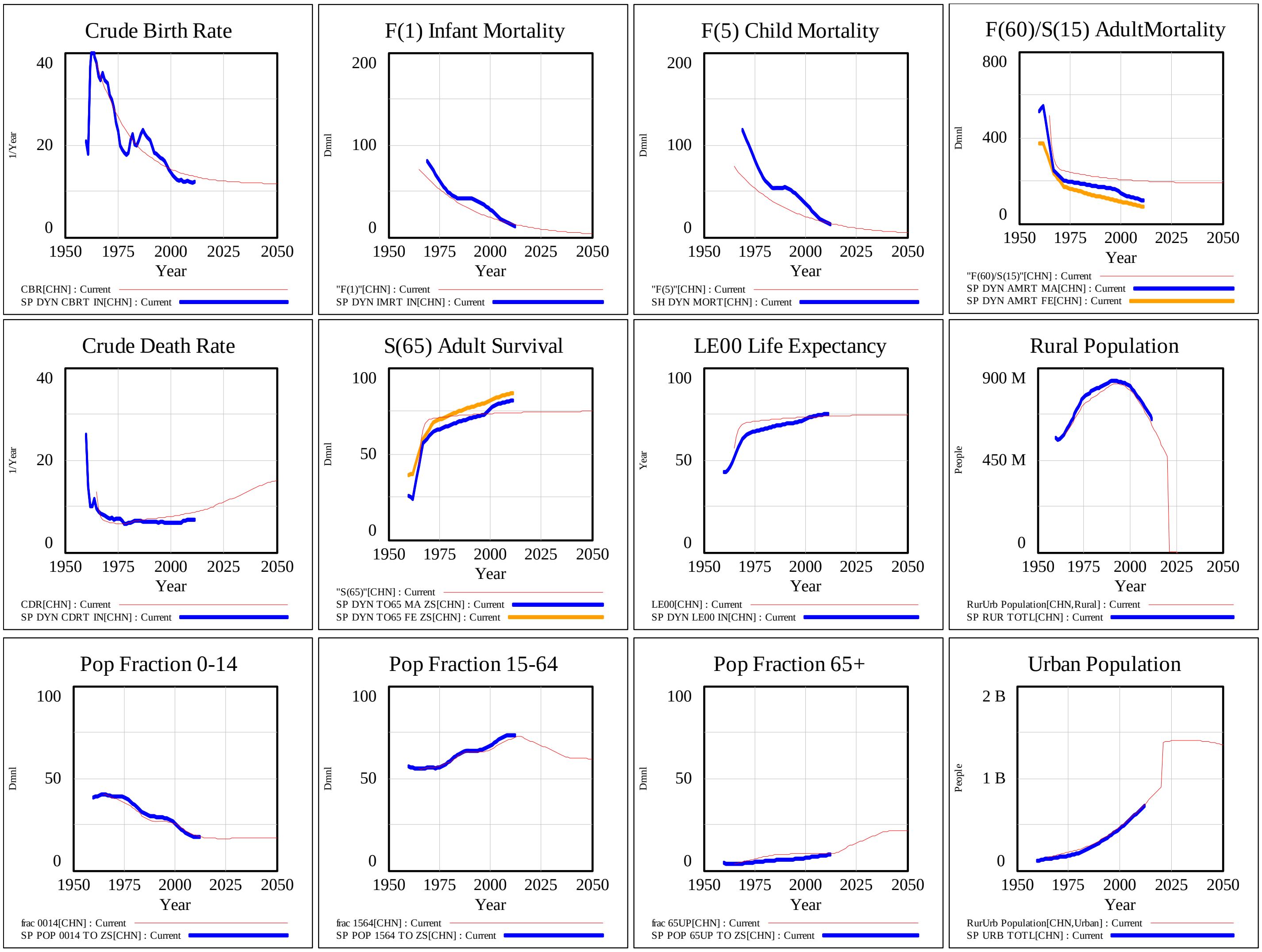


Figure 112: Population dashboard given stress test of instant relocation of rural popoulation to urban area in 2020.

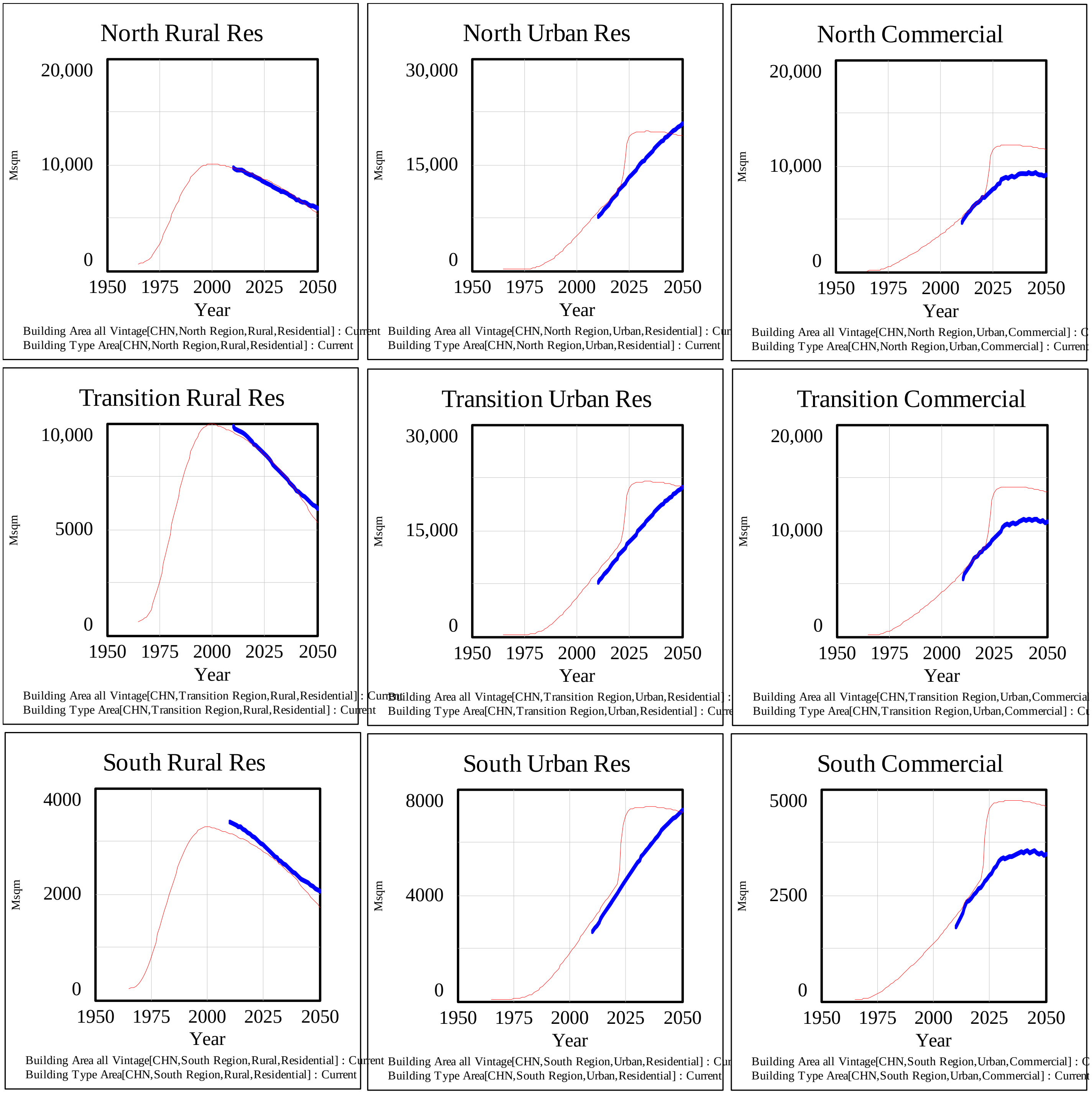


Figure 113: Building dashboard given stress test of instant relocation of rural popoulation to urban area in 2020.

12.5 Savings Rate Doubled

Figure 114 shows the equations for the stress test imposing a doubling of the savings rate. Figure 115 shows the savings rate doubling in 2020, and Figure 116 shows the GDP dashboard reflecting the result. Specifically, the doubling the savings rate causes the capital to increase exponentially, with corresponding increase in GDP and GDP per capita. All building area subgroups (by region, urban or rural, building type and age group) remain positive, as expected.


```
"Savings Doubled, Buildings Positive"[region,ClimateZone,RurUrb,BuildingType,Building Age Group]
:THE CONDITION:
Savings Fraction[region] =
RC STEP( Savings Fraction[region], 2,RC START TIME )
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 114: Equations for stress test: savings rate doubling.

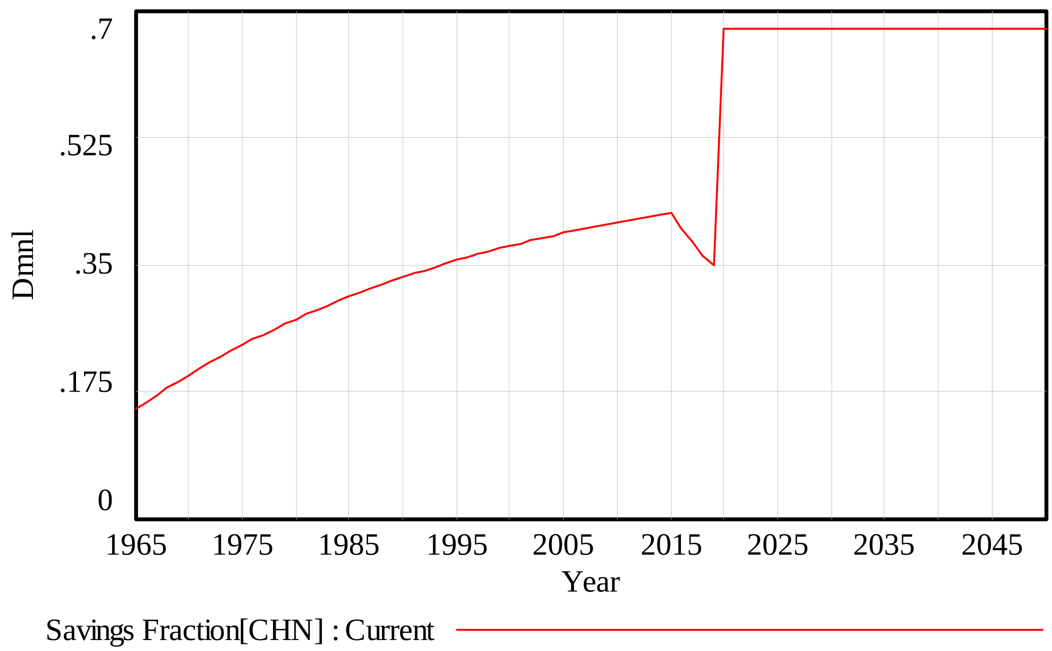


Figure 115: Test input for stress test, showing savings rate doubling at 2020.

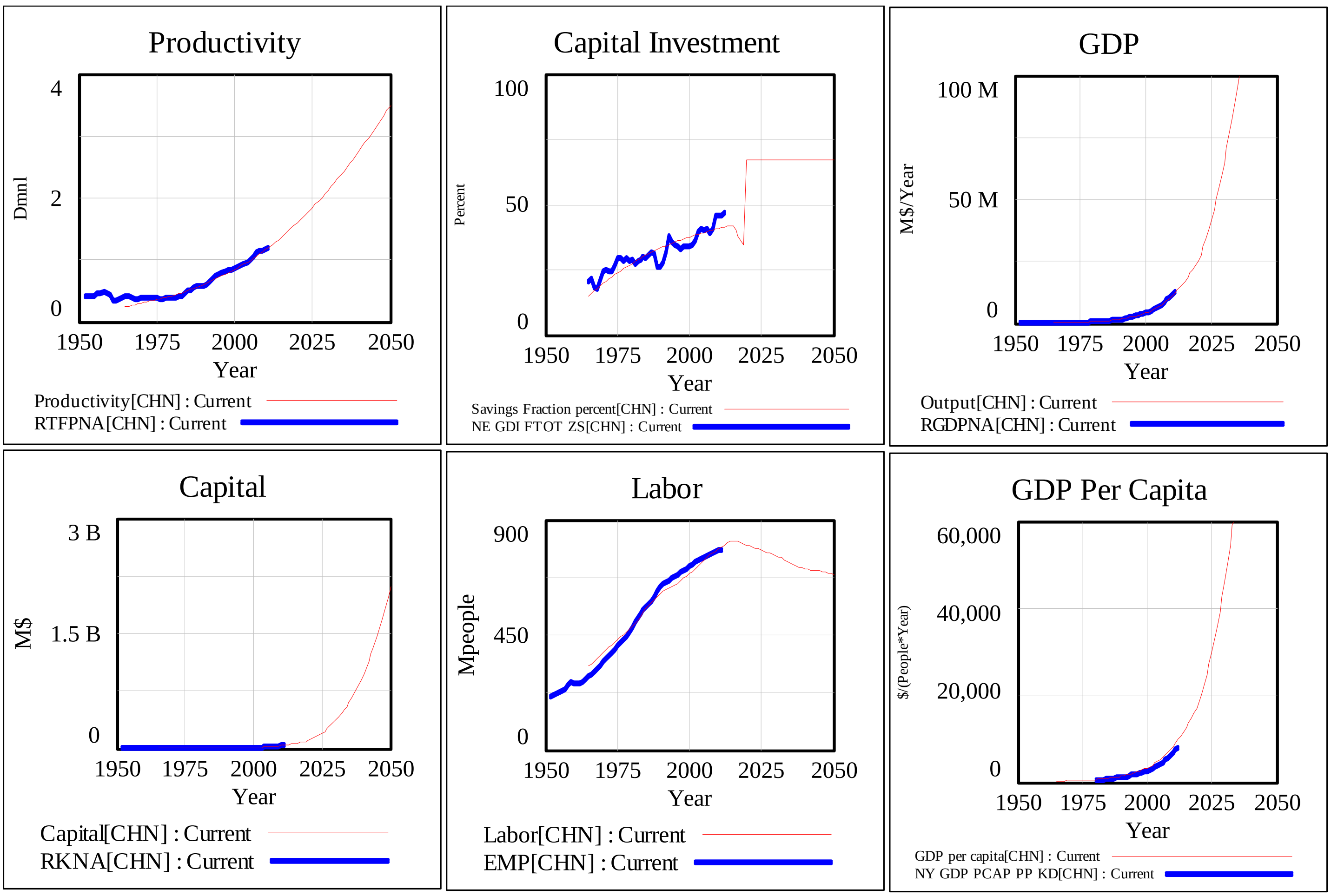


Figure 116: GDP dashboard given stress test of savings rate doubling at 2020.

12.6 Savings Rate Zero

Figure 117 shows the equations for the stress test forcing the savings rate to zero. Figure 118 shows the savings rate vanishing in 2020, and Figure 119 shows the GDP dashboard reflecting the result. Specifically, the dropping the savings rate to zero causes the capital to decay with the capital lifetime. GDP and GDP per capita continue to increase because it is assume that productivity continues to increase. All building area subgroups (by region, urban or rural, building type and age group) remain positive, as expected.


```
"Savings Zero, Buildings Positive"[region,ClimateZone,RurUrb,BuildingType,Building Age Group]
:THE CONDITION:
Savings Fraction[region] =
RC STEP( Savings Fraction[region], 0,RC START TIME )
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 117: Equations for stress test: savings rate zero starting 2020.

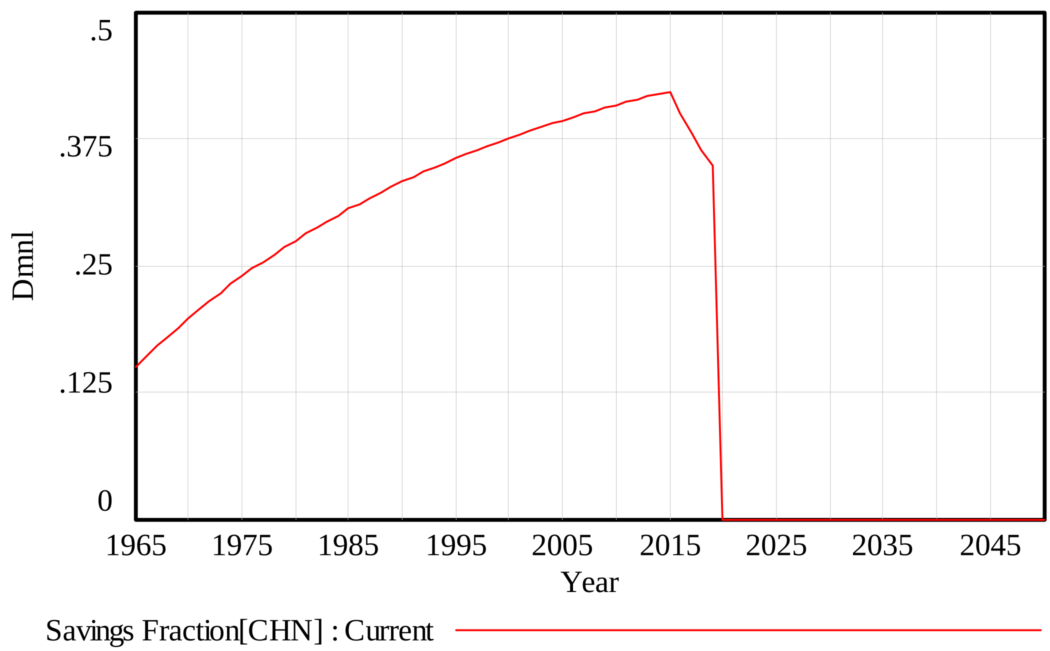


Figure 118: Test input for stress test, showing savings rate zero starting 2020.

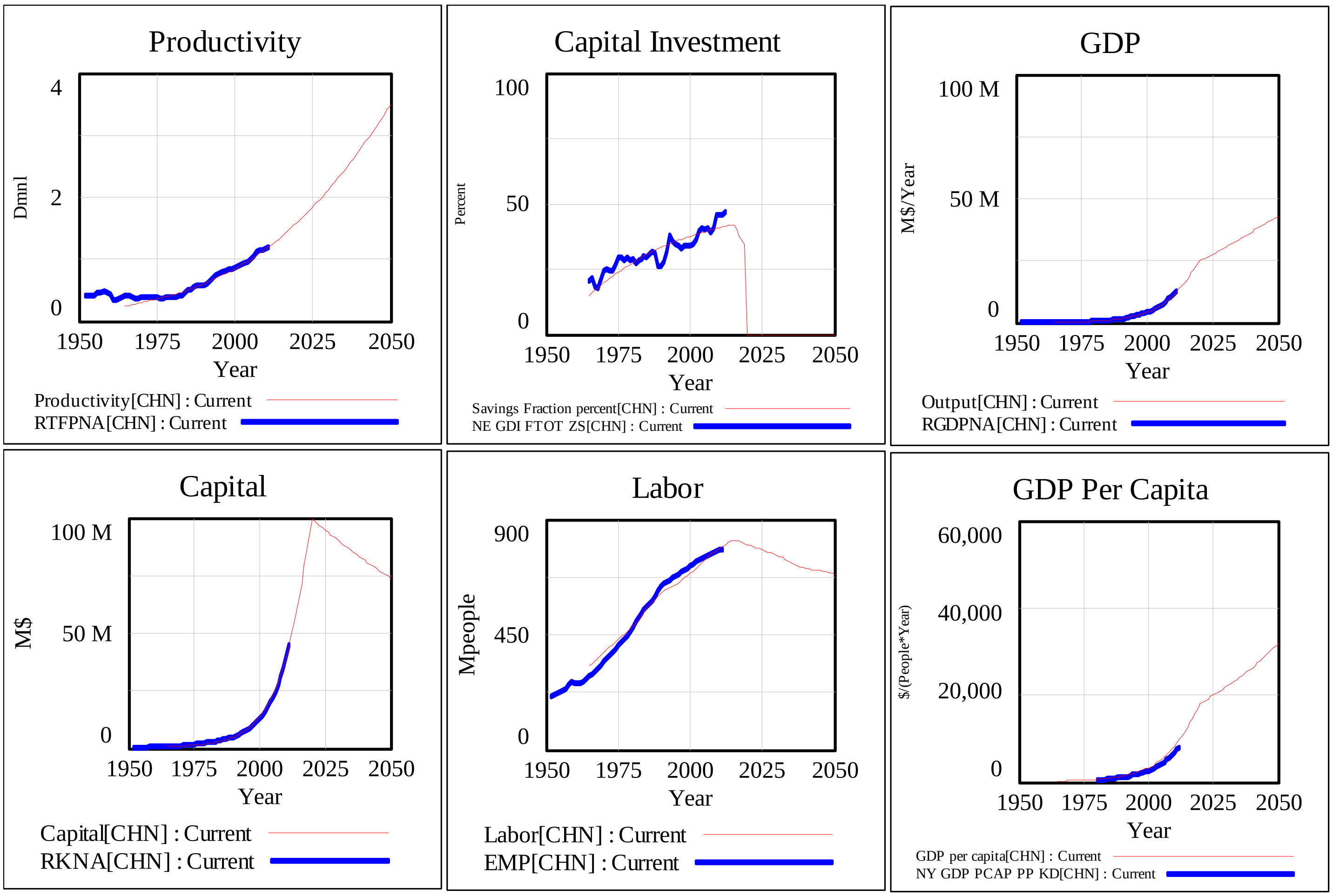


Figure 119: GDP dashboard given stress test of savings rate going to zero starting 2020.

12.7 Productivity Halved

Figure 120 shows the equations for the stress test forcing the total factor productivity in half. Figure 121 shows the total factor productivity dropping in half in 2020, and Figure 122 shows the GDP dashboard reflecting the result. Specifically, the dropping the productivity in half drops the GDP and GDP per capita in half, and subsequently causes the increase in capital to occur at a slower rate. All building area subgroups (by region, urban or rural, building type and age group) remain positive, as expected.


```
"TFP Halved, Buildings Positive"[region,ClimateZone,RurUrb,BuildingType,Building Age Group]
:THE CONDITION:
Productivity[region] =
RC STEP( Productivity[region], 0.5,RC START TIME )
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 120: Equations for stress test: total factor productivity halved.

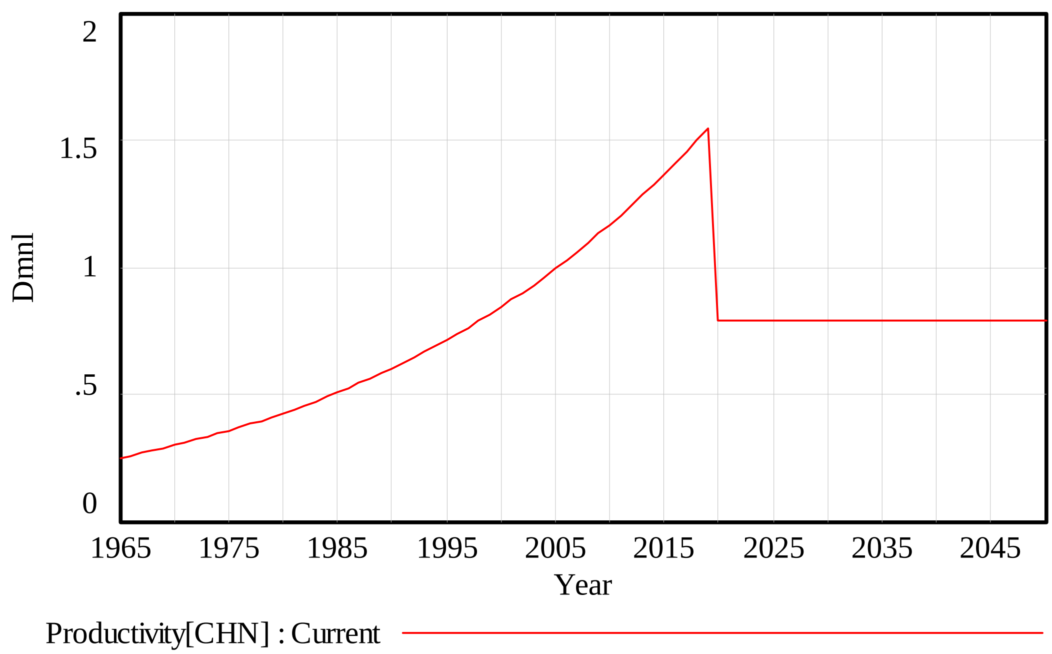


Figure 121: Test input for stress test, showing total factor productivity halved starting 2020.

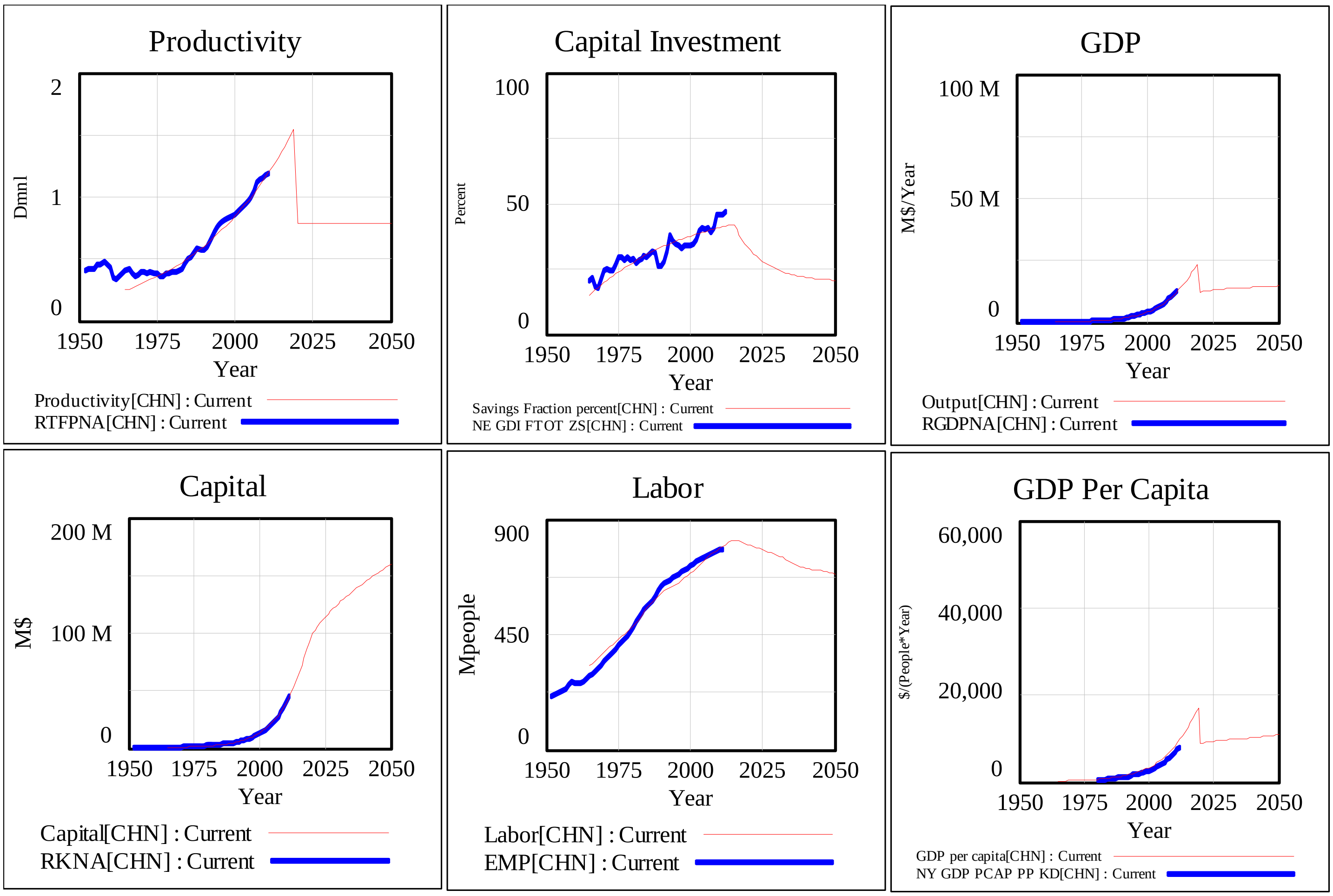


Figure 122: GDP dashboard given stress test of total factor productivity halved starting 2020.

12.8 Area Per Capita Halved

Figure 123 shows the equations for the stress test forcing the indicated area per capita in half. Figure 124 shows the indicated area per capita dropping in half in 2020, and Figure 125 shows the resulting drop in construction starts. Figure 126 shows the buildings dashboard reflecting the result. Specifically, the dropping the indicate area per capita means no new buildings are required, and existing buildings area decreases with the natural rate of demolition according to the building lifespan. All building area subgroups (by region, urban or rural, building type and age group) remain positive, as expected.


```
"Area pcap Halved, Buildings Positive"[region,ClimateZone,RurUrb,BuildingType,Building Age Group]
:THE CONDITION:
Area Income Factor[region,RurUrb,BuildingType] =
RC STEP( Area Income Factor[region,RurUrb,BuildingType], 0.5,RC START TIME )
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 123: Equations for stress test: indicated area per capita halved.

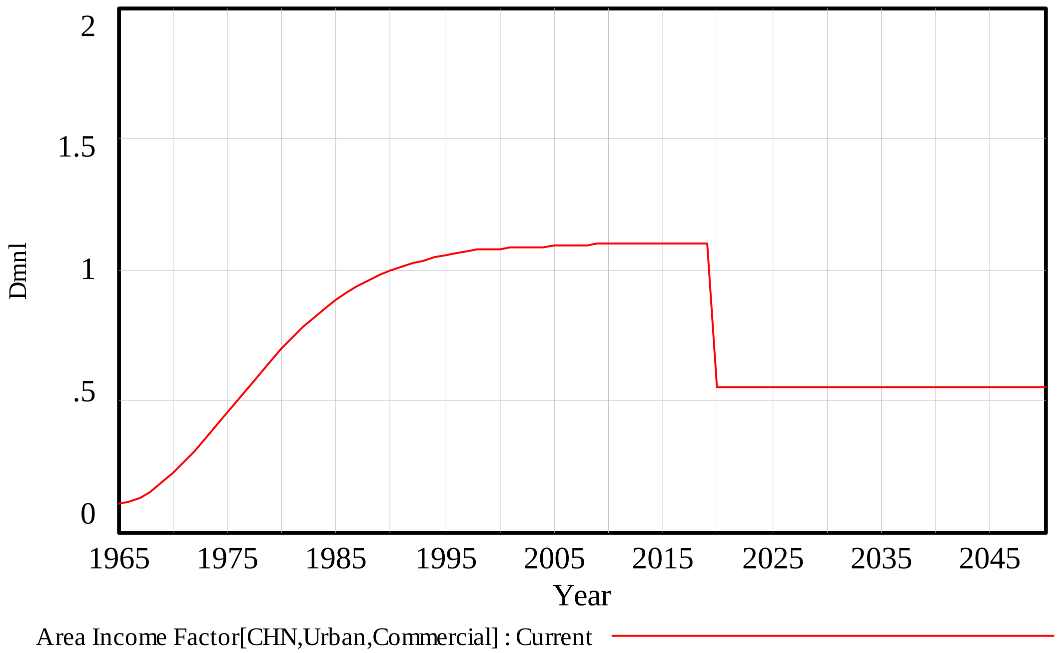


Figure 124: Test input for stress test, showing indicated area per capita halved starting 2020.

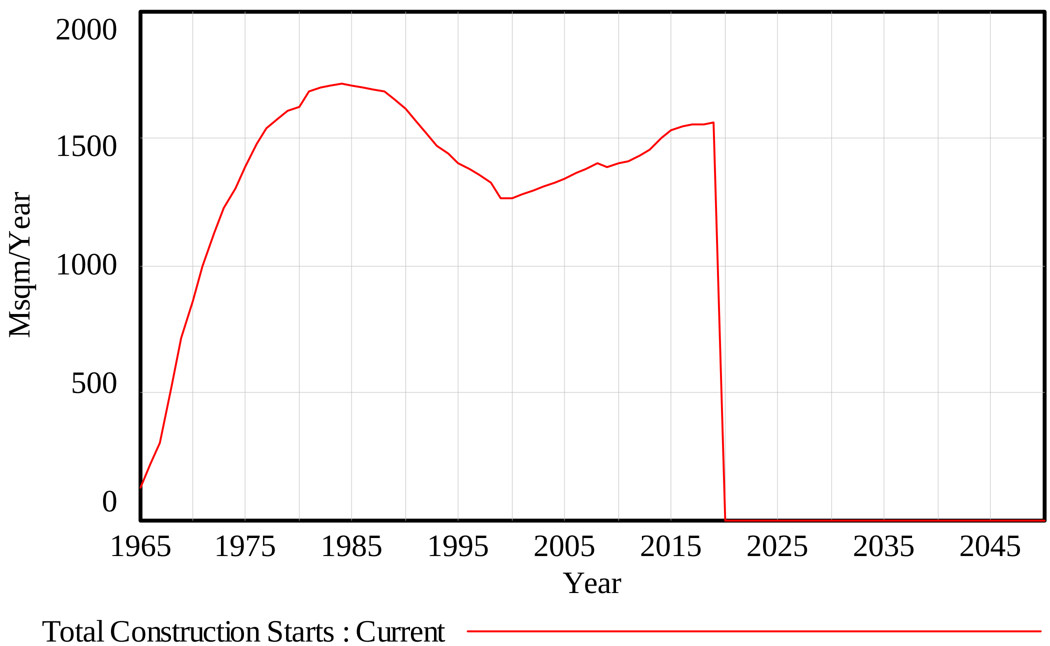


Figure 125: New construction for stress test, showing that halving indicated area per capita starting 2020 drives new construction to zero for several decades.

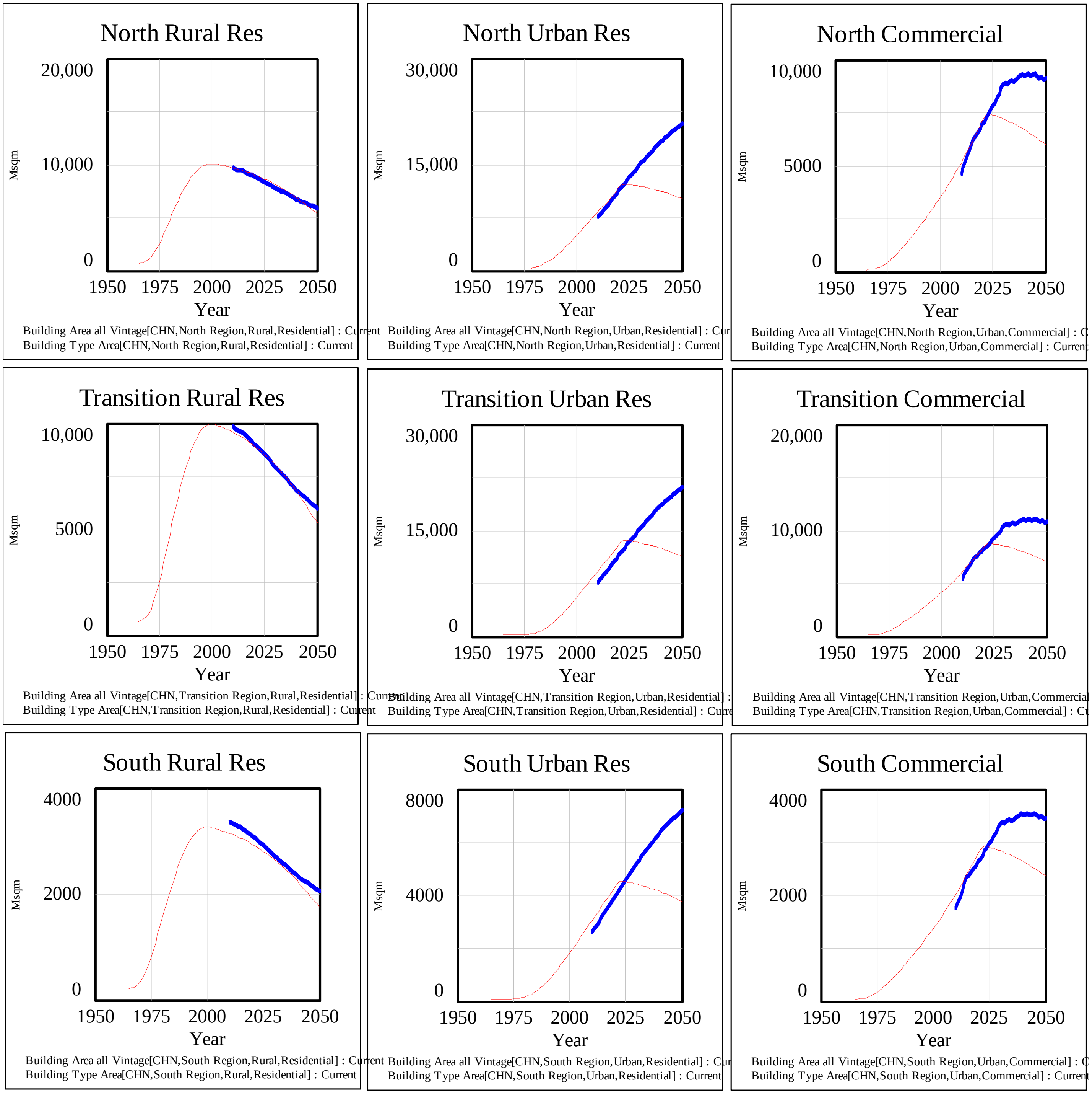
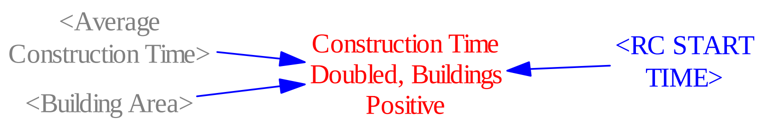


Figure 126: Buildings dashboard given stress test of halving indicated area per capita starting 2020.

12.9 Construction Time Doubled

Figure 127 shows the equations for the stress test forcing the average construction time to double. Figure 128 shows both construction completions lagging construction starts by the average construction time. The average construction time doubles in 2020, causing the rate of completions to drop in half. After the buildings in construction backlog is cleared, construction completions again match construction starts with the longer lag time. All building area subgroups (by region, urban or rural, building type and age group) remain positive, as expected.




```
"Construction Time Doubled, Buildings Positive"[region,ClimateZone,RurUrb,BuildingType\
,Building Age Group]
:THE CONDITION:
Average Construction Time =
RC STEP( Average Construction Time, 2,RC START TIME )
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 127: Equations for stress test: doubling construction time .

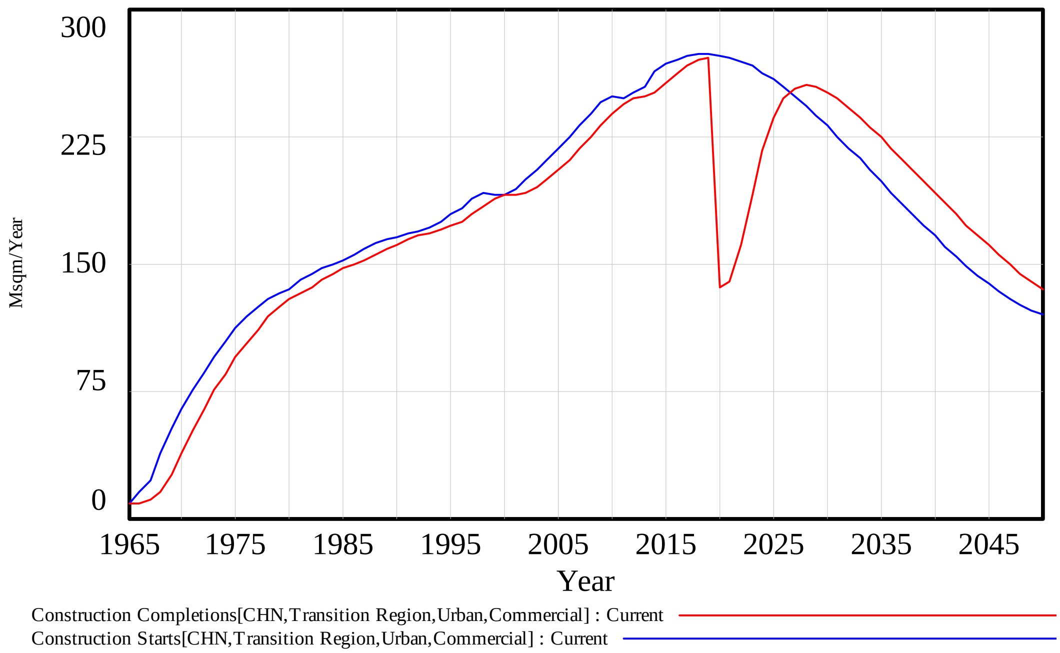


Figure 128: Test input for stress test, showing doubling of construction time starting 2020. Buildings in construction is preserved, and after an adjustment construction completions matches construction starts but with a longer lag.

12.10 Building Lifespan Infinite

Figure 129 shows the equations for the stress test forcing the building life to infinity, i.e. dropping the demolition rate to zero. Figure 130 shows both total demolition rate vanishing in 2020, with a corresponging drop in new construction (that would have been needed to replace the urban buildings demolished). Figure 131 shows the building dashboard. Urban built area is largely unaffected by dropping demolition rate to zero, but rural buildings area now remains constant. All building area subgroups (by region, urban or rural, building type and age group) remain positive, as expected.


```
"Building Life Infinite, Buildings Positive"
[region,ClimateZone,RurUrb,BuildingType,Building Age Group]
:THE CONDITION:
Building Hazard Func[Building Age Group] =
RC STEP( Building Hazard Func[Building Age Group], 0,RC START TIME )
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 129: Equations for stress test: building life span becomes infinite (demolition rate vanishes).

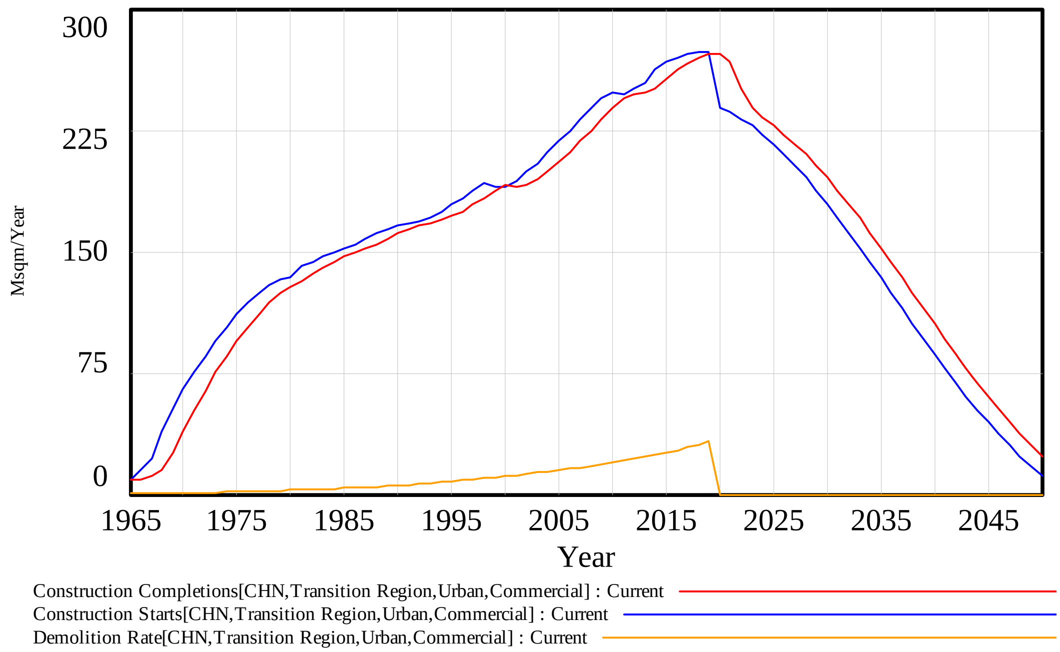


Figure 130: Test input for for stress test, showing that making building life infinite drives the demolition rate to zero starting 2020.

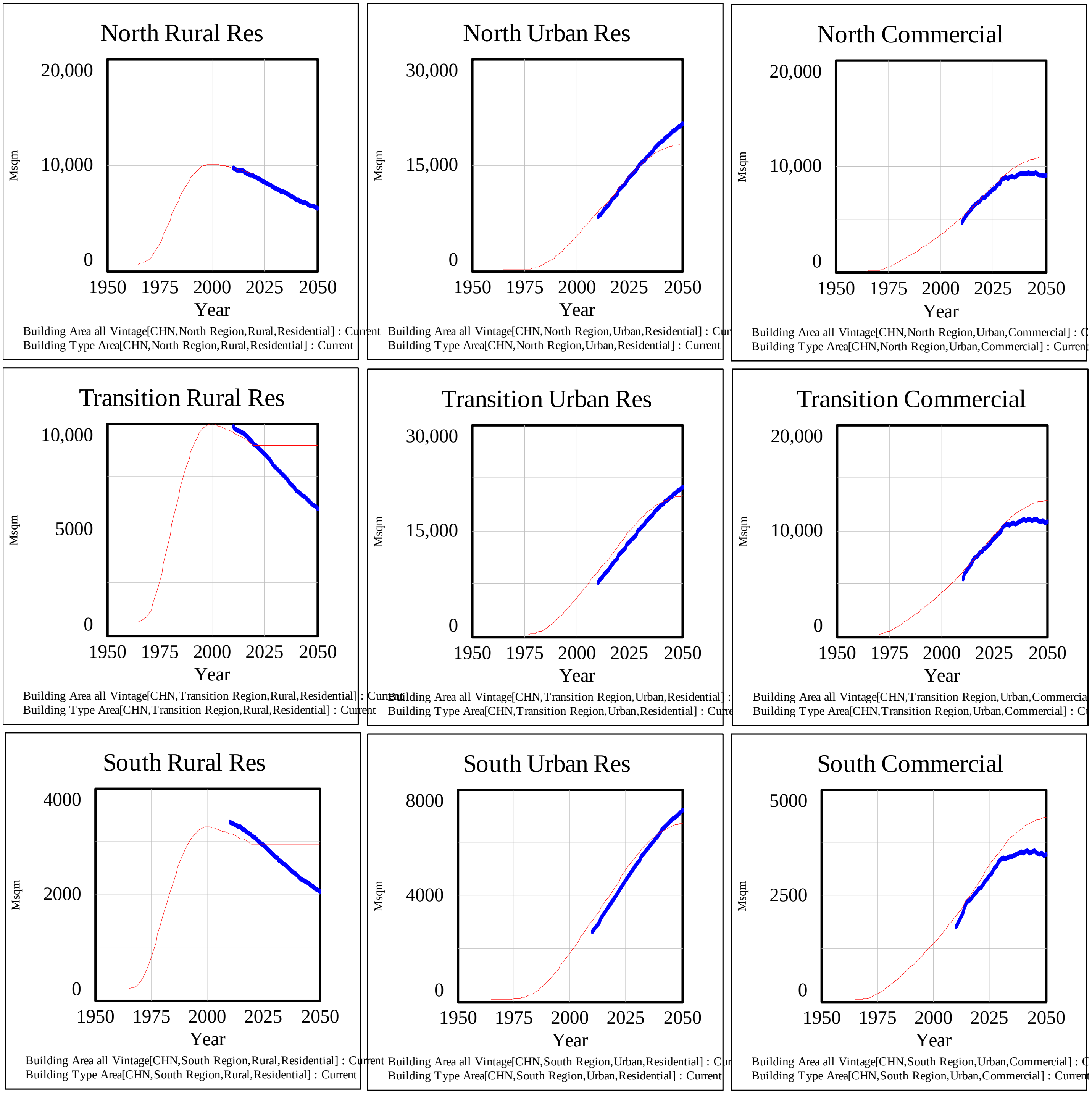


Figure 131: Buildings dashboard given stress test of zero demolition rate starting 2020.

12.11 Building Lifespan Halved

Figure 132 shows the equations for the stress test dropping the building life in half, i.e. doubling the demolition rate. Figure 133 shows both total demolition rate doubling in 2020, with a corresponging rise in new construction (needed to replace the urban buildings demolished). Figure 134 shows the building dashboard. Urban built area is largely unaffected by doubling the demolition rate to zero, but rural buildings area now decay faster than before. All building area subgroups (by region, urban or rural, building type and age group) remain positive, as expected.


```
"Building Life Halved, Buildings Positive"
[region,ClimateZone,RurUrb,BuildingType,Building Age Group]
:THE CONDITION:
Building Hazard Func[Building Age Group] =
RC STEP( Building Hazard Func[Building Age Group], 2,RC START TIME )
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 132: Equations for stress test halving building lifespan.

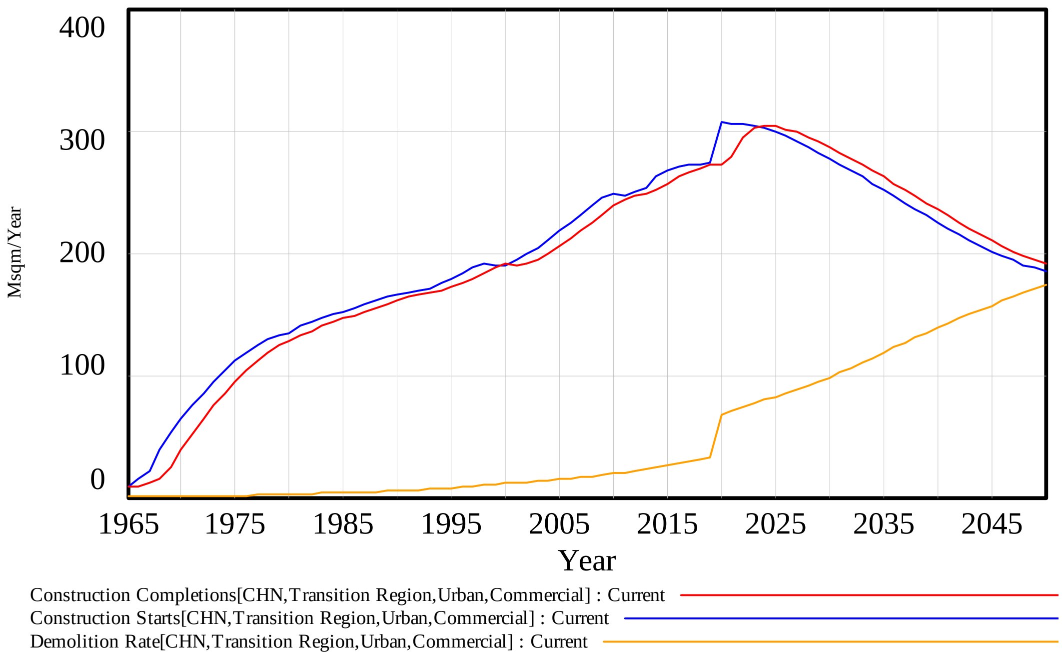


Figure 133: Test input for for stress test, showing halving of building life 2020.

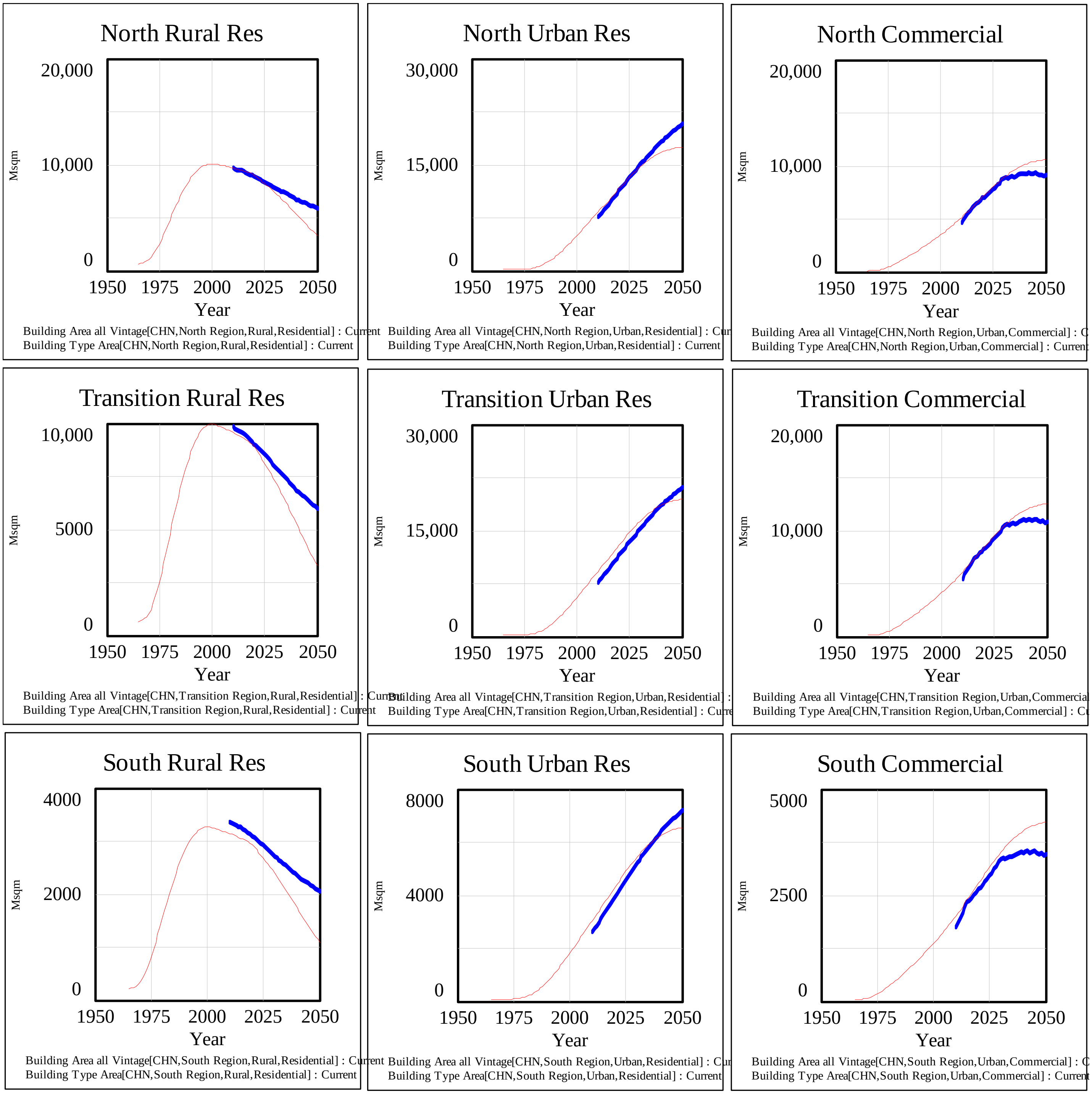


Figure 134: Buildings dashboard given stress test of builing lifespan halved starting 2020.

12.12 Building Retrofit Eliminated

Figure 135 shows the equations for the stress test dropping the rate of retrofit to zero. Figure 136 shows both constructions starts and total retrofit rate, with total retrofit rate vanishing in 2020. Figure 137 shows the building area by generation of retrofit. After 2020 the area of buildings that have previously undergone retrofit no longer increases: only the group with no retrofits at all continues to increase. All building area subgroups (by region, urban or rural, building type and age group) remain positive, as expected.


```
"Retrofit Eliminated, Buildings Positive"
[region,ClimateZone,RurUrb,BuildingType,Building Age Group]
:THE CONDITION:
Time between Retro =
RC STEP( Time between Retro, 100,RC START TIME )
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 135: Equations for stress test: eliminating building retrofit.

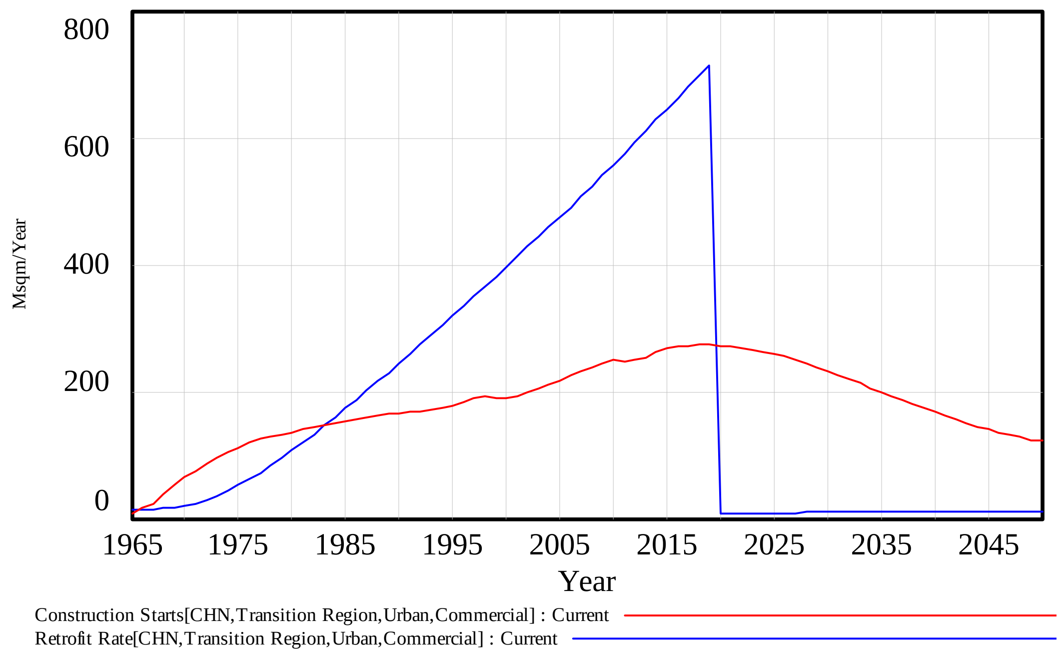


Figure 136: Test input for stress test, retrofit rate of buildings vanishing starting 2020.

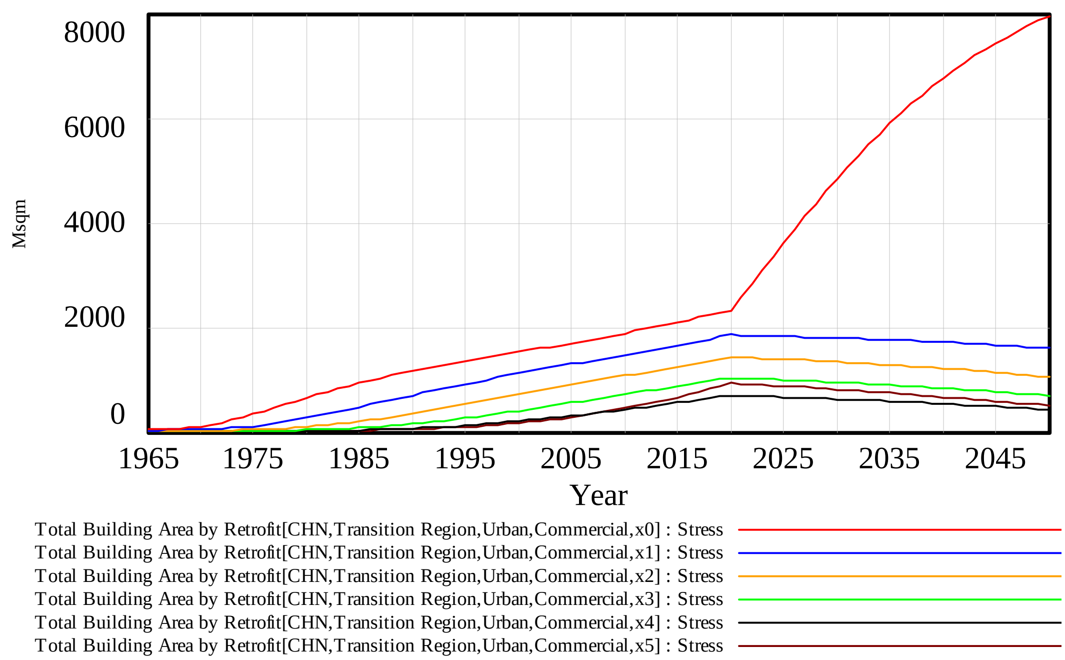


Figure 137: Output for stress test, showing that as retrofit rate vanishes, populations of buildings who have had a retrofit stagnate.

12.13 Building Retrofit Annually

Figure 138 shows the equations for the stress test forcing retrofits to occur annually. Figure 139 shows both constructions starts and total demolition, with total demolition vanishing in 2020. Figure 140 shows the building area by generation of retrofit. After 2020 the rapid rate of retrofit move all building areas into the category with five retrofits. Since the model assumes that buildings retrofit no more than five times, once most buildings are in this category the retrofit rate subsequently drops. All building area subgroups (by region, urban or rural, building type and age group) remain positive, as expected.


```
"Retrofit Annual, Buildings Positive"
[region,ClimateZone,RurUrb,BuildingType,Building Age Group]
:THE CONDITION:
Time between Retro =
RC STEP( Time between Retro, 0.1,RC START TIME )
:IMPLIES:
Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] >=
RC STEP CHECK(0, Building Area[region,ClimateZone,RurUrb,BuildingType,Building Age Group] , 0)
~
~ |
```

Figure 138: Equations for stress test: annual retrofit.

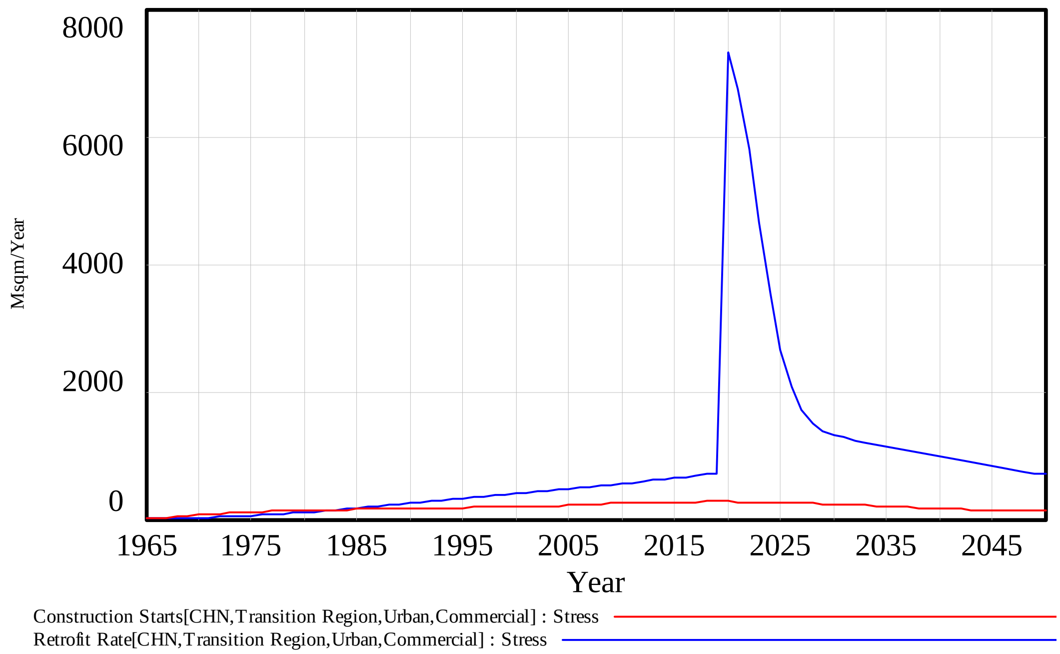


Figure 139: Test input for stress test, annual retrofit rate of buildings starting 2020.

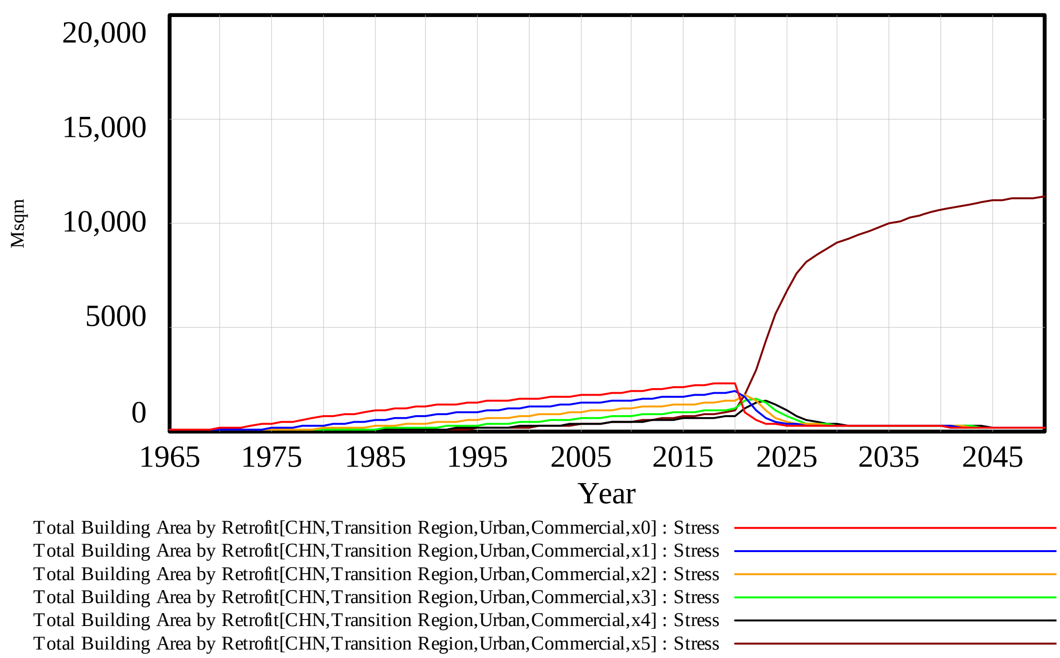


Figure 140: Output for stress test, showing that as retrofit rate increases, all buildings quickly accumulate five retrofits (assumed maximum number of the model).

12.14 CERC Technology Negative Infinite Price

Figure 141 shows the equations for the stress test forcing the CERC-BEE technologies to be introduce with negative infinite price. Figure 142 shows the total building energy consumption for the scenarios where the price is identical to the conventional alternatives as well as the stress test scenario. The negative infinite price means that as soon as the CERC-BEE technology is introduced it commands 100% sales market shaer, thus this stress test also serves to demonstrate the maximum possible energy savings of the CERC-BEE technology portfolio. All building area technology subgroups (by region, urban or rural, climate zone, building type and technology type) remain positive, as expected.


```
"Tech Price Infinite, Installed Area Positive"[region,ClimateZone,RurUrb,BuildingType\
,BEE Tech Adv]:THE CONDITION:
iU[ClimateZone,RurUrb,BuildingType,BEE Tech Adv] = RC STEP(1, RC Tech Price Mult, RC START TIME\
):IMPLIES:Installed Tech Area[region
,ClimateZone,RurUrb,BuildingType,BEE Tech Adv] >=
RC STEP CHECK(0, Installed Tech Area[region,ClimateZone,RurUrb,BuildingType,BEE Tech Adv\
] , 0)
~
~ |

RC Tech Price Mult=
-1e+006
~ Dmnl
~ |
```

Figure 141: Equations for stress test: CERC BEE technologies are introduced with negative infinite price.

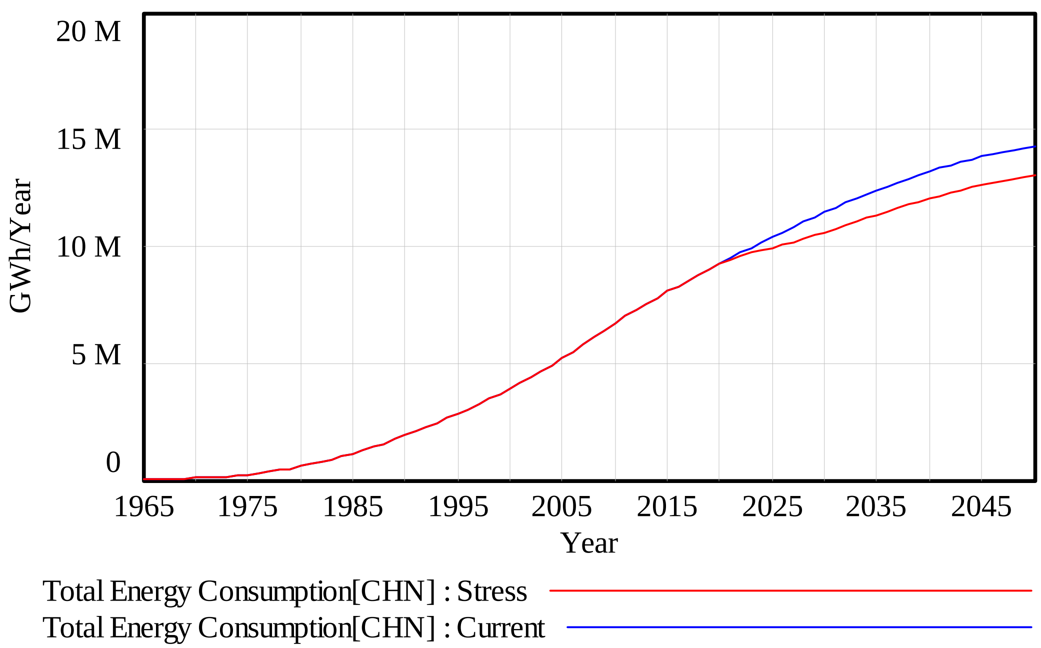


Figure 142: Results of stress test, introducing CERC BEE technologies with negative infinte price. Technologies gain instant 100% sales market share, saving energy compared with baseline scenario.

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