Appendix A: Detailed Explanation of Lessons for the Study

Stage 1 & 2: Enactive & Iconic (linear)

Activity 1: (Motion) The teacher will use a motion detector activity, where a motion detector is connected to a computer projection unit and will have students move in front of the detector to produce several graphs of linear function patterns described in short paragraphs. Questions will be asked (in the handout) to determine if the students can identify key graphical concepts that are related to the motion (i.e., connection between y-intercept and starting position of the person who will walk in front of the motion detector, connection between slope and velocity, the characteristic of the motion that makes the graph linear, etc.) Students will be expected to predict/sketch graphical result before actual data is captured. Students will also be expected to create velocity graphs from position graphs and position graphs from velocity graphs. Finally, students will be asked to create their own motion descriptions that will produce graphs composed of linear segments to move a person from point A to point B (on the graph). The researcher will "grade" this original motion problem the students have produced.

Stage 3: Symbolic (linear)

Activity 2: Students will be shown the STELLA software icons and, using the software, the teacher will build one linear model of motion in class. Students will then use the classroom set of computer netbooks to build various linear models based on problem descriptions. They will be expected to use the model to make predictions and answer questions about the problem scenario. Students will be expected to create an original linear model of their choice, label the icons appropriately (with units), produce a graph that is linear, and explain why the graph produced is linear, the meaning of the slope and y-intercept for their scenario. The researcher will "grade" this original model.

Stage 1: Enactive (exponential)

Activity 3: (Floor stock/flow activity: Linear functions and exponential functions) This activity will consist of using masking tape to form a 5ft. by 5ft. rectangle on the floor of the classroom. There will also be an inflow path, with an arrow pointing toward the rectangle (created by masking tape on the floor). Students will walk inside the arrow path, into the rectangle, in certain constant flow patterns (used to quickly review creating a linear function). The number of people in the stock will be recorded in a table. Student will be asked to come up with ideas to produce linear decay.

Then the teacher will have one student start in the stock and have him (her) take out his (her) cell phone. A strip of masking tape will be placed on the floor connecting the stock back to the flow (in a curved pattern). The teacher will take out his (her) cell phone and, standing with his (her) back to the stock pretend to contact the person in the stock using the phone, asking how many people are in the stock. Then the teacher will ask that same number of people in the class to move into the stock. The teacher then asks (mimicking a call on his (her) cell phone) the person in the stock (with the cell phone) to tell him (her) how many people are in the stock.

When he (she) receives that information he (she) sends in that number of new people, from the class, into the stock. (A table of the number of people in the stock will be recorded, each turn, on the board.) This process continues for two more turns. Students will come up with original ideas for descriptions that will produce a similar pattern of increase in the number of people in the stock.

Stage 2: Iconic (linear and exponential)

Activity 4: Students will graph the values that were recorded in the tables that were produced in stage 1 (for exponential functions). Students will be given some tables of values and asked to determine if they represent linear or exponential growth and if so explain how they know.

Stage 3: Symbolic (exponential)

Activity 5: Students will be reminded of the STELLA software icons and the teacher will build one exponential model of an interest-bearing bank account in class. Students will then use the classroom set of netbooks and build various exponential models, using the STELLA software, based on problem descriptions given in their handout. They will be expected to use the model to make predictions and answer questions about the problem scenario. Students will be expected to create an original exponential STELLA model of their choice, label the icons appropriately (with units), and explain why the graph produced is exponential.

Day after activity 5: The teacher will build a bank account model with constant withdrawals with the aid of students in the class, asking students how to build the model and asking them to predict behavior based on changing values in the model.

Activity 6: A random selection of one pair[1] of students will be removed from each experimental class when the rest of the class is working on the drug model lesson. Each pair will build a drug model. These students will be videotaped, by the researcher, as they try to determine how to build the model and answer the questions. The student pairs will build the model identified as the IV drug model on a computer netbook provided in an empty classroom. The rest of the class will build the same model. Each student pair will be asked to: 1. explain the behavior (graph) of the model, 2. Explain how they would modify the model to include a change in the story scenario, and 3. anticipate the behavior of the modified model. The researcher will "grade" this packet.

[1] Student names will be placed in pairs on small papers and selected at random from a container. The pairing is designed to maintain student comfort in working with another student they usually choose to work with, if possible, on the task for activity 3.

Appendix B: Detailed Explanation of Pre- and Post-Assessments for the Study

1. Question 1: Given 4 straight line graphs with different vertical scale designations (only min and max numbers specified) and the same horizontal scale designations students are asked to determine the graph that has the largest slope.

2. Question 2: A description of filling or emptying a tub or glass is given and students are to draw a graph of the resultant volume over time.

3. Question 3: An ecosystem with a deer herd that is growing or declining is described. Students are to designate the relationship between the birth and death rates of the deer herd over time.

4. Question 4: A description of a person moving in front of a motion detector is given with four graphs to choose from showing potential representations of the described motion.

5. Question 5: A description of a scenario that produces a linear graph, and the linear graph produced. Students are to designate a label for the dependent and independent variables for the graph.

6. Question 6: Asking students how the slope of an exponential graph changes over time.

7. Question 7. Given two straight-line graphs, on the same scale, one of the inflow and the other of outflow for contents of a container, describe how the quantity in the container is changing over time.

8. Comparing the growth or the decay amounts of two items that are growing/decaying exponentially where one change rate is exactly twice the other change rate.

9. Given a STELLA linear function diagram with icons described, but no values indicated, determine if the stock value will increase or decrease and determine the pattern of change that will occur.

10. Determine if linear growth/decay of a money scenario is more efficient/productive in reaching the goal over 100 years compared to an exponential growth/decay.

11. Determine whether slope or y-intercept is altered if a line is shifted up/down or right/left.

12. Given 5 scenarios determine if the growth/decay is linear or exponential.

Appendix C: Videotape of Two Student Pairs, One From Each Experimental Class, Building the Drug Model

1. Overall Purpose 1: To capture the thinking of the students as they decide how to construct the stock/flow diagram to capture the behavior of the following problem:

   "You are continuing your work as a medical resident at a local hospital. You are again working in the emergency room when a patient arrives and needs medical attention." "For this patient you decide you must insert an IV drip into the person's arm in order to administer a therapeutic drug. You set the IV drip so it will allow a constant inflow of 1 g/min of the drug into the person's blood system. The patient, you estimate, will eliminate 0.55 % of the drug in his system each minute. (Be careful, 0.55% is less than 1%)."

   a. A subordinate purpose was to determine if the students could construct a stock/flow diagram that had the students select a stock and identify it as the amount of drug in the body, and construct a constant inflow toward the stock and an exponential outflow from the stock. Note that the generic exponential growth and exponential decay stock/flow diagrams were shown in a boxed display at the top of the paper, for student reference.

      The following sentence in the lesson indicated how the structure should be constructed:

      "Modify the basic exponential STELLA diagram to incorporate the constant inflow of drug and the exponential outflow. Draw the STELLA diagram you now have in the space below. Label each icon to match the situation described. Be sure to place the correct value or formula in each icon."

      (Note: it would have been *much more useful* to the analysis not to have specified that the inflow was constant and the outflow was exponential. It would have been better to see if the students could have performed this construction merely from the description of the scenario given above.)

      An unintended bias on the part of the researcher was that, if the students did not get the model structure constructed correctly they could not proceed with the rest of the lesson that was to enhance and experiment with the model.

      Another unintended bias on the part of the researcher was that, since the researcher was videotaping the two students working on the Drug Model lesson at the same time the classroom teacher was having the rest of the class work on constructing and exercising the Drug Model in class, the flow description would help the teacher troubleshoot questions, making this part of the lesson easier, since students could not proceed without producing at least a correct basic model.

As it turns out the two videotaped pairs did not proceed in the same way at this point. In fact the first pair overlooked the "Modify the basic exponential…" sentence completely, that specified that the inflow should be constant and the outflow should be exponential.

Team from Teacher 1's class build the correct structure immediately:
S1 = student 1, S2 = student 2

S1: [reads the problem description] …be careful .55 is less than one. Modify the STELLA diagram. So we're supposed to draw the STELLA diagram different or the same and just label it?
S2: So stock is like medicine in the system and then inflow
S1: So it's inflow or outflow cause it's…
S2: Well there's both.
S1: So there's one going in and one going out?
S2: Because there's 1 gram a minute going in and then you lose .55% of it.
S1: And there's a connection on the outflow right because its .55%?
S2: Yeah.
S1: Of the drug it's one gram per minute so
S1: Because it's .5% so would be .0055
S2: Yeah.
S1: And then the initial is, there is no initial, so it's zero and then for the inflow it's one. What's next? …

Team from Teacher 2's class build the incorrect inflow structure initially, then correct their mistake later:
P1 = pupil 1, P2 = pupil 2

P1: [reads the problem description] …be careful 0.55 percent is less than one percent. Modify the basic exponential STELLA diagram to incorporate the constant inflow of drug and the exponential outflow. Draw the STELLA diagram you now have in the space below. Label each icon to match the situation described. Be sure to place the correct value or formula in each icon.
P2: So we make this one first.
P1: All right. So we're making a model.
[Instead of drawing the structure first, as indicated in the lesson, the students go directly to the computer to build the model.]
P2: We pull down the stock.
P1: Name it.
P2: I'm just going to name it later. Okay, I'll name it now. So what would be the name of the stock? Drug in system, wouldn't it be?
P1: I guess, yes.
P2: What percentage of drug is in the system? That's what it would be right. [P2 labels the stock Drug in System] How much should we start of with?

P1: That's a good question. Hold on, let's just get all this stuff down first. [P1 draws an inflow]
P2: Wouldn't it be an outflow for how much is leaving or would it be both?
P1: Well, this is the IV drip thing [P1 is drawing an exponential structure for the inflow]
P2: But it's also saying
P1: There's stuff going out of it.
P2: Yeah. There's how much is getting out of the system.
P1: [P1 is now drawing an exponential outflow]
P2: There's one percent going in.
P1: There's less than 1 percent going in.
P2: No, there's one percent going in and less than .55% going out. Yeah.
P1: I got you. I got you.
P2: So this is the rate of drug going into the system [P2 is pointing go the converter on the inflow side.]
P1: So that's one. You said there was a constant inflow of one g per minute of drug going into the blood stream.
P2:[P2 is renaming the converter on the inflow side as Rate of Drug Entering] And this is the drug entering the system. [P2 is now labeling the inflow Amount of drug entering system] This is the amount, the rate of drug leaving the system. [P2 is labeling the converter on the outflow side as Rate of Drug Leaving System and then labels the outflow Amount of Drug Leaving System]. P2 now goes back to the converter on the inflow side of the diagram.] So the rate of drug entering the system is 1% so we have to put that as a decimal right?
P1: It's not 1% it's 1 gram a minute.
P2: Oh, just kidding.
P1: So that's not a exponential that would be
P2: That would be linear.
P1: [P1 starts erasing the converter on the inflow and tries to erase the flow from the stock to the inflow but is unsuccessful.]
Researcher: put the arrow point in the little circle tail, in the little circle tail, and hit backspace. [Researcher explains how to erase a connection on the diagram]
P1: Adios. So this is one [P1 is defining the inflow value] gram a minutes.
P2: Yeah.
P1: [P1 is now double-clicking on the outflow converter] this would be the
P2: That would be the
P1: So it's less than 1% so would be .0055?
P2: No.
P1: Because .55
P2: Yeah, yeah, yeah. You move it over two it's .0055. Yeah. So we start out with zero in the drug in the system.
P1: [P1 double-clicks on the outflow] and this would be drug in system times rate of drug leaving system.
P2: Okay, so we have to draw that on here [referring to drawing the stock/flow structure they just built on the computer, in the packet].

So the team from teacher 2 started out with an incorrect inflow structure but realized, since the inflow was a constant they need to remove the exponential inflow structure and replace it with a constant inflow structure. They completed this alteration correctly. They also defined the model components correctly.

2. Overall Purpose 2: To determine if students can explain what is happening with the dynamics of the problem that causes the shape of the stock graph to be produced when the model is simulated? That is, can the students interpret the model output to the real world problem?

   a. It is typical in System Dynamics modeling lessons to request that students anticipate model behavior before simulation runs are executed. Most students, initially, have a great deal of trouble doing this, as it is not something that is typically asked of them in their math classes. This situation is no different.

      - The first pair of students drew an exponential growth curve that was incorrect.
        S1: I think it's going to start really slow because it's removing little of the body. Then as it gets bigger it's going to be, like, would it be growing or decreasing?
        S2: It be growing, I think.
        S1: Yeah, because it's adding one and .05%, so the amount leaving would be bigger but it would be more in the body
        S2: So wouldn't it stay like
        S1: It be starting flat and then get steeper over time, right?
        S2: Yeah, I think so.

      - The second pair creates a linear graph where they move by blocks, in time, and increase the next point vertically by 1 block but then subtract .5 of a block from that, creating a line that has essentially a slope of .5 of a block. This is incorrect on many levels.
        P2: Oh yeah, the prediction. So there is going to be more drugs going into the system than leaving. It goes in one and only leaves at half a percent, or whatever.
        P1: So for every one it goes up it goes back down.5, yes?
        P2: What?
        P1: It goes up one g.
        P2: It goes up one, so.
        P1: And then half of that so it goes up one and leaves half [P1 goes to the first block on the grid on the lesson packet (that is actually at 360 min) and puts a dot half way up the block from the horizontal axis. Then he goes to the next block (actually at 720 min), and increases by one block and then reduces half a block and places a dot there] and then one and then half and then one and then half [P1 places another dot at the 3rd block -at 1080 min- at the 1.5 block mark up from the horizontal axis. P1 shows P2 what he is doing. He finished by placing a dot half-way up the vertical axis - i.e. at the top to the second box at the far right of

the grid -the 1440 min time. Then P1 connects his dots with a straight line.]

b. Students are now asked how the simulation produced the graph shown on the computer (whose appearance shows an exponential convergence upward from 0 to about 173 g, reaching steady state about 1/3 the way through the simulation).

- The first pair of students reaches a reasonable conclusion:

S2: So the inflow is
S1: Equal to the percentage going out, like, over time. Or closer to being equal to.
S2: Yeah.
S1: Cool.
S2: so we should do this [he selects and places the table in the modeling window and double clicks to define the table. He selects "medicine in the body" and moves it to the selected window, and moves elimination of medicine and medicine entering to the selected window] let's see how it changes as we run the thing. The elimination is eight it's slowly getting closer to it. I'm going to start writing because we already know what happens the medicine is eliminated slowly approaches the value of the medicine entering till it let's see how far it gets.
S1: We're only at 400. We need to get to, what? We're only a third of the way there.
S2: I'm going to hit on fast-forward what's going on there so it's slowly reaches it but it never actually gets there you know that rule you know
S1: Will never actually reach a point where it's decreasing
S2: It's staying at .99

In the packet the students wrote: The medicine being eliminated slowly approaches the value of the medicine entering but never will reach the same value or decrease.

Unfortunately, they do not explain why the medicine level should not decrease. But it is a valuable insight that they did consider this option.

- The second pair, after exclaiming how different the simulation graph was from their prediction, comes to a reasonable explanation for the shape of the simulation graph.

The students observe the graph, then set up a table with the drug in the system, the inflow value and the outflow value.

P1: So we have one g per minute entering the system and then 55% of it, point 55% of it leaving the system.
P2: So the same goes in every single time, 1 g, right? So how much is leaving? So then .55% of the drug is leaving each time. So eventually, like it starts to catch up, see. like now. .96 of a gram or whatnot is leaving but only one more is going in so even though that's like .55% of the entire drug in the system it's almost like the same, that's why it flattened out of the top.
P1: I got it. So, yeah, it will eventually catch up to like, one. So this is when I start not

giving him the drug, so like, here.

P2: Yeah. So even, like .55%, is really a really small amount of the entire drug

P1: Since it's exponential it will catch up pretty quickly.

P2: Yeah, because this is constant and it doesn't get higher.

P1: So now it's caught up and this is kind of flat line, hopefully not like the patient. So what are we writing

P2: I don't know how to put into words the inflow is the same amount but the outflow is exponential so eventually it will catch up and be the same amount as the inflow and it will be as if no new drugs are entering. It eventually won't. Does it get to be more than it or no?

P1: No, this won't become more than this [pointing first to the outflow value in the chart and then to the inflow value in the chart]. That's it. That's all the drug going in.

P2: Well, yeah, but eventually. Like, no, that makes sense. When they stop giving him drugs he'll start [unintelligible]. That makes sense so the inflow is constant the outflow is [unintelligible] so as more drugs enter the system the outflow of .55%, at a rate of .55%. Eventually the outflow will grow to be the same as the inflow …

In the packet the students wrote: The inflow is constant but the outflow is exponential so as more drugs enter the system the outflow at a rate of .55% the outflow will grow to be the same as the inflow.

3. Overall Purpose 3: The final objective was to determine if the students are becoming comfortable enough understanding the model dynamics that they can correctly predict what would happen to the graphical model output if a modification of the model is made. The following description is given to the students:

   "A complication occurs with this patient about 8 hours after the IV is administered. One of his kidneys quits functioning, causing the elimination rate to reduce to half. Predict what you think will happen to the drug level in the patient's body, recording your prediction on the grid below."

   Both pairs correctly predict the new behavior and are able to identify where the model needs to be modified, and how the definition of the selected component needs to be modified. Although they do not know exactly how to get the modification to happen at 8 hours and not before, the research tells them the command to use, as this is not a command the students have learned.

The first pair of students:

S1: We have that [referring to the first simulation graph recorded on his paper]. That's not the new one

S2: It's like that one for eight hours.

S1: Then it's going to get really flat again. Or is it actually going to go back down. No.
S2: Yeah. It would go back down wouldn't it and then it would go?
S1: But it's decreasing by half so wouldn't it go back up?
S2: The medicine in the system would go up and then back down to whatever.
S1: Well, it's half so as soon as it hits eight hours it would go back up again. It would be back just like.
S2: So at eight hours it would start going up again.
S1: Yeah. What would it go up to? 400? Yeah. It would go up to 400. Let's see what happens.
[We ran out of time to see the simulation run, so they could check their prediction. Both students drew prediction graphs in their lesson packet that indicated an initial jump upward at 8 hours followed by a leveling off of the drug at a higher equilibrium. This is the correct behavior the model would have produced.]

The second pair of students:

P2: It's going to continue growing [P2 is pointing at the original Drug in System graph starting at about 720 minutes and pointing her finger to rise above the Therapeutic Maximum line that is at 200 g] and it's going to go quicker [P2 pointing to the initial increase in the Drug in the System graph] because it's not going to have
P1: Well, it's after eight hours. How much is eight hours in minutes?
P2: Well, there's 60 minutes, so it's 60 times eight, which is 480, so would be, like
P1: So it be around here-ish. The kidney's going to fail, so I think it'll, like, keep going along this [P1 is pointing to the original Drug in System graph] until it gets here [pointing to the place on the graph that is at about 480 minutes] and then [he points as if the graph would start increasing quite a bit].
P2: It will spike up, yeah. [P2 indicates a curving of the increased prediction.]
P1: Yeah. It'll still be curved.
P2: It's about a quarter of the way-in so, it's doing its thing, and once it gets there it's going to spike. How far did you say it was going to go up?

These students drew both the prediction (using a dotted curve) and the actual simulation run (using a solid curve). The dotted curve increased in a more gradual fashion, indicating a smoother upward transition at 8 hours, than the simulation graph that made a more pronounced upward jump at 8 hours.

The transcripts indicate that the students were able to mathematize the problem without much difficulty, even after a faulty start with the second pair. They were able to use the software to explain the dynamic behavior represented in the simulation graph of the initial model. That was a key point. Once they had an understanding of the cause of the dynamic behavior pattern of the original model they were able to modify the model and were reasonably successful predicting the new behavior of the modified model. All of this work was on a model whose behavior was not typical of functions they had seen in class at that point.

The student problem solving scenario captured in these two videotapes supports the claim that students are able to mathematize new scenarios that are combinations of behaviors they already know and are able to analyze and modify and reanalyze the problem with relative ease. The videotaping lasted about 33 minutes for each pair of students.
This analysis was done comparing three problems across both student pairs (horizontal analysis) and also analyzing the improvement in thinking over the full time frame (vertical analysis) for each pair of students.