Dynamic of fixed capital – different points of view

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Abstract

The paper deals with the implications of the capital theory and national accounts statistics for the system dynamics models. In system dynamics' modelling terminology the retirement and depreciation of the fixed capital are some kinds of delay. We compare the common structures for the production capacity depreciation with the fixed capital indicators and show the interconnections and interpretation.

We analyse the data from building approvals to quantify the construction delay as common part of investment behaviour. For these purposes we interconnected two databases. This connection allowed us to find the parameters for the non-residential buildings from two points of view – the industry and type of construction.

Keywords: Fixed capital, investment, average service life, retirement function, perpetual inventory method, construction

Introduction

Is it possible that something that is fixed by the name is also dynamic? In this paper we will focus on the fixed capital and its forms as it is understood for the purposes of the national accounts. The ways of measuring capital has long history; it was and still is the important topic for the theoretical discussion, econometric analysis, statistics of national account, moreover, the capital is part of many system dynamics models. The points of view are often different. Nevertheless, the comparison and identification of data sources could affect the parametrisation of the simulation model and interpretation of results, furthermore, due to the existing detailed data the proper knowledge could simplify the process of the model parametrisation.

Our motivation for this paper lies in the simplification and categorisation of part of the modelling process that we frequently meet. Despite the fact that the model structure is commonly more important than parameters (Meadows, 2008, Forrester 1987b), the proper parameters could help to improve the trust of the modeller and problem owner (whether it is model for study of commercial purposes). Moreover, the knowledge of the existing parameters could simplify and speed up the process. Providing the common understanding between different fields is one of the original goals of systems theory (Bertalanffy, 1968), therefore, we find the value added in the intersection of statistics focused on capital and system dynamics modelling practice.

Together with labour, the capital represents the most important production factor (Solow 1957, Abramovitz 1993, OECD 2001). However, the measurement of the capital shows lot of difficulties in comparison to the labour. Cambridge capital controversy grew from the criticism of neoclassic approach to measurement of capital (Robinson 1953, Harrod and Sraffa 1961, Mata 2004). The critique aimed the aggregation and expression of value of capital as the production factor. The expression as list of capital goods, as labour aggregation, purchase prices or terms of future earning – all these way had its flaws. Despite we can find authors that doesn't consider the debate as solved but as calmed mainly due to the death of the main protagonists of one side (Cohen and Harcourt 2003), the OECD counties measure the capital on the basis of neoclassical capital theory (OECD 2009). Nevertheless, the official approach respects the consequences of Cambridge capital controversy and tries to reflect 'dual nature of capital which is both the storage of wealth and a source of capital services in production' (OECD 2009, p. 11).

Nowadays, the balances of fixed capital are standard component of national accounts statistics of developed countries (United Nations *et al.* 2009). In these balances, the fixed capital is represented by various indicators that are strictly identified as flows or stocks.

The fixed capital consists of assets that can be repeatedly used for more than one year (United Nations *et al.* 2009). That 'repeatedly' stresses that the durability is not enough and e.g. tinned food on the stock belongs to inventory and not to the capital stock. In simulation models the terminology could be different. But as the fixed capital represents the assets from computers, cars and machinery to dwellings, non-residential buildings or cultivated assets (OECD 2009) the corresponding model variables are not always called "capital" but very often the variables are named "capacity" or "production capacity" (Sterman 2000, Bossel 2007).

The first problem that occurs in economic analysis and national accounts statistics (but the national accounts solution could simplify the parametrisation of simulation models as consequence) is that the stock values in book keeping are not suitable for the economic analysis (Pigou 1935). The book keeping aggregates the assets in prices from different periods without the necessary revaluation and depreciation is frequently based on the law, accounting rules and owner's decision, therefore the assets commonly reaches the zero value in book keeping but are still in use and has the market price (Hulten and Wykoff 1996, Diewert 2005).

As a result, the statistical offices use the model estimation of stocks. The Perpetual Inventory Method (PIM) is based on data survey of inflows and estimation of outflows (OECD 2009). The main (and surveyable) inflow is gross fixed capital formation (acquisition less disposals of assets, i.e. mainly investment). In our paper we focus on two kinds of capital stock – the gross and net fixed capital stock. The gross fixed capital stock is represented by 'assets surviving from investment and revaluated to at the purchasers' prices of current period' (United Nations *et al.* 2009, p. 125). The main outflow of the gross fixed capital are retired assets. The net stock of fixed capital (wealth capital) reflects the decrease of the price of assets. The main outflow of such stock is consumption of fixed capital (depreciation[1]), which depicts the moral obsolescence and physical deterioration of assets.

[1] The term depreciation is common model variable, but could mislead to the book keeping data. Consumption of fixed capital should be based on the real service lives of assets and the proper depreciation profile.

In both cases the outflow is modelled on the basis of service lives. In case the model isn't in the prices of the basic year but the assets are revaluated to prices of the current period, the necessary flow is also the holding gain or loss. That indicator could be inflow (in case of assets price increase) or outflow (in case price decrease). The last important flow are 'other changes in volume of assets', which (beside the statistical reclassification) contain 'exceptional, unanticipated events' (Eurostat 2013) e.g. catastrophes, therefore other changes could be considered as naturaly exogenous variables for the simulation models. Moulton (2004, p. 261) stresses that the introduction of other changes in international standard System of National Accounts 1993 'provide a complete reconciliation between the stocks and flows in the system'.

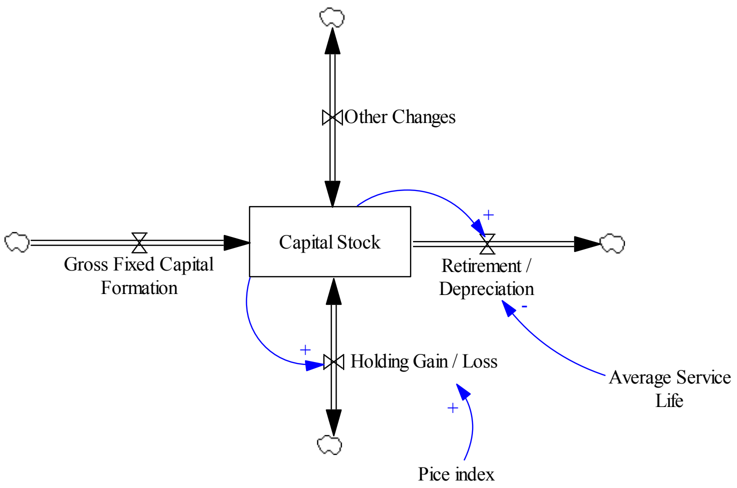

Figure 1: Basic structure of fixed capital in current prices

There is also possibility that the average service life is not just the constant parameter as in figure 1. Some statistical offices assume the change of the service life (mainly decrease) as the implication of modernisation process (Schmalwasser and Schidlowski 2006, OECD 2009).

The important information for the modeller and reader of this paper is that the average service lives are the matter of survey of statistical offices and are simply accessible[2] in various classifications (type of asset, industry). On the other hand, the common first order material delay from (1) does not necessarily fit any of the outflows correctly.

$$Depreciation = \frac{Capital\ Stock}{Average\ Service\ Life}, \tag{1}$$

In this paper, we show the possible intersection of official statistics, capital theory and system dynamics modelling. We describe the differences and the implications for the interpretation of the model. The retirement and depreciation could be simply represented by elementary modelling structures and specified kind of delay in the outflow of the capital stock. Moreover, we identify the common delay that has the impact on the inflow. Due to the fact that the inflow

[2] Parameters for the selected countries are in the OECD manual (OECD 2009), otherwise, contact the national statistical office and ask if they use Perpetual Inventory Method.

delay is not necessary for the official statistics and therefore the parameters are missing, we connect two different databases and categorise the parameters for the delay.

Material and Methods

Depreciation and retirement

The model of gross fixed capital stock shows the similar behaviour as the demography models and could be transformed into aging chain. Instead of the mortality function, which states the share of deaths in population, the capital decreases on the basis of retirement function with same implication – the part of the population ages to higher cohort, the rest retire and isn't used for the production anymore. The most used retirement functions are some kind of bell-shaped distribution (OECD 2009).[3]

We will show the net capital stock on two most used depreciation profiles. Equation (2) shows the straight-line depreciation. Variable p_n represents the value of asset in age of n years p_0 is value of new asset and T states for average service life:

$$\frac{p_n}{p_0} = 1 - \frac{n}{T}, \qquad n = 0,1,\ldots,T. \tag{2}$$

In case of most recommended geometric depreciation profile the net value of assets is obtained from (3) where δ represents depreciation rate (this is same as exponential decay used in system dynamics models, just the national accounts apply discrete approach):

$$\frac{p_n}{p_0} = (1-\delta)^n. \tag{3}$$

The depreciation rate δ isn't just the $1/T$ as in (1). Besides the surveys on second hand markets, the common approach to express the relation between the service life and the depreciation rate is the double declining balance (Diewert 2005, OECD 2009). With assumption that both depreciation profiles are correct and investment is on the level of one unit in constant prices the equilibrium net capital stock value under straight-line depreciation

$$1 + \frac{T-1}{T} + \cdots + \frac{2}{T} + \frac{1}{T} = \frac{T(T+1)}{2T} = \frac{T+1}{2}. \tag{4}$$

And long-run equilibrium value under same conditions and geometric depreciation is

[3] We already used the model with the decreasing average age, therefore the retirement share wasn't fixed. According to the Czech Statistical Office (2002) we used the Log-normal retirement function and therefore to model the retirement function the approximation of Gauss error function was necessary. In that case, we used the handy approximation by Winitzki (2008).

$$1 + (1-\delta) + (1-\delta)^2 + (1-\delta)^3 + \cdots = \frac{1}{1-(1-\delta)} = \frac{1}{\delta}. \qquad (5)$$

Therefore, the depreciation rate is obtained from equality of stocks from (4) and (5):

$$\delta = \frac{2}{T+1}. \qquad (6)$$

The double declining balance approach commonly simplify the denominator only to average service life (without the '+1'). However, the analysis focused on more accurate estimation of relation between average service life and depreciation rate (7) could show differences from the nominator equal to 2.

$$\delta = \frac{X}{T}. \qquad (7)$$

Hulten and Wykoff (1996) finds X to be 0.91 for non-residential buildings and 1.86 for durable equipment. On the other hand, different surveys supports the $X = 2$, e.g. max X for Canada is 2.3 for Machinery and Equipment (excluding software) (Micro-economic Analysis Division 2007, Baldwin *et al.* 2015). The selected depreciation rates δ for various types of assets in U.S. and Canada are again in OECD (2009, p.208-216).

Investment delay

The investment behaviour is affected by many feedbacks. The capital renewal was part of the investment function analysis from the very beginning (Solow 1960, Jorgenson 1963, 1966). On the other hand, the basic critique of the neoclassical investment function is that the neoclassical function assumes the equity between desired and actual capital stock. System dynamics has opposite point of view and assumes the discrepancy between actual and desired capital stocks (Senge 1978, Forrester 1987b).

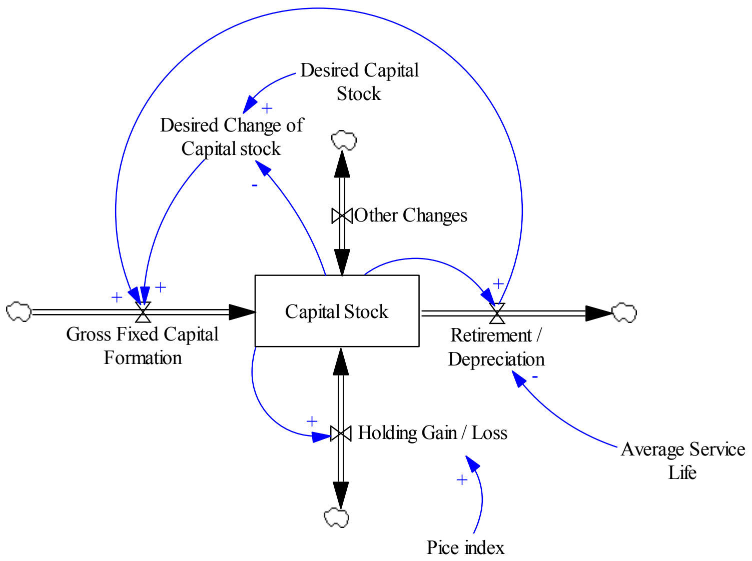

Figure 2: Impact of actual stock on investment behaviour

There could be many causes of the discrepancy (e.g. actual vs perceived needs) that are subjective for the decision maker. But we can identify one natural source of that discrepancy that is common for many investment projects – the construction period. Therefore, the figure 2 for the buildings is expanded to the figure 3.

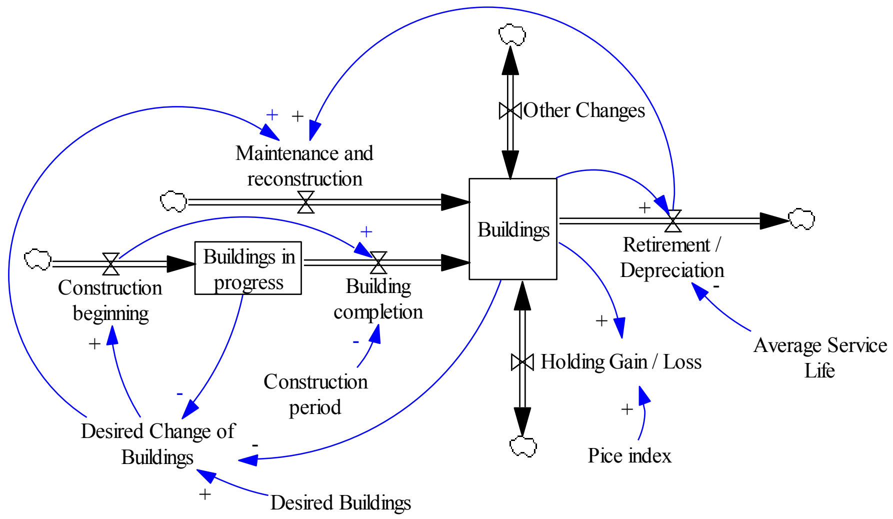

Figure 3: Construction period in case of buildings

To parametrise the construction period in terms of material delay, we at first interconnected two databases: the business register (Czech Statistical Office 2016a) and the statistics of building approvals (Czech Statistical Office 2016b). This connection allows us to categorise 9,084 records (years 2009-2013) from two points of view – type of the building and industry. To express the construction period we use date of two legal acts: building permit and building approval. From that, 542 records were excluded from the sample because the construction period was equal to zero (these were building permits renewals and immediate approvals). Consecutively, the construction period was tested by Kolmogorov–Smirnov test (Arnold and

Emerson 2011) for goodness of fit on Erlang distribution i.e. material delay in system dynamics models (Hamilton 1976). For the statistical testing, we use Statgraphics Centurion XVII.

Results and discussion

Figure 4 compares the impact of different depreciation profiles on example of investment of 100 units in year 0. The average service life *T*=5. Horizontal axis represents the age; vertical is the value of capital stock. The retirement function is lognormal with standard deviation of service life *s*=2.9.

Only the black curve shows the behaviour of gross stock, all other curves represent the development of net capital stock. The graph contains also the combination of retirement function and depreciation profile. In such case the investment is divided into groups with different service lives according to the retirement function. In case of straight-line depreciation and retirement function the modelling process demandingness significantly grows (*M* aging chains where *M* is maximum service life) without significant difference from geometric depreciation.

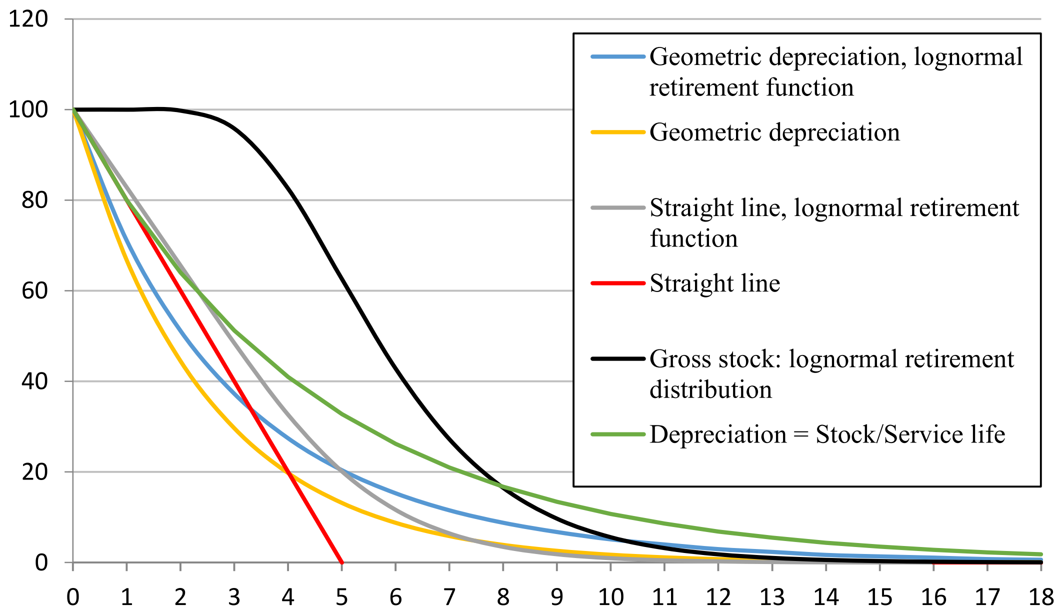

Figure 4: Capital stock – comparison of different approaches to capital estimation

It is clear from the comparison of practice of capital statistics and basic modelling structures that the outflow with first order material delay reflects the depreciation of net fixed capital stock. The green line is the result of the simple calculation from (1). It shows the overestimation of the net capital stock. As the geometric depreciation is based on the double declining balance ((7) where *X*=2), the depreciation from (1) must be of half value.

The modelling of depreciation on the basis of (1) would be most appropriate for the model that needs to express the "private non-residential structures" and "residential capital" (both in U.S.) as these have the declining balance rates X from range 0.89-0.97 (OECD, 2009,p.209). Otherwise, the depreciation (decrease of production capacity) would be under underestimated, consequently the net capital stock (stock that represents the wealth) would be overestimated. From figure 2 and 3 it is clear that the overestimated capital stock leads to the underestimation of the investment in simulation model.

The average construction delay for the all 8,542 non-residential buildings is first order material delay with the average equal to 6.07 year. Nevertheless, the Kolmogorov–Smirnov test rejects the hypothesis on the Erlang distribution (p-value < 0.001).

Table 1 shows the parameters of the construction delay according to the type of the construction. Classification code respects the Eurostat classification (Eurostat 1998). P-value is included only in case that the Kolmogorov–Smirnov test can't reject the hypothesis that the delay comes from the Erlang distribution on the level of significance higher than 0.001.

Classification code	Type of construction	Average delay (years)	Order	*p*-value	Sample size
1211	Hotel buildings	3.83	2		111
	Restaurants and bars	3.86	1		166
1212	Other short-stay accommodation buildings (excluding cottages)	8.57	1		2702
	Cottages	9.24	1		436
1220	Buildings of financial institutions	1.36	3		10
	Public administration and post buildings	3.85	1		46
	Other administrative buildings	2.77	2		367
1230	Department stores	1.70	2		164
	Buildings for trade and services	2.86	2		1253
1241	Communication buildings, stations, terminals and associated buildings	1.95	7		11
1242	Garage buildings	9.93	1		976
1251	Buildings used for industrial production (factories, workshops…)	2.83	2		490
	Buildings for energetics (excluding power stations), sewage and water treatment plants	3.85	1		46
1252	Reservoirs, silos and warehouses	2.71	1		320
1261	Public entertainment buildings	3.77	1		52
1262	Museums and libraries	3.66	1		12
1263	School, university and research buildings	2.18	3		114
1264	Hospital buildings	2.29	2	0.157	70
	Health resorts, institutional care buildings	2.15	2	0.604	14
1265	Sports halls	3.42	1		181
1271	Non-residential farm buildings	4.29	3		22
	Storage and processing buildings used for agriculture farming, silos	6.64	1		97
	Buildings for animal production	2.99	1		21
1272	Buildings used as places of worship and for religious activities	3.07	3	0.990	29
1274	Other non-residential buildings not elsewhere classified	4.98	1		832

Table 1: Construction period, parameters of material delay – type of construction

Table 2 shows the parameters of the construction delay according to the major economic activity of the owner – industry classification NACE rev. 2 (Eurostat 2008). The main body of the sample are individuals without Company registration number, therefore, the connection of the databases does not lead to the identification of the industry (economic activity).

NACE	**Industry**	**Average delay (years)**	**Order**	***p*-value**	**Sample size**
---	Individuals	7.26	1		6008
A	Agriculture, forestry and fishing	3.44	1		42
C	Manufacturing	2.26	2		392
CA	Manufacture of food products, beverages and tobacco products	1.69	5		31
CC	Manufacture of wood	3.38	2		21
	Manufacture of paper products, printing	1.80	5		19
CG	Manufacture of rubber and plastics products, and other non-metallic mineral products	2.19	2	0.148	55
	Manufacture of other non-metallic mineral products	2.75	2	0.050	30
CH	Manufacture of basic metals and fabricated metal products, except machinery and equipment	2.51	1		90
CI	Manufacture of computer, electronic and optical products	2.11	2	0.543	41
CK	Manufacture of machinery and equipment	1.93	2	0.032	48
CL	Manufacture of transport equipment	2.31	2	0.074	17
CM	Other manufacturing, and repair and installation of machinery and equipment	1.55	3		22
D, E	Electricity, gas, steam and air-conditioning supply and Water supply, sewerage, waste management and remediation	1.66	3		15
F	Construction	2.87	2		156
G	Wholesale and retail trade, repair of motor vehicles and motorcycles	2.02	2		549
H	Transportation and storage	3.32	2		41
I	Accommodation and food service activities	2.75	1		52
J, K	Information and communication and Financial and insurance activities	1.45	4		23
L, M, N	Real estate activities, Professional, scientific, technical, activities and Administrative and support service activities	2.16	2		463
O	Public administration and defence, compulsory social security	4.20	1		219
P	Education	3.16	2		15
Q	Human health services and Residential care and social work activities	2.71	2	0.001	29
S	Other services	3.19	2		164

Table 2: Construction period, parameters of material delay – industry classification

The Kolmogorov–Smirnov test can't reject the hypothesis that the delay comes from the Erlang distribution only in few cases. Figure 5 contains graphs with the rejected hypothesis on delay distribution. The horizontal axis represents the time (years), blue line shows the theoretical Erlang distribution. It's up to the reader to decide whether the average and order fits the purposes of her/his modelling purposes.

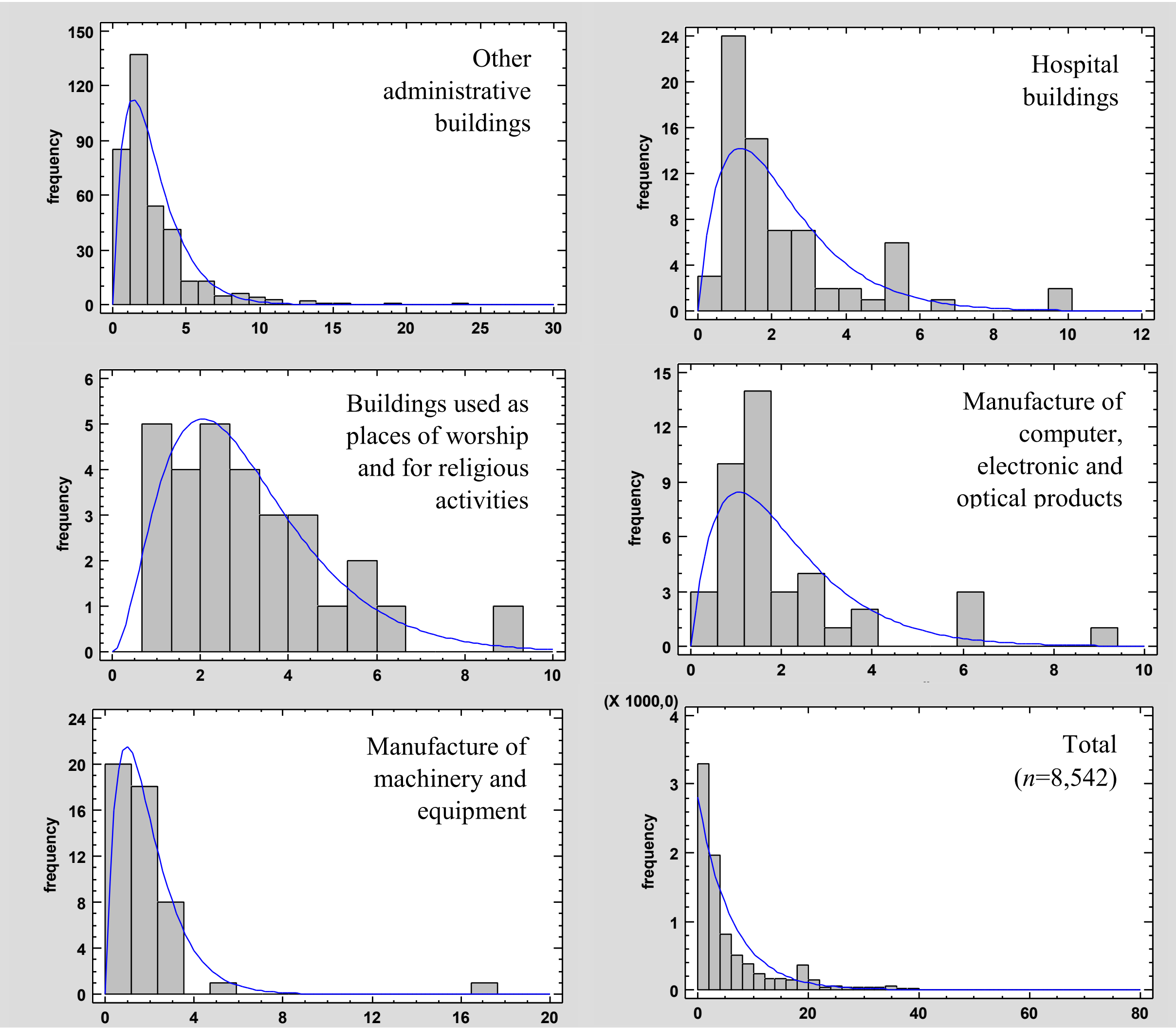

Figure 5: Selected examples of construction delay

Conclusion

In this article, we compared different points of view on the fixed capital. The national accounts statistics of OECD countries could provide the important parameters - service lives for the many types of fixed assets that are inseparably connected with the production activity.

The comparison shows that the first order material delay structure could be interpreted as net (wealth) capital. Nevertheless, the empirical research shows that the simple division by service lives could lead to overestimation of the stock value and consequently the underestimation of investment in the simulation model. In case the depreciation rate is not clear, the double declining balance provides the possible solution to depreciation rate quantification.

As the investment behaviour could be frequently characterised by the discrepancy between the actual and desired capital stock, we quantify the common delay in investment behaviour – the construction period. The interconnected databases allowed us to find the parameters for the non-residential buildings from two points of view – the industry and type of construction.

For the future, we plan to elaborate the interconnection between value of the building and the construction period as it is possible to assume that bigger and more time demanding buildings also cost more. Despite the higher detail will necessarily lead to smaller sample we also plan the categorisation from both points of view together industry x type of construction.

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