The Analysis of Characteristic Dynamics of Stock Management Structure with a First Order Supply Line Delay

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Abstract

Stock management is a dynamic task which is often found in managerial, physical, and biological systems. The aim in stock management is to bring a stock at a desired level and maintain it at that level by taking corrective actions. Stock management task imposes difficulties to the decision maker, which results in unwanted oscillations. In this paper, we carry out complete parametric analysis of stock management problems with first order continuous delay aiming to obtain the range of values for different characteristic dynamics of stock. For parametric analysis, we use control theoretic approaches. We first provide the stock management structure modeled using stock-flow diagrams of system dynamics methodology. Secondly, we obtain the corresponding simplified differential equations of the stock management model. Thirdly, we convert simplified differential equations of the model from time domain to s-domain using Laplace transformation technique and obtain the transfer function. Fourthly, the characteristic equation of the transfer function is determined. Finally, we determine the critical values of the decision parameters at which a qualitative change in dynamics is observed by analyzing the roots of the characteristic equation. The critical values that are reported in this paper are valid for all durations of the delay between the corrective actions and their eventual results on the stock. We also obtained a few counterintuitive results such as increasing the level of aggressiveness in stock corrections can completely eliminate oscillations in one of the cases.

Introduction

Stock management is a dynamic task that requires continuous effort. The aim in stock management is to bring a stock at a desired level and maintain it at that level by taking corrective actions (Mutallip, 2013; Sterman, 2000). Stock management structures are often found in managerial, physical, and biological systems. For example, inventory control appears in managerial systems (Barlas, 2002; Barlas and Ozevin, 2004; Riddalls and Bennett, 2002a, Sipahi and Delice, 2010), the control of the level of production capacity also appears in managerial systems (Paich and Sterman, 1993; Chapter 20 in Sterman, 2000; Vlachos *et al.*, 2007), the temperature of a room that is regulated by an air-conditioning device constitutes a physical system, the control of the angular position of a helicopter is a result of human-physical system interactions (Şeker and Yasarcan, 2010), and the regulation of blood glucose in a healthy person is a part of a biological system (Herdem and Yasarcan, 2010).

Delays in Stock Management

Stock management is subject to many studies (Angerhofer at al., 2000; Akkermans and Vos, 2003; Edali and Yasarcan, 2014; Chaharsooghi *et al.*, 2008; Größler *et al.*, 2008; Sterman, 1989; Yasarcan and Barlas, 2005a; Yasarcan, 2010 and 2011). According to the experimental studies carried out with human participants, stock management task imposes difficulties to the decision maker (or to the controller), which results in unwanted oscillations and costs associated with these dynamics (Barlas and Ozevin, 2004; Diehl and Sterman, 1995; Moxnes, 2000; Sengupta *et al.*, 1999; Sterman, 1989; Yasarcan, 2010). The intrinsic cause of difficulty in stock management is the existence of delays because time delays separate causes and their effects in time and, perhaps, also in space. Regardless of the increase in the level of complexity caused by the existence of a delay, it must be represented in the model of a dynamic system because models containing time delays are more realistic compared to the models that do not represent delay causing structures, which is also valid for the models of stock management tasks (Barlas, 2002; Chapter 11 in Sterman, 2000; Yasarcan, 2011).

The existence of a delay causing structure brings difficulty to the stock management task that may result in excessive stock, stock deficit, or intolerable oscillations around the desired level. These undesirable dynamics bring about additional costs (Barlas and Gündüz, 2011; Li and Liu, 2013; Mutallip and Yasarcan, 2014; Riddalls and Bennett, 2002a; Rydzak and Sawicka, 2008; Sengupta, *et al.*, 1999; Sipahi and Delice, 2010; Sterman, 1989; Yasarcan, 2011). For example, in inventory control, overshoot of the inventory level leads to an increase in holding costs while undershoot of the inventory level results in an increase in backlog costs or lost sales costs (Edali and Yasarcan, 2015; IE 413, Unpublished Lecture Notes; Sterman, 1989). As the aim in stock management is to bring a stock at a desired level and maintain it at that level and as it is costly to have oscillatory dynamics, eliminating oscillations is suggested as a solution (Edali and Yasarcan, 2015; Riddalls and Bennett, 2002a and 2002b; Sipahi and Delice, 2010; Sterman, 1987, 1989; Yasarcan and Barlas, 2005a; Yasarcan, 2011).

Physical and Decision Parameters

A stock management model usually represents two aspects of the stock management task: (i) the physical process, (ii) the decision making process. Sterman (1989) suggests using anchor-and-adjust heuristic to represent the decision making processes of human participants and he claims that the suggested heuristic is a good representation. Therefore, we represent the decision making process using the anchor-and-adjust heuristic. In this paper, the parameters related to the physical aspect of the stock management task are called *physical parameters* and the parameters related to the decision making processes, the parameters of the anchor-and-adjust heuristic, are called *decision parameters*.

Stock Management Studies and Motivation for this Study

Sterman (1989) suggests optimum values for two decision parameters for each individual stock management task in The Beer Game; there exists an inventory management problem for each echelon on the supply-chain in the game. Each one of these cascading stock management problems involves a discrete supply line delay (i.e., pure supply line delay; infinite order supply line delay). Therefore, the suggested optimum values are only valid for such problems. Yasarcan and Barlas (2005a) use one of those optimum values to obtain non-oscillatory dynamics in stock management with different structural types of delays (material delay, information delay, control via a secondary stock, and composite delay). Mutallip and Yasarcan (2014) optimizes one of the parameter values for stock management problems with continuous delays (first, second, third, fourth, and eight order delays) using a simple penalty function that accumulates the differences between the stock and its desired level. They also suggest ways to select meaningful values for the other decision parameter. In addition to the results given for continuous delay cases, they validate the suggested values by Sterman (1989) for a discrete delay case. Riddalls and Bennett (2002a) analyze a stock management problem with a discrete delay from a control theoretic perspective, where the results obtained by them are also in accordance with the results reported by Sterman (1989). Barlas and Ozevin (2004) uses two stock management tasks (one with a first order delay and the other one with a discrete delay) in their experiments where they use human participants as decision makers. They fit different ordering rules to the data obtained from the participants aiming to evaluate the adequacy of literature suggested ordering polices. According to their results, the widely used anchor-and-adjust heuristic can represent the orders placed by participants who make relative smoother adjustments, but the same heuristic is weak in representing the orders placed by participants who have sudden jumps in their orders. Mikati (2010) studies the problem from a different angle and he caries out simulation experiments in determining the effect of batch sizes on the average duration of delay under different operational conditions. Yasarcan and Barlas (2005b) diverts from the rest literature because they assume that the loss from the stock depends proportionally on the stock itself. However, none of these studies makes a through stability analysis for a stock management problem with a continuous delay. Note that discrete delay (pure delay, infinite order delay) has a simple mathematical expression. Thus, it is widely used in representing delays. However, continuous delays can more realistically represent a delay causing structure compared to discrete delay (Mutallip and Yasarcan, 2014). Mikati (2010) used a first order material delay structure in representing delays in a production-inventory system and argued that, for their case, it is a better representation compared to discrete delay. Venkateswaren and Son

(2004) suggested using a higher order continuous delay structure rather than using a discrete delay to obtain a more correct behavioral representation of delay.

The motivation for this paper is to make a first step towards closing this gap in the literature by carrying out a complete parametric analysis of a stock management problem with a first order continuous supply line delay aiming to obtain the range of values for different characteristic dynamics of stock such as goal-seeking behavior (i.e., asymptotic approach to the desired level of stock) and stable oscillations.

Methodology

In the paper, the decision parameters of the stock management structure, which is modeled using system dynamics (SD) methodology, are analyzed using control theoretic approaches. We first provide the stock management structure modeled using stock-flow diagram of SD methodology. Secondly, we obtain the corresponding simplified differential equations of the SD model. Thirdly, we convert simplified differential equations of the model from time domain to s-domain using Laplace transformation technique and obtain the transfer function. Fourthly, the characteristic equation of the transfer function is determined. Finally, we determine the critical values of the decision parameters at which a qualitative change in dynamics is observed by analyzing the roots of the characteristic equation.

In this paper, we modeled the stock management structure using SD methodology because (i) SD is often used in modeling stock management structures, and (ii) SD has a strong focus on the validity of the constructed models, which is partially achieved by explicitly representing all problem related elements of the system in the model. Although SD has a strong focus on constructing valid models, it's a simulation based approach because analytical analysis is either hard or impossible as some basic and most non-basic SD models involve complexities such as nonlinear relations. Accordingly, analytical analysis of a dynamic model is not a main issue in most SD studies. Thus, control theoretic analytical analysis is carried out in this paper. The differences between the two systems approach, SD and CT, are caused by the focus of the studies carried out in the two fields. The main focus of the studies using system dynamics is to either make structural changes to the system and/or develop control policies in managing it (see, for example, Sterman, 2000; Yasarcan, 2011). However, the studies using control theory focus on either optimizing parameter values to obtain the optimal response from the system and/or come up with values that produce stable dynamics (see, for example, Riddalls and Bennett, 2002a; Sipahi and Delice, 2010; Zhou and Disney, 2005).

System Dynamics Model of the Stock Management Structure with a First Order Supply Line Delay

Stock-flow diagram of a stock management structure with a first order supply line delay is given in Figure 1.

The model equations are 1-5.

$$CF = LF + SA + SLA \tag{1}$$

$$SA = \frac{S^* - S}{sat} \tag{2}$$

$$SLA = wsl \times \frac{SL^* - SL}{sat} \tag{3}$$

$$SL^* = adt \times LF \tag{4}$$

$$AF = \frac{SL}{adt} \tag{5}$$

where *adt* stands for "Acquisition Delay Time", *AF* stands for "Acquisition Flow", *CF* stands for "Control Flow", *LF* stands for "Loss Flow", *S* stands for "Stock", *S** stands for "Desired Stock", *SA* stands for "Stock Adjustment", *sat* stands for "Stock Adjustment

Time”, *SL* stands for “Supply Line”, *SL** stands for “Desired Supply Line”, *SLA* stands for “Supply Line Adjustment”, and *wsl* stands for “Weight of Supply Line”.

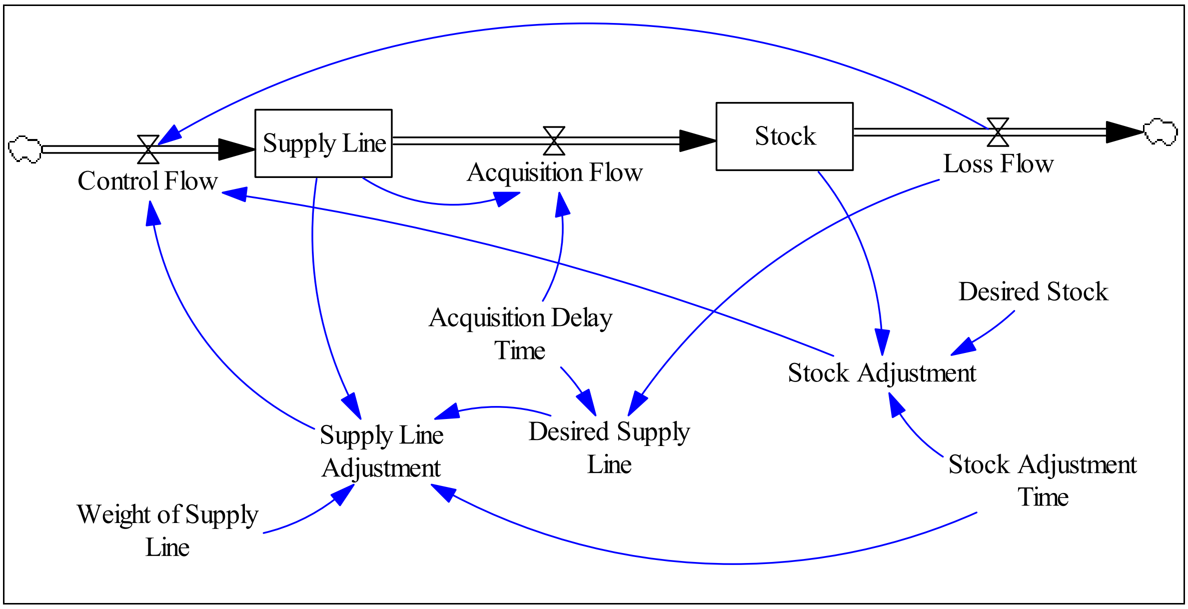

Figure 1. Stock-flow diagram of the stock management structure with a first order supply line delay.

The diagram in Figure 1 and equations 1-5 define a stock management structure with a first order supply line delay. See Appendix for the block diagram of this structure.

The Physical Process Depicted by the Stock Management Model

The physical process of the stock management task depicted by the model diagram given in Figure 1 consists of *adt*, *AF*, *LF*, *S*, and *SL*. *S* and *SL* are the state variables (i.e., stock variables) of this model, where *S* is the main variable of concern that is subject to control and *SL* represents the past decisions that have not yet reached the *S*. *LF* is the outflow of *S* and it drains S. In realistic systems, *LF* cannot be instantaneously measured and reported. Therefore, it is usually estimated with a smoothing method. However, in this paper, we are not concerned with the difficulties in the estimation of *LF*. Therefore, estimation formation is not included in the model. *LF* is still an important aspect of the model because it serves as the input to the model.

adt is the duration of the delay between the corrective actions and their eventual results on the stock and its value is strictly greater than zero. In this study, we assume that *adt* cannot be controlled by the decision maker; it is inherent to the physical process. *AF* is a delayed version of *CF* and it represents the decisions that arrive to and have an effect on *S* (Equation 5). The physical process is defined by the stock-flow relations that are present in Figure 1 and Equation 5.

The Decision Making Process Depicted by the Stock Management Model

The decision making process depicted by the model given in Figure 1 consists of *CF*, *SA*, *sat*, *SLA*, *S**, *SL**, and *wsl*. *CF* represents the decisions; it is the output of the decision making process and it is input to the physical process of the stock management task (Equation 1). *SA* and *SLA* are the adjustment terms (equations 2 and 3). *SL** is the desired level of *SL* that is necessary to maintain an outflow (i.e., *AF*) equal to *LF* so as to be able to maintain *S* at its desired level. If *SL** is not correctly calculated, a steady-state error will be obtained (Equation 4). *S** is an input to the decision making process, which is assumed to be determined by processes that are excluded from this study; *S** is a predetermined constant value.

LF, *S*, and *SL* are also the inputs to the decision making process. The decision making process represented in the model is known as the anchor-and-adjust heuristic (Sterman, 1989). The decision making process is defined by equations 1-4.

The two important decision parameters that can be controlled by the decision maker are *sat* and *wsl*; *sat* is the intended time to close the gap between the stock and its desired level and *wsl* is the relative importance given to the supply line compared to the stock. The order of the delay, *adt*, *sat*, and *wsl* determine the nature of the dynamics observed in stock. As the order of the delay and *adt* are assumed to be inherent to the physical process, the decision maker needs to decide on the values of *sat* and *wsl* to obtain the desired dynamics in *S*, which is the main concern of this paper.

Simplified Set of Differential Equations of the Stock Management Structure with a First Order Supply Line Delay

The simplified set of differential equations that corresponds to the stock management structure with a first order supply line delay structure is given in equations 6 and 7.

$$\frac{dS}{dt} = AF - LF = \frac{SL}{adt} - LF \tag{6}$$

$$\frac{dSL}{dt} = CF - AF = LF + \frac{S^* - S}{sat} + wsl \times \frac{adt \times LF - SL}{sat} - \frac{SL}{adt} \tag{7}$$

Analysis of the Dynamics of Stock Management Structure with a First Order Supply Line Delay

Laplace transform of the simplified set of differential equation of the stock management structure with a first order supply line delay (equations 6 and 7), by assuming S^* equals to zero, is given in equations 8 and 9.

$$s \times S(s) - S(0) = \frac{SL(s)}{adt} - LF(s) \tag{8}$$

$$s \times SL(s) - SL(0) = LF(s) - \frac{S(s)}{sat} + \frac{adt \times LF(s) - SL(s)}{sat} \times wsl - \frac{SL(s)}{adt} \tag{9}$$

Assuming $S(0) = 0$ and $SL(0) = 0$, and arranging equations 8 and 9, equations 10 and 11 are obtained.

$$SL(s) = adt \times s \times S(s) + adt \times LF(s) \tag{10}$$

$$SL(s) \times \left(s + \frac{wsl}{sat} + \frac{1}{adt} \right) = LF(s) - \frac{S(s)}{sat} + \frac{wsl \times adt}{sat} \times LF(s) \tag{11}$$

Inserting Equation 10 into Equation 11 and arranging, Equation 12 is obtained.

$$\left[adt \times s^2 + \left(\frac{adt \times wsl}{sat} + 1 \right) \times s + \frac{1}{sat} \right] \times S(s) = -adt \times s \times LF(s) \tag{12}$$

Transfer function of this system is the ratio of output variable ($S(s)$) to the input variable ($LF(s)$) in the Laplace domain, which is given in Equation 13.

$$\frac{S(s)}{LF(s)} = -\frac{adt \times s}{adt \times s^2 + \left(\frac{adt \times wsl}{sat} + 1 \right) \times s + \frac{1}{sat}} \tag{13}$$

The denominator of the transfer function gives characteristic equation of the system (Equation 14).

$$adt \times s^2 + \left(\frac{adt \times wsl}{sat} + 1 \right) \times s + \frac{1}{sat} = 0 \tag{14}$$

Equation 14 is solved in MATLAB and characteristic roots of the characteristic equation are given in equations 15 and 16.

$$s_1 = -\frac{sat + adt \times wsl + \sqrt{adt^2 \times wsl^2 + 2 \times adt \times sat \times wsl - 4 \times adt \times sat + sat^2}}{2 \times adt \times sat} \tag{15}$$

$$s_2 = -\frac{sat + adt \times wsl - \sqrt{adt^2 \times wsl^2 + 2 \times adt \times sat \times wsl - 4 \times adt \times sat + sat^2}}{2 \times adt \times sat} \tag{16}$$

For stability property, real parts of the characteristic roots must be analyzed. If real parts all of the roots are negative (all of the roots are located in the left-half of the s-plane), the system is stable. If real part of the at least one of the roots is positive (at least one of the root is located in the right-half of the s-plane), the system is unstable.

The characteristic roots of the characteristic equation are complex conjugate (equations 15 and 16), therefore, have the same real parts. Since *adt*, *sat* and *wsl* are always positive parameters, the real parts of the characteristic roots are always negative, which is given in Equation 17.

$$-\frac{sat + adt \times wsl}{2 \times adt \times sat} < 0 \tag{17}$$

Therefore, the system is always stable which is also consistent with Yasarcan (2003).

To determine the parameter values that make stock show an oscillatory behavior, the term in the square root of the characteristic roots must be analyzed. If the term in the square root is negative, the value of characteristic root becomes complex and the stock starts to oscillate. An oscillatory behavior is obtained if the condition given in Equation 18 is satisfied.

$$adt^2 \times wsl^2 + 2 \times adt \times sat \times wsl - 4 \times adt \times sat + sat^2 < 0 \tag{18}$$

Relative Aggressiveness

A new parameter that is first introduced by (Mutallip and Yasarcan, 2014), namely, "Relative Aggressiveness" (*ra*), is defined as the ratio of *adt* to *sat* (Equation 19). Both *sat* and *adt* are strictly positive parameters, thus, ra is strictly greater than zero too. *ra* can be used together with *wsl* to determine the nature of the stock behavior (i.e., the characteristic dynamics). In this way, the analysis becomes simpler as the three parameter space (*wsl*, *sat*, *adt*) is reduced to two (*wsl*, *ra*). Accordingly, Equation 18 can be rewritten in terms of *ra* and is given in Equation 20.

$$ra = \frac{adt}{sat} \tag{19}$$

$$ra^2 \times wsl^2 + 2 \times ra \times (wsl - 2) + 1 < 0 \tag{20}$$

Results

For different *wsl* values between 0 and 1, critical values of *ra* are obtained by equating the right hand side of the inequality given in Equation 20 to zero and solving it. The results are reported in Table 1.

Table 1. Critical *ra* values obtained for different *wsl* values.

wsl	ra_1	ra_2
0.00	0.250	-
0.01	0.251	39,800
0.02	0.253	9,900
0.05	0.256	1,560
0.10	0.263	379.7
0.15	0.271	164.2
0.20	0.279	89.72
0.30	0.296	37.48
0.40	0.318	19.68
0.50	0.343	11.66
0.60	0.375	7.403
0.70	0.417	4.889
0.80	0.477	3.273
0.90	0.577	2.139
0.95	0.668	1.659
0.97	0.727	1.463
0.99	0.826	1.235
1.00	-	-

For *ra* values below or equal to ra_1 or above or equal to ra_2, stock shows no oscillations; it shows a goal seeking behavior (i.e., asymptotic approach to the desired level of stock). If a selected *ra* value is between ra_1 and ra_2, stock shows oscillatory behavior in a stable manner. For wsl equal to 0, stable oscillations can be observed for all ra values greater than 0.25. For *wsl* above or equal to 1, there are no real nonnegative values of *ra* that can make the stock oscillate. Therefore, there are no oscillations for *wsl* above or equal to 1. For *wsl* values less than 1, as *wsl* increases, the range of interval of *ra* in which stock shows oscillatory behavior decreases. In the section named *The Values of the Decision*

Parameters that will Produce the Desired Dynamics, it is explained how *ra* values can be used to obtain *sat* values as *sat* is the decision parameter and *ra* is only used to simplify the analysis.

Validity of Results for All Durations of Delay

The results (i.e., the characteristic behaviors) presented in previous section only depend on *wsl* (the relative importance given to the supply line compared to the stock) and *ra* (the ratio of *adt* to *sat*) and they are valid for all *adt* values because Equation 20 does not explicitly contain *adt* (i.e., the duration of delay between the corrective actions and their eventual results on the stock). Note that a stock management structure with a first order supply line delay is capable of producing only non-oscillatory behavior (i.e., goal seeking) and stable oscillations. However, a stock management structure with a higher order delay is capable to produce richer dynamics including unstable oscillations. Note that the analysis carried out in this paper and additional analyses carried out for higher order delays are present in Mehmet (2015).

The Values of the Decision Parameters that will Produce the Desired Dynamics

In this section, we explain how Table 1 can be used to obtain values for the decision parameters (*wsl*, *sat*) that will produce the desired dynamics. The results provided in this paper are valid for all *adt* values and the value of the decision parameter *sat* can be obtained for a given *adt* using Equation 21.

$$sat = \frac{adt}{ra} \tag{21}$$

The duration of delay between the corrective actions and their eventual results on the stock, *adt*, is a physical parameter and belongs to the physical process. Thus, we assume that the decision maker does not have a control over it. After a decision maker determines the desired dynamics, if she wants to choose a set value for the two decision parameters, *wsl* and *sat*, she must first use Table 1. Later, she must use the *adt* value present in the physical process, the *wsl* and *ra* values obtained from Table 1, and Equation 21 to obtain a value for *sat*. Thus, as a result, a set of *wsl* and *sat* values is obtained.

Example Decision Parameter Value Selection for a Stock Management Model with a First Order Supply Line Delay

We assume a stock management model with a first order supply line delay and with duration of 5 units of time. We arbitrarily select *wsl* as 0.2. According to Table 1, if *ra* is selected below 0.279, stock shows a goal seeking behavior and the approach of the stock to its desired level gets faster as *ra* increases; see stock dynamics in Figure 2 for *ra* values of 0.15 (Line 2), 0.2 (Line 3), and 0.25 (Line 4).

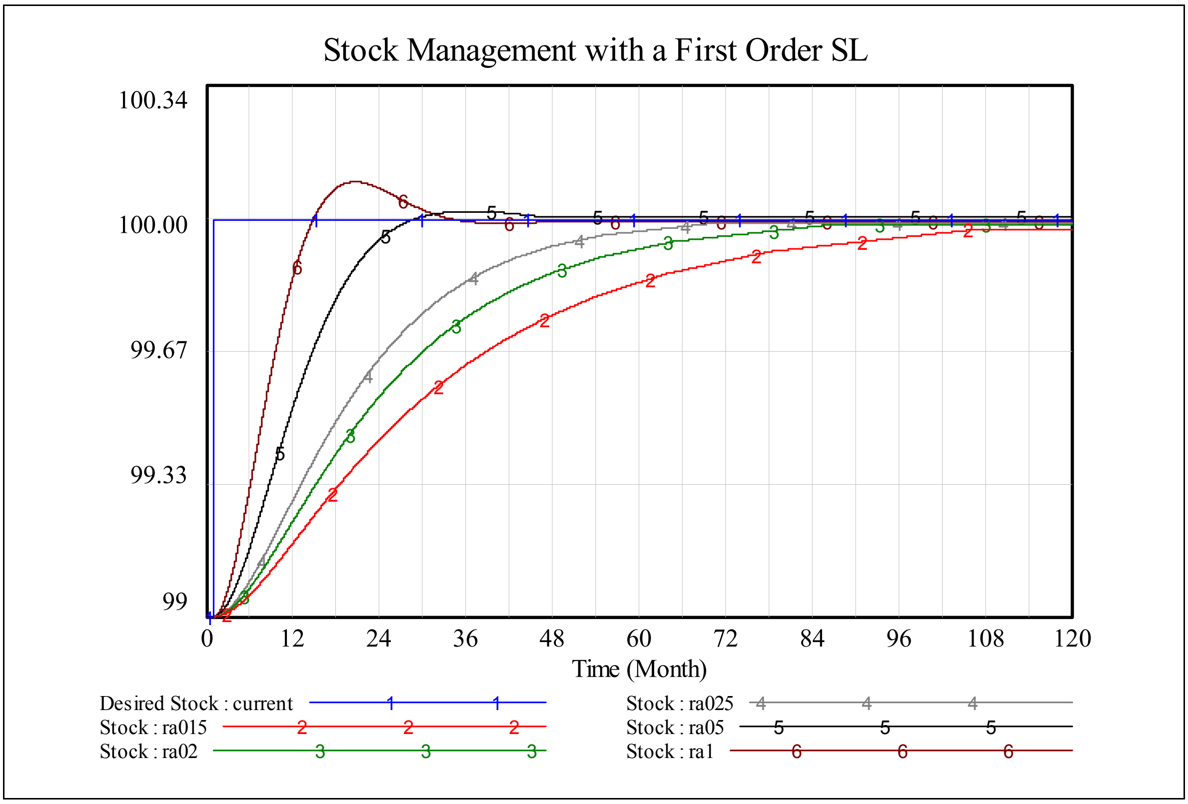

Figure 2. Different stock dynamics for *ra* values of 0.15, 0.2, 0.25, 0.5, and 1.

Line 1 in Figure 2, Figure 3, Figure 4, Figure 5, Figure 8, Figure 9, Figure 12, represents "Desired Stock" for all experiments in those figures. If *ra* is equal to or bigger than 0.279 and smaller than 89.72, stock shows oscillatory behavior in a stable manner; see stock dynamics in Figure 2 for *ra* values of 0.5 (Line 5) and 1 (Line 6).

After oscillatory behavior is observed, the highest value that stock attains starts to increase as *ra* increases, but the period of oscillations shortens; see stock dynamics in Figure 3 for *ra* values of 0.75 (Line 2), 1 (Line 3), 2 (Line 4), and 3 (Line 5).

The highest value that stock attains increases until *ra* becomes equal to 5, after that value, it starts to decrease as *ra* increases (Figure 4). However, the period of oscillations continues to shorten as *ra* increases.

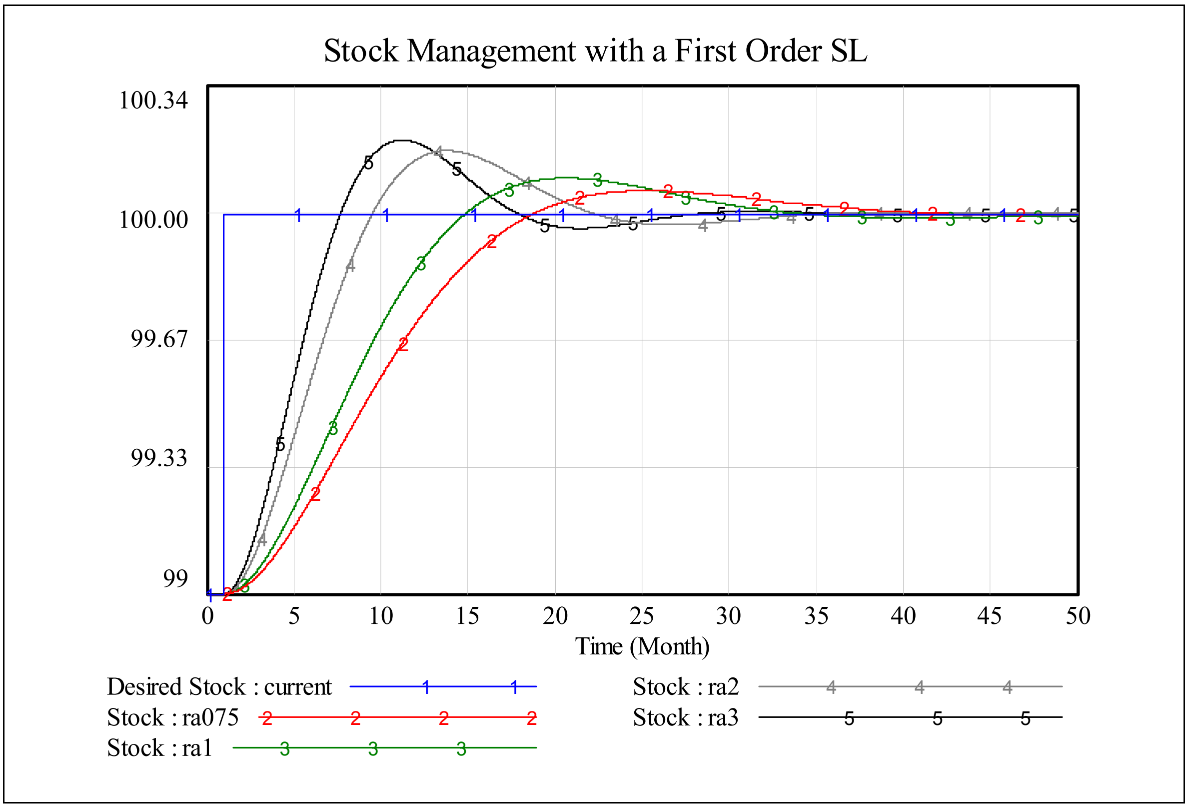

Figure 3. Different stock dynamics for *ra* values of 0.75, 1, 2, and 3.

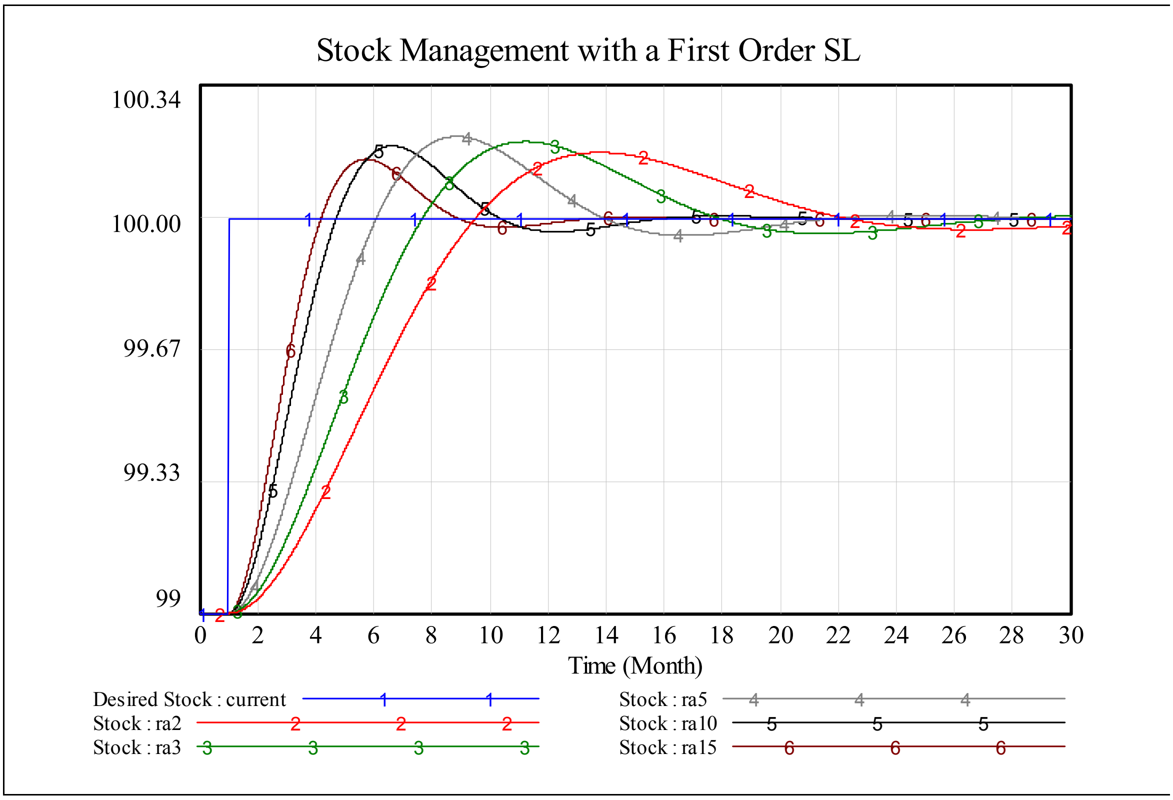

Figure 4. Different stock dynamics for *ra* values of 2, 3, 5, 10, and 15.

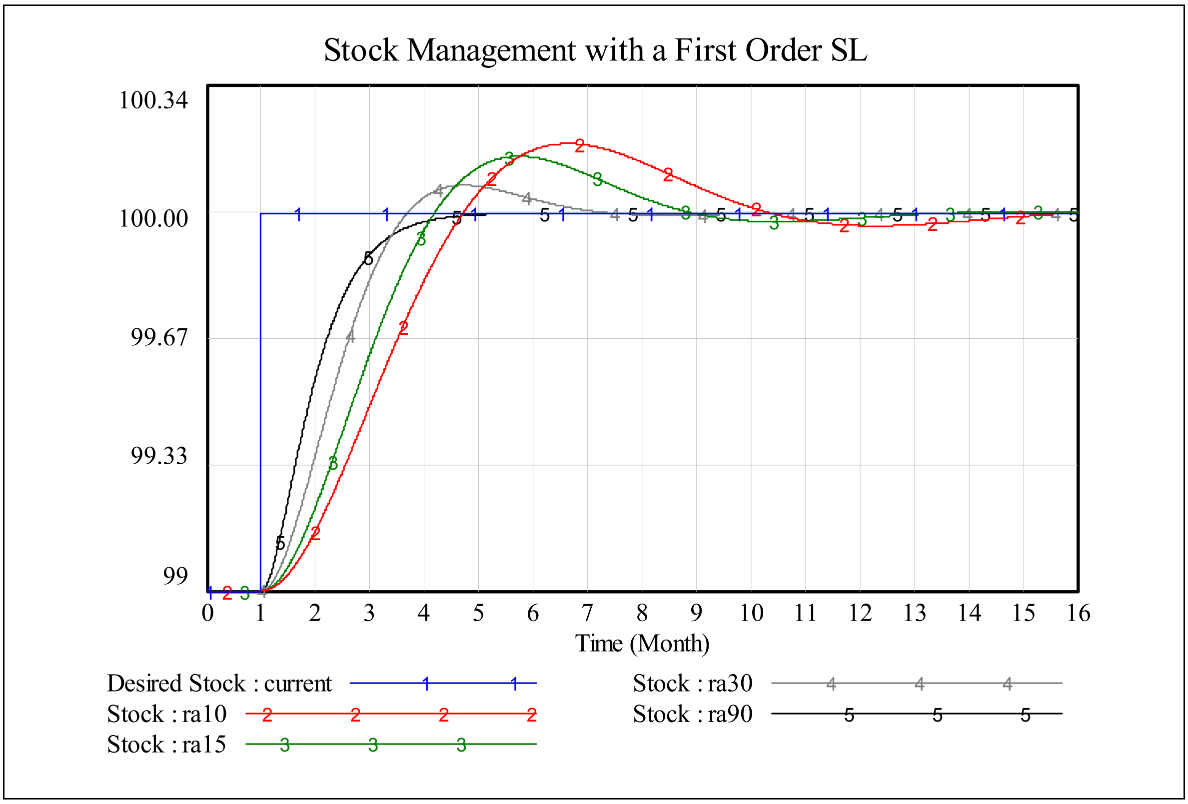

Figure 5. Different stock dynamics for *ra* values of 10, 15, 30, and 90.

The highest value that stock attains decreases as *ra* further increases (Figure 5). If *ra* is selected equal to or greater than 89.72, stock shows a goal seeking behavior again. Period of the oscillations always shortens as *ra* increases, which can be seen in Figure 2, Figure 3, Figure 4, and Figure 5.

If the desired dynamics is stable oscillations, according to Table 1, there are infinitely many alternatives of set of *wsl* and *ra* values for stable oscillations. For *wsl* equals to 0.2, if the desired dynamics is stable oscillations with comparatively higher amplitude values, *ra* can be chosen equal to 5. Since *adt* is 5, using Equation 21, *sat* is obtained as 1. Thus, the set of values for the decision parameters, *wsl* and *sat*, are 0.2 and 1, respectively. If the desired dynamics is a goal seeking behavior with a fast approach to the desired level of stock, *ra* can be chosen equal to 90. Since *adt* is 5, using Equation 21, *sat* is obtained as $0.0\overline{5}$. Therefore, the set of values for the decision parameters, *wsl* and *sat*, are 0.2 and $0.0\overline{5}$, respectively.

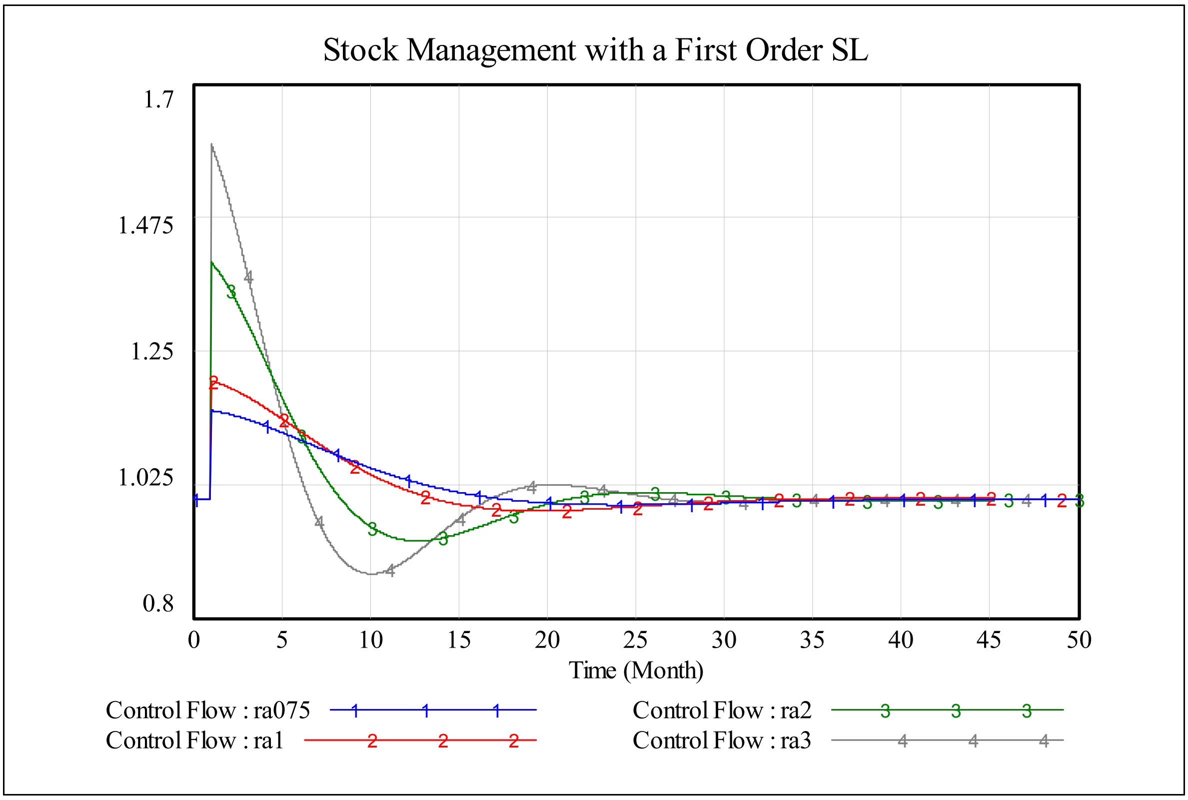

Figure 6. Different control flow dynamics for *ra* values of 0.75, 1, 2, and 3.

The initial value of *Control flow* increases as *ra* increases (i.e., as *sat* decreases), which can be seen in Figure 6 and Figure 7.

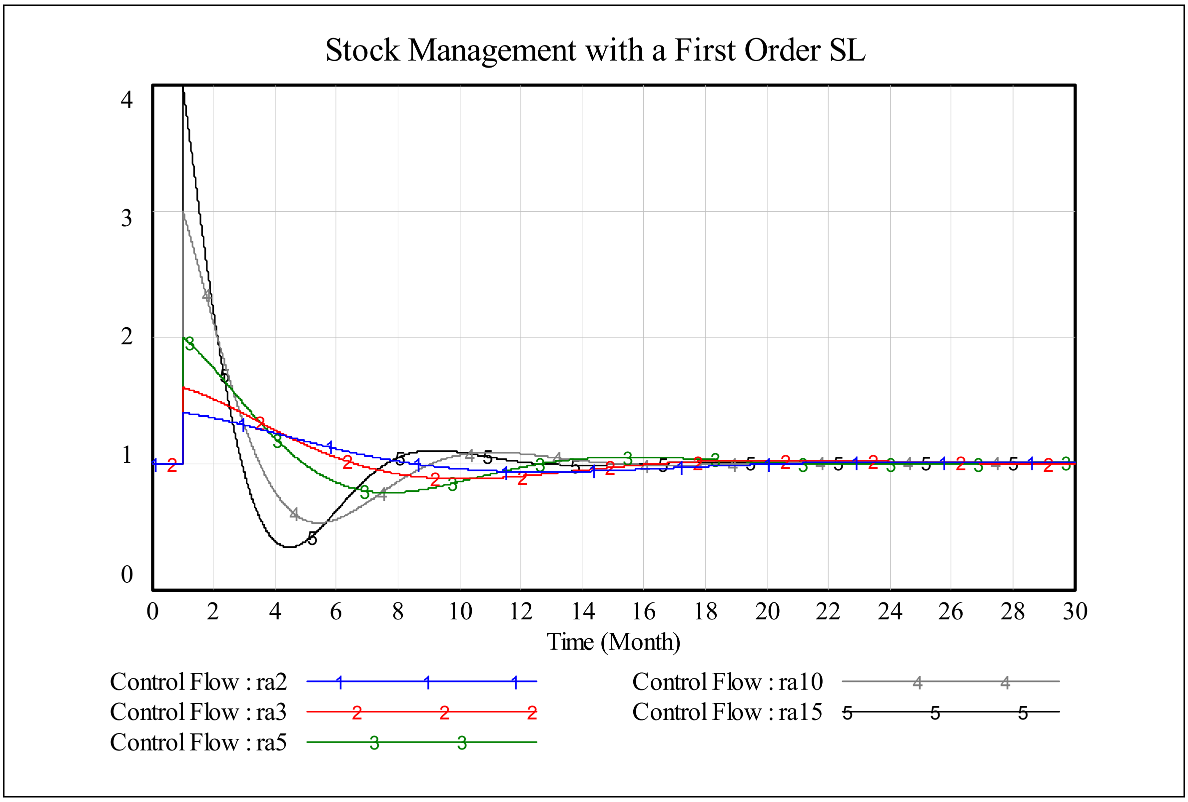

Figure 7. Different control flow dynamics for *ra* values of 2, 3, 5, 10, and 15.

Comparison of the Dynamics with the Literature Suggested Values

For *wsl* equals to 1, stock always shows a goal seeking behavior regardless of the level of aggressiveness in corrections, which is the main reason behind selecting *wsl* as 1 in many studies (Sterman, 1989; Yasarcan and Barlas, 2005a; Yasarcan, 2011). If *wsl* is 1, the approach of the stock to its desired level gets faster as *ra* increases; see stock dynamics in Figure 8 for *ra* values of 1 (Line 2), 5 (Line 3), 90 (Line 4), and 150 (Line 5). However, this increase in the rate of approach becomes negligible after a point; for example, compare the dynamics for *ra* values of 90 (Line 4) and 150 (Line 5) in Figure 8.

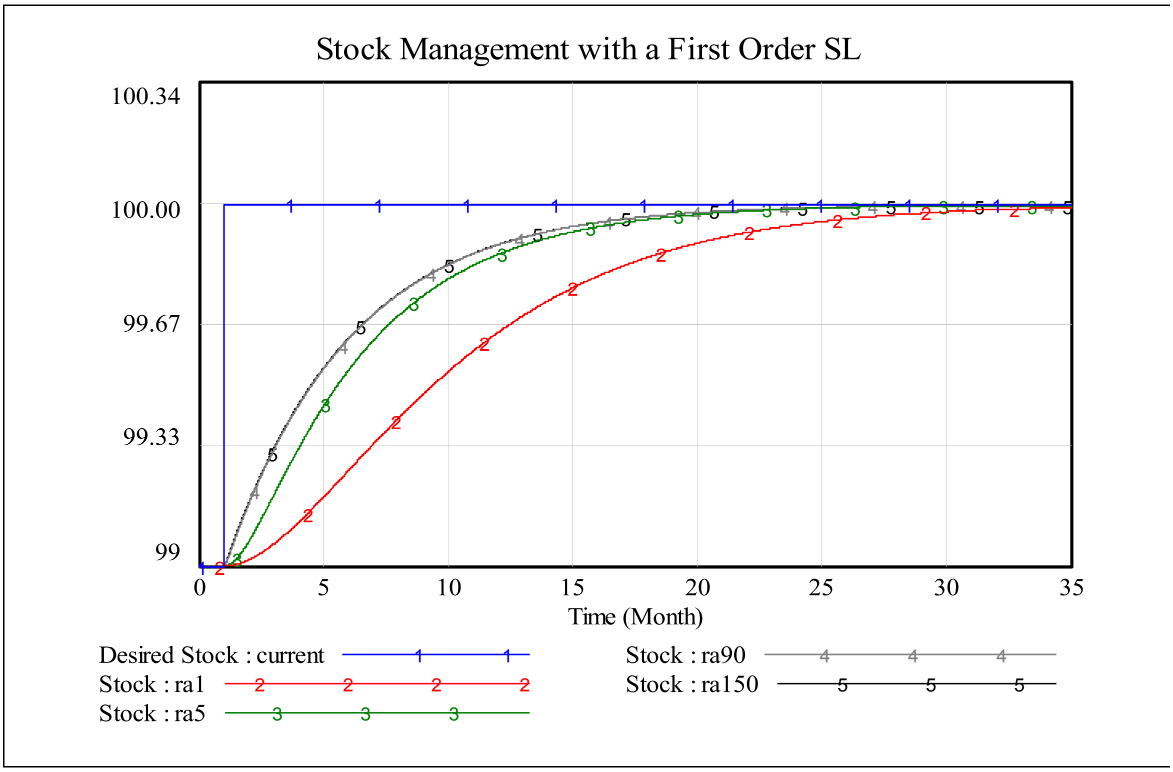

Figure 8. Different stock dynamics for *wsl* equals to 1 and *ra* values of 1, 5, 90, and 150.

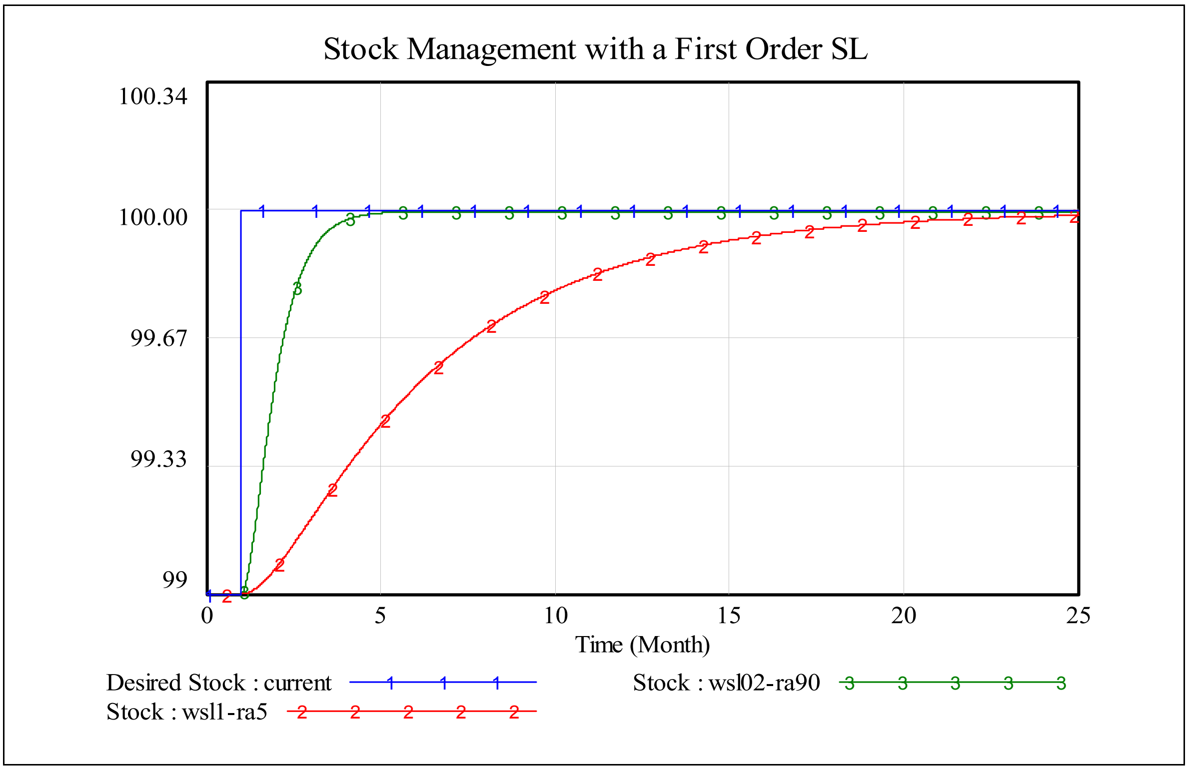

Figure 9. Comparison of different stock dynamics generated by literature suggested values (wsl = 1 and sat = 1) and (wsl = 0.2 and sat = $0.0\overline{5}$).

wsl equals to 1 and *sat* equals to 1 (*ra* equals to 5 in this case) are the decision parameter values suggested by the literature for optimum stock behavior (Sterman, 1989). However, for *wsl* equals to 0.2 and *sat* equals to $0.0\overline{5}$ (*ra* equals to 90), stock approaches to its desired level faster than the literature suggested *wsl* and *sat* values, which can be seen in Figure 9. This result is surprising because ignoring most of the past decisions (i.e., supply line) and making extremely aggressive corrections produce a better behavior than the literature suggested decision parameter values that suggest that supply line must fully be considered and aggressive corrections must be avoided. Surprisingly a fast and stable approach of stock to its desired level is obtained by using wsl = 0.2 and sat = $0.0\overline{5}$.

For *wsl* equals to 0.2 and *sat* equals to $0.0\overline{5}$ (*ra* equals to 90), first value of the control flow is higher than for *wsl* equals to 1 and *sat* equals to 1 (*ra* equals to 5) which is shown in Figure 10. Although the approach of stock to its desired level is faster for wsl = 0.2 and sat = $0.0\overline{5}$, control flow is more smooth for *wsl* equals to 1 and *sat* equals to 1.

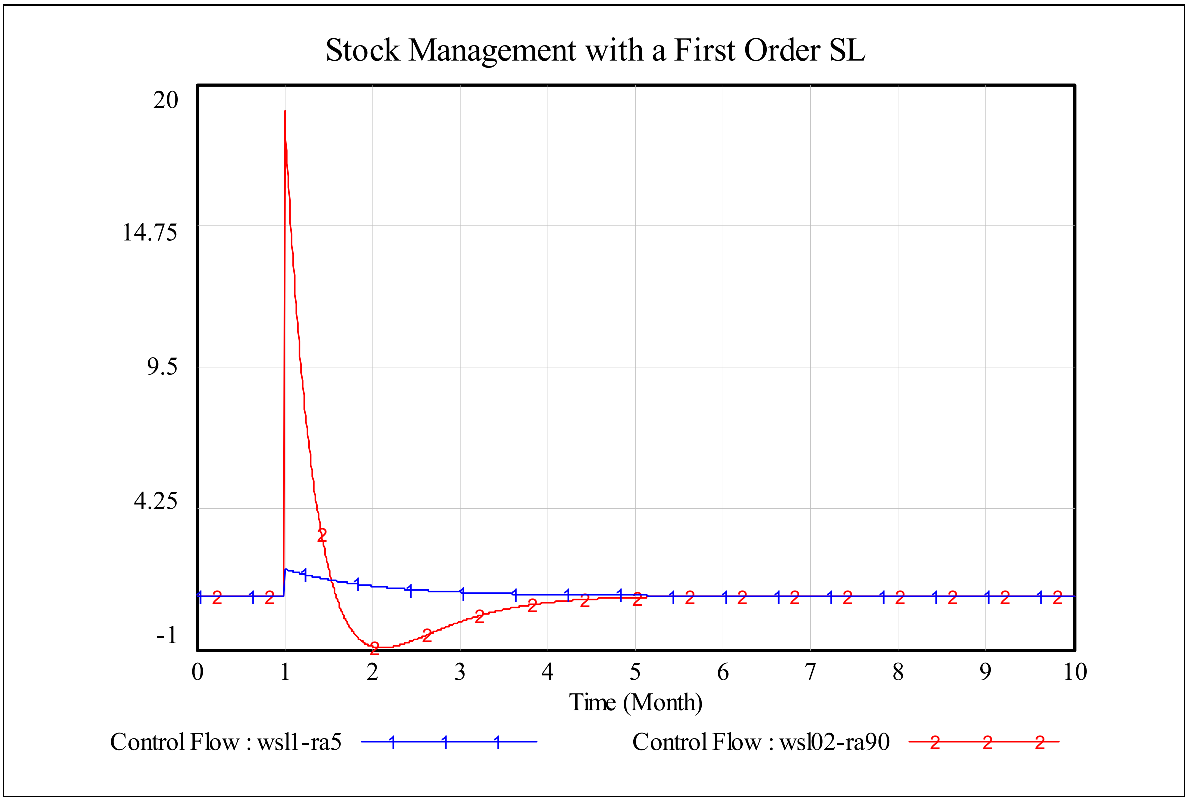

Figure 10. Comparison of different control flow dynamics generated by literature suggested values (wsl = 1 and sat = 1) and (wsl = 0.2 and sat = $0.0\overline{5}$).

For continuous delays, it is known that if *wsl* equals to 1, stock cannot show oscillatory behavior and *sat* can be chosen any value greater than 0 including very low *sat* values, which corresponds to very aggressive corrections. However, *wsl* equals to 1 cannot give as fast approach as *wsl* equals to 0.2 even when *sat* is chosen very small (even if a higher *ra* value for *wsl* = 1 is used). For example, initial sizes of the control flow are the same for *wsl* equals to 1 and *ra* equals to 90 and *wsl* equals to 0.2 and *ra* equals to 90 (Figure 11).

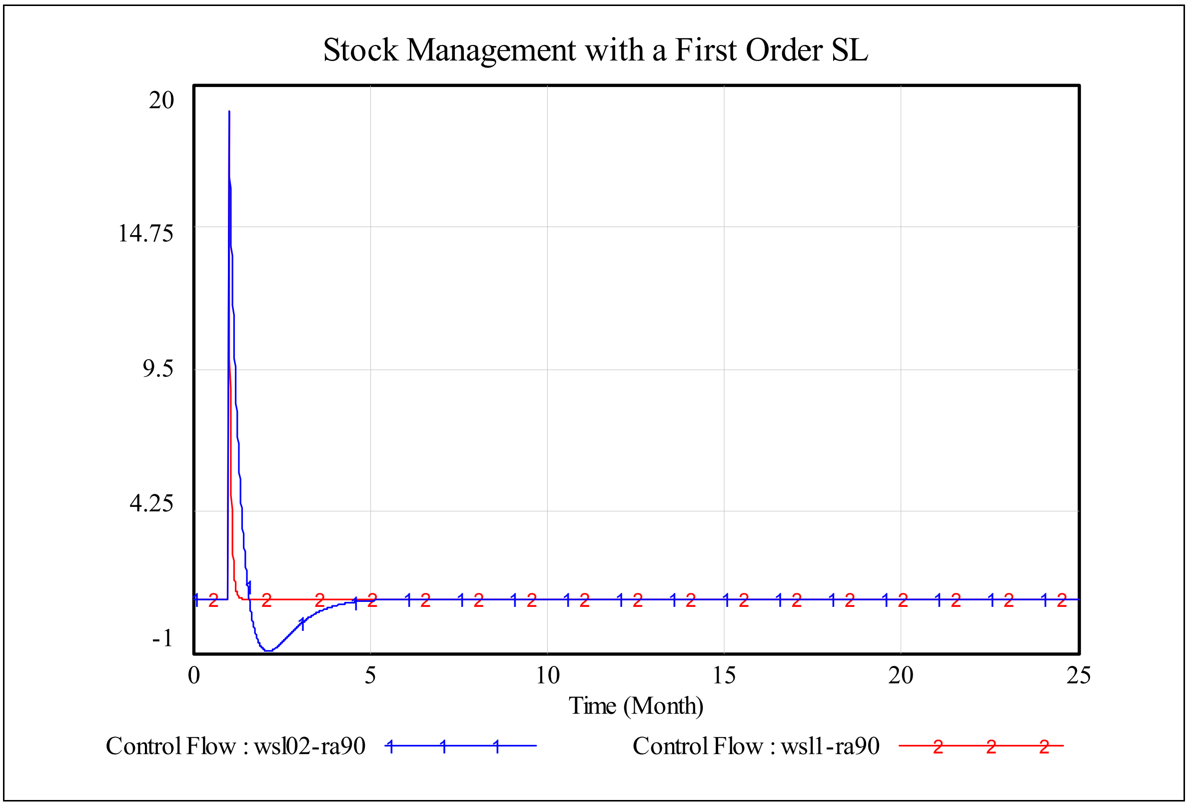

Figure 11. Same initial sizes of control flow for *wsl* equals to 1 and *wsl* equals to 0.2.

For the same initial size of the control flow, *wsl* equals to 0.2 can give faster approach to the desired level of the stock (Figure 12).

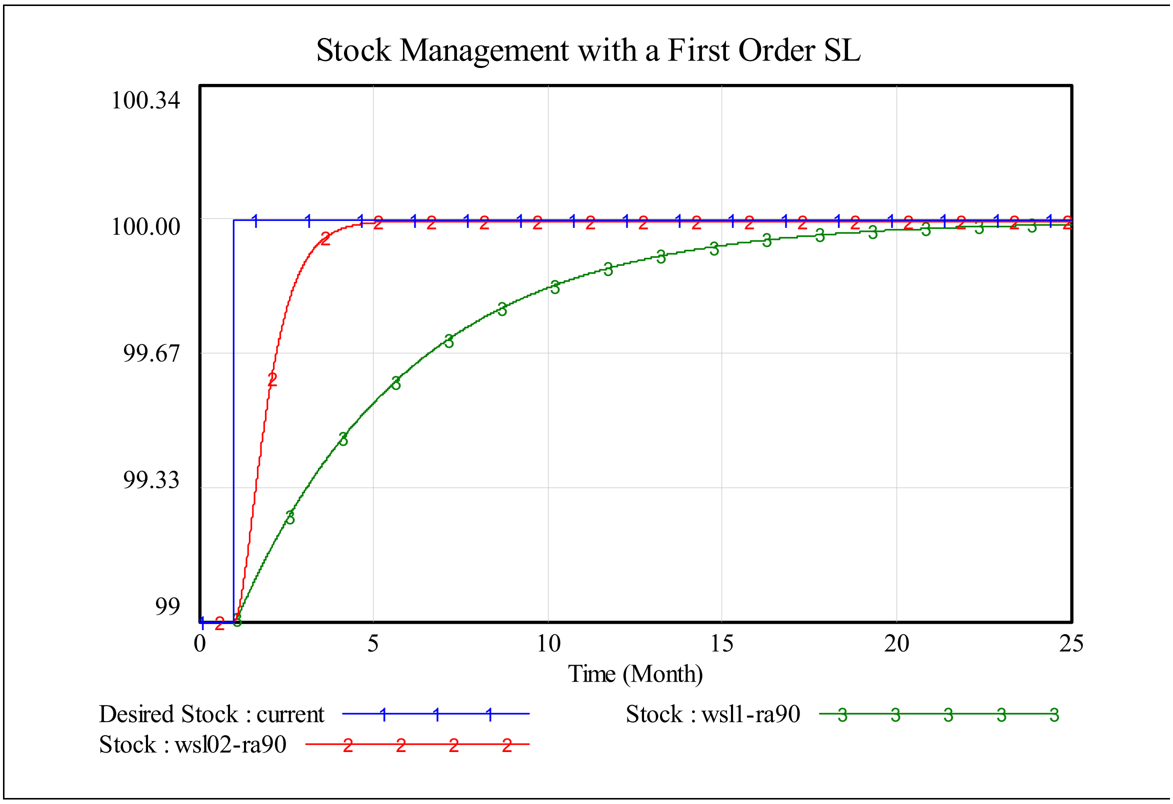

Figure 12. Stock dynamics for the same initial sizes of control flow for *wsl* equals to 1 and *wsl* equals to 0.2.

Conclusion

In this paper, we carried out complete parametric analysis of the stock management problem with a first order supply line delay and obtained the range of values for different characteristic dynamics of stock. The stock of *the stock management structure with a first order supply line delay* can only show either goal seeking behavior or stable oscillations, which confirms the results reported in the literature. However, we also obtained a counterintuitive result for this structure. Assume that the stock is producing stable oscillations for a given set of acquisition delay time (*adt*; the duration of delay between the corrective actions and their eventual results on the stock), weight of supply line (*wsl*; the relative importance given to the supply line compared to the stock), and stock adjustment time (*sat*; the intended time to close the gap between the stock and its desired level) values. It is expected that decreasing *sat* (i.e., increasing aggressiveness in making corrections) would strengthen the existing oscillations. In spite of this intuitive expectation, our results show that this relationship between the strength of oscillations and *sat* exists up to a point and decreasing *sat* below that point, makes oscillations disappear and, surprisingly, stock starts to show a goal seeking behavior; very aggressive corrections can completely eliminate oscillations.

Another surprising result is obtained by ignoring most of the past decisions and making extremely aggressive corrections, which contradicts the literature that suggests that supply line must fully be considered and aggressive corrections must be avoided. We show that a selected set of decision parameters values gives a faster approach of stock to its desired level compared to the literature suggested values without sacrificing the stability in stock dynamics.

The results reported in this paper can be used to obtain goal seeking behavior or stable oscillations for the stock management structure with a first order continuous supply line delay. The results that we obtained are valid for all durations of delay between the corrective actions and their eventual results on the stock.

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APPENDIX: Block Diagram of the Stock Management Structure with a First Order Supply Line Delay

Block diagram of the stock management structure with a first order supply line delay is given in Figure 13, which represents all the details present in the SD model given in this paper (Figure 1 and equations 1-5).

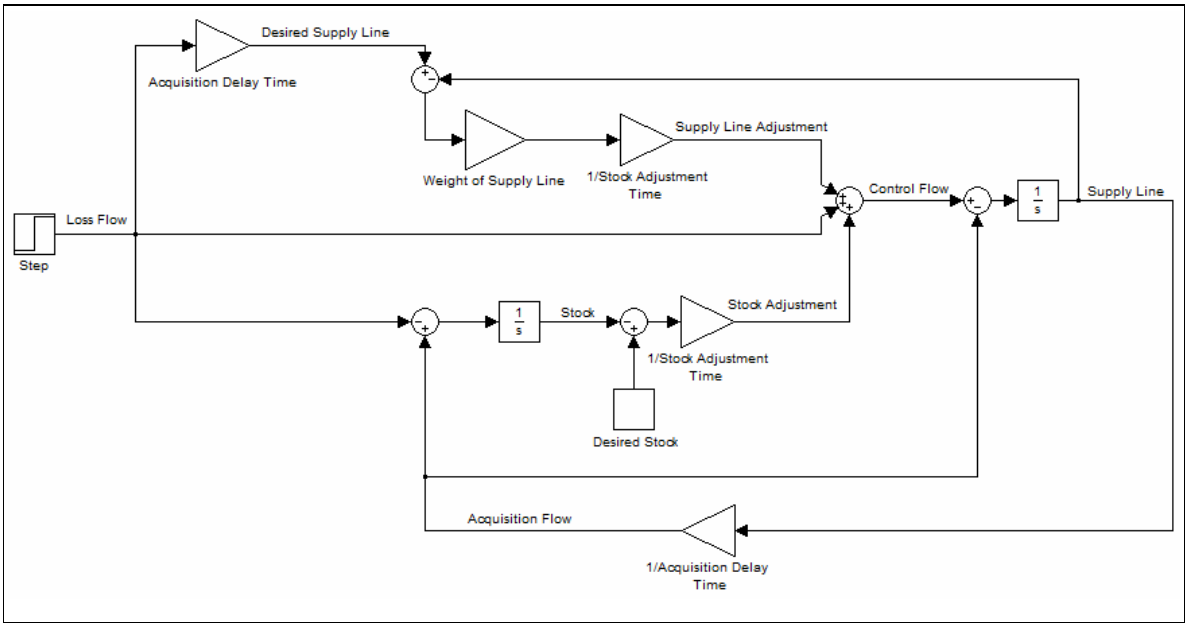

Figure 13. Block diagram of the stock management structure with a first order supply line delay.

For block diagrams of generic SD models, see Mehmet and Yasarcan (2015).