An Enhancement for the Textbook's Models of Natural Resources and Economic Growth

©Alexander V. RYZHENKOV

Economic Faculty
Novosibirsk State University
2 Pirogov street Novosibirsk 630090 Russia
Institute of Economics and Industrial Engineering
Siberian Branch of Russian Academy of Sciences
17 Academician Lavrentiev Avenue Novosibirsk 630090 Russia
E-mail address: ryzhenko@ieie.nsc.ru

Appendix A[1]
Additional information on the models' properties and simulation runs

Summary on three patterns of behavior in NM-1 whereby $\hat{F} = -b$

Table A-1. Attractors in NM-1 for $t \to \infty$ depending on parameter b[▫]

Indicator (variable)	Expression for		
	$b \geq b_c$	$b < b_c$	$b = -n$
Growth rate of net output	$\hat{P}_c = -\beta b + (1-\alpha-\beta)d \leq 0$	$\hat{P}_c < \hat{P}_b = d - \frac{\beta}{1-\alpha}(d+b) < d$	$d > \hat{P}_n = d - \frac{\beta}{1-\alpha}g > \hat{P}_b$
Output-fixed capital ratio	$m_c = 0$	$m_c < m_b = \frac{-\beta b + (1-\alpha-\beta)d}{c(1-\alpha)}$	$m_n = \frac{\beta n + (1-\alpha-\beta)d}{c(1-\alpha)} > m_b$
Growth rate of output-fixed capital ratio	$\hat{m}_c = \hat{P}_c \leq 0$	0	0
Growth rate of output per worker	$\hat{a}_c = -\beta b + (1-\alpha-\beta)d - n \leq 0$	$\hat{a}_c < \hat{a}_b = \hat{P}_b - n$	$\hat{a}_n = \hat{P}_n - n > \hat{a}_b$
Growth rate of output-proved mineral reserves ratio	$d + b > \hat{q}_c = \hat{P}_c + b = -\beta b + (1-\alpha-\beta)d + b > 0 \geq \hat{P}_c$	$\hat{q}_c < \hat{q}_b = \hat{P}_b + b > \hat{P}_b$	$0 < \hat{q}_n = \hat{a}_n < \hat{q}_b$
Output-proved mineral reserves ratio	$q_c = \infty$	$q_b = \infty$	$q_n = \infty$
Unit depletion of proved mineral reserves	$e_c = 0$	$e_b = 0$	$e_n = 0$
Growth rate of unit depletion of proved mineral reserves	$-\hat{q}_c < 0$	$-\hat{q}_b < 0$	$-\hat{q}_n < 0$
Proved mineral reserves-depletion ratio	$\frac{F}{Z} = const = \frac{1}{b}$	$\frac{F}{Z} = const = \frac{1}{b}$	$\frac{F}{Z} = const = \frac{1}{eq} < \infty$

[▫] $d = g + n > 0$, $0 < b_c = (1-\alpha-\beta)d/\beta$.

[1] The decimal sign is comma in this Appendix, contrasting with dot in the main text.

Table A-2. Examples of attractors in NM-1 whereby $\hat{F} = -b$ for $t \to \infty$ depending on b

Indicator (variable)	Indicators' magnitudes for		
	$b = 0{,}2285 > b_c = 0{,}2085$	$b = 0{,}0271 < b_c = 0{,}2085$	$b = -n = -0{,}005$
Growth rate of proved mineral reserves	$\hat{F}_c = -0{,}2285$	$\hat{F}_b = -0{,}0271$	$\hat{F}_n = 0{,}005$
Output-fixed capital ratio	$m_c = 0$	$m_b = 0{,}3554$	$m_n = 0{,}4183$
Output-proved mineral reserves ratio	$q_c = \infty$	$q_b = \infty$	$q_n = \infty$
Growth rate of net output	$\hat{P}_c = -0{,}0017$	$\hat{P}_b = 0{,}0179$	$\hat{P}_n = 0{,}0211$
Growth rate of output per worker	$\hat{a}_c = -0{,}0067$	$\hat{a}_b = 0{,}0129$	$\hat{a}_n = 0{,}0161$
Growth rate of output-proved mineral reserves ratio	$\hat{q}_c = 0{,}2267$	$\hat{q}_b = 0{,}0450$	$\hat{q}_n = 0{,}0161$
Growth rate of fixed capital	$\hat{K}_c = 0$	$\hat{K}_b = 0{,}0179$	$\hat{K}_n = 0{,}0211$
Growth rate of unit depletion of proved mineral reserves	$\hat{e}_c = -0{,}2267$	$\hat{e}_b = -0{,}0450$	$\hat{e}_n = -0{,}0161$
Unit depletion of proved mineral reserves	$e_c = 0$	$e_b = 0$	$e_n = 0$
Proved mineral reserves-depletion ratio	$\frac{F}{Z} = const = 4{,}38$	$\frac{F}{Z} = const = 36{,}87$	$\frac{F}{Z} = const = 10{,}99$

Illustrations on unbalanced growth in NM-1 for $m_0 > m_b$, $b = 0{,}0271 < b_c = 0{,}2085$

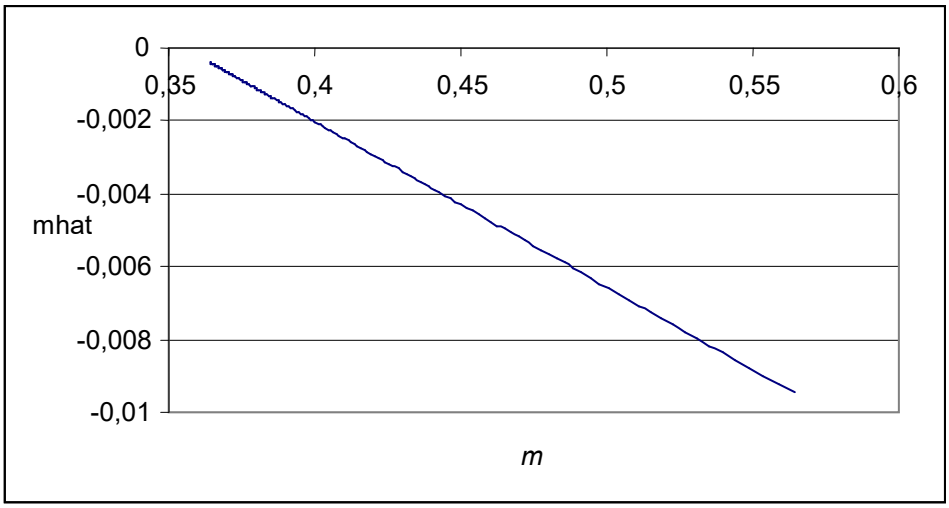

Figure A-1 – A linear negative dependence of the rate of change of output-fixed capital ratio on this ratio m, 1991–2161

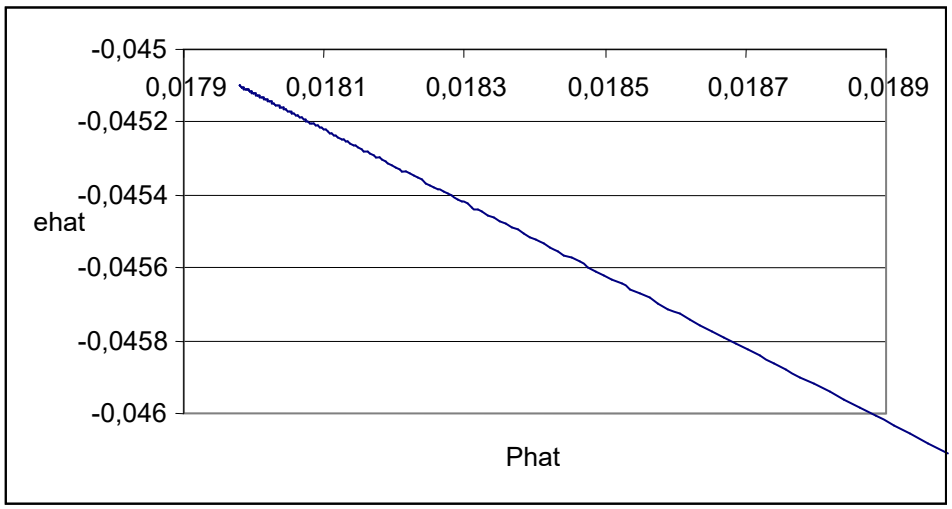

Figure A-2 – A linear negative dependence of the rate of change of unit depletion of proved mineral reserves on the growth rate of net output, 1991–2161

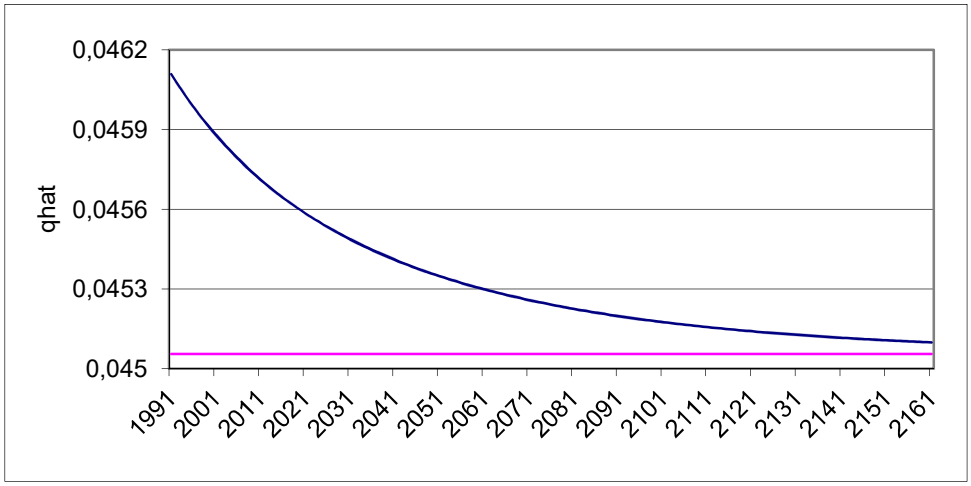

Figure A-3 – The growth rate of output-proved mineral reserves ratio on the transient to its attractor beyond real barriers, 1991–2161

Illustrations on unbalanced growth in NM-1 for $m_0 > m_n$, $b = -n = -0{,}005$

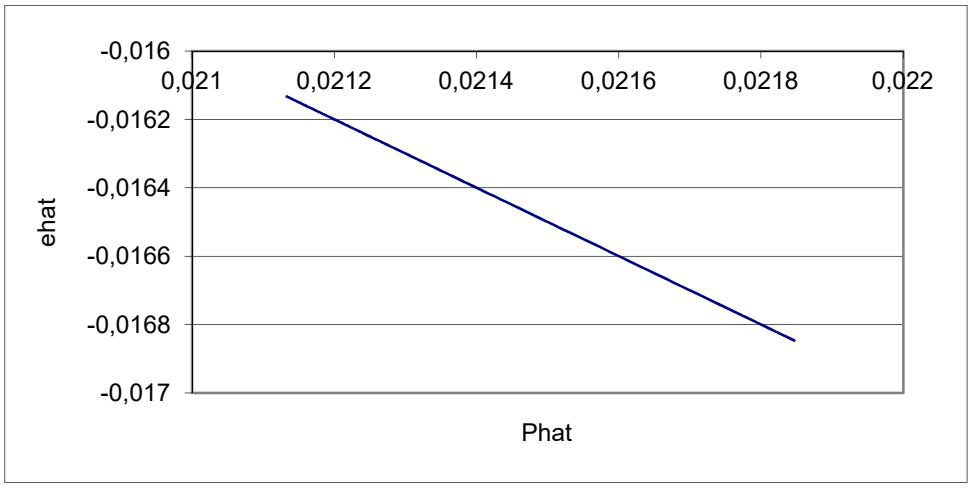

Figure A-4 – A linear negative dependence of the rate of change of unit depletion of proved mineral reserves on the growth rate of net output, 1991–2161

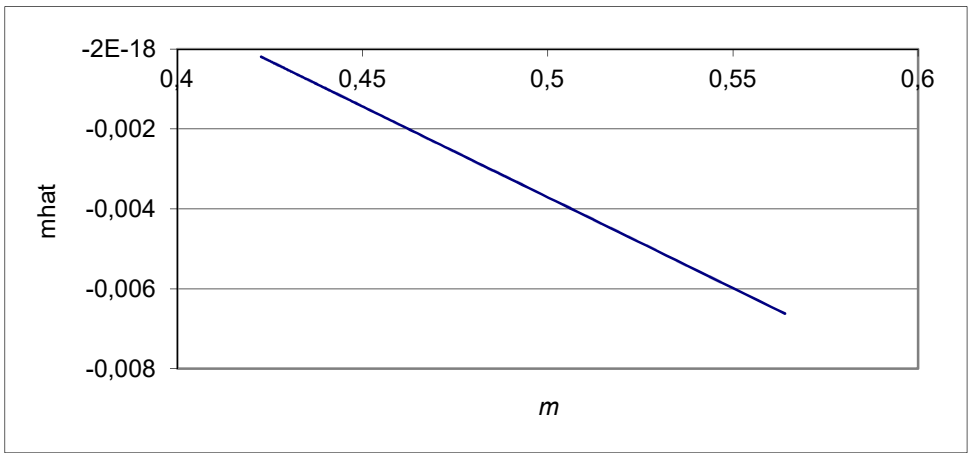

Figure A-5 – A linear negative dependence of the rate of change of output-fixed capital ratio on this ratio *m*, 1991–2161

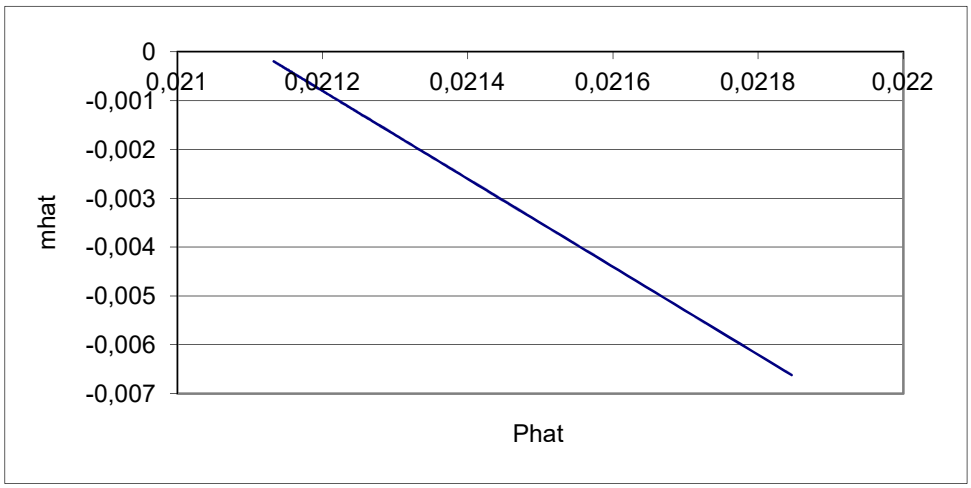

Figure A-6 – A linear negative dependence of the rate of change of output-fixed capital ratio on the growth rate of net output, 1991–2161

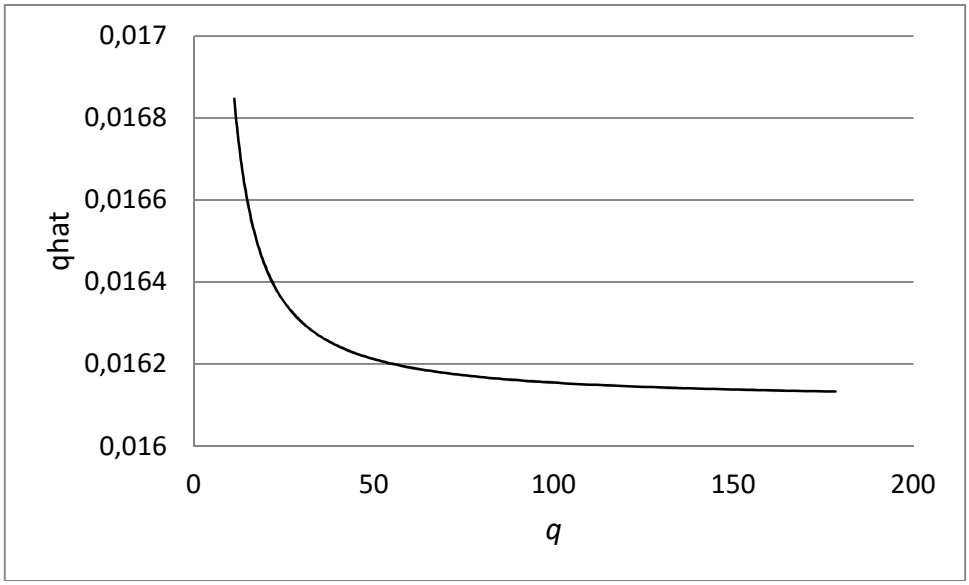

Figure A-7 – A non-linear dependence of the growth rate of output-proved mineral reserves ratio on this ratio *q* beyond real barriers, 1991–2161

Illustration on tension between owners of proved reserves and other capitalists that tends to violate the Hotelling rule in NM-1

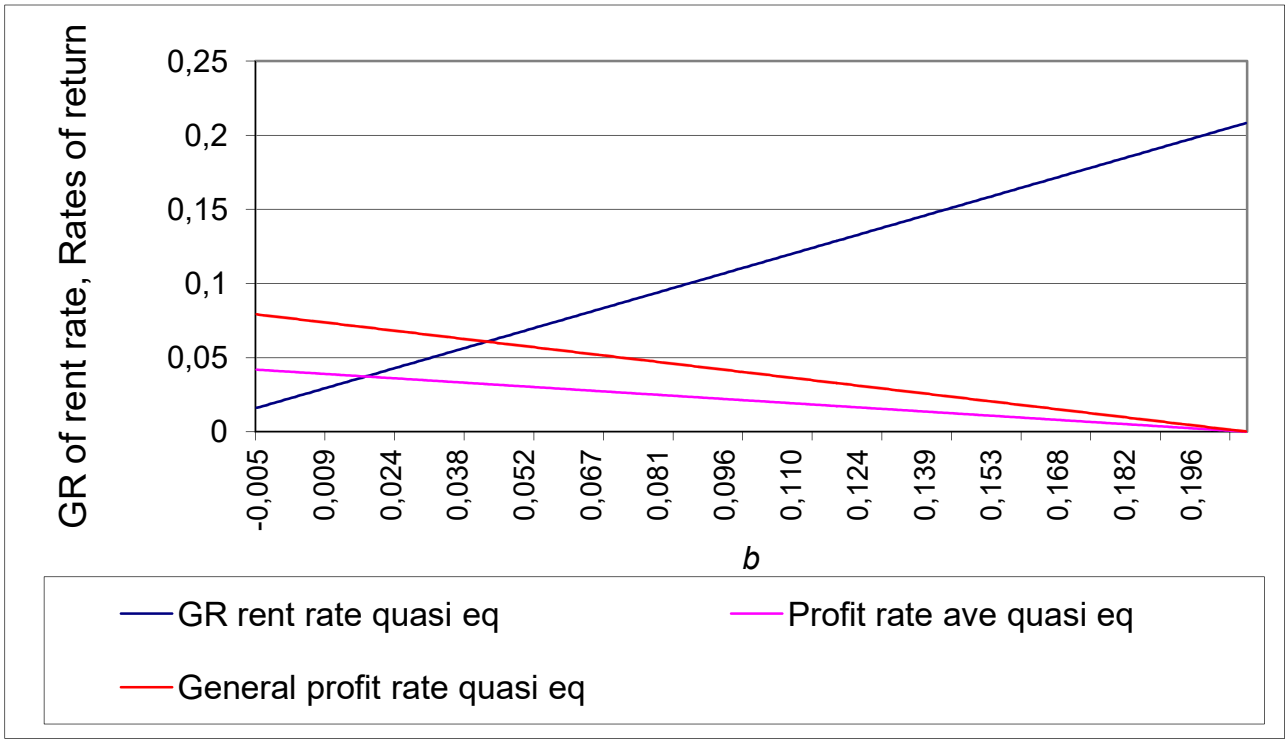

Figure A-8 – Positive linear dependence of the quasi-stationary growth rate of rent rate on parameter b, negative linear dependence of the quasi-stationary average and general profit rates on parameter b where $b_n = -n = -0{,}005 < b_H = 0{,}0184 < b_g = 0{,}0446 < b_c = 0{,}2085$, $b \in [b_n, b_c]$

Illustration on quasi-stationary economic growth rate depending on parameter b in NM-1

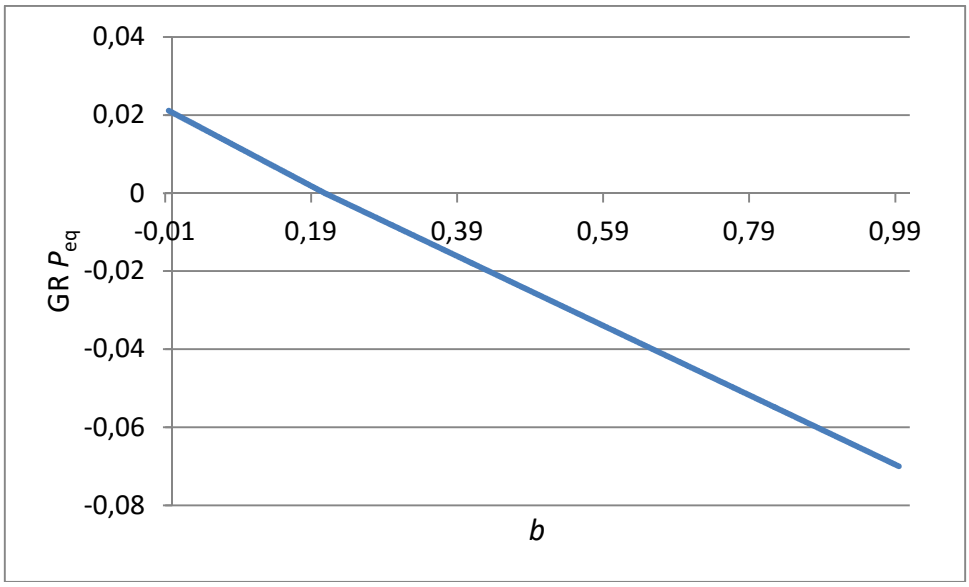

Figure A-9 – A weak negative dependence of the quasi-stationary growth rate of net output P on parameter b, where $b \in [b_n, 0{,}990]$, $b_n = -n = -0{,}005$ and critical value $b_c = 0{,}2085$

Table A-3. Attractors in basal NM-2 and in its damaged form depending on y and $\hat{F}$

Indicator (variable)	Expressions for y and $\hat{F}$		
	$y = 0$, $\hat{F} = -eq < 0$ for $t \to t_c - \Delta$	$y = const > e_0 > e_1$, $\hat{F} = (y - e)q > 0$ for $t \to \infty$	$y = e + n/q > e > e_1$, $\hat{F} = n \geq 0$ for $t \to \infty$
	(1)	(2)	(3)
Growth rate of net output	$\hat{P} \to \hat{P}_{t_c-\Delta} << 0$	$\hat{P}_e = d > \hat{P}_n$	$\hat{P}_n = d - \frac{\beta}{1-\alpha} g > \hat{P}_b$
Growth rate of fixed capital	$\hat{K}_a > 0$	$\hat{K}_e = d$	$\hat{K}_n = \hat{P}_n$
Growth rate of proved mineral reserves	$\hat{F} \to \hat{F}_{t_c-\Delta} << 0$	$\hat{F}_e = d > \hat{P}_n$	$\hat{F}_n = n < \hat{F}_e$
Output-fixed capital ratio	$m \to m_{t_c-\Delta} < m_0$	$m_e = d/c > m_n$	$m_n = \frac{\beta n + (1-\alpha-\beta)d}{c(1-\alpha)} > m_b$
Growth rate of output-fixed capital ratio	$\hat{m} \to \hat{m}_{t_c-\Delta} = << 0$	$\hat{m}_e = 0$	$\hat{m}_n = 0$
Growth rate of output per worker	$\hat{a} \to \hat{a}_{t_c-\Delta} << 0$	$\hat{a}_n < \hat{a}_e = g$	$\hat{a}_n = g - \frac{\beta}{1-\alpha} g > \hat{a}_b$
Growth rate of output-proved mineral reserves ratio	$\hat{q} \to \hat{q}_{t_c-\Delta} >> 0$	$\hat{q}_e = 0$	$0 < \hat{q}_n = \hat{a}_n < \hat{q}_b$
Output-proved mineral reserves ratio	$q \to q_{t_c-\Delta} >> q_0$	$q_e = \frac{1}{f_e} = \frac{d}{y - e_1}$	$q_n = \infty$
Unit depletion of proved mineral reserves	$e \to e_{t_c-\Delta} > e_0$	$e_e = e_1 > 0$	$e_n = e_e = e_1 > 0$
Growth rate of unit depletion of proved mineral reserves	$\hat{e} \to \hat{e}_{t_c-\Delta} >> 0$	0	0
Proved mineral reserves-depletion ratio	$\frac{F}{Z} \to \frac{F_{t_c-\Delta}}{Z_{t_c-\Delta}} << \frac{F_0}{Z_0}$	$\frac{F_e}{Z_e} = \frac{1}{e_1 q_n} = \frac{f_e}{e_1} = \frac{y - e_1}{e_1}\frac{1}{d}$	$\frac{F_n}{Z_n} = \frac{1}{e_1 q_n} = 0$
Gross rent per resource unit extracted (upper frontier to potential unit royalty)	$\frac{R}{Z} \to \frac{\beta}{e_{t_c-\Delta}} < \frac{\beta}{e_0}$	$\frac{R_e}{Z_e} = \frac{\beta}{e_1} > \frac{\beta}{e_0}$	$\frac{R_n}{Z_n} = \frac{\beta}{e_1} > \frac{\beta}{e_0}$

Note. Two possible patterns of evolution in basal NM-2: (1) – aggravation mode for $t \leq t_c - \Delta$, (2) – sustainable development. The additional pattern (3) in damaged NM-2 is not feasible in the long term.

Table A-4. Examples of attractors in basal NM-2 and in its damaged form for $t \to \infty$ compared with attractors in NM-1 under the Hotelling rule $\hat{q}_H = \alpha\, m_H$

Indicator (variable)	Indicators for			
	Basal NM-2 with $y = 0{,}0114 > e_0 > e_1 > 0$, $\hat{F} = (y - e)q > 0$	Basal NM-2 with $y_e = 0{,}0503$	Damaged NM-2 with $y = e + n/q > e > e_1$, $\hat{F} = n \geq 0$	NM-1 with $b_H = 0{,}0184$
Output-fixed capital ratio	$m_e = 0{,}4534$	$m_e = 0{,}4534$	$m_n = 0{,}4183$	$m_H = 0{,}3723$
Average profit rate	$\alpha m_e = 0{,}0453$	$\alpha m_e = 0{,}0453$	$\alpha m_n = 0{,}0418$	$\alpha m_H = 0{,}0372$
Output-proved mineral reserves ratio	$q_e = 3{,}787$	$q_e = 0{,}5094$	$q_n = \infty$	$q_H = \infty$
Gross rent rate	$\beta\, q_e = 0{,}3371$	$\beta\, q_e = 0{,}0453$	$\beta\, q_n = \infty$	$\beta\, q_H = \infty$
Net rent rate	$(\beta - e_1)\, q_e = 0{,}3166$	$(\beta - e_1)\, q_e = 0{,}0426$	$(\beta - e_n)\, q_n = \infty$	$(\beta - e)\, q_H = \infty$
General profit rate	$(\alpha + \beta)/(s_e + f_e) = 0{,}0765$	$(\alpha + \beta)/(s_e + f_e) = 0{,}0453$	$(\alpha + \beta)/(s_n + f_n) = 0{,}0791$	$(\alpha + \beta)/(s_H + f_H) = 0{,}0703$
Growth rate of net output	$\hat{P}_e = 0{,}0229$	$\hat{P}_e = 0{,}0229$	$\hat{P}_n = 0{,}0211$	$\hat{P}_H = 0{,}0187$
Growth rate of fixed capital	$\hat{K}_e = 0{,}0229$	$\hat{K}_e = 0{,}0229$	$\hat{K}_n = 0{,}0211$	$\hat{K}_H = 0{,}0187$
Growth rate of output per worker	$\hat{a}_e = 0{,}0179$	$\hat{a}_e = 0{,}0179$	$\hat{a}_n = 0{,}0161$	$\hat{a}_H = 0{,}0137$
Growth rate of proved mineral reserves	$\hat{F}_e = d = 0{,}0229$	$\hat{F}_e = 0{,}0229$	$\hat{F}_n = 0{,}005$	$\hat{F}_H = -0{,}0184$
Growth rate of output-proved mineral reserves ratio	$\hat{q}_e = 0$	$\hat{q}_e = 0$	$\hat{q}_n = 0{,}0161$	$\hat{q}_H = 0{,}0372$
Growth rate of unit depletion of proved mineral reserves	$\hat{e}_e = 0$	$\hat{e}_e = 0$	$\hat{e}_n = 0$	$\hat{e}_H = -0{,}0372$
Unit depletion of proved mineral reserves	$e_1 = 0{,}0054$	$e_1 = 0{,}0054$	$e_1 = 0{,}0054$	$e_H = 0$
Proved mineral reserves-depletion ratio	$\frac{F_e}{Z_e} = 48{,}88$	$\frac{F_e}{Z_e} = 363{,}52$	$\frac{F_n}{Z_n} = 0$	$\frac{F_H}{Z_H} = 54{,}20$

Note. Systematically higher gross and net rates of rent than average profitability in NM-1 and in NM-2 with $e_1 < y < y_e$ are theoretically possible because of absolute rent in the non-renewable resource sector in both models. The gross and net rates of rent are unbounded in NM-1 contrasting with the bounded gross and net rates of rent in NM-2 (except aggravation mode below).

Illustrations on sustainable development in NM-2 for $m_0 > m_e$, $y = 0{,}0114 > e_0 = 0{,}0082 > e_1 = 0{,}0054$

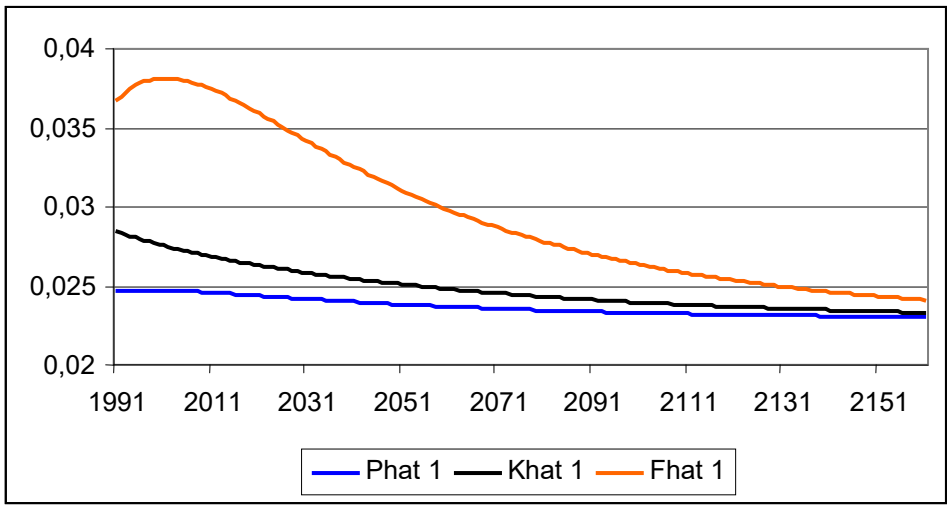

Figure A-10 – The growth rates of net output P, of fixed capital K and of proved mineral reserves F on the transient to their common attractor d, 1991–2161

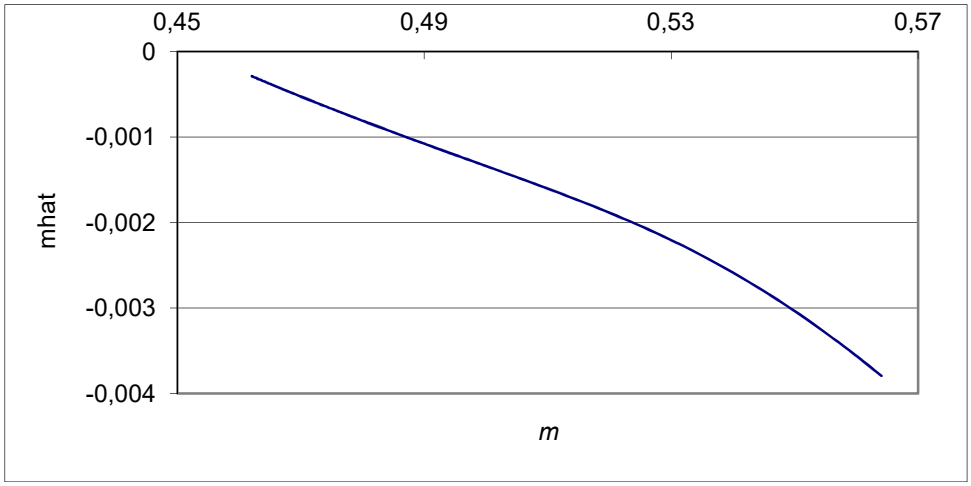

Figure A-11 – A non-linear negative dependence of rate of change of output-fixed capital ratio on this ratio m, 1991–2161

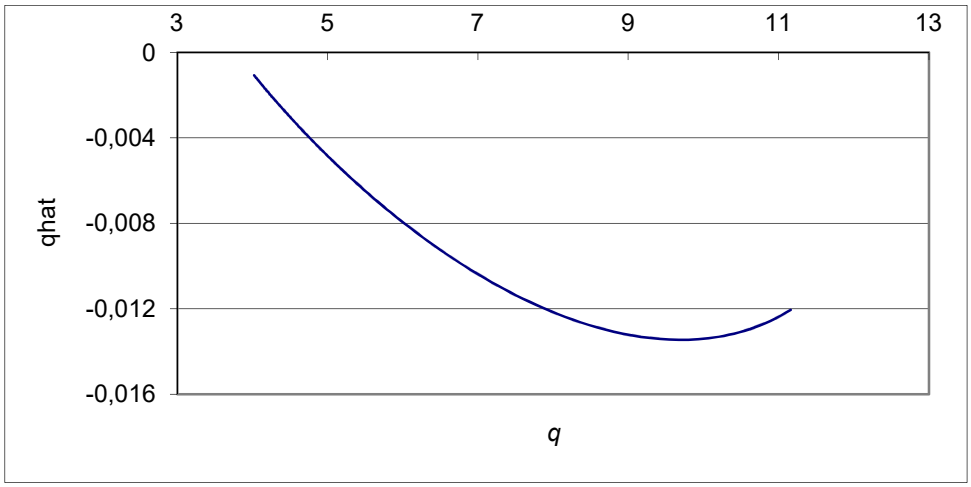

Figure A-12 – A non-linear dependence of rate of change of output-proved mineral reserves ratio on this ratio q, 1991–2161

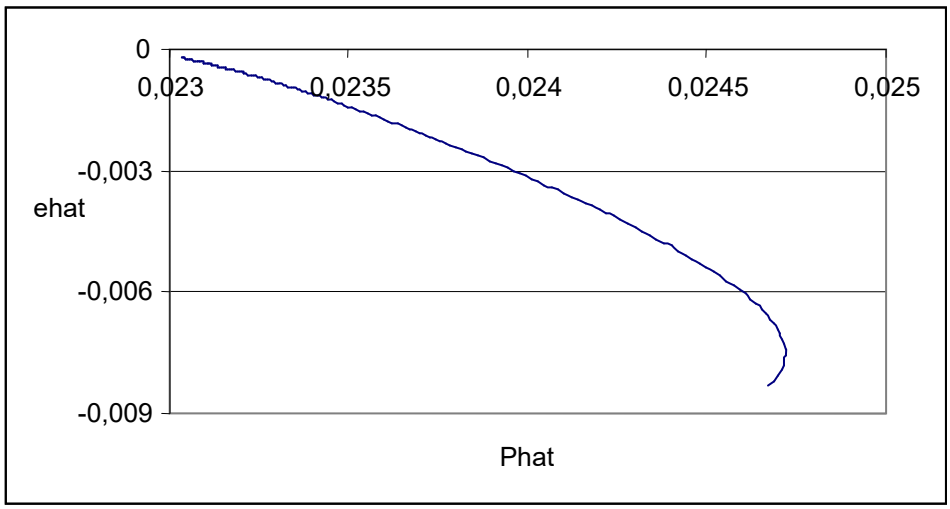

Figure A-13 – A non-linear dependence of rate of change of unit depletion of proved mineral reserves *e* on growth rate of net output *P*, 1991–2161

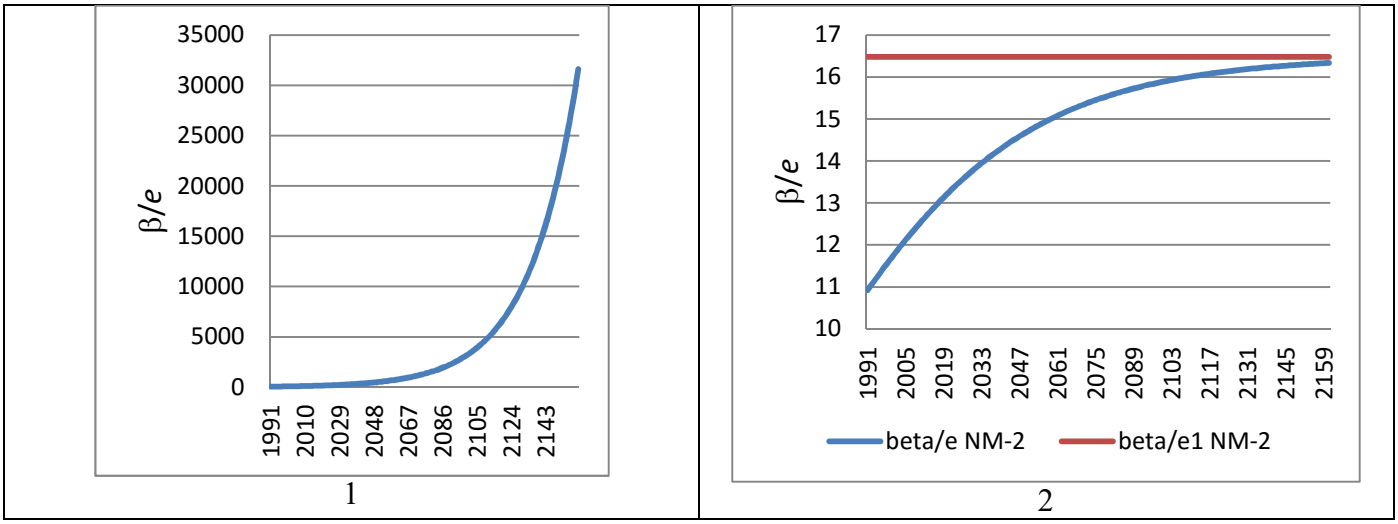

Figure A-14 – Rent per unit of resource extracted β/*e*, 1991–2161.
Panel 1 – in NM-1 under the asymptotic Hotelling rule (unbounded potential unit royalty).
Panel 2 – bounded in NM-2 on the transient to stationary state
for $y_e > y > e_1$ (upper frontier to potential unit royalty)

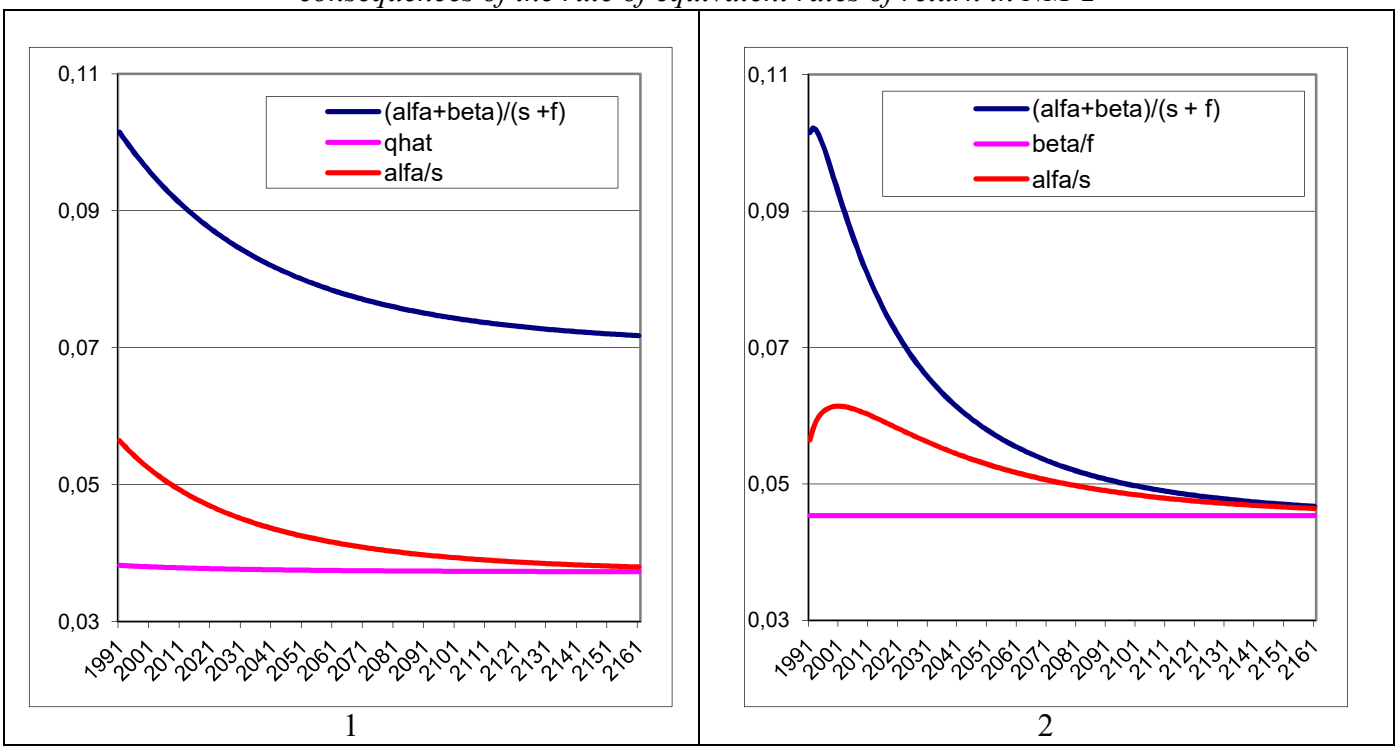

Figure A-15 – Asymptotic equilibrium conditions in NM-1 and in NM-2, 1991–2161.
Panel 1 – the Hotelling rule valid asymptotically in NM-1 for b_H;
Panel 2 – the rule of equivalent rates of return valid asymptotically in NM-2

for $y_e = 0{,}0503$ and $k = 0{,}5046 > \dfrac{\alpha}{1-\alpha-\beta} = 0{,}1233$

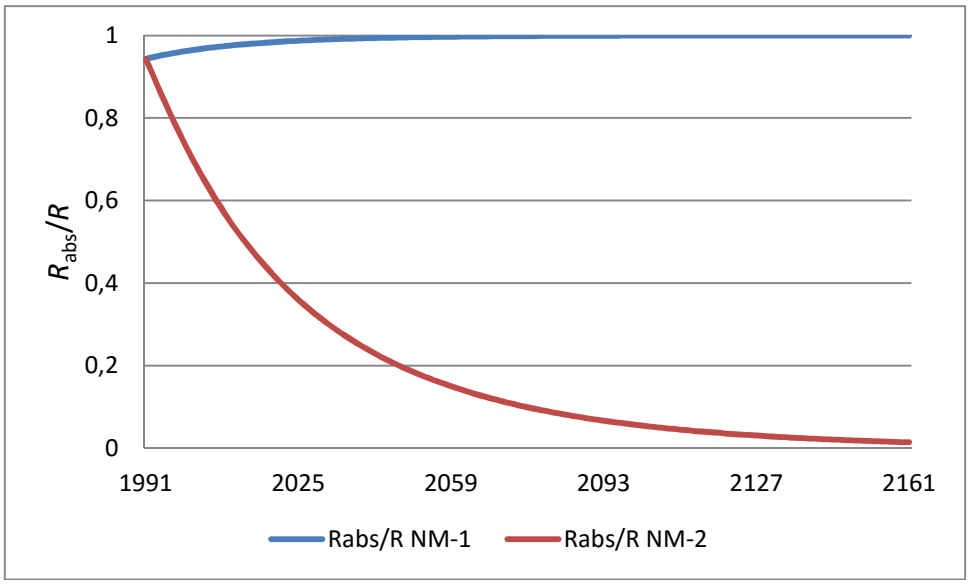

Figure A-16 – The shares of absolute rent in gross rent R_{abs}/R in NM-1 for b_H and in NM-2 for y_e, 1991–2161

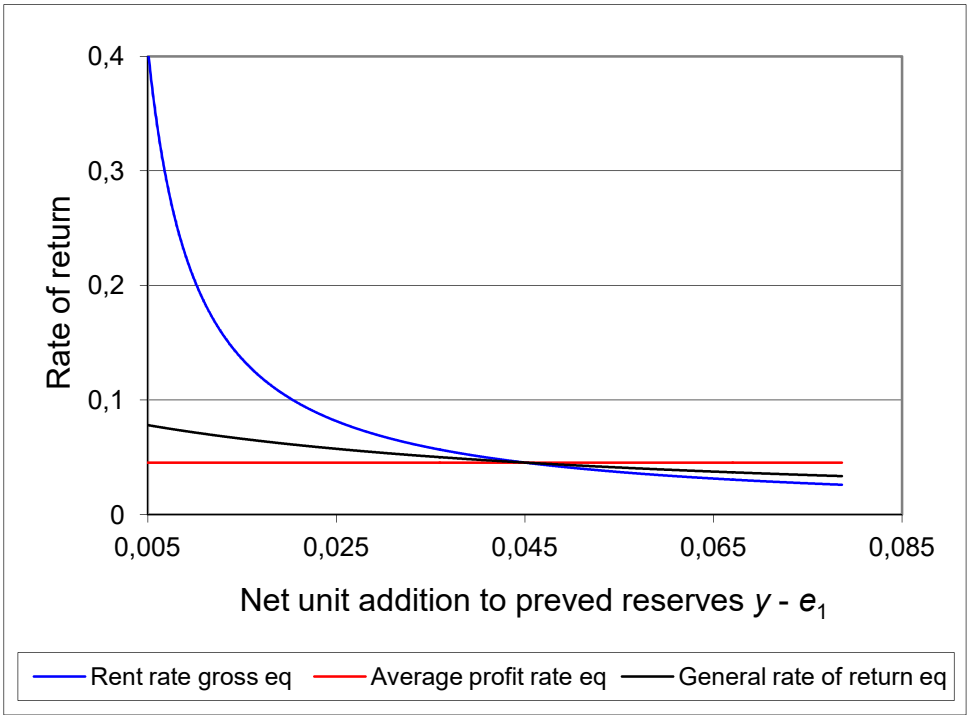

Figure A-17 – The stationary rates of return depending on net unit addition to proved reserves $y - e_1$ in NM-2

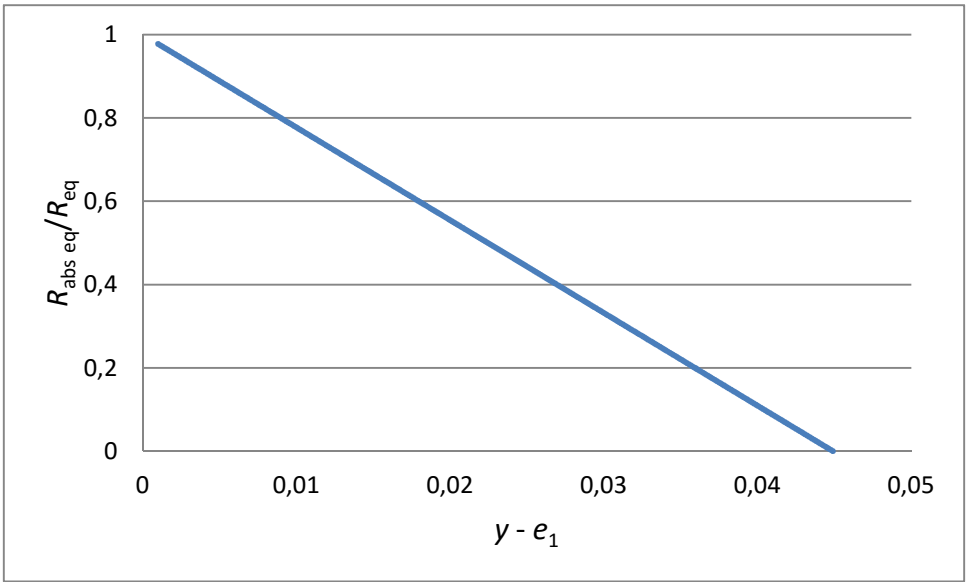

Figure A-18 – The stationary share of absolute rent in gross rent $R_{\text{abs eq}}/R_{\text{eq}}$ depending on net unit addition to proved reserves $y - e_1 < y_e - e_1$ in NM-2

Illustrations
on moderate economic slow-down and decline in NM-1 for $m_0 > m_b$, $b = 0{,}2285 > b_c = 0{,}2085$,
and on aggravation mode (break-down) in NM-2 for $m_0 > m_e$, $e_0 = 0{,}02047 > e_1 = 0{,}0054 > y = 0$

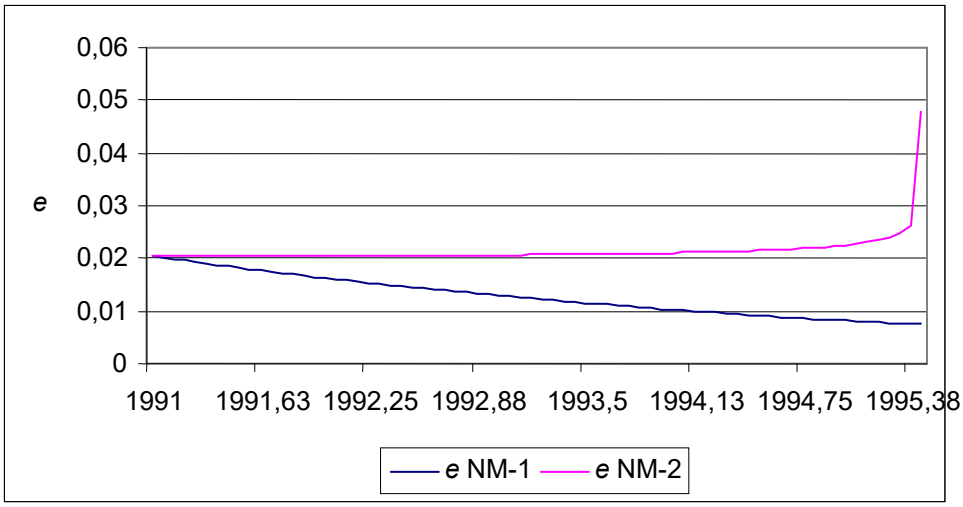

Figure A-19 – Unit depletion of proved mineral reserves *e* in NM-1 and NM-2, 1991–1995.38

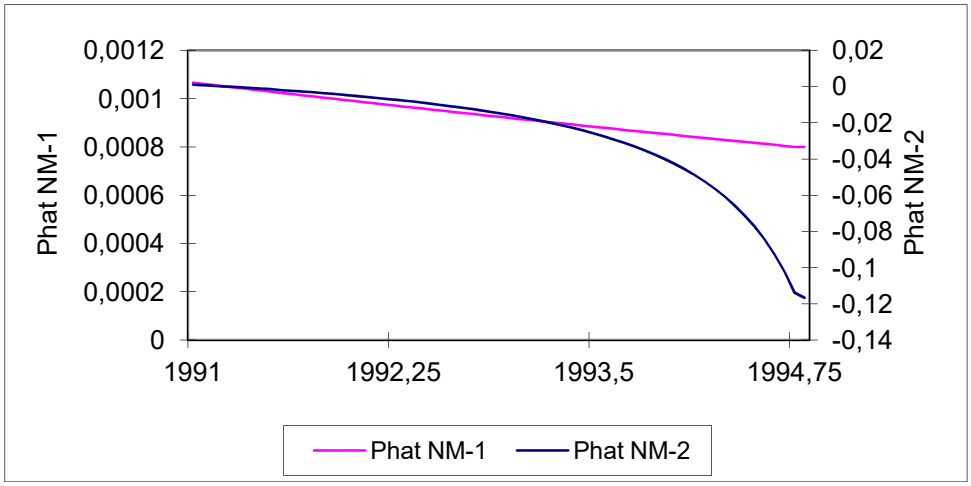

Figure A-20 – The rates of change of net output *P* in NM-1 and NM-2, 1991–1994,75

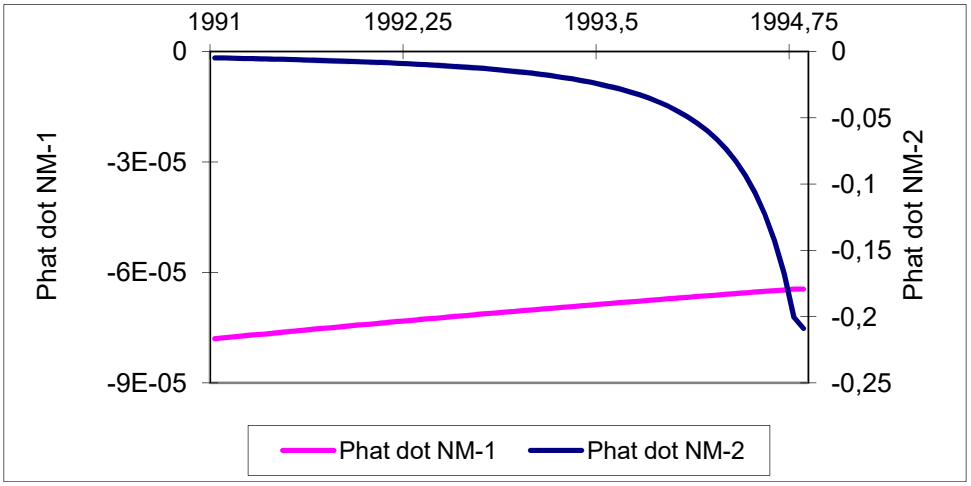

Figure A-21 – The first derivatives of growth rate of net output *P* with respect to time in NM-1 and NM-2, 1991–1994,75

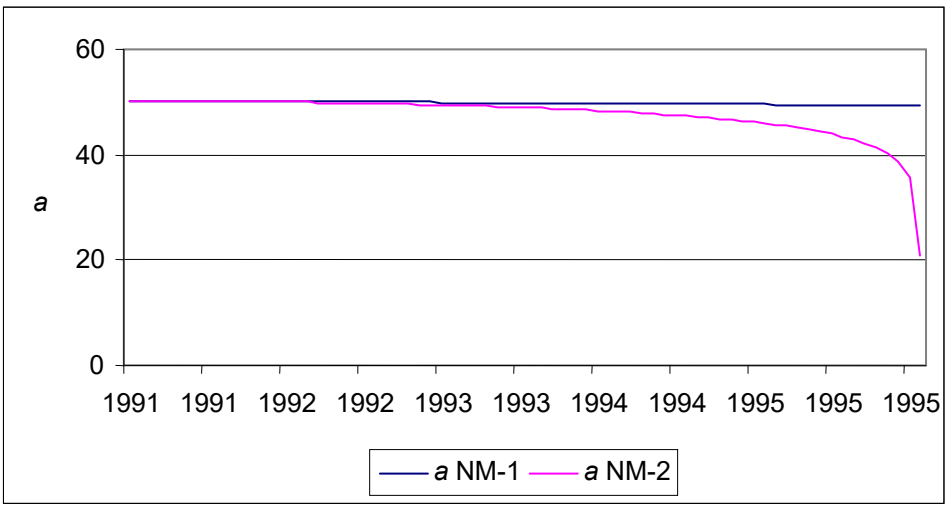

Figure A-22 – Output per worker a in NM-1 and NM-2, 1991–1995,38

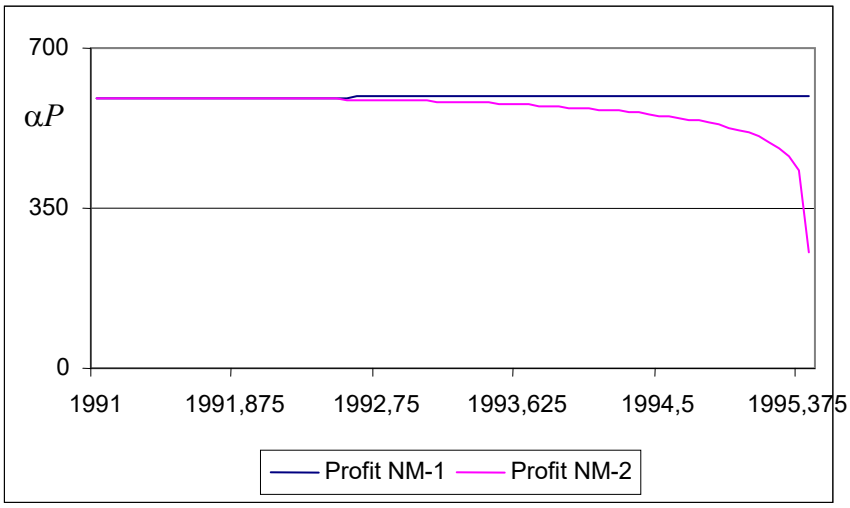

Figure A-23 – Profit αP in NM-1 and NM-2, 1991–1995,38

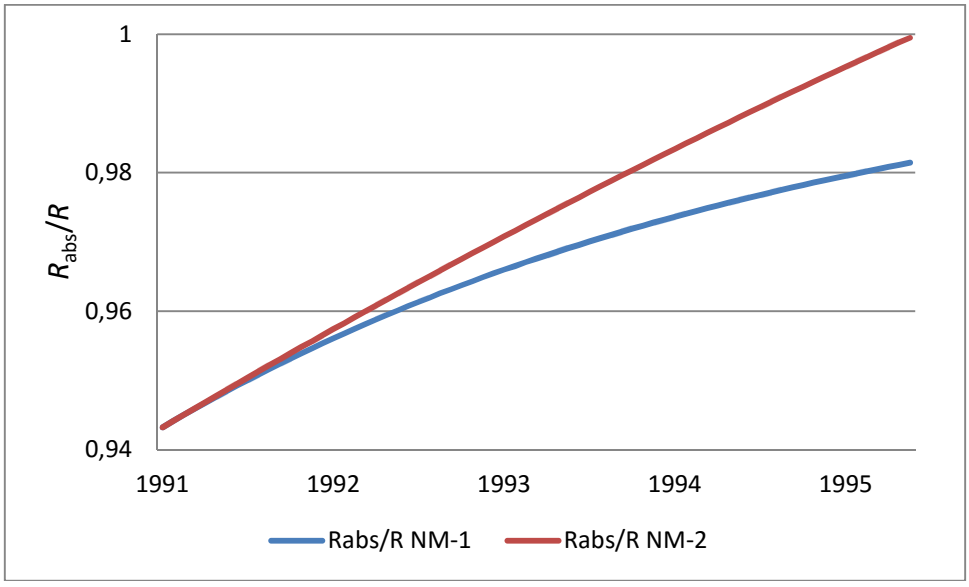

Figure A-24 – The shares of absolute rent in gross rent $R_{abs}/R = 1 - \alpha m/\beta q$ in NM-1 and NM-2, 1991–1995,38

Illustrations on unfeasible growth in damaged NM-2 for $m_0 > m_e$, $y = e + n/q > e > e_1$

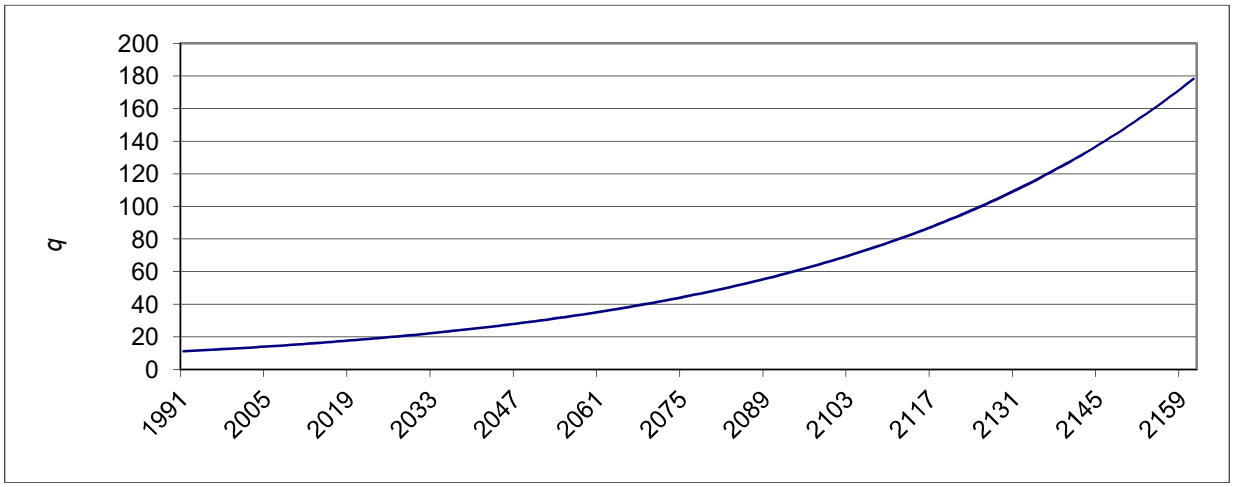

Figure A-25 – The uninterrupted growth of output-proved mineral reserves ratio q beyond real barriers, 1991–2161

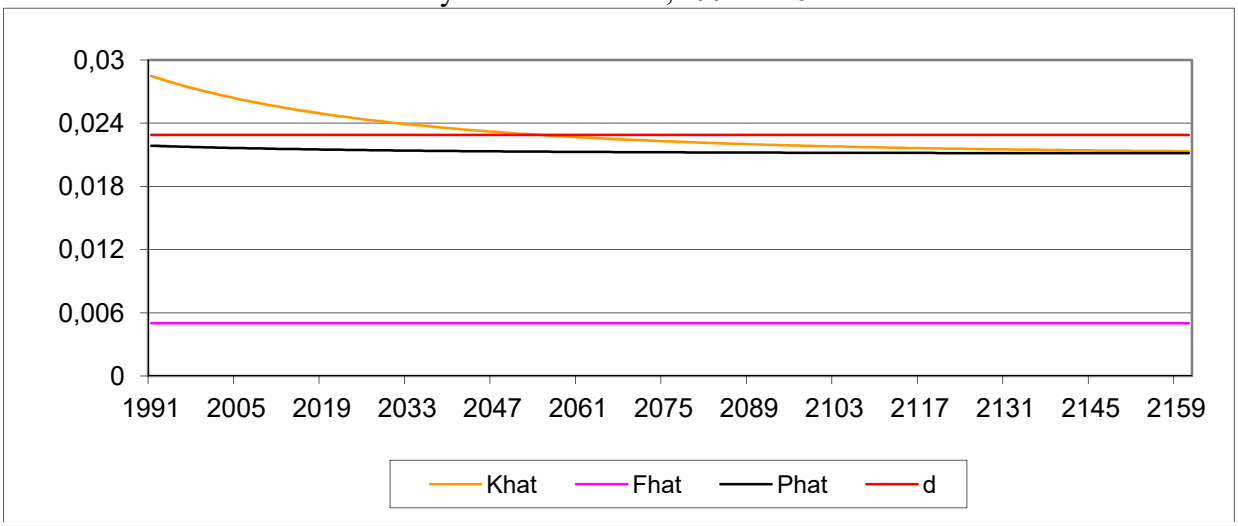

Figure A-26 – The growth rates of fixed capital K, proved mineral reserves F and net output P in subordinate long-term relation to d, 1991–2161

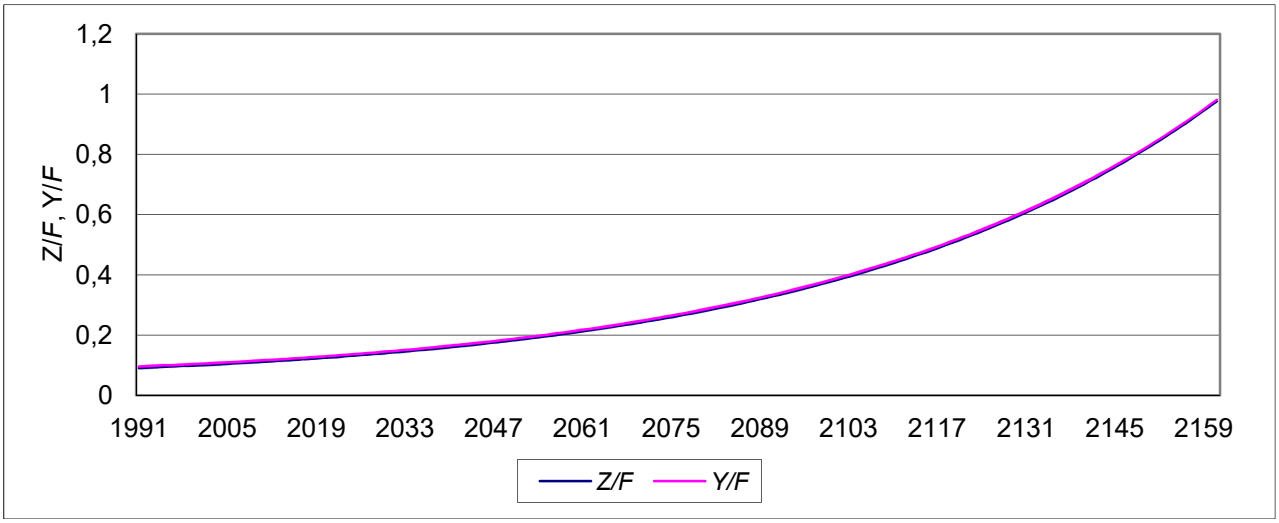

Figure A-27 – The tendency to unfeasible ratio of depletion of proved mineral reserves to their stock Z/F and to unfeasible ratio of gross additions to these reserves to their stock Y/F, 1991–2161

Economic contraction in NM-1 for $b > b_c$ vs aggravation mode in NM-2 for $y = 0$

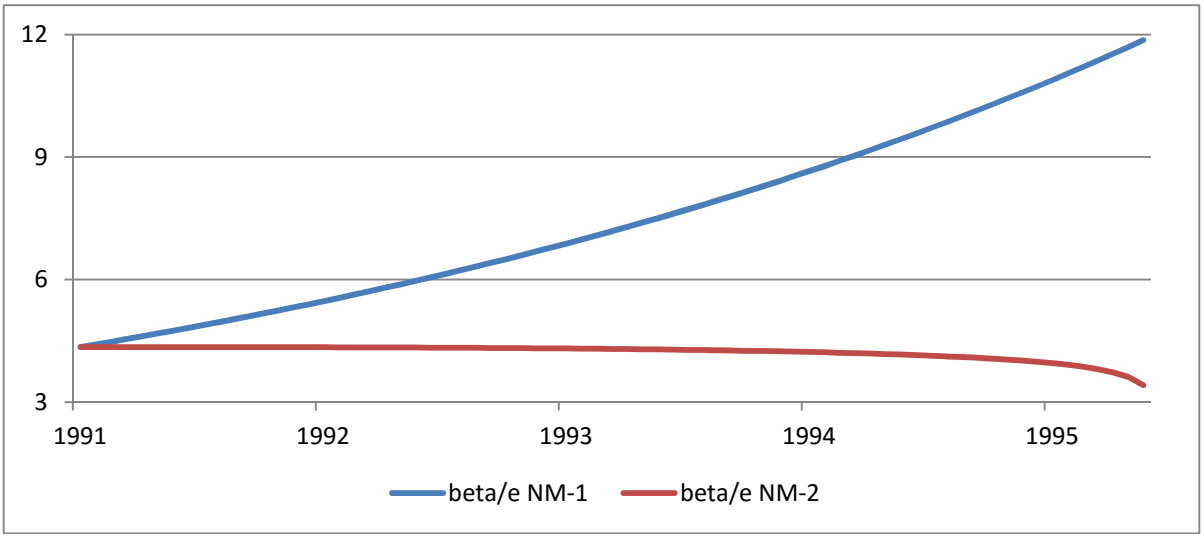

Figure A-28 – Rent per unit of resource extracted β/*e* (upper frontier to potential unit royalty) in NM-1 for $b = 0{,}2285 > b_c = 0{,}2085$ and in NM-2 for $y = 0 < e_1 = 0{,}0054$, 1991–1995,38.

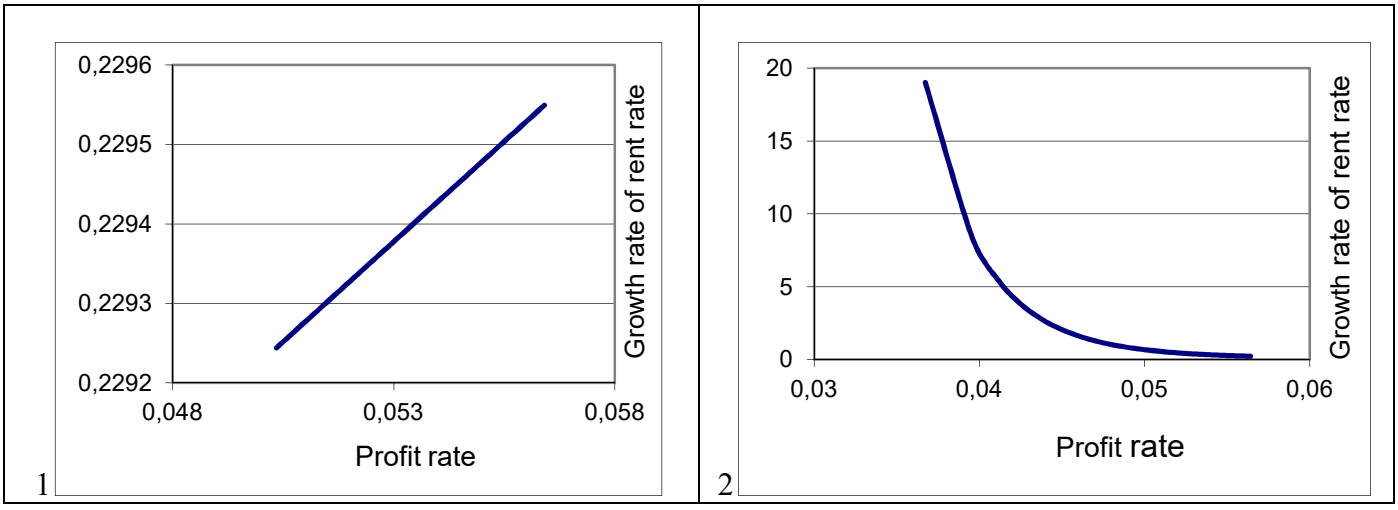

Figure A-29 – Market failures and violations of the Hotelling rule.
Average profit rate αm [1/y.] vs growth rate of rent rate $\hat{q}$ [1/y.], 1991–1995,38.
Panel 1 – NM-1 for $b = 0{,}2285 > b_c = 0{,}2085$, Panel 2 – NM-2 for $y = 0 < e_1 = 0{,}0054$

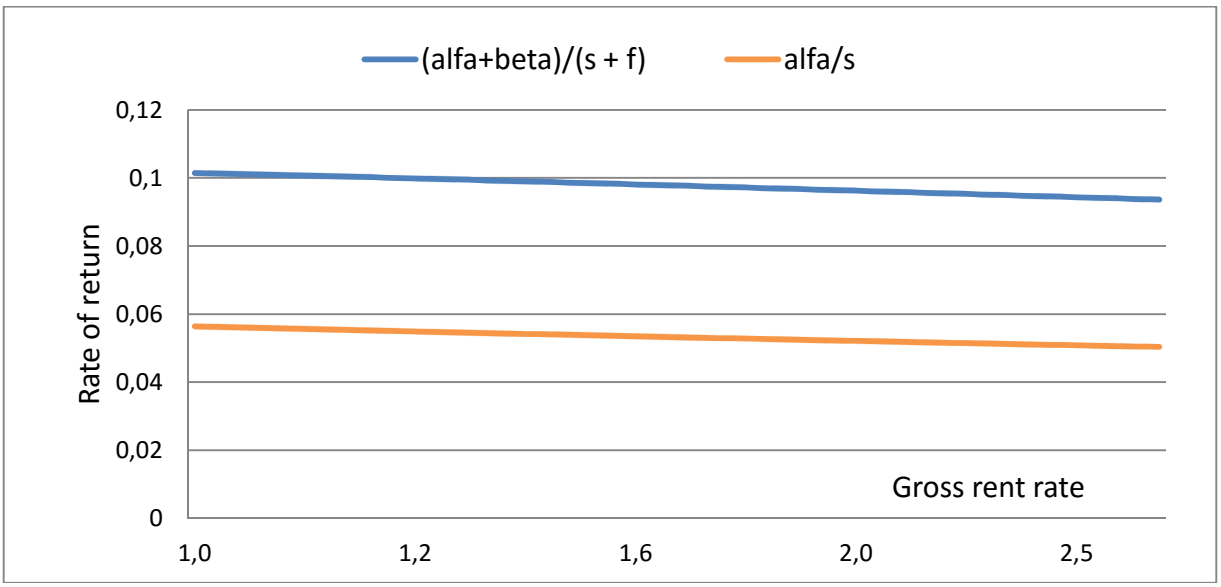

Figure A-30 – Market failures and violations of the rule of equivalent rates of return in NM-1 for $b = 0{,}2285 > b_c = 0{,}2085$, 1991–1995,38: scatter of average profit rate α/s and general rate of return to fixed assets and proved reserves $(\alpha+\beta)/(s+f)$ against gross rent rate β/f

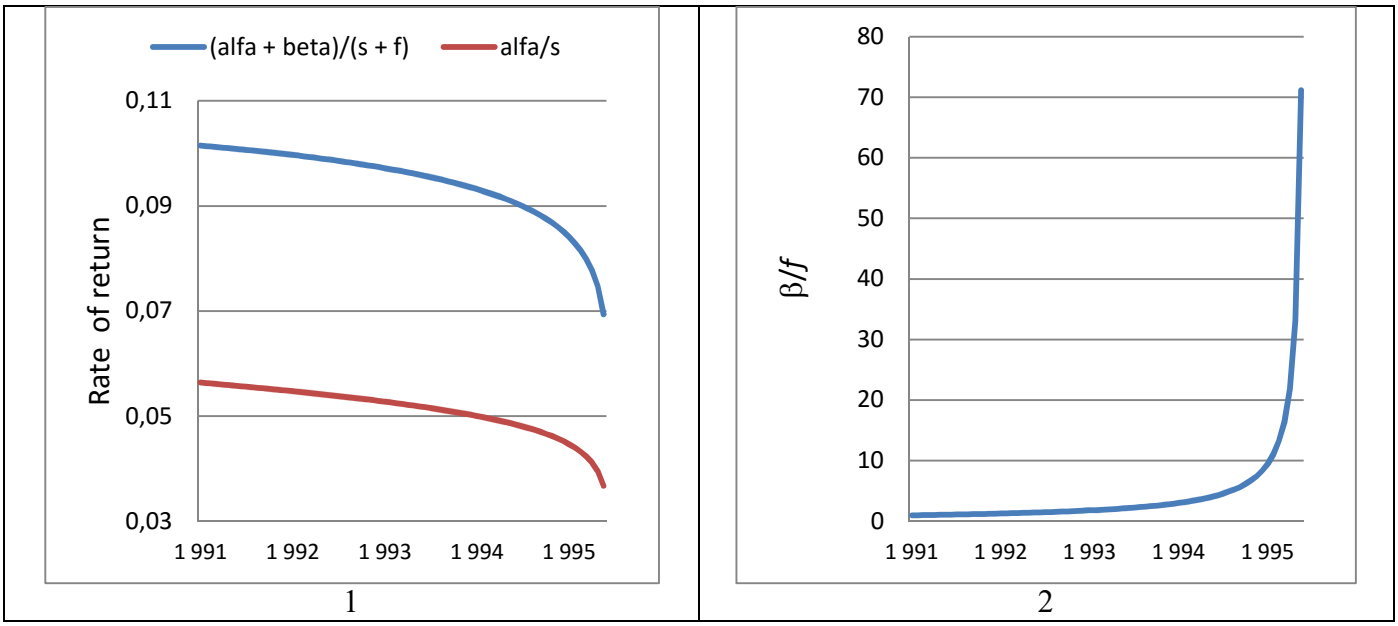

Figure A-31 – Stronger market failures and bigger violations of the rule of equivalent rates of return in NM-2 for $y = 0$ than in NM-1 for $b = 0{,}2285 > b_c = 0{,}2085$, 1991–1995,38.
Panel 1 – average profit rate α/s [1/y.] and general profit rate $(\alpha+\beta)/(s+f)$ [1/y.].
Panel 2 – gross rent rate β/f [1/y.]

Appendix B
Three hypothetic scenarios of capital accumulation in the U.S.
based HL-1 (updated Ryzhenkov 2007)

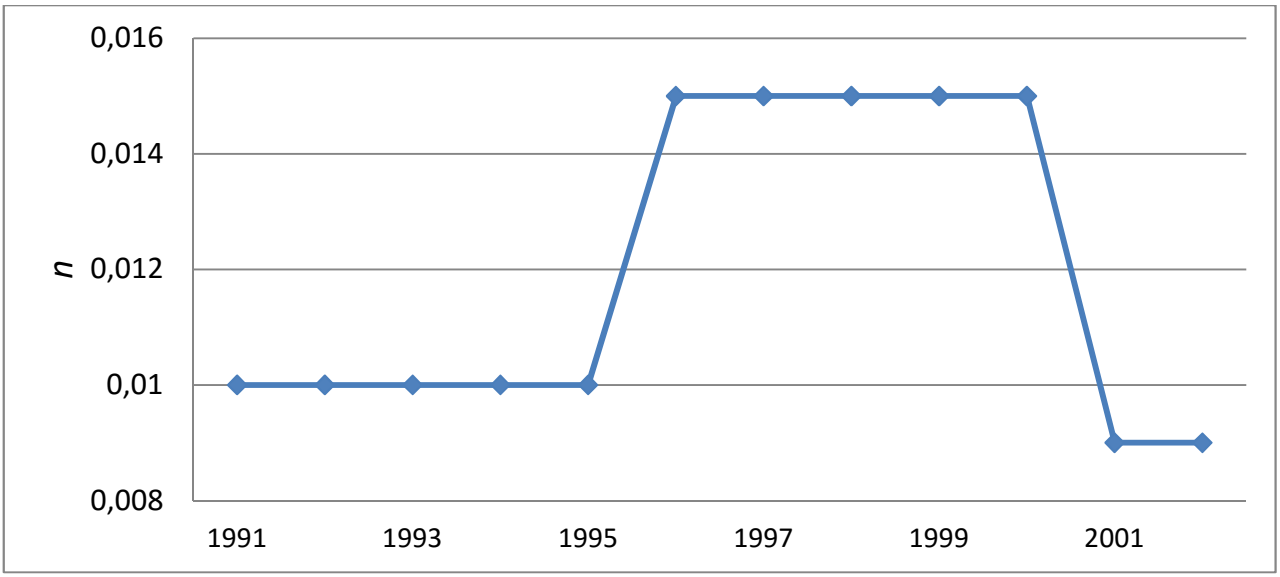

Figure B-0 – Exogenous growth rate of labour force *n*
in aggravation mode, inertia and normative scenarios, HL-1, U.S., 1991–2002

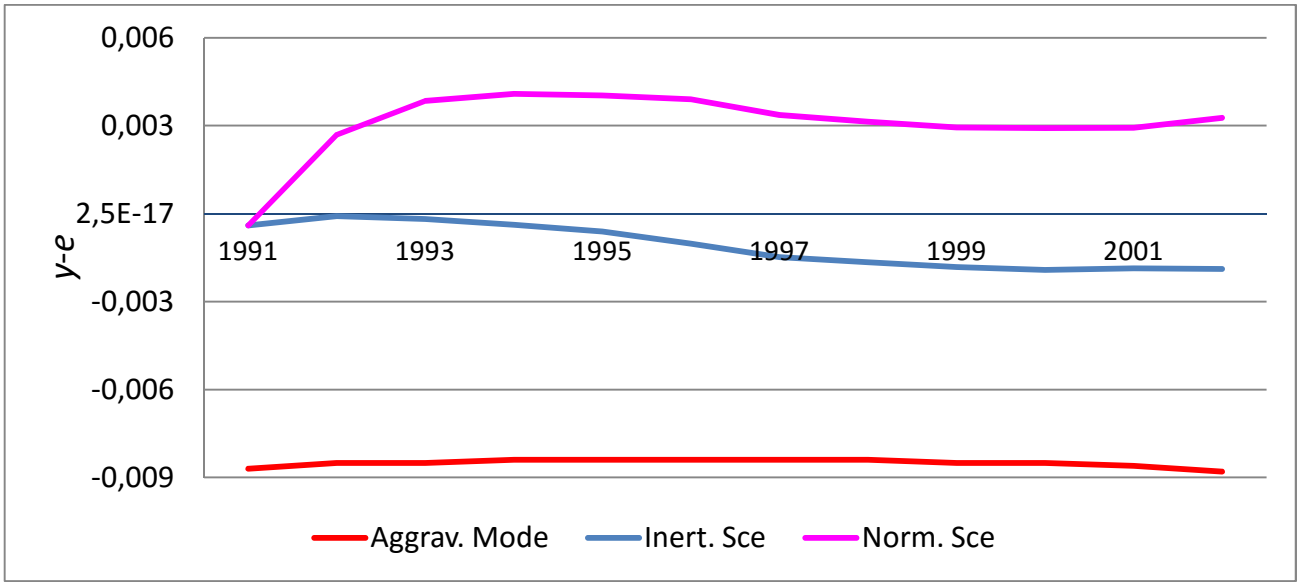

Figure B-1 – Net unit addition to proved reserves *y* – *e*
in aggravation mode (*y* = 0), inertia and normative scenarios, HL-1, U.S., 1991–2002

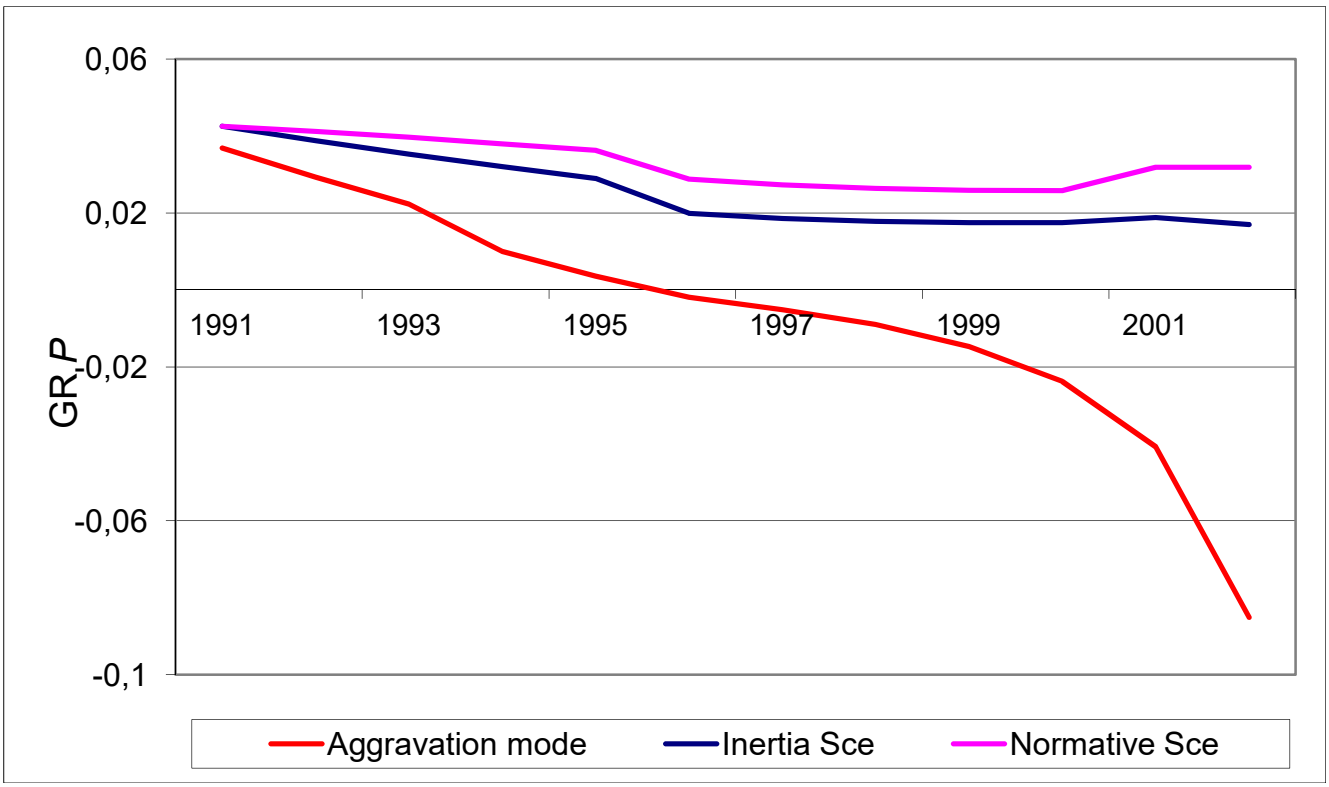

Figure B-2 – Growth rate of net output P in aggravation mode ($y = 0$), inertia and normative scenarios, HL-1, U.S., 1991–2002

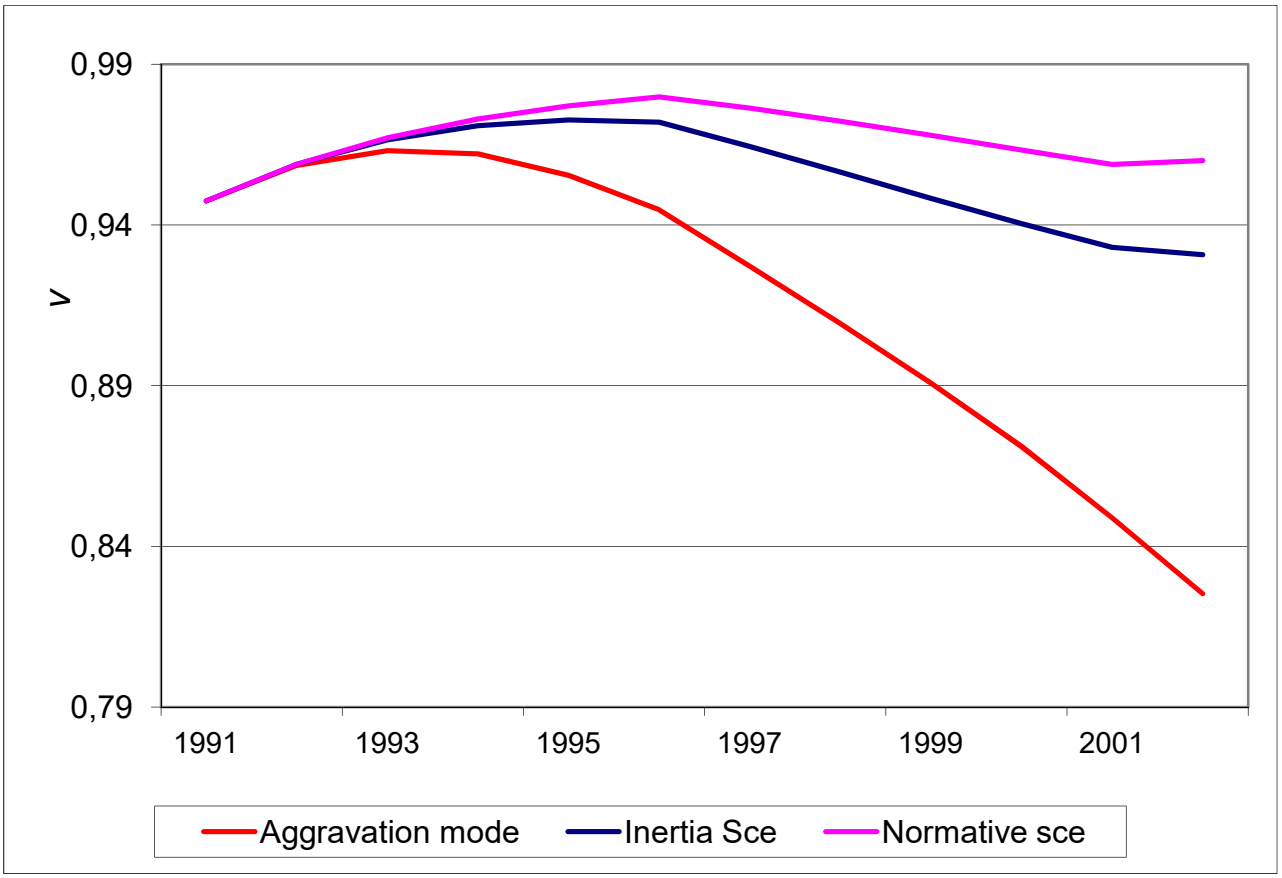

Figure B-3 – Employment ratio v in aggravation mode ($y = 0$), inertia and normative scenarios, HL-1, U.S., 1991–2002

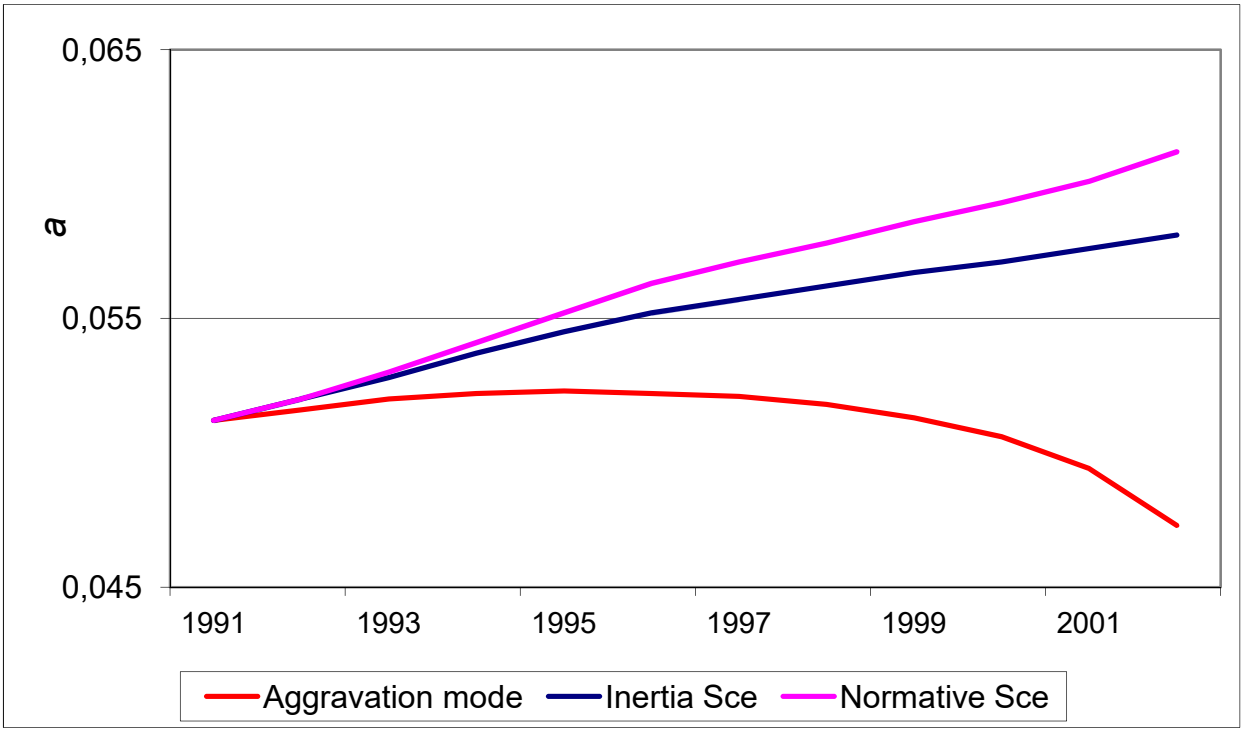

Figure B-4 – Output per worker *a* [mln 1996 $/(year*worker)] in aggravation mode ($y = 0$), inertia and normative scenarios, HL-1, U.S., 1991–2002