The classical Cobweb Theorem  
and real commodity market behavior:  
Modeling delayed adjustment of supply in industrial metals’ markets

Authors:  
Simon Glöser  
Johannes Hartwig

Fraunhofer Institute for Systems and Innovation Research (ISI), Karlsruhe Germany

Contact:  
simon.gloeser@isi.fraunhofer.de  
johannes.hartwig@isi.fraunhofer.de

www.r-cubed-research.eu

This conference paper is a shortened version of the original manuscript. The original paper was submitted to a peer reviewed journal. Therefore, in the conference proceedings, we only provide this abridged version.

Abstract:
In this paper we analyze how System Dynamics can help to better understand the dynamics of raw material markets and commodity price fluctuations caused by delayed adjustment of supply. We drew on the classical Cobweb Theorem and showed how this theorem can be successfully implemented into the System Dynamics approach. Regarding industrial metal markets, we systematically develop a simple model which helps to achieve a better understanding of dynamic market behavior and may be used as a flexible forecasting tool. On the example of the global copper market, we demonstrate how this simple market model can be merged with a physical material flow model capturing both market dynamics and technical aspects of raw material processing, recycling and substitution. This provides an explanation why linking physical material flows to a market model can add a substantial benefit for the systemic understanding. 
In a comparison of econometric forecasting methods we found System Dynamics models to be more intuitive and better suited to catch the structural market fundamentals.
Introduction: problem statement

High volatility in raw material prices imposes a planning problem for many manufacturing companies as material costs accounts on average for 43% of gross production value (Angerer et al. 2009). Especially over the last decade, prices for industrial metals were unstable: after a sharp increase before 2008 the prices dropped dramatically, partly due to the financial crisis in 2009. Since the last peak in 2011, prices for most industrial metals are momentarily continuously decreasing. Due to longer-term delivery contracts in manufacturing this volatility either results in direct profit losses if the company does not hedge or in an increased risk premium for futures or other commodity derivatives.

A major reason for mid to long term commodity market fluctuations in the field of industrial metals can be found in the delayed adjustment of supply (Humphreys 2012, Rosenau-Tornow et al. 2009): new mining projects have long lead times in planning phases and construction periods (5-10 years). This makes an instantaneous reaction on unexpectedly increasing demand impossible, leading to rapidly increasing prices in times of undersupply. On the other side, continuous production processes such as ore milling and concentration have got limited possibilities in adjusting their output volume. Therefore, production volumes are maintained in times of decreasing prices as long as the variable costs (cash costs) are covered by current market prices. The fact that production capacities need to be planned many years in advance based on future market expectations poses severe challenges to mining corporations. In times of high prices, expectations of future market prospects are higher, and so are the investments in new production capacities, while in times of low metal pricing investments tend to be declined, leading to an increasing risk of undersupply in future. Hence, we claim that due to adaptive expectations of market participants they cannot account correctly for time lags in the system that arises e.g. from production lead times. From a neoclassical point of view this problem simply should not exist: rational expectations include not making any systematic error in forecasting and eventual ignorance of the exact future equilibrium prices is normally distributed over all market participants. Furthermore, the prices for options should correctly account for these deviations and reflect market fundamentals, according to the efficient market theory (Fama 1970). Finally, the static general equilibrium theory does simply not account for delayed adjustments (Morishima 1996) despite the fact that a time lag between supply or demand reactions may constitute an important factor in determining market equilibrium states (Leontieff 1966 p.142 ff). Models of overlapping generations, which could account for this fact, may have pareto suboptimal equilibria or even an empty core (Geanakoplos, 2008).

We will try to show in this article, whether or not it is possible to match historical data on raw material prices with a simple model accounting for the delivery delays deemed to establish the structural reason for the convergence failures in the cobweb model.

As methodology we chose System Dynamics, since the combination of time delays and feedback effects in a stock and flow model enables an advanced analysis of dynamic market behavior (Sterman 2000 p. 182).

Previous studies of commodity markets using a System Dynamics approach (Auping et al. 2012, van Vuuren et al. 1999, Sverdrup et al. 2014, Kwakkel et al. 2012) have proven the advantages of SD for dynamic market modeling. However, the focus of those studies did not lie in the dynamics of capacity building delays of raw material producers, something that has been proposed as a main cause for cyclical price behavior (see for example Humphreys 2012, Rosenau-Tornow 2009). This phenomenon is commonly labeled as the Cobweb
The classical Cobweb Theorem and delays in adjusting supply on industrial metals’ markets

Theorem (Waugh 1964) and constitutes one focus of this paper. Early System Dynamics studies on the Cobweb Theorem were developed by Meadows (1969) and have been applied to livestock markets which formed the origin of this theorem in the early 20th century (Benner 1876, Hanau 1928, Kaldor 1934, Ezekiel 1938).

After discussing the basic principles of the classic Cobweb Theorem, we systematically develop a simple System Dynamics model aiming at being capable to reproduce real market behavior of industrial metals in the following sections. An enhanced model of the global copper market is provided by linking a model of physical material flows with the dynamics and feedbacks of a commodity market model, forming the structural reasons for the price behavior of the system. This enables a more detailed analysis of the copper market including recycling and substitution effects.

**Modeling the classical Cobweb Theorem within a System Dynamics environment**

First approaches of describing market fluctuations and price volatility due to delayed supply adjustment go back to the analysis of agricultural markets in the late 19th century (Benner 1876). The apparent inconsistency between these observed cycles and the claimed convergence towards an equilibrium posited by Walrasian economic theory were further analyzed in the early 20th century, particularly on agricultural markets, and were named after the commodities being analyzed, e.g. the “hog cycle” or the “corn cycle” (Hanau 1928, Ezekiel 1938).

Arthur Hanau (1928) analyzed pork prices in Berlin in the late 1920s and used the observed periodically recurring price peaks to forecast future price developments. He described the “hog cycle” as shown in Figure 1.

![Figure 1 Classical hog cycle as described by Hanau (1928). Hanau analyzed the pork market in Berlin in the early 20th century and identified periodical cycles of 3-4 years length.](image)

The theoretical explanation to these observed cycles in micro economic market theory has become known as the “Cobweb Theorem” (Harlow 1960, Larson 1964, Holt und Craig 2006). This term is based on the appearance of the supply and demand combinations showing the shape of a cobweb (cf. Figure 2). Ezekiel (1938) provides an overview of different cases of...
The classical Cobweb Theorem and delays in adjusting supply on industrial metals’ markets

supply and demand functions (see Figure 2a) leading to (1) continuous fluctuation, (2) divergent fluctuation or (3) convergent fluctuation which until today forms the explanation to market fluctuations due to lags in supply adjustment in micro economic theory (Chauhan 2009, Pashigian 2008).

The classical Cobweb Theorem is based on a discontinuous adjustment on the supply side and an instantaneous reaction to price changes on the side of demand (Ezekiel 1938). This behavior can be simply modeled with a one stock System Dynamics model including a delay function as displayed in Figure 2b.

Figure 2 Implementation of the classical Cobweb Theorem in a system dynamics environment. a: Different cases leading to continuous, divergent and convergent fluctuations on the example of an initial supply shock (the initial disequilibrium could also result from an abrupt demand alteration). b: Single stock price model to simulate the classical cobweb fluctuations as described in a. c: Results depending on the course of the supply function (see b right side).
The classical Cobweb Theorem and delays in adjusting supply on industrial metals' markets

The resulting market fluctuations depending on the profile of the supply function (cf. see Figure 2b right side) are illustrated in Figure 2c.
While these simple theoretical cobweb models capture the core structure of commodity cycles—which is the delayed response of supply to demand changes—these models are not sufficient to describe real market behavior. As pointed out by Sterman (2000 p.182 ff.), they do not adequately represent both the delay and feedback structure of real commodity markets, which can be better represented by a stock and flow structure, taking into account price levels, inventories, capacity building and work in progress. Furthermore, the mathematical properties of the classical Cobweb Theorem depend on using discrete time steps and are not robust with higher integration methods.

**A simple System Dynamics market model with delayed capacity building**

Introducing the modeling of mid-term developments on commodity markets with delayed capacity adjustment, we present a simple model which captures the basic features of a commodity market. Figure 3 displays the structure and results of this simple conceptual SD model. The price alteration in each time step is calculated from the relative difference of real supply and theoretical demand which forms the basis for real market behavior as described by micro economic theory (oversupply leading to decreasing prices, undersupply to increasing prices). In contrast to the classical Cobweb Theorem described in the previous section, this model is capable to simulate continuously increasing (exogenous) demand and the delayed adjustment of supply due to planning and construction phases leading to periodical price fluctuations.

![Figure 3 Structure and behavior of a simple System Dynamics model to simulate price cycles due to delayed capacity building.](image)

While this model already describes the continuous fluctuations on early agricultural markets as described by Hanau (1928, see Figure 1) or Benner (1876), current commodity markets price cycles are less periodical. Their dynamic behavior is more complex to capture since an adjustment to periodical cycles for market participants would be quite straightforward. Nonetheless, today’s commodity markets—be it the crude oil market or industrial metal markets—face exactly the same problem of delayed supply adjustment as described by this simple model; however, due to irregular development of demand, economic crisis, unexpected supply disruptions, political exertion of influence or other market shocks, the price peaks and market fluctuations are not as periodical as resulting from the theoretical model displayed in Figure 3. These unpredictable, stochastic external influences on real
commodity markets are the main external reasons why -even though suppliers adapt their capacity planning to the expected future demand- a perfectly balanced market is almost never achieved. In addition, further endogenous reasons like an increased propensity to substitute with higher prices or decoupling of raw material use with economic development are also prevalent.

As the following sections will demonstrate, the flexibility of the SD approach with its broad possibilities in simulating delays and feedback effects makes System Dynamics a suitable modeling environment to develop market forecast tools and perform advanced scenario analysis which may help stakeholders to get a better understanding of past, current and, ultimately, future commodity markets.

**Metal markets’ characteristics and behavior**

In this section we present some basic information about metal markets and behavior of mining and exploration companies regarding investments in new capacities. This forms the basis for different dynamic hypotheses used to model the global copper market (cf. following section).

The delayed capacity building on the supply side is seen as a major reason for price fluctuations (cf. introduction) even though real market behavior is not as periodical as observed on agricultural markets in the early 20th century (cf. Figure 1) and as resulting from conceptual System Dynamics models (cf. Figure 3). However, because planning and construction phases require long lead times in the realization of new mining capacities, the problem of delayed adjustment of supply remains dominant. Even though mining companies are expected to plan their future capacities based on their expectations of future demand, there is a very high correlation between current commodity prices and investments in new mining and exploration activities.

That is, in times of high metal pricing (such as the period before the financial crisis in 2009) the expenditure for new production capacities is also very high. With a delay of around 7 years, this might lead to overproduction resulting in decreasing prices which is currently observable in metal markets. The time spread between the final decision of project realization and the beginning of production is around 7 years, while 5 years before production begins, the investment commitments have reached a level after which a stop of the project seems very unlikely. Hence, mining companies have to plan their future capacities around 8 years before first raw material production is realized (cf. for example Rosenau-Tornow 2009, Buchert 2009, Gocht 1983).

The general effects of an imbalance in an industrial metal’s market (temporary scarcity), be it (in addition to the described endogenous explanations) due to a supply shock or due to unexpected high demand, has been described by Solar et al. (2009). While supply shocks stemming from disruptive actions are hardly predictable, they ought to be viewed as exogenous to the system. Adjustment to demand changes, on the other hand, are truly endogenous in that respect.

**Combining physical material flows and market dynamics: An enhanced model of the global copper market**

Before providing a detailed description of the copper model developed in this study, we give an overview of major studies using a System Dynamics approach to model the copper market. Auping et al. (2012) have undertaken extensive studies of the uncertainties
regarding copper demand and copper price development using System Dynamics. They spot main causes for high copper prices in an energy transition-transitioned demand shift and an overall rising demand due to the catch-up of developing countries like China. While they refer to copper supply in two sources, mining and recycling, their main focus does not lie in the dynamics for capacity building delays of copper producers, something that has been identified as a main cause for cyclical behavior in the markets for industrial metals (see for example Humphreys 2012). Kwakkel et al. (2014) compare different bottom-up approaches for modeling extraction capacity and the role of inherent uncertainty in modeling of supply fluctuations. They furthermore distinguish between conventional and unconventional mining capacities. While their main focus was on the quantitative comparison of different models and on exploring the attached uncertainties in these models, they do not make a case for the relationship between prices and capacity investment demand. Further studies using a System Dynamics approach to model metal markets (Sverdrup et al. 2014; van Vuuren et al. 1999) focus on very long perspectives of copper availability regarding the potential depletion time of metal resources. Resource availability does play a role, but not in the near future and has supposedly minor impacts on current price developments (cf. Buchholz 2012). Depletion-related aspects, though widely discussed in the System Dynamics literature, are therefore only of minor importance to the goal of this study focusing on mid-term market developments (5 to 10 years).

The Global Copper Flow Model

The global copper flow model simulates copper flows from mining and refining over the fabrication of semi-finished goods to the end use of copper over product life spans and the recycling of copper scrap based on historical production data going back 100 years. A detailed description of this material flow model is provided by Göse et al. (2013a+b). In this section we only provide a short overview of this System Dynamics model used to simulate material accumulation over time and model the global anthropogenic copper cycle. While this model uses the delay structure of stocks and flows to simulate different forms of lifetimes, it does not include any direct feedback loop. Hence, as described below, system dynamics software has proved high suitability to model these dynamic material cycles (cf. Glesor 2013b, Bornhofft 2013, Sprecher 2015) even though the stock and flow structure is only used to simulate anthropogenic material stocks and flows over time without making use of real feedback loops resulting in systems of differential equations.

The classical Cobweb Theorem
and delays in adjusting supply on industrial metals’ markets

Basic concept of keeping a closed mass balance over time: the collection rate of metal scrap is adjusted to production data

Recycled metal from EoL scrap = Total metal use - Mining - New scrap
Recycled metal from EoL scrap = Total waste • CR • SR • RE

Calculation of the collection rate to close the mass balance:

CR = \[ \frac{\text{Total metal use}}{\text{Total waste}} \times \text{Mining} \times \text{New scrap} \]

CR: Collection Rate SR: Separation Rate RE: Refining Efficiency

Figure 4 Enabling a closed mass balance over time by defining the global collection rate of EoL (End-of-Life) copper scrap as a function of reported global production data.
The classical Cobweb Theorem
and delays in adjusting supply on industrial metals’ markets

The basic concept of simulating material life cycles and accumulation over product life spans and enabling of closed mass balance of a global metal cycle over time is described in Figure 4. A dataset from the international copper association (ICA) was used to separating the global copper market into 17 different end-users. Based on historic semis (semi finished copper products), refinery and mining data (going back to 1910), the global copper life cycle is simulated. The collection rate of end-of-life copper scrap as a function of reported historic refinery production (cf. Figure 4) enables a closed mass balance over time. From the field of quality and safety engineering different types of typical left and right skewed product lifetime distributions are available (see supplementary data to this paper), while for the copper flow model, due to a lack of detailed lifetime data, only Gaussian distributions as were used (cf. Gloeser et al. 2013a+b).

The total structure of the copper flow model including further information about the anthropogenic global copper cycle is provided in the supplementary data to this paper. Herein, we focus on the enhancement of the retrospective (only based on historic data) physical material flow model by market dynamics in order to develop a tool enabling the analysis of future market scenarios.

Copper market model including physical material flows

The copper market model combines the material flow model described above with the major feedback effects on raw material. As each copper end-use sector has got different substitution potentials and thus reacts differently to price changes, the demand in each sector is calculated individually based on exogenous GDP development and the substitution intensity which is directly influenced by the price.

The material flow model provides specific supply data regarding both recycling flows together with technical aspects of recycling taking into account total scrap availability and mining data influenced by the delayed capacity building from investments in new mining projects. This combination of physical material flows with dynamic aspects of market adjustment mechanisms forms a flexible tool for advanced market analyses.

The model was calibrated to historic copper price and mining development in order to analyze its ability to reproduce real market behavior based on past GDP development. The development of copper mining in the previous 20 years can be divided into periods of strong growth and periods of weak growth (see also Humphreys 2012). While prices tend to decrease in periods of stronger growth, they tend to increase in times of weak growth. As a matter of course, price developments can not only be explained by regarding the supply side (see for example price drop in a period of weak growth in the early 1990s due to low economic development), however, the probability of a temporary undersupply is higher in times of weak growth. This trend of stronger and weaker development phases is typical for cyclical markets and is already shown in the simple example in Figure 3.

We performed a first copper price forecast based on the expected global economic development published by the World Bank (Global Economic Prospects 2015). The phase of overcapacity and decreasing prices is expected to not last longer than 2015 as both decreasing additional mining capacities (which is a delayed effect of lower investments during the financial crisis in 2009) and slightly increasing economic prospects (cf. World Bank 2015) make a further price drop unlikely. However, as proposed by the model, a strong recovery of prices in the coming three years is unlikely. This is due to expected further increasing mine capacities resulting from higher investments in 2010 and 2011. However, then, a significant decrease in new mining capacities can be expected. This increases the
The classical Cobweb Theorem and delays in adjusting supply on industrial metals’ markets

probability of a severe price recovery after 2018. However, this assumption is based on a continuously increasing global GDP (positive growth rates) which is the major external input to the model. Needless to say, this highly simplified model only captures few relations and correspondences in the copper market; hence the results must be treated with some reservation. However, it underlines that the System Dynamics approach combining time delays and feedback effects is highly suitable to model raw material markets taking into account physical material flows and market dynamics realized with different forms of feedback effects.

Comparison of conventional time series analyses and System Dynamics

Additionally to the market analysis with System Dynamics, we provide a comparison of the aforementioned models with econometric time series analysis, since this paradigm is currently dominant in raw material market analysis. Most studies dealing with commodity cycles and price developments are based on time series analysis (Labys et al. 1999, Labys et al. 2000 Davutyan und Roberts 1994, Roberts 2009). Common tools for forecasting prices using econometrics are the so-called ARIMA models. This acronym stands for autoregressive integrated moving average and is a form of time series analysis where time lags in the data can account for seasonality. They belong to the so-called Box-Jenkins methods and the introduction of lags constitutes the dynamics in econometrics. However, as Sims (1980) has noted, it is indispensable in presence of rational expectations to have a priori knowledge of lag lengths for their identification. System Dynamics does at least help to better control for the various parameters setting the model boundaries. So the standard objection put forth by rational expectation theory (see Muth 1961 for details), namely that cycles in a dynamic system ought to exhibit a fairly stable period, is counteracted here. ARIMA-models are able to include cycles of the same wave length and amplitude, but fail if those cycles are of other type. Lyneis (2000) has elaborated on the advantages of System Dynamics in understanding dynamics (which is quite distinct from the econometrics understanding of dynamics, as we have shown), namely the identification of sensitive parameters, a better range of forecast scenarios and early-warning indicators.

Conclusions and recommendations for future research work

In this paper we have tried to argue for adaptive price expectation formulation in raw material markets, causing cyclical investment strategies in mining and production capacities. Needless to say, real commodity markets are influenced by far more factors than those covered by the simple models described in this paper, but already comparatively simple System Dynamics models are able to reproduce main market behavior. As demonstrated in this paper, the System Dynamics Approach offers possibilities to combine physical material flows with market dynamics in a single model. This enables an enhanced analysis of raw material markets. Both primary (mining) and secondary (recycling) material flows can be simulated, taking into account physical limits (e.g. total amount of scrap availability through detailed life cycle simulation, total mining capacity and delayed capacity building) and technical aspects of material processing and potential substitution as a feedback effect to high price levels. This combination of technical aspects with general market dynamics within a single SD model provides a flexible tool for different mid and longer term market analyses. For example, the effect of disproportionately high economic
development on raw material markets and the expected adaptation of different sectors to increasing price levels due to different price sensitivities can be analyzed. This enables a better understanding of market behavior and may support market forecasts which – in contrast to single time series analyses – take into account feedback effects and individual judgment of exogenous influences such as global economic development.

**Literature**

Angerer, Gerhard; Erdmann, Lorenz; Marscheider-Weidemann, Frank; Scharp, Michael; Lüllmann, Arne; Handke, Volker; Marwede, Max (2009): Rohstoffe für Zukunftstechnologien. Einfluss des branchenspezifischen Rohstoffbedarfs in rohstoffintensiven Zukunftstechnologien auf die zukünftige Rohstoffnachfrage. Fraunhofer Verlag Stuttgart.


Auping, Willem; Pruyt, Erik; Kwakkel, Jan (2012): The uncertain future of copper: An Exploratory System Dynamics Model and Analysis of the global copper system in the next 40 years. Proceedings of the 30th International Conference of the System Dynamics Society, St. Gallen, Switzerland.


Glöser, Simon; Soulier, Marcel; Tercero Espinoza, Luis A. (2013a): Dynamic analysis of global copper flows. Global stocks, postconsumer material flows, recycling indicators &
The classical Cobweb Theorem and delays in adjusting supply on industrial metals' markets


The classical Cobweb Theorem
and delays in adjusting supply on industrial metals’ markets


Schmitz M, Buchholz P, Huy D, Sievers H. Rohstoffversicherung und Beschaffung bei steigenden Preisen und zunehmenden Versorgungsrisiken; Presentation IHK 2012.


