# WHY DO WE SLIP IN THE BATHTUB? - EXPLANATION OF STOCK-FLOW FAILURE BASED ON SYSTEMS WITH LIFE

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**ABSTRACT**

*Booth Sweeney and Sterman [2] through their experiments with Bathtub Dynamics, relating to stock and flow models, report that irrespective of educational level, business background the subjects have a poor level of understanding of stock and flow relationships and time delays. Recently, Cronin, Gonzalez and Sterman [5] ask the question, 'Why don't well-educated adults understand accumulation?' Though accumulation is a fundamental process in dynamic systems, behavior of simple stock-flow situations are not understood well by even students trained in mathematics or physics. Through a series of experiments Cronin et al. show that the poor performance is not explicable by inability to interpret graphs, lack of contextual knowledge, motivation and lack of feedback or cognitive capacity. They also show the presence of an erroneous heuristics called 'correlation heuristics'.*

*In this paper we explain this phenomenon applying what is called the Missing Middle Hypothesis (MMH) arising out of autopoietic understanding of cognition as put forward by Maturana and Varela [19]. Missing middle hypothesis deals with the question why human memory does not record the middle of the distribution of its experience in its environment. We also suggest how MMH can explain other human decision making pitfalls noticed earlier by Kahneman and Tversky [13].*

*Keywords: Bathtub paradox; Stock-Flow failure; Autopoiesis; Systems with life; Missing middle Hypothesis; Human decision making*

## 1. Introduction

System dynamic modeling has at its core the stock and flow metaphor, which is fundamental to describe a basic process in different levels of human organisations, starting with the individual life to communities at large [4, 28, 29]. The fields of applications of system dynamics also vary from personal financial insights to carbon foot prints to global warming; supply chain management to reverse logistics; patient care to disease prevention and immunisation, to mention a few [34]. As noted by Sterman[27], "the resource-based view expanded the definition of a firm's resources beyond tangible stocks such plant, equipment, cash, and other traditional balance sheet items to include less obvious but more important stocks underlying firm capabilities, such as employee skills, customer loyalty, and other forms of intangible human, social, and political capital". The underlying structure of all stock-flow systems can be given the simple bathtub analogy: Consider a bathtub. Water flows in at a certain rate, and exits through the drain at another rate. The stock at any time corresponds to the water in the bathtub. So, stock at time t equals Initial stock plus the total inflow minus total outflow. With some reflection, it is clear that water level rises only when the net inflow exceeds outflow, falls only when the outflow exceeds inflow and remains unaltered only when inflow equals outflow. Notwithstanding the simplicity of everyday bathtub experience, stock-flow dynamics seems to be difficult for people from different walks of life with different science, technology, engineering, and mathematics (STEM) backgrounds. Booth Sweeney and Sterman [2] in 2000 presented their initial results of a systems thinking inventory, on this paradoxical inability of humans to comprehend bathtub dynamics. Since then several explanations exploring reasons for the stock-flow failure and psychological processes operating behind them have been offered. Among them, cognitive capacity limitation and working memory [1], information display, cognitive burden of calculation involved [24], inadequate motivation, lack of feedback, unfamiliarity of task context, inability to interpret graphs [21] were considered by Cronin et al.[5] to experimentally test the efficacy of the different alternative explanations offered for stock-flow failure. They report the results of five experiments conducted for this purpose, and find stock-flow failure is a robust phenomenon that could not be explained for any of those reasons. Hence, the title of their paper not only makes them question, 'Why don't well-educated adults understand accumulation?' but also throw a challenge to researchers, educators and citizens alike.

In this paper we have a fresh look at the stock-flow failure from a more fundamental perspective of living and being human and offer an explanation based on autopoiesis, cognition, and human experience developed and studied by Maturana and Varela [18, 19, 20]. We introduce what is called the Missing Middle Hypothesis (MMH) arising out of autopoietic understanding of human experience. Missing middle hypothesis deals with the question: why humans have difficulty in using the middle of the distribution of their experience in their environment, for their decision making. We apply MMH to explain the root cause of stock-flow failure. We also note how MMH can explain some of the other human decision making pitfalls noticed earlier by researchers [6, 7, 13].

The rest of the paper is organised as follows: Section 2 summarises the experimental results from Cronin et al. after briefing the stock-flow inventory from [5]. Section 3 introduces briefly the concepts from autopoiesis and cognition. Section 4 deals with the missing middle hypothesis and its manifestations in everyday life and other management and scientific fields. Section 5 offers the new explanation for stock-flow failure. Section 6 deals with discussions and concluding remarks.

## 2. Stock-Flow Failure Phenomenon

Booth Sweeney and Sterman [2] in 2000 reported the findings on some experiments with highly educated graduate students on basic stock-flow tasks. The task for the participants is to draw the trajectory of water stock in a tub using the given history, in the form of a line graph, showing the flow into and flow out of the tub at different discrete time points.

They also considered different contexts of dynamics, namely, bathtub or cash flow. Also they considered different simple flow patterns, square wave or saw-tooth. The experimental layout is like:

| | Flow Pattern / Task | Square wave | Saw-tooth |
|---|---|---|---|
| Group 1 | Bathtub | | |
| | Cash Flow | | |
| Group 2 | Bathtub | | |
| | Cash Flow | | |

The disappointing results and conclusions from the initial study can be summarised using excerpts from [2]:

"... We use the inventory to assess understanding of basic systems concepts in subjects with little prior exposure to systems thinking. The subject pool ... are highly educated and possess an unusually strong background in mathematics and the sciences compared to the public at large."

"As we show, the performance of these students was quite poor and the students exhibited persistent, systematic errors in their understanding of these basic building blocks of complex systems."

"It appears that we should spend considerable time on the basics of stocks and flows, time delays, and feedback, with an emphasis on developing intuition rather than the mathematics."

"For a large fraction of the subjects, training and experience with calculus and mathematics did not translate into an intuitive appreciation of accumulations, of stocks and flows."

The authors expressed their surprise why the participants fail to comprehend the 'simple logic' behind stock accumulation. This came to be known as stock-flow failure or paradox.

Subsequently, Sterman in 2002 describes a departmental store setting for the stock-flow task providing a graph showing the number of people entering and leaving a departmental store over 30-minute period (See Appendix 1) [30]. The participants have to answer the four questions: During which minute did the most people enter the store? During which minute did the most people leave the store? During which minute were the most people in the store? And During which minute were the fewest people in the store?

Here the stock is the number of people in store, inflow is the number entering and outflow is the number leaving. This departmental store task forms the baseline experiment for a later study described in the following subsection.

### Experimental Verification of Usual Explanations of Stock-flow Failure

Recently, Cronin et al. [5] investigate the sources of and psychological processes involved in stock-flow failure. They address many usually cited reasons for performance failure in dynamic decision-making studies and situations demanding bathtub dynamics reasoning. The experiments were conducted in different times among different participants; however ensuring the homogeneity of the population (students with STEM background) from which the participants were drawn, except for comparison among subgroups of participants. The details of the study can be found in [5]. Table 1 gives a brief summary of these experiments, to the extent needed for following the rest of the paper. In Table 1, column 1 gives the experiment name or number, column 2 gives a short description of the experimental design, column 3 gives the explanation(s) being tested and the last column gives the findings.

<table>
<tr><th colspan="4">Table 1: Summary of the Cronin et al. experiments on Stock-Flow Failure</th></tr>
<tr><th>Experiment</th><th>Details</th><th>Explanation(s) Tested</th><th>Results</th></tr>
<tr><td>Baseline</td><td>Departmental Store Task (see Appendix 1, Four questions were asked.)<br>Sample size: 173<br>Volunteers from MIT or Harvard graduate students with Science, Technology, engineering, mathematics (STEM) or economics background.</td><td>To assess the performance with Stock and Flow task by students with calculus and strong mathematics training for baseline comparison.</td><td>
<ul><li>Question Order made no difference.</li></ul>
<table>
<tr><th colspan="2">Correct answers</th></tr>
<tr><th>Question</th><th>Percent Correct</th></tr>
<tr><td>Q1</td><td>96%</td></tr>
<tr><td>Q2</td><td>95%</td></tr>
<tr><td>Q3</td><td>44%</td></tr>
<tr><td>Q4</td><td>41%</td></tr>
</table>
<ul><li>Correlational reasoning around 30%</li></ul>
</td></tr>
<tr><td>Experiment 1</td><td>Cognitive burden and data display (see Appendix 2, Four different displays of the department store task)<br>Sample size: 271<br>Volunteers from a subsequent term similar with baseline experiment background. Participants randomly assigned to one of four data presentation modes.</td><td>To assess whether performance will improve if (a)data are presented in tabular or textual form, (b) bar chart is used instead of line chart, (c) tabular form is presented instead of text and (d) number of data points are reduced (cognitive burden: 30 reduced to 12 data points).</td><td>
<ul><li>No support for difficulties in interpreting graphs as responsible for SFF.</li>
<li>Performance on Q1 &Q2 was significantly better with graphs than with table or text.</li>
<li>No significant difference in performance with line or bar graph presentation.</li>
<li>No significant difference in performance with tabular or text</li></ul>
</td></tr>
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<table>
<tr><td></td><td></td><td></td><td>presentation.<ul><li>Simpler version of the task did not improve performance.</li></ul></td></tr>
<tr><td colspan="4">Table 1: Summary of the Cronin et al. experiments on Stock-Flow Failure (Continued)</td></tr>
<tr><td>Experiment 2</td><td>Task Context<br>(Tub, Car, or Stores three contexts compared)<br>Sample size: 47<br>Volunteers from Carnegie Mellon University received $5 compensation for their time. Participants randomly assigned to one of tub, car or store task.</td><td>To assess whether performance will improve if tub and car tasks are used instead of stores task, due to familiarity with the task condition.</td><td><table>
<tr><th colspan="2">Correct answers</th></tr>
<tr><th>Question</th><th>Percent Correct</th></tr>
<tr><td>Q1</td><td>96%</td></tr>
<tr><td>Q2</td><td>94%</td></tr>
<tr><td>Q3</td><td>28%</td></tr>
<tr><td>Q4</td><td>26%</td></tr>
</table><ul><li>No statistically significant difference in performance on stock questions across the three contexts.</li></ul></td></tr>
<tr><td>Experiment 3</td><td>Motivation and Feedback<br>(role of incentives and feedback on performance)<br>Sample size: 69<br>Recruited from George Mason University School of management. 32 received motivation and feedback, the rest 37 were used as control group.</td><td>To assess whether performance will improve if (a) motivation to do correctly and leave early or (b) frequent feedback on current performance is provided.</td><td><ul><li>Motivation did not improve significantly the performance.</li><li>Feedback did improve performance eventually.</li><li>No significant difference in success rate with feedback.</li><li>See table in Appendix 3 for the slow improvement over attempts.</li></ul></td></tr>
<tr><td>Experiment 4</td><td>Priming Stock-flow Knowledge<br>Sample size: 37<br>Recruited from George Mason University.32 received motivation and feedback, the rest 37 were used as control group.</td><td>To assess whether priming participants to notice stock flow structure and behaviour will improve the performance.</td><td><ul><li>Priming did improve Performance.</li><li>But did not eliminate the SFF.</li></ul></td></tr>
<tr><td>Experiment 5</td><td>The Correlation heuristic<br>Sample size: 282<br>Volunteers from a subsequent term similar with baseline and experiment 1 background. Participants randomly assigned to one of eight different graphs showing flows of people entering and leaving of varying degrees of complexity.</td><td>To assess the fraction of participants erroneously selecting the maximum net inflow (outflow) for the maximum (minimum) of the stock respectively will be the greatest in the line graph display and the lowest in the tabular and text based display.</td><td><ul><li>Overall, only 54% drew correct patterns varying in the range 19 to 83% for the 8 different flow patterns.</li><li>Overall, 71% of the incorrect responses show paths for the stock identical to that of the inflow or net flow.</li></ul></td></tr>
</table>

In line with the reported studies on stock-flow failure among highly educated adults [2,14-17,22, 26, 30], prior to Cronin et al., results from experiments 1-4 and the baseline comparison demonstrate the human inability to understand stock and flows, especially the accumulation process of stock arising from inflow and outflow overtime. In the words of the authors [5], "… the error reflects serious misunderstanding of the basic principles of accumulation."

One of the major finding from the stock-flow failure data is the fact that the subjects use what is called, 'Correlation reasoning'. Decision maker assuming that the output (the stock here) should look like the input (the flow or the net flow here) is called the correlation heuristic by Cronin et al. [5].

But such a heuristics, though may lead to correct decision in some situations, can be seen absurd in the context we are now in, namely the bathtub dynamics. Correlation heuristics implies that a bathtub continuously filled with water faster than it drains will never overflow. But such conclusions are shockingly made by policy makers and matured adults when it comes to national debt or greenhouse gas concentrations [3, 31- 33].

Experiment 5 in the study by Cronin et al. tests the prevalence of correlation heuristics. From the results of Experiment 5, we find, over a range of experiments and participant groups, consistent use of correlational thinking, while violating basic accumulation principles.

Thus, this elaborate and valuable study calls for a deeper understanding of 'why we fail in the bathtub dynamics?' So we move on to examine the phenomenon from a fresh point of view from the perspective of systems with life.

## 3. Brief Introduction to Autopoiesis, Cognition and Human experience

A living biological organism, even a single cell amoeba, apart from having a structure composed of components, possesses what we call organization. To be organized is having or consisting of parts acting in co-ordination: having the nature of a unified whole: organic. Maturana and Varela [19] characterize the organization of the living as 'autopoietic'. Encarta Dictionary (English (U.K)) defines autopoiesis (noun) as "- a process whereby a system, organization, or organism produces and replaces its own components and distinguishes itself from its environment - self-maintaining system, organisation or organism".

An autopoietic system is rigorously defined as [19]:

"…A network of processes of production {transformation and destruction} of components that produces components that: 1) through their interactions and transformations continuously regenerate the network of processes {relations} that produced them; and 2) constitute it {the machine} as a concrete unity in the space in which they {the components} exist by specifying the topological domain of its realization as such a network."

The main idea is that the organism maintains itself as a unity not by its components and the relationship among its parts. The term structure refers to the details within a given entity, i.e. the physical properties of the components and the roles of the components – their actual relationships within the system. To maintain this organization the organism requires a medium in which it can get its supply of necessary building blocks (molecules). Living things are energetically and materially open to the environment but operationally they must be closed. 'The structure may change all the time within a given organism, when the organization changes it spells the end of that lifetime. Whereas, the structure is physical or molecular, organization is conceptual [9]. A distinguishing feature of autopoiesis is that organisms are not subject to environmental or natural selection, organisms interact with themselves. An organism inter-acts recursively in the same network that produces it. So they are not open to their medium but closed, enabling them to interact with themselves. How then are we interacting with our surrounding medium – which includes one another? An organism exists only in its connection with the medium and that connection is actually its history of interaction. This history of interaction will continue as long as the organism can maintain its organization – then it ceases. Autonomy means that the organism subordinates all changes in the environment to the maintenance of its organization no matter how its structure may have to do this. 'The relationship between organisms and with their environment is a particular kind of structural coupling in which changes within the organism and changes in its surrounding medium are interlocked; they trigger and select one another from the available possibilities, maintaining a structural congruence as long as the relationship exists [9].' We say the organism and the medium are structurally coupled. This might appear to an observer as if 'the organism is encoding and decoding information from the outside.'

Any perturbation in the medium provides an opportunity for the organism to come to terms with the medium for now and to come to terms with the perturbation itself, by the history of interaction with the medium. However if the perturbation is such that the organism cannot maintain its organization the organism disintegrates; it is no more a unified whole. It may die or become some other organism(s).

In addition to having interactions with the medium of their existence (this planet), human beings interact with one another. This gives rise to a consensual domain, according to Maturana. A language exists in a community, and is continually regenerated through the members engaging in 'languaging' and the structural coupling generated by that activity. Maturana refers to the behaviour in a consensual domain as 'linguistic behaviour.'

Maturana and Varela replace two notions that are very important in understanding what we call decision making in organizations. Firstly, we are not processing information and there is no representation of the world that is perceived in our mind. Secondly, without our linguistic activity in a community we do not have a mind or consciousness.

## A New Perspective on Cognition [36, 37]

Maturana [9,20,18] while explaining what transpires in the linguistic interaction between two humans says, "The basic function of language as a system of orienting behaviour is not the transition of information or the description of an independent universe about which we can talk, but the creation of a consensual domain of behaviour between linguistically interacting systems through the development of a cooperative domain of interactions." The neuronal connections, which are closed to the outside, neither process symbols nor represent the outside in our nervous system. But our symbol creation and processing is taking place in the culture in which the symbols are constantly used and regenerated. The part of brain that learns and uses the symbols is not giving meaning to it. "The meaning has its anchor elsewhere down below. … ."[8]

It is interesting, our 'body being' is structurally coupled to the environment, while our 'mind-being' is symbolically coupled to other similar 'mind-beings' in our environment, and together we ensure our organization for life. "Humans thus bridge two worlds. We are hybrids, half analogizers, with direct experience of the world, and half symbolizers, embedded in a cultural web. During our evolution we somehow supplemented the analogue capacities built into our brains over hundreds of millions of years with a symbolic loop through culture [8]."

Winograd and Flores [35] from another perspective, while paraphrasing Heidegger's philosophy say, "Meaning is fundamentally social and cannot be reduced to the meaning-giving activity of individual subjects. The rationalistic view of cognition is individual-centered."

Thus we are embarking on a new understanding of cognition. This is different from the cognitive science fabricated by the marriage of theories of human thought and formal languages, residing on rationalistic landscape. Winograd and Flores [35] summarize three assumptions behind the traditional cognitive science as: 1) all cognitive systems are symbol systems. 2) All cognitive systems share a basic underlying set of symbol manipulating processes and 3) a theory of cognition can be couched as a program in an appropriate symbolic formulation such that the program when run in the appropriate environment will produce the observed behaviour. This metaphor is also present in neuroscience, and Maturana and Varela[19]observe, "…This puts the burden of knowledge on pre-given items in the world and leaves no place for the significance and meaning proper to the autonomy of the living." We cannot deny the fact that computational understanding of the mind has yielded remarkable results in Artificial Intelligence.

## 4. Missing Middle Hypothesis[1]

The history of interactions with the medium produces internal operations in an autopoietic entity by which it learns to come to terms with the perturbations in its medium or

[1] See Appendix 4 for conference presentations and seminars given by the author on *Missing Middle Hypothesis.*

environment. This in turn may give rise to rules or principles: if situation x do operation y or follow procedure z. This way the entity comes to terms with the perturbation itself and so it is unperturbed when it recurs. Understanding is essentially a feeling of coming to terms with the perturbations in the boundary of a living entity. How to sense whether the perturbation is altogether new or one of the known categories is another type of perturbation or irresolution requiring monitoring and learning, possibly in another level.

An organism when facing a perturbation in the medium, it just has to feel whether the changes in the medium are within the bounds. If it is out of bounds and the pattern is familiar it follows a known structural change possibly 'marked' by its history. If it works the bounds are revised. The bounds are learned through the history of interaction with the medium. Otherwise the perturbation could destroy its organization so it strives to do all that is possible, to maintain its organization, like redefining the bounds, or even to modify its structure (internal arrangement) in a totally new way.

A human continuing in an environment obeys this pattern of coming to terms with the perturbations in the environment and the experience registered has kind of only the bounds and the distribution in the middle is not 'marked' in her memory. This leads us to define what we call the Missing Middle Hypothesis:

A human experiential record/marking contains the changes in the medium, only if it amounts to a perturbation, and in that the blind spot is the middle of the distribution of such perturbations.

Though this may be true for any living system, (say biological/ community/ organisation), here we restrict our attention to human experience.

## Manifestations of Missing Middle Hypothesis

In everyday life in management situations, political arena and even in academia we see manifestations of missing middle hypothesis. For brevity, we choose not to be exhaustive in citing such examples here.

Consider if you are asked to describe your life in the past few months, what comes to your mind from your autobiographical memory? Unless you have a Proustean2 memory, you would have recalled a few extreme happenings in those days. For instance, students remember extremely good teachers, the bad ones or those with extreme characteristics or mannerisms; similarly, teachers remember such extreme students, even after many years.

Consider experiment 1: The participant is given a sequence of 20 cards, each containing two-digit numbers. Each card is dealt with the numbered face up, with some time gap, as the participant feels adequate to process the number. The participant is not doing any

[2] Marcel Proust (1871-1922), French novelist is best known for his extended memoir/autobiographical novel, *À la recherché du temps perdu*, (In search of lost time) translated in English as "Remembrance of Things Past", which runs into seven volumes.

computation or writing.  Then the next card is placed face up covering the earlier ones dealt. The task is to report the average of the 20 numbers dealt.

Consider experiment 2: The setting is the same as experiment 1 but the task is to report the maximum and the minimum of those numbers. Assume you are the participant. 1) In which one of the two experiments would you have taken less time to answer? 2) In which one of the two experiments are you more likely to get the correct answer?

'Two.' is the almost unanimous response received for these two experiments conducted in different forums consisting of psychologists, distributed software developers, students from tertiary educational institutes from different part of the world and participants from other walks of life.

Consider what has become a folk puzzle in the speed-reading community, if you can make out what the following passage reads you know why:

"Aoccdrnig to a rscheearch at Cmabrigde Uinervtisy, it doesn't mttaer in waht oredr the ltteers in a wrod are, the olny imporetnt tihng is taht the frist and lsat ltteer be at the rghit pclae. The rset can be a total mses and you can sitll raed it wthouit porbelm. Tihs is bcuseae the huamn mnid deos not raed ervey lteter by istlef, but the wrod as a wlohe."

In this we find the missing middle (jumbled middle) does not matter if one is already familiar with reading English; on the other hand, when one pays attention to each word, like a proof reader does, one is amazed at this capability of our brain.  For a reader new to the language this passage however would pose many situations of irresolution. It is interesting to note that Rawlinson remarks, "This reminds me of my PhD at Nottingham University (1976) [23], which showed that randomising letters in the middle of words had little or no effect on the ability of skilled readers to understand the text. Indeed one rapid reader noticed only four or five errors in an A4 page of muddled text"[3].

Consider managers in an organisation brainstorming a process needing attention (activities of repetitive nature) and resolution of an issue.  More often than not, it is observed (by the author and other applied researchers) that they bring only this middle-less experience about the process. So they exaggerate the extreme events they have noticed, even if they may be rare. Hardly do they dwell on the location or spread, unless they are habituated to use external support from data summaries produced by calculation.

## Witness for the Missing Middle

Our belief/opinion/understanding about our experience of the world around is formed without our being conscious about this blind spot, namely the missing middle. And this causes considerable conflict in our conversations, through which we construct/bring forth our

[3] http://www.mrc-cbu.cam.ac.uk/people/matt-davis/cmabridge/

worlds. And this pitfall is deep rooted and has got to do with our survival or autopoietic organizational requirement. If so what can be the available alternative? Like a prosthetic extension, we need the help from an artefact, which we can call the witness for the missing middle. Such devices are common in process monitoring, flying, the stock market, climate and natural disaster monitoring. The purpose of the artefact could be early warning, calling attention, providing summary principles and procedures and decision support.

## 5. Explaining Stock-Flow Failure using Missing Middle Hypothesis

A Model is a tool we use to exhibit our current understanding of a system we intend to understand. And so new observations of the system may change our current understanding of the system and so calls for a change of our model or our intention. Thus the dynamics of modeling is an adaptive process of feedbacks, intentions and understandings.

Language (mathematical, natural, computer, or other symbol manipulating system) is used to model our understanding, but when it limits we have the need to use other tools to enhance our understanding. A mathematician's hand waving might help some students to understand, but for some the hand waving will not constitute a proof or verification and so might require further explanation or formal proof. In understanding systems that are complex verbal descriptions due to the linearity of written account (or for that matter a monologue), at times are not suitable for explicating the interconnectedness. Pictorial rendering is tried at times to aid the understanding. Systems thinking is a visual language that uses, causal loops, graphs and stock and flow diagrams to communicate our understanding of complex systems and their dynamics.

An interesting aspect of systems is they exhibit behaviors not found in the components so a study of the components is neither sufficient nor necessary to understand the behavior of the system. For example knowledge about the hydrogen and oxygen atoms is not sufficient to know how water behaves, though they are the components of water. Similarly not knowing the components did not stop human race from using water from time immemorial for its own change.

The consensus domain of system dynamists, the "languaging" or the symbols (causal loops, feedback, delays and stock flow diagrams and so on) are relatively newly brought forth; though in the domain of mathematics, the symbolization used in calculus already captured concepts and representation of rate of change, integration or accumulation and dynamical equations. For students exposed to calculus but not to system thinking or system dynamics, these new symbolism is a perturbation in their mental and perceptual space. With this understanding of system dynamics tasks and models, we move on to explain the results of bathtub dynamics experiments discussed earlier using missing middle hypothesis.

### Missing Middle Hypothesis and Bathtub Dynamics

In order to see how MMH can be used to explain SFF, we need to identify the organism, medium and the perturbation first. In the bathtub dynamics tasks, the medium is the graduate school environment at large (including the physical, learning, social and other dimensions) and the interactions between students and educators. The organism is the well-educated student participant. The perturbation is any one of the bathtub dynamics tasks (in the departmental store setting, the four questions that need to be answered are - Q1: During which minute did the most people enter the store? Q2: During which minute did the most people leave the store? Q3: During which minute were the most people in the store? And Q4: During which minute were the fewest people in the store? (See Appendix 1 for details.) The observer is the experimenter, or the team including some assistants.

From the observer's perspective: The observer reports for the questions Q1 & Q2 the organism behaved as expected, but for Q3 & Q4 the behaviour of the organism perplexed the observer.

From the organism's perspective: Q1 & Q2 are not perturbations that lead to a situation of irresolution, as the organism has a known trick or principle or rule to apply. Especially this is so because the organism, even with the missing middle of the experience of having interactions with the display of history of arrivals and departures, the organism can identify the extremes (that is, time when maximum number of people arriving or similarly when minimum number are arriving). On the other hand Q3 & Q4 as perturbations pose a situation of irresolution because these tasks cannot be answered with the missing middle. So as noted earlier the organism either gives up (that is fails to find a structural change to survive the task) or tries one of the known tricks or principles or rules to apply. In this regard we see in the reported studies, different participants coming up with different answers mostly absurd as per the observer. And the predominant principle picked up by the participants turns out to be the correlation heuristics, as reported. Now why this particular heuristics was the favourite can also be explained using missing middle hypothesis. For the organism, the easily available part of the experience, of having interaction with the display of history of arrivals and departures, is the extremes. So it tries to pick a principle that could produce a solution as a function of this available input. And, if correlational reasoning in the repertoire of the organism, it is likely to be picked up for answering Q3 & Q4 dealing with the extremes with respect to stock, using available input (dealing with extremes of the experience from the display).

More detailed discussion on the various results obtained and concerns expressed in these experiments from the missing middle perspective will be taken up elsewhere.

The missing middle hypothesis is also useful in understanding the why of the celebrated Herbert Simon's bounded rationality of human decision makers [25, 12]. Other anomalies noticed with respect to probabilistic reasoning and human judgement [6, 7, 10 -13] also may have bearing on missing middle hypothesis.

## 6. Conclusions and Future Directions of Research

This paper offers a fresh explanation for what is called the stock flow failure in system dynamics literature. Booth Sweeney and Sterman [2] through their experiments with Bathtub Dynamics, relating to stock and flow models, report that irrespective of educational level, business background the subjects have a poor level of understanding of stock and flow relationships and time delays. Recently, Cronin, Gonzalez and Sterman [5] ask the question, 'Why don't well-educated adults understand accumulation?' Though accumulation is a fundamental process in dynamic systems, behaviour of simple stock-flow situations are not understood well by even students trained in mathematics or physics. Through a series of experiments Cronin et al. show that the poor performance is not explicable by inability to interpret graphs, lack of contextual knowledge, motivation, and lack of feedback or cognitive capacity. They also show the presence of an erroneous heuristics called 'correlation heuristics'. A summarization of these results is also presented for understanding the experiments and the consequences discussed in Cronin et al.

In this paper we explain this phenomenon applying what is called the Missing Middle Hypothesis (MMH) arising out of an understanding of cognition as put forward by Maturana and Varela. The paper presents a brief account of autopoiesis, cognition and human experience, so as to understand the concept of missing middle hypothesis. It deals with the question: why human experience does not record the middle of the distribution of its interactions with its environment, while facing perturbations in the environment. We also mention that MMH can explain other human decision making pitfalls noticed by researchers engaged in human decision making and paradoxical behaviour deviations from the expectations of rationalist theories. Especially showing that the availability bias [13] is a manifestation of MMH is promising for future research.

Other future research directions include, experimenting with missing middle hypothesis and its manifestations at different levels (neurological, psychological, learning & pedagogical and problem solving) and in different contexts. This might lead to the development of appropriate artefacts - witness for missing middle - in the context of bathtub dynamics or accumulation tasks involved in other decision situations. Hopefully, Cronin et al.'s [5] wish `of finding effective methods to improve performance on stock flow problems, improving our ability to understand and manage the complex systems affecting our personal lives, organizations, and society', will be fulfilled through such witness for the missing middle - artefacts developed.

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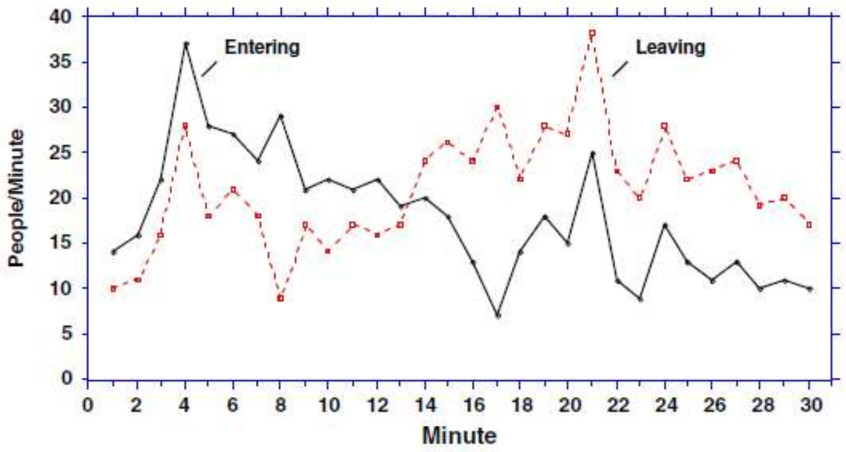
## Appendix 1 (Excerpted from: Cronin et al. [5])

*M.A. Cronin et al./Organizational Behavior and Human Decision Processes 108 (2009) 116–130*

The graph below shows the number of people *entering* and *leaving* a department store over a 30-minute period.




Please answer the following questions.

Check the box if the answer cannot be determined from the information provided.

1. During which minute did the most people enter the store?

Minute ________ ❑ Can't be determined

2. During which minute did the most people leave the store?

Minute ________ ❑ Can't be determined

3. During which minute were the most people in the store?

Minute ________ ❑ Can't be determined

4. During which minute were the fewest people in the store?

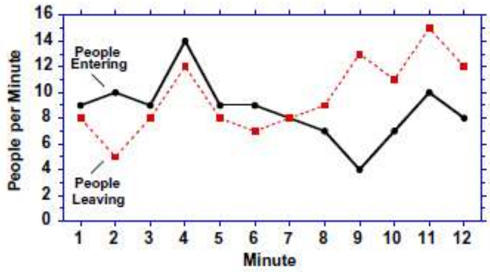
Minute ________ ❑ Can't be determined

**Fig. 1.** Department store task.

## Appendix 2 (Excerpted from: Cronin et al. [5])

*M.A. Cronin et al. / Organizational Behavior and Human Decision Processes 108 (2009) 116–130*

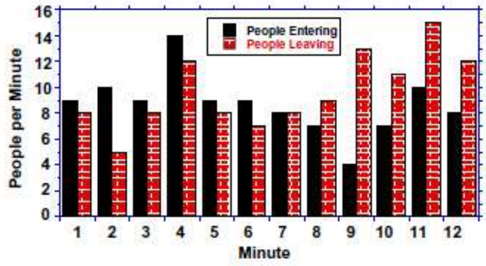
*A. Line graph*




*C. Table*

| Minute | People Entering | People Leaving |
|---|---|---|
| 1 | 9 | 8 |
| 2 | 10 | 5 |
| 3 | 9 | 8 |
| 4 | 14 | 12 |
| 5 | 9 | 8 |
| 6 | 9 | 7 |
| 7 | 8 | 8 |
| 8 | 7 | 9 |
| 9 | 4 | 13 |
| 10 | 7 | 11 |
| 11 | 10 | 15 |
| 12 | 8 | 12 |

*B. Bar Graph*




*D. Text*

In the first minute, 9 people enter and 8 leave. In the second minute, 10 people enter and 5 leave. In the third minute, 9 people enter and 8 leave. In the fourth minute, 14 people enter and 12 leave. In the fifth minute, 9 people enter and 8 leave. In the sixth minute, 9 people enter and 7 leave. In the seventh minute, 8 people enter and 8 leave. In the eighth minute, 7 people enter and 9 leave. In the ninth minute, 4 people enter and 13 leave. In the tenth minute, 7 people enter and 11 leave. In the eleventh minute, 10 people enter and 15 leave. In the twelfth minute, 8 people enter and 12 leave.

**Fig. 2.** Experiment 1: Visual isomorphs for the simpler department store task.

## Appendix 3 (Excerpted from [5])

**Table 4A**
Experiment 3: Effect of motivation and feedback on success rates for task 1

| | Question 1: Most entering? | Question 2: Most leaving? | Question 3: Most in store? | Question 4: Fewest in store? |
|---|---|---|---|---|
| No motivation/no feedback condition ($n = 37$) | 100% | 86.5% | 18.9% | 21.6% |
| Motivation/feedback condition ($n = 32$): Attempt 1 | 93.8% | 96.9% | 15.6% | 12.5% |
| Exact test, $p$ | .211 | .205 | .761 | .359 |
| Attempt 2 | 100% | 100% | 28.1% | 25.0% |
| Attempt 3 | | | 56.3% | 50.0% |
| Attempt 4 | | | 65.6% | 62.5% |
| Attempt 5 | | | 68.8% | 71.9% |
| Attempt 6 | | | 71.9% | 81.3% |
| Attempt 7 | | | 81.3% | 84.4% |
| Attempt 8 | | | 81.3% | 84.4% |
| Attempt 9 | | | 81.3% | 84.4% |

## Appendix 4 [Missing Middle Hypothesis related Conference Presentations and Seminars]

1. Arthanari, T.S., seminar on Foundations of a new Decision Science, on 7th June 2005, at Department of ISOM, University of Auckland.
2. Arthanari, T.S., invited presentation on Human Decision Making and Missing Middle hypothesis in June 2006, at The Psychology Research Forum of the Department of Psychology, University of Auckland.
3. Arthanari, T.S., Making Decisions within the Space of Seven Breaths, In Sylnovie Merchant (ed.), Proceedings of 4th Global Conference on Business & Economics, Oxford, UK, June 26-28, 1-18, 2005.
4. Arthanari, T.S., Invited talk on “Missing Middle Hypothesis and Statistical thinking” organised by CR Rao AIMSCS, Hyderabad, 1 April, 2009.
5. Arthanari, T.S., Invited talk on “Missing Middle Hypothesis and Statistical Thinking” as a lead panellist in the Panel Discussion, in the International Conference on Present Practices and Future Trends in Quality and Reliability [ICONQR08], 25 January 2008, Panel on ‘Statistical Thinking: Industrial Perspective’ Chairperson : Prof B.K. Sinha, ISI Kolkata other panellists: Prof C.R. Rao, FRS, Prof S.P. Mukherjee, Calcutta University, Dr A. Dharmadhikari, Tata Nano Project, and Prof J.K. Ghosh, Purdue University (Discussant).
6. Arthanari, T.S., invited talk to top 25 senior executives at BHEL, Trichy, India on 17th January 2008, on Missing Middle and New Management concepts.
7. Arthanari, T.S., invited talk (session 2) to 50 executives and senior level officers of BHEL, Trichy, India on 17th January 2008, on Missing Middle and New Concepts problem solving.
8. Arthanari, T.S., invited talk to software developers and system architects at CORENT Technology Private Limited, on 17 March, 2008, on Missing Middle – New concepts in distributed software development.
9. Arthanari, T.S., An Explanation of Stock and Flow Paradox -Why do we slip in the bathtub, at the Second Annual Oceania Regional Workshop on Supply Chain Management, University of Auckland, 26 November, 2010.
10. Arthanari, T.S., invited talk to business school staff and students of Indian Institute of Management, Indore, India, on Missing Middle Hypothesis, 9 July, 2012.