Win-lose, Lose-lose and Win-win Stabilization Policies for a Growth Cycle

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Abstract. This paper considers the Fanti and Manfredi Goodwinian two-dimensional model that stabilizes growth cycle by profit-sharing, although a long term employment rate declines, whereas the stationary relative wage is not affected. For checking robustness of profit-sharing, flexible capacity utilization is included. The Phillips – Wolfstetter – Flaschel 'capricious' investment function destroys stability of a non-trivial stationary state. Adding 'neoclassically' balanced government taxes and expenditures results in attaining stable stationary state again in a three-dimensional model. Yet stationary labour share (even gross) and stationary employment ratio becomes lower than in the initial model.

This paper revises the preceding equations. The first non-linear three-dimensional model implements proportional and derivative control over growth rate of profit. This rate depends on a gap between the indicated and current employment ratios and on growth rate of this ratio. The second four-dimensional model redefines this combined control applying excess income levy that equals subsidy. The previous models enable extreme condition tests for these non-Goodwinian models. Parametric policy optimization supported by *Vensim* shortens a transient to a deliberately high target employment ratio without lowering stationary relative wage against the Goodwinian models. The proposed policies enhance stability and efficiency of capital accumulation; they also provide stronger gains for workers' well-being.

Introduction

The valuable achievement of the Fanti and Manfredi paper (1998) is a warning on possible detrimental effects of standard profit-sharing as demonstrated in (Ryzhenkov 2013). The latter paper revises the equations for profit-sharing and bargained wage terms in the two substantially non-linear four-dimensional models of capital accumulation. The model built for closed economy (FM) for a specific accumulation rate (k = 1) when capitalists invest total profit in fixed capital is generalised for broader region of k; consequently, the original propositions are reconsidered.

An abrupt drop of the rate of accumulation can be destroyer for stabilisation policy through standard profit-sharing. So models with endogenous rate of accumulation are required. The Lordon (1995) paper implicitly and Ryzhenkov papers explicitly (2008, 2010, 2012) offered such Goodwinian models.

Still the models that hide an endogenous rate of capital accumulation under a veil of government taxes and expenditures represent a great interest for a deeper inquiry into the subject matter. Such models are developed at different corners of the world. The most well-known of them belongs to Wolfstetter (1982), the other have been presented by Yoshida and Asada (2007), later – by Flaschel (2009).

The mystery in these works has been waiting for explanation: why government expenditures balanced by taxes in Goodwinian or semi-Goodwinian models reduce stationary gross and net labour shares in national income and can even lessen long term employment ratio if standard profit-sharing is added? The voluminous literature on crowding-out effects, including the paper (Spencer, Yohe 1970) and subsequent ones, over-looks this fundamental problem and other important aspects of alternative stabilization policies.

The rest of this paper is organized in the following manner. Section 1 reviews briefly properties of a model of cyclical dynamics with profit sharing proposed in (Fanti and Manfredi 1998). This model is abbreviated as FM.

The FM extensive form is condensed into two-dimensional system of non-linear ODEs, local asymptotical stability of its non-trivial stationary state is exposed. Although the stationary relative wage remains the same as in the Goodwin model (GM), the stationary employment ratio declines. The stabilization policy through standard profit-sharing is win-lose for this reason. The Phillips (1961) – Wolfstetter (1982) – Flaschel (2009) 'capricious' investment function destroys stability of a non-trivial stationary state in a modified model (EFM). Explosive investment behaviour and breakdown (Zusammenbruch) because of dominant positive feedback loops mean that profit-sharing becomes (de)stabilizing lose-lose policy.

Section 2 explores how balanced taxes and expenditures narrow workers profit-sharing in an attempt to enforce stability of capital accumulation in a model (WFM) where balanced government taxes and expenditures taken from (Wolfstetter 1982) complement the standard profit-sharing. Unfortunately not only the stationary employment ratio declines in relation to GM, the stationary relative wage also shrinks.

With a sufficiently strong profit-sharing not only 'neoclassical' but also 'Keynesian' policy stabilizes a non-trivial stationary state. The difference between both is connected in the first place to a multiplier – negative in the former and positive in the latter – at a gap between stationary and current employment ratios in an equation for a variable tax rate. It is proved, that at a critical positive magnitude of this parameter a simple Andronov – Hopf bifurcation with a period longer than in GM happens and closed orbits are generated in the phase space.

The same 'capricious' investment function worsens the systemic risk and complicates the problem of stabilization again in extended WFM. It is demonstrated that for sufficiently strong 'neoclassical' policy the asymptotical stability of a non-trivial stationary state is maintained. A possibility of singularity in this model because of a specific magnitude of the same control parameter is found out. These uncovered properties heavily restrict opportunities of 'neoclassical' win-lose stabilization policy and practically exclude successful 'Keynesian' stabilization policy in extended WFM.

Section 3 offers a modernized model (MM) with two mathematically equivalent forms of reinforced employment-centred stabilization policy that is win-win policy unlike the previous ones. The first of them upgrades profit-sharing applying a combination of proportional and derivative control over a growth rate of profit, the second maintains capital accumulation, relative wage and employment through excess income levy that equals subsidy. 'Parasitic' (in the terminology of Wolfstetter 1982) government taxes and expenditures are not required in MM at all for a more efficient stabilization than in the preceding models.¹ A stationary relative wage is the same as in GM (higher than in WFM), stationary employment ratio is deliberately higher than in GM, whereas stationary relative excess income levy equals zero.

Even Maxwell's demon would be happy with such accomplishments of the proposed feedback and feed-forward control. A booklover may recall Shakespeare's "Taming of the Shrew".

1. A model of cyclical dynamics with profit sharing

1.1. The model extensive form

I shortly review a simplified two-dimensional form of the four-dimensional model presented in (Fanti and Manfredi 1998) and critically analysed in (Ryzhenkov 2013). Throughout this text a variable's time derivative is denoted by a dot, its growth rate – by a hat over the variable's sign.

Labourers are advancing capitalists as they receive wage after a particular circuit of capital is finished. Having abstracted from the public sector and foreign economic relations, FM consists of the following equations:

P = Km;	(1.1)
a = P/L;	(1.2)
u = w/a, 0 < u < 1;	(1.3)
$\hat{a} = h > 0,$	(1.4)
m = const < 1;	(1.5)
v = L/N, 0 < v < 1;	(1.6)
$N = N_0 e^{nt}, n = const \ge 0, N_0 > 0;$	(1.7)
$\hat{w} = -g + rv + e(1 - u)m, g > 0, r > 0, 0 < e < kh/d;$	(1.8)
$P = C + \dot{K} = wL + (1 - k)S + \dot{K};$	(1.9)
$\dot{K} = kS = k(1 - u)P, d/m < k \le 1.$	(1.10)

Equation (1.1) specifies a technical-economic relationship between the fixed capital K and the net output P. Output-capital ratio is denoted by m. Equation (1.2) expresses output per worker a as a ratio of net output P to employment L. Equation (1.3) describes the relative wage as the labour share in net output u. Equation (1.4) assumes a constant exogenous growth rate of output per worker a that equals to the growth rate of capital intensity K/L, whereas output-capital ratio m remains constant according to equation (1.5).

Equation (1.6) defines the employment ratio v as a result of the sale of the labour power. According to equation (1.7), the growth rate of labour force N is equal to a constant n. Equation (1.8) links the growth rate of real unit wage (w) with employment ratio v and profit rate (1 - u)m.

¹ This section challenges, in particular, the view in (Wolfstetter 1982: 376) "that polemically speaking, the "state" ... is ... a "macroparasite" who preys upon the public without providing direct benefits to any subgroup." We may see the task of government action in providing social benefits greater (and no less) than social costs especially when 'invisible hand' fumbles.

The use of current profit reflects absence of information lags for labourers regarding the actual relative wage. In other words, capitalists and workers receive information on relative wage in real time.

A growth rate of wage is represented as the sum of bargained \hat{w}^m and profit sharing \hat{w}^b terms

$$\hat{w} = \hat{w}^m + \hat{w}^b, \qquad (1.8)$$

where the first is determined by employment ratio v as in a linear Phillips equation

$$\hat{w}^m = -g + rv, \tag{1.8a}$$

and the second – by the profit rate

$$\hat{w}^b = e\left(1 - u\right)m,\tag{1.8b}$$

here e > 0 is a profit sharing index.

Balance equation (1.9) shows the end use of the net output P, where C is a private consumption, \dot{K} is net fixed capital formation in the equation (1.10). Investment delays are not taken into account.

It is expected that the surplus product *S* that equals total profit *M* can be not only invested, but also be used to cover personal expenses of the bourgeoisie. Consequently, the rate of accumulation k, or share of investments in surplus product, is such that $d/m < k \le 1$. The left boundary is set to avoid a non-positive stationary relative wage.

The presence of d/m as a lower boundary for the rate of accumulation is a drawback of both GM and FM, since in reality, relative wage remains positive even when $d/m \ge k$. This means they do not pass this particular extreme condition test (cf. Sterman, 2000: 337). Models in the articles (Ryzhenkov 2008 and 2010) contain endogenous capital-output ratio and endogenous rate of accumulation in the absence of the specified lower bound as a real necessity. Long term decline in this ratio mitigates the tendency of profit rate to fall in Italy and in the USA as well.

Figure 1.1 and Table 1.1 present a causal loop structure of FM. Loop B1 is inherited from GM, loop B2 is due to the profit sharing rule. Besides these, FM includes two minor feedback loops for relative wage u and employment ratio v with alternating polarity.



Figure 1.1. A causal structure of FM without information delay in relative wage

Loop B1 of length 9	Loop B2 of length 9
Wage share $u \xrightarrow{-}$	Wage share $u \xrightarrow{-}$
Profit rate	Profit rate
Growth rate of fixed capital	Growth rate of profit sharing wage term
Growth rate of employment ratio	Growth rate of wage
Net change of <i>v</i>	Growth rate of wage share
Employment ratio v	Net change of <i>u</i>
Growth rate of bargained wage term	
Growth rate of wage	
Growth rate of wage share	
Net change of <i>u</i>	

Table 1.1. Two main negative feedback loops in FM

Note. Only a negative first partial derivative is explicitly shown as an arrow. All other first partial derivatives are positive.

1.2. The model intensive form and properties of a stationary state

As shown in (Fanti and Manfredi 1998), an intensive form of deterministic FM without information delays consists of two non-linear ordinary differential equations. Here is this system in a generalised form for $d/m < k \le 1$ in relation to the original form (for k = 1):

$$\dot{u} = [-g + rv + e(1 - u)m - h]u \tag{1.11}$$

$$v = [k(1-u)m-d]v.$$
 (1.12)

A positive stationary state of the system (1.11–1.14) is defined as

$$E_a = (u_a, v_a), \tag{1.13}$$

where $u_a = u_G = 1 - \frac{d}{km}$, $v_a = v_G - \frac{e}{k} \frac{d}{r} < v_G = (g + h)/r$.

A stationary growth rate of output per worker and growth rate of wage equals h. A stationary growth rate of fixed capital and net output is $\hat{K}_a = \hat{P}_a = d = h + n$, $d \ge h$. A stationary rate of surplus value is $m_G' = (1 - u_a)/u_a$. A stationary profit rate is $(1 - u_a)m_a = d/k$.

Let's pay also attention to a very important positive dependence of the stationary relative wage and employment ratio on the rate of accumulation: $\frac{\partial u_a}{\partial k} = \frac{d}{mk^2} > 0$, $\frac{\partial v_a}{\partial k} = \frac{ed}{rk^2} > 0$. As we see below (section 2.1) this property surprisingly explains effects of government taxes and expenditures on stationary relative wage and employment ratio in subsequent models.

For the stationary state (1.13) equation (1.14) defines the Jacoby matrix in FM:

$$J(E_a) = \frac{-emu_a}{-kmv_a} \frac{ru_a}{0}$$
(1.14)

It has a negative trace and positive determinant. Therefore the stationary state E_a is a stable node or focus in this model depending on parameters (Fanti and Manfredi 1998). The right panel of Figure 1.2 reflects a stable focus in FM replacing neutral centre in GM on the left panel.

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FM establishes divide between long term steady state growth and jobs creation that deserved careful consideration in (Ryzhenkov 2013). One of the model main paradoxes resides in stabilisation policy that governs economy to lower employment ratio in the long term than before the policy onset. For the same parameters, the stationary employment ratio is lower in FM than in GM: $v_a < v_G$. The relative decline of stationary employment ratio after onset of the stabilisation policy is $\frac{v_a - v_G}{v_G} = -\frac{e}{k} \frac{d}{g+h}$ in agreement with definition (1.13). This has other harmful consequences.



Figure 1.2. A neutral centre in GM (l.) and stable focus in FM (r.), clock-wise, 1958–2158: initial conditions: $u_0 = 0.8788$, $v_0 = 0.5180$; common parameters: g = 1, h = 0.02, k = 1, m = 0.33, n = 0.02, r = 2, additionally e = 0.1 in FM, stationary: $u_a = u_G \approx 0.8788$ and $v_a \approx 0.5080 < v_G = 0.51$, periods: $T_{FM} = 11.57 > T_G = 11.55$

The profit sharing rule does not alter the stationary relative wage u_G inherited from GM. Other stationary magnitudes (ratios and growth rates) also coincide. The long run distribution is left inalterable only in *relative* terms! As the proposed stabilisation policy reduces long run employment ratio v of steady growing labour force N, the employment L, net output P, surplus value S/a, total wage wL, consumption per head wv and profit M are, as a rule, lower that they would be in GM. This policy worsens reproduction and use of economic (first of all – labour) potential in the long term and typically even in the middle term. In particular, the higher profit sharing index e, the lower are the long term and usually even middle term output and employment.² This standard profitsharing is therefore a win-lose stabilization policy.

A proportional control in FM

An equivalent for equation (1.11) takes the form of combined proportional control over the net change of relative wage defined by deviations of employment ratio and profit rate from their stationary magnitudes

$$\dot{u} = r(v - v_a)u + e\left[(1 - u)m - \frac{d}{k}\right]u$$
 (1.11a)

An equivalent for equation (1.12) reflects proportional control over the net change of employment ratio

$$v = k \left[(1-u)m - \frac{d}{k} \right]$$
$$= km(u_G - u).$$
(1.12a)

The equations for two terms of the growth rate of wage can be equivalently presented as manifestation of combined proportional control in the respective elementary forms

$$\hat{w}^m = r(v - v_a) + h - e\frac{d}{k},$$
 (1.8c)

$$\hat{w}^{b} = e \left[(1-u)m - \frac{d}{k} \right] + e \frac{d}{k} .$$
(1.8d)

This proportional control is embryonic since there is no conscious targeting of a stationary magnitude of employment ratio. More developed forms of social control are offered below.

1.3. Explosive investment behaviour and breakdown (Zusammenbruch)

An abrupt drop of the rate of accumulation can be destroyer for this stabilisation policy. So models with investment functions are required. Papers of Lordon (1995), Ryzhenkov (2008, 2010, 2012) offered such Goodwinian models.

A particularly challenging case of investment function is that in (Wolfstetter 1982: 387, equation 3.17 and Flaschel 2009: 135, equations 4.30 and 4.32). It was proposed for explaining how a degree of capacity utilization is determined. Wolfstetter stipulated that firms consider a certain amount of excess capacity as desirable, since it allows them to cash in on demand fluctuations.

Instead of the equation (1.1) we have now

$$P = mKx, \tag{1.1a}$$

where the new variable x is a rate of capacity utilization (see Wolfstetter 1982 and Flaschel 2009 for details). A profit rate is now (1-u)mx.

² Notice that for a particular closed orbit in GM, the average magnitudes of relative wage and employment ratio are practically the same as their stationary counterparts. A similar correspondence is weaker for a stable focus or node in FM.

Wolfstetter (1982) added two further assumptions: 1) firms are uncertain concerning the deviation of the short-run (γ) from the long-run rate of growth in aggregate demand (d), i.e. the expected value of ($\gamma - d$) is zero; 2) a simple exponential error adjustment process with finite speed of response, ε . Since the assumed existence of a steady state solution d equals the natural rate of economic growth, $\gamma_n = h + n$, the following equation defines the proposed investment function (Wolfstetter 1982: 387, Flaschel 2009: 135)³:

$$\hat{K} = \varepsilon [d - \hat{K} + (x/x_a - 1)\theta] = \varepsilon (d - \hat{K} - 1 + x) = km [\dot{x} (1 - u) - x \dot{u}], \qquad (1.15)$$

where the growth rate of fixed capital is $\hat{K} = kmx(1-u)$, ε is an adjustment parameter, x and $x_a = 1$ denote the actual and the desired (exogenously given stationary) degree of capacity utilization; a conversion factor $\theta = 1 \left[\frac{1}{\text{year}} \right]$ is added for conformity of units of measurement. These both (θ and

 x_a) are omitted in the final form of the equation (1.15) and in equations below for brevity.

It follows from the equation (1.15) that

$$\dot{x} = [\varepsilon(d-1+x-\hat{K}) + \dot{u} \frac{\hat{K}}{1-u}] \frac{x}{\hat{K}} = \\ \{\varepsilon[d-1+x-kmx(1-u)] + kmx\dot{u}\} \frac{x}{kmx(1-u)} = \\ \varepsilon[d-1+x-kmx(1-u)] \frac{1}{km(1-u)} + \frac{\dot{u}}{1-u}x.$$
(1.16)

The latter equation is a generalization of a time derivative of capacity utilization rate for the particular investment function (Flaschel 2009: 135, equations 4.32 and 4.33) with $0 \le k = \text{const} \le 1$. An equivalent form (1.16a) presents itself below. Now we have all the elements for a death spiral.

The intensive form of extended FM (EFM) and instability of stationary state

Besides equation (1.16) the EFM intensive form contains the following two non-linear differential equations

$$\dot{u} = [-g + rv + emx(1 - u) - h]u$$
, (1.11b)

$$\dot{v} = [kmx(1-u) - h - n + \hat{x}]v.$$
 (1.12b)

A positive stationary state of the system EFM is defined as

$$E_a = (u_a, v_a, x_a),$$
(1.17)

where $x_a = 1$ and as before $u_a = u_G = 1 - \frac{d}{km}$, $v_a = v_G - \frac{e}{k}\frac{d}{r} < v_G$.

Here a stationary growth rate of output per worker and growth rate of wage equals h. A stationary growth rate of fixed capital and net output is $\hat{K}_a = \hat{P}_a = d = h + n$, $d \ge h$. A stationary rate of surplus value is $m_a' = (1 - u_a)/u_a$. A stationary profit rate is $(1 - u_a)m = d/k$.

For the stationary state E_a (1.17) equation (1.18) defines the Jacoby matrix for k = const

³ In the business cycle literature a similar investment function had been applied by Phillips (1961).

	$-emu_a < 0$	$ru_a > 0$	$em(1-u_a)u_a>0$	
$J_a =$	$\frac{\varepsilon - d - emu_a}{1 - u_a} v_a > 0$	$r\frac{u_a v_a}{1 - u_a} > 0$	$\left(d + \frac{\varepsilon}{d} - \varepsilon + emu_a\right)v_a > 0$	(1.18)
	$\frac{\varepsilon - emu_a}{1 - u_a} > 0$	$r\frac{u_a}{1-u_a} > 0$	$\frac{\varepsilon}{d} - \varepsilon + emu_a > 0$	

A trace at the stationary state in EFM is positive – hence the stationary state is always unstable in this system:

$$\operatorname{Trace}(J_a) = r \frac{u_a v_a}{1 - u_a} + \frac{\varepsilon}{d} - \varepsilon > 0.$$
(1.19)

I omit a formal proof that an unstable stationary state (1.17) is typically saddle-focus.

Independently of how strong is profit-sharing measured by index e, it is not able to stabilize capital accumulation with the chosen investment function. Like subprime lenders in the recent financial bubble, investors in productive assets in my story "ate their own cooking, and got poisoned" collectively if not individually. In this worst-case scenario the economy is undermined by rational irrationality.⁴



Figure 1.3. A condensed causal loop structure of EFM with polarity typical for a vicinity of a stationary state; total number of feedback loops – 8, among them:
1st order – 3 (1 – negative, 2 – *positive*), 2nd order – 3 *positive*, 3rd order – 2 *positive*

In the first instance this is due to the positive dependence of \dot{x} on x (cf. Phillips 1961, Flaschel 2009: 136) and, secondly, because of a positive dependence of \dot{v} on v. This model can even be with co-operation between u, v, x, no competition for jobs, and with only 'intra-specific' competition in u. In particular, at the stationary state $\frac{\partial \dot{v}}{\partial u} > 0$ as a rule, since typically $\varepsilon > d + emu_a$. Strictly speaking this is not Goodwinian (predator-prey) model. In particular, for e = 0 as in GM without profit sharing a stationary state is utterly unstable too.

⁴ This metaphor and the term *rational irrationality* are taken from Cassidy (2009: 273, 329). Rational irrationality – objectively determined behaviour that, on the individual level, is perfectly reasonable but that, when aggregated in the marketplace, produces calamity (Ryzhenkov 2000: 25–43).

We see that profit-sharing under the stipulated investment behaviour becomes *a fix that fails* due to the dominant positive feedback loops that propel the model economy to death. The former win-lose profit-sharing suddenly turns into lose-lose (de)stabilization policy. The nightmare of complete breakdown (*Zusammenbruch*) is awakened.

Still we should not succumb to a vision of this dangerous opportunity. Questions for the system dynamics specialists suggest themselves. First, how can we convert the dominant positive feedback loops into negative? More technically, second, how do we turn two positive partial derivatives $\frac{\partial \dot{v}}{\partial v}$ and $\frac{\partial \dot{x}}{\partial x}$ for a stationary state into negative for curing the self-reinforcing explosive processes? Third, what will be a 'price' of this conversion?

Third, what will be a price of this conversion?

Narrowing workers' profit-sharing by balanced taxes and expenditures

2.1. A growth cycle with a cyclically non-neutral government sector

J. Cassidy (2009: 233-234) observes disproving an ode to the invisible hand: "...what usually enables modern economies to «right themselves» is prompt government action".

To discuss the effects of fiscal policies, Wolfstetter (1982) introduced a public sector into the model. His definition of a tax base is flawed due to confusion between productive and fictitious capital (debt in form of bonds). The same drawback is the characteristic of the Flaschel (2009, Chapter 4) update of the Wolfstetter model.⁵ It is possible still to extract a valuable element of their considerations for a balanced budget when this confusion does not matter. This refinement leads to a technically more complicated model than proposed by Wolfstetter (1982) and Flaschel (2009).

Let *T* and *G* denote government taxes and expenditures, respectively. They are quantitatively the same by my assumption. Equation (2.1) describes a government policy rule that implies a constant proportion $\delta \ge 0$ of government spending and taxing in national income for the steady state

$$T = \delta_1 P = G = \delta P + \mu (\nu^* - \nu) P, \qquad (2.1)$$

where rather fuzzy still quite plausible bounds may be set as $0.5 \ge \delta_1 \ge 0$, $0.5 \ge \delta \ge 0$, $|\mu| \le 3$. Non-vague bounds for specific values of parameters δ and μ can be determined only with a help of computer simulations. The case $\delta_1 = 0$ considered above will be generalised.

Equation (2.1) implies $\delta_1 = G/P = T/P = \delta + \mu(v^* - v)$. Notice that $\dot{\delta}_1 = -\mu \dot{v}$.

A 'Keynesian' policy, according to Wolfstetter, would attempt to counteract the cycle in vain by choosing $\mu > 0$, whereas a 'neoclassical' policy would reduce expenditure in the slump and is thus characterized by $\mu < 0$ (see Wolfstetter (1982: 379–383) for further details).

My central result in this section: 'Keynesian' fiscal policy will be destabilizing while a sufficiently strong 'neoclassical' policy will do the opposite. This is already observed in Wolfstetter (1982) and Flaschel (2009), although for different models that are theoretically less accurate than the present one because of their confusion between productive and fictitious capital as noticed already above.

⁵ A corrected definition of a tax base without noticing its incongruity in (Wolfstetter 1982: 386, equation (3.8)) was proposed in (Yoshida and Asada 2007: 444) for a similar framework. This refinement was not taken into account in (Flaschel 2009).

Consider, first, government stabilization policy together with profit sharing in a model setting without the above potentially implosive investment behaviour in a modified two-dimensional model without an explicit rate of capacity utilization and without investment function.

National income is the sum of gross incomes of workers and capitalists:

$$vL + M = P. \tag{2.2}$$

After government taxes and expenditures are added consumption of workers and capitalists is presented as

$$C = C_w + C_c, \tag{2.3}$$

where

$$C_w = w(1 - \delta_1)L,$$

$$C_c = (1 - k)M(1 - \delta_1) = (1 - k)(1 - \delta_1)(1 - u)P_s$$

Consider budget constraints. Start with the workers budget constraint

$$Pu(1 - \delta_1) - C_w = 0, \tag{2.4}$$

u = w/a is the wage share in national income before taxes on wages and profits are paid.

Turn to the capitalists budget constraint

$$P(1-u)(1-\delta_1) - \dot{K} - C_c = 0, \qquad (2.5)$$

where investment are equal to net fixed capital formation $\dot{K} = kM(1 - \delta_1)$, $0 < d/m < k(1 - \delta) = const \le 1$, $\hat{K} = k(1-\delta_1)(1-u)m$. The last definition is a modification of the similar one in section 1.1.

The previous literature with an implicit rate of capital accumulation overlooked the effect of the tax rate δ_1 on this key variable. Now the rate of capital accumulation can be defined as $c = k(1 - \delta_1) < k$ for $\delta_1 > 0$ assumed here and below. This lowering affects strongly the main characteristics of macro-economic dynamics.

Product market equilibrium requires

$$P = C + \dot{K} + G = w(1 - \delta_1)L + (1 - k)M(1 - \delta_1) + \dot{K} + G.$$
(2.6)

After rearranging and substituting net capital formation is obeying the material balance

$$\dot{K} = P(1-\delta) - [w(1-\delta_1)L + (1-k)M(1-\delta_1)] - \mu (v^*-v)P.$$
(2.7)

Figure 2.1 displays a causal-loop structure of the extended model. The only new feedback loop reflected in Table 2.1 can be stabilizing or destabilizing depending on $sgn(\mu)$.



Figure 2.1. A causal structure of WFM

Table 2.1. A new main first-order feedback loop in WFM

-	
Loop R1 ($\mu > 0$) or B3 ($\mu < 0$) of length 4	
Growth rate of employment ratio	
Net change of v	
Employment ratio $v \xrightarrow{-}$ for $\mu > 0$ or $\xrightarrow{+}$ for $\mu < 0$	
Tax rate $\delta_1 \xrightarrow{-}$ Growth rate of fixed capital	

The model intensive form can be easily derived. We assume gross profit-sharing as in FM since production and primary distribution of national income lie at very heart of workers-capitalists relations. The first ODE for the relative wage is given by equation (1.11). The second ODE is for the employment ratio

$$v = (\hat{K} - d)v = [k(1 - \delta_1)(1 - u)m - d]v = \{k[1 - \delta - \mu(v^* - v)](1 - u)m - d\}v,$$
(1.12b)

where $1 > \delta > 0$, $0 < \delta_1 = \delta + \mu(v^* - v) < 1$.

It is important to define a time derivative of the true rate of capital accumulation $c = k(1-\delta_1)$: $\dot{c} = -k\dot{\delta}_1 = k\mu\dot{v}$. So the time derivative of the rate of capital accumulation is directly connected with the time derivative of the employment ratio if $\mu > 0$. For $\mu < 0$, the higher is \dot{v} the lower is \dot{c} . Therefore the following interpretation suggests itself: the 'Keynesian' policy with $\mu > 0$ is procyclical with respect to the rate of capital accumulation whereas the 'neoclassical' policy with $\mu < 0$ is counter-cyclical.⁶

A positive stationary state of the system (1.11–1.12b) is defined as

$$E_b = (u_b, v_b), \tag{1.13a}$$

where

$$0 < u_b = 1 - \frac{d}{k(1-\delta)m} < u_a = u_G < 1, \ 0 < v_b = v_G - \frac{e}{k(1-\delta)}\frac{d}{r} < v_a < v_G < 1.$$
 The stationary rate of

surplus value before taxes is $m_b' = \frac{1 - u_b}{u_b} > m_a'$.

Basically expressions for u_b and u_a as well as for v_a and v_b are *qualitatively* the same because they use the same expression for the stationary relative wage $1 - \frac{d}{c_b m}$ and the same expression for

⁶ An opposite (honestly, rather superfluous) interpretation disconnected from the rate of capital accumulation in a model (similar to Wolfstetter's model) was given in (Yoshida and Asada 2007: 445): "Government's fiscal policy is counter-cyclical when $\mu > 0$, while it is pro-cyclical when $\mu < 0$." The authors of that paper did not discuss the effects of stabilization policies on a stationary rate of capital accumulation, stationary employment ratio and stationary relative wage.

A deeper analysis of the Yoshida and Asada contributions goes beyond a limited scope of my paper that does not emphasize effective demand and effects of the policy lag on macroeconomic stability that are in the focus of their research.

the stationary employment ratio $v_G - \frac{e}{c_b} \frac{d}{r}$. The discrepancy between u_b and u_a as well as between v_a and v_b are based on the difference in the stationary rate of capital accumulation: $c_a = k$ in the first case and $c_b = k(1 - \delta)$ in the second.

The stationary relative wage u_b and stationary employment ratio v_b for e > 0 are both lower than previous ones (respectively, u_a and v_a) because the current stationary rate of capital accumulation $k(1 - \delta)$ is lower than the previous one (k). This explains the mystery stated in *Introduction*.

Restrictions on the stationary rate of accumulation seem stronger in the present model (WFM) than in FM and EFM: $1 \ge k > \frac{d}{m(1-\delta)} > 0$. Still for a generalized stationary rate of capital accumu-

lation c_b they remain the same as before: $1 \ge c_b > \frac{d}{m} > 0$. A failure to pass extreme condition test

still remains: the requirement $0 < u_b$ is violated for $k \le \frac{d}{(1-\delta)m}$ or $c_b \le \frac{d}{m}$.

The stationary net relative wage is defined quite independently of profit-sharing index e

$$u_b(1-\delta) = 1-\delta - \frac{d}{km} = u_G - \delta < u_a = u_G.$$

We would like to emphasize growing inequality in distribution of net output against GM as result of government taxes and expenditures. The stationary net relative wage declines and stationary gross relative wage has become lower too. There is also worsening of the long term employment ratio. Consumption a head = $wv(1 - \delta_1)$ is also *ceteris paribus* lower than in FM.

For the system of the two ODEs (1.11)-(1.12b) equation (1.14a) defines the Jacoby matrix in WFM. We see this model still belongs to Goodwin-type (predator-prey) models.

$$J = \frac{\hat{u} - emu}{-km[1 - \delta + \mu(v - v_b)]v < 0} \frac{ru > 0}{\hat{v} + km\mu(1 - u)v}.$$
 (1.14a)

A Jacoby matrix (1.14b) for stationary state (1.13a) is defined as

$$J_{b} = \frac{-emu_{b} < 0 \quad ru_{b} > 0}{-km(1-\delta)v_{b} < 0 \quad km\mu(1-u_{b})v_{b}} . (1.14b)$$

This matrix shows besides 'intra-specific' competition for relative wage an appearing of stabilizing 'intra-specific' competition for jobs for $\mu < 0$ additionally as both $\frac{\partial \dot{u}}{\partial u} < 0$ and $\frac{\partial \dot{v}}{\partial v} < 0$ at the stationary state.

2.2 A local stability analysis

The characteristic equation connected to the Jacoby matrix J_b is defined as

$$a_0 \lambda^2 + a_1 \lambda + a_2 = \lambda^2 + [emu_b - km\mu(1 - u_b)v_b]\lambda + kmu_b v_b [(1 - \delta)r - em\mu(1 - u_b)] = 0, \qquad (2.8)$$

13

where $a_0 = 1$,

$$a_1 = -Trace(J_b) = emu_b - km\mu(1 - u_b)v_b,$$
(2.9)

$$a_{2} = det(J_{b}) = -emu_{b}km\mu(1-u_{b})v_{b} + km(1-\delta)v_{b}ru_{b} = kmu_{b}v_{b}[(1-\delta)r - em\mu(1-u_{b})].$$
(2.10)

The stationary state is asymptotically locally stable for $a_1 > 0$ and $a_2 > 0$ iff

$$emu_b - km\mu(1 - u_b)v_b > 0$$
 (2.11)

and

$$[(1-\delta)r - em\mu(1-u_b)] > 0.$$
 (2.12)

The inequalities (2.11) and (2.12) are clearly satisfied for $\mu < 0$ even if e = 0 (no profit sharing). Still their satisfaction is possible for a restricted interval with $\mu > 0$ and e > 0 as well. We explore this issue deeper.

The roots of the characteristic equation (2.8) are

$$\lambda_{1,2} = \frac{km\mu(1-u_b)v_b - emu_b}{2} \pm \sqrt{\left[\frac{km\mu(1-u_b)v_b - emu_b}{2}\right]^2 + kmu_bv_b[em\mu(1-u_b) - (1-\delta)r]} = \frac{d\mu v_b}{2(1-\delta)} - \frac{emu_b}{2} \pm \sqrt{\left[\frac{d\mu v_b}{2(1-\delta)} - \frac{emu_b}{2}\right]^2 + e\mu\frac{dmu_bv_b}{1-\delta} - \frac{drv_bu_b}{1-u_b}}$$
(2.11).
For a focus a period of fluctuations is
$$T_c = 2\pi / \sqrt{\frac{drv_bu_b}{1-u_b} - e\mu\frac{dmu_bv_b}{1-\delta} - \left[\frac{d\mu v_b}{2(1-\delta)} - \frac{emu_b}{2}\right]^2}.$$
(2.12)

Notice that this period is typically longer than the period of conservative fluctuations in the GM: $T_c > T_G = 2\pi / \sqrt{\frac{drv_G u_G}{1-u_G}}$. For example, for parameters that are the same as before for FM above and additionally $\delta = 0.12$, $\mu = -0.5$, $0.1155 \le \delta_1 \le 0.1251$, we get $T_c = 12.46 > T_{FM} = 11.57 > T_G = 11.55$ for $u_b = 1 - \frac{d}{k(1-\delta)m} = 0.8622 < u_G = 1 - \frac{d}{km} = 0.8787 < 1$, $v_b = 0.5077 < v_a = v_G = 0.51$.

It could be easily shown that a simple Andronov – Hopf bifurcation may happen in this system of two non-linear ODEs. Consider μ as the bifurcation parameter. The parameter of the characteristic equation (2.8) $a_1 = 0$ for

$$\mu_g = \frac{emu_b}{km(1 - u_b)v_b} = \frac{eu_b}{k(1 - u_b)v_b}.$$
(2.13)

Then the transversality condition of the Hopf theorem is satisfied at μ_g :

$$\frac{\partial \operatorname{Re}\lambda_{1}(\mu)}{\partial \mu} = \frac{\partial Tr(Jeq)/2}{\partial \mu} = \frac{km(1-u_{b})v_{b}}{2} = \frac{dv_{b}}{2(1-\delta)} > 0.$$
(2.14)

For the same set of illustrative data $\mu_g = 1.232 < \mu_{max} = 3$, $0.1073 \le \delta_1 \le 0.1323$ with period $T_{LC} \approx 2\pi/\text{Im}(\lambda_1) = 12.48$ (Figure 2.2.). This Andronov – Hopf bifurcation is super-critical.

The restriction $\mu < 0$ is necessary and sufficient for stability of a stationary state in my model without profit-sharing (e = 0) as in simple models in (Wolfstetter 1982) and (Flaschel 2009). We see that a successful 'Keynesian' stabilization policy is also possible hereby entirely due to profit sharing for e > 0 if $0 < \mu < \mu_g$. This result is new in relation to (Wolfstetter 1982), (Yoshida and Asada 2007) and (Flaschel 2009) that do not take profit-sharing into account at all in their characteristics of 'Keynesian' and 'neoclassical' stabilization policies. The mixture of the standard profit-sharing and 'Keynesian' or 'neoclassical' stabilization policies is also a variety of win-lose policies.



Figure 2.2. Comparison of conservative fluctuations in GM with self-sustained fluctuations in WFM, 1958–2021: on the left Panel – relative wage – gross u and net $u(1 - \delta_1)$, on the right Panel – employment ratio v

It follows from an economic restriction on e < kh/d (Ryzhenkov 2013) that maximal $\mu_g < \frac{h}{d} \frac{u_b}{(1-u_b)v_b}$. For the same parameters as before maximal $\mu_g \approx 6.162$ that is outside the permissible (roughly determined) economic interval, $-3 \le \mu \le \mu_{max} = 3$ ($\mu_g > \mu_{max}$). Thus profit-sharing can typi-

cally stabilize instability resulting from 'Keynesian' policy with $0 < \mu \le 3$ (taken for certainty k = 1, $0.1 \le \delta \le 0.5$). Still we ought not to forget about the above 'capricious' investment function and flexible rate of capacity utilization that aggravate the difficulty of the stabilization problem in a model that follows.

2.3. Complication of the stabilization problem due to 'capricious' investment function

Government taxes and expenditures introduced in the previous section affect the former investment function. This becomes more complicated. In result the complexity of a newly constructed model is grown tremendously.

Now the intensive form of the modified model including investment function consists of the following three non-linear ODEs.

$$\dot{u} = [-g + rv + emx(1 - u) - h]u$$
, (1.11b)

$$v = (\hat{K} - d + \hat{x})v = [kx(1 - \delta_1)(1 - u)m - d + \hat{x}]v, \qquad (1.12c)$$

$$\dot{x} = [\varepsilon(d-1+x-\hat{K}) + \dot{u}(1-\delta_1)kmx - (1-u)kmx\mu(\hat{K}-d)v]/\{(1-u)km[(1-\delta_1) + \mu v]\} = 0$$

$$\frac{1}{(1-u)(1-\delta_1+\mu v)} \left[\dot{u}(1-\delta_1) - (1-u)\mu(\hat{K}-d)v + \frac{\varepsilon}{kmx}(d-1+x-\hat{K}) \right] x.$$
(1.16a)

Notice that the important feature of the equation (1.16a) is a possibility of singularity detrimental for stability when $1-\delta_1 + \mu v = 0$ if $\mu < 0$. Therefore defining δ and μ should be made within safe and sound margins. It is hardly possible without prior simulations, here policy tests by Vensim are very helpful indeed.

The system (1.11b), (1.12c) and (1.16a) has a stationary state defined as

$$E_c = (u_b, v_b, 1),$$

(1.13b)

where u_b and v_b are the same as those in (1.13a), $x_b = 1$.

Partial derivatives of the growth rate of fixed capital for $\delta_1 = \delta + \mu (v^* - v)$ and $\dot{\delta}_1 = -\mu \dot{v}$ with $(\partial \hat{K})$

respect to the phase variables in Tables 2.1 and 2.2 will be helpful. Notice that $sgn\left(\frac{\partial \hat{K}}{\partial v}\right) = sgn(\mu)$.

$\partial \hat{K}$	$\partial \hat{K}$	$\partial \hat{K}$
∂u	∂v	∂x
$-k(1-\delta_1)xm < 0$	$kx\mu(1-u)m$	$k(1-\delta_1)(1-u)m > 0$

Table 2.1. Partial derivatives of growth rate of fixed capital $\hat{K} = k(1-u)(1-\delta_1)mx$

Table 2.2. Partial derivatives of growth rate of fixed capital $\hat{K} = k(1-u)(1-\delta_1)mx$ at the stationary state (1.13b)

$rac{\partial \hat{K}}{\partial u}$	$\frac{\partial \hat{K}}{\partial v}$	$\frac{\partial \hat{K}}{\partial x}$
$-k(1-\delta)m = -d\frac{1}{1-u_b} < 0$	$\mu d \frac{1}{1-\delta}$	<i>d</i> > 0

The Jacoby matrix for the stationary state (1.13b) of the system (1.11b), (1.12c) and (1.16a) is defined as

$$J_{c} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} . (2.15)$$

The particular elements of this matrix follow together with their typical signs:

$$J_{11} = -emu_a < 0, J_{12} = ru_a > 0, J_{13} = em(1-u_a)u_a > 0,$$

$$J_{21} = -k(1-\delta)mv_b + \frac{\partial \hat{x}}{\partial u}v_b =$$

$$-k(1-\delta)mv_b + \frac{1}{(1-u_a)(1-\delta+\mu v_b)} \left[-emu_a(1-\delta) + \mu dv_b + \varepsilon(1-\delta)\right]v_b < 0,$$

$$J_{22} = km\mu(1-u_a)v_b + \frac{\partial \hat{x}}{\partial v}v_b =$$

$$km\mu(1-u_a)v_b + \frac{1}{(1-u_a)(1-\delta+\mu v_b)} \left[ru_a(1-\delta) - (1-u_a)\frac{d}{1-\delta}v_b\mu^2 - \varepsilon\mu(1-u_a)\right]v_b < 0,$$

$$J_{23} = k(1-\delta)(1-u_{a})mv_{b} + \frac{\partial \hat{x}}{\partial x}v_{b} = k(1-\delta)(1-u_{a})mv_{b} + \frac{1}{(1-u_{a})(1-\delta+\mu v_{b})} \left[e\frac{d}{k}u_{a} - (1-u_{a})\mu dv_{b} + \frac{\varepsilon}{km}(1-d) \right]v_{b} < 0$$

$$J_{31} = \frac{1}{(1-u_{a})(1-\delta+\mu v_{b})} \left[\frac{\partial \hat{u}}{\partial u}(1-\delta) - (1-u_{a})\mu \frac{\partial \hat{K}}{\partial u}v_{b} - \frac{\varepsilon}{kmx}\frac{\partial \hat{K}}{\partial u} \right] = \frac{1}{(1-u_{a})(1-\delta+\mu v_{b})} \left[-emu_{a}(1-\delta) + (1-u_{a})\mu d\frac{1}{1-u_{a}}v_{b} + \frac{\varepsilon}{kmx}d\frac{1}{1-u_{a}} \right] = \frac{1}{(1-u_{a})(1-\delta+\mu v_{b})} \left[-emu_{a}(1-\delta) + (1-u_{a})\mu d\frac{1}{1-\delta}v_{b} - \frac{\varepsilon}{kmx}\frac{\partial \hat{K}}{\partial v} \right] = \frac{1}{(1-u_{a})(1-\delta+\mu v_{b})} \left[\frac{\partial \hat{u}}{\partial v}(1-\delta) - (1-u_{a})\mu \frac{\partial \hat{K}}{\partial v}v_{b} - \frac{\varepsilon}{kmx}\frac{\partial \hat{K}}{\partial v} \right] = \frac{1}{(1-u_{a})(1-\delta+\mu v_{b})} \left[ru_{a}(1-\delta) - (1-u_{a})\mu d\frac{1}{1-\delta}v_{b} - \frac{\varepsilon}{kmx}\mu d\frac{1}{1-\delta} \right] = \frac{1}{(1-u_{a})(1-\delta+\mu v_{b})} \left[ru_{a}(1-\delta) - (1-u_{a})\mu d\frac{1}{1-\delta}v_{b} - \frac{\varepsilon}{kmx}\mu d\frac{1}{1-\delta} \right] = \frac{1}{(1-u_{a})(1-\delta+\mu v_{b})} \left[\frac{\partial \hat{u}}{\partial x}(1-\delta) - (1-u_{a})\mu d\frac{\partial \hat{K}}{\partial x}v_{b} + \frac{\varepsilon}{km}(1-\delta) \right] < 0,$$

Figure 2.3 reflects the condensed causal loop structure of the extended model with polarity typical for a vicinity of the stationary state (1.13b).



Figure 2.3. A condensed causal loop structure of the extended model near a stationary state; total number of feedback loops - 8, among them: 1st order - 3 negative, 2nd order - 3 (2 negative, 1 positive), 3rd order - 2 positive

The elements of this Jacoby matrix including those on the main diagonal are negative except the pair in the first row ($J_{12} > 0$ and $J_{13} > 0$). The Routh-Hurwitz necessary and sufficient conditions for local asymptotical stability of the stationary state (1.13b) are satisfied.⁷ Thus the problem stated at the end of section 1.3 is solved as anticipated.

Instability in this extended model is cured mostly by appearance of the negative multiplier $\frac{1}{1-\delta+\mu v_b}$ on the main diagonal in the Jacoby matrix J_c . Singularity mentioned above reveals itself

again in all the elements of the Jacoby matrix J_c already for μ infinitesimally close to

$$\mu_{\text{critical}} = \frac{\delta - 1}{v_{\text{b}}} < 0. \tag{2.16}$$

Ultra-instability is typical for $\mu = \mu_{critical}$. Therefore in the region of stability μ is such that $-\mu_{max} = -3 < \mu < \mu_{critical} < 0 < \mu_{max} = 3$.

Simulations have revealed that although the two parameters of a characteristic equation a_1 , a_2 can be positive for $\mu > 0$, the third one $a_0 < 0$ – therefore stability of a stationary state is not amenable to a 'Keynesian' policy even under profit-sharing. A policy optimization enabled to find sub-optimal magnitudes of the control parameters: $\mu = -2 < \mu_{critical} \approx -1.3804$, $\delta = 0.3$, $\varepsilon = 1$ for $u_0 = u_b \approx 0.8268 > u_{b net} \approx 0.5787$, $v_0 = 0.5180 > v_b \approx 0.5071$, $x_0 = 1.2 > x_b = 1$, whereas the magnitudes of the other parameters remain the same. It is checked that the tax rate (0.1855 $\leq \delta_1 \leq 0.3562$) lies in the roughly permissible segment [0, 0.5]. The restriction $|\mu| \leq 3$ is also satisfied.

Simulations in a rather broad space of parameters magnitudes reveal that $\mu_{critical}$ from equation (2.16) defines extreme upper bound for stability range of μ . Only sufficiently careful and strong 'anti-Keynesian' policy for $-\mu_{max} < \mu < \mu_{critical}$ is able to solve the task of stabilization. Thus this paper disproves the claim (Wolfstetter 1982: 388): "The sign of μ does not matter. In other words, governments may subscribe to either "Keynesian" or "classical" views concerning effective fiscal stabilization; what matters alone for successful stabilization is the strength, and not the kind of response."

At a deeper level of this critical analysis, we come to a more profound assertion. Both types of stabilization policy ('Keynesian' and 'anti-Keynesian') are rather deficient and win-lose because they lead to the stationary employment ratio and relative wage that are both below than their counterparts in GM. These policies are not appropriate instruments for solving the problem of dynamic inefficiency of capitalism more fairly and successfully.

The previous authors possibly perceive this result as inevitable. No conscious vigorous attempt was made in their cited works for attaining a target employment ratio $X > v_b$.

The next section denies this unacceptable passivity and opportunism that is socially detrimental. We need stronger protection of public interests. Forestalling economic calamities like the Great Depression or the Great Recession before they start unfolding again is urgent. "The aim must be to prevent the emergence of rationally irrational behaviour. Unless some restrictions are placed on people's actions, they will inevitably revert to it" (Cassidy 2009: 344).

⁷ A rather tedious formal proof is skipped for brevity. I omit in particular a formal proof that a stable stationary state (1.13b) is typically focus-node.

3. Employment-centred stabilization of capital accumulation

The paper (Ryzhenkov 2013) has proposed a stabilization policy and reinforced stabilization policy in two models abbreviated as AM and MM. This section demonstrates that these policies can be developed further for curing the instability generated by the above 'capricious' investment function without aggravating inequality and unemployment.

3.1 An alternative design of profit-sharing and reinforced stabilization policy

According to the AM key assumption, owners of capital, state officials under pressure of workers parties, trade-unions and grass-root organisations set a target growth rate of profit depending on the difference between the indicated (X_1) and current (v) employment ratios (now taking into account the growth rate of capacity utilization \hat{x}):

$$\hat{M} = -\frac{\dot{u}}{1-u} + \hat{K} + \hat{x} = c_2(X_1 - v), \qquad (3.1)$$

where $c_2 > 0$, $v < X_1 = X + \frac{d}{c_2}$, X denotes a target employment ratio, absent in the opponents mod-

els, d is a stationary economic growth rate as before. Information delays are not taken into account again.

A *reinforced* stabilisation policy in MM modifies the latter equation by adding an element of derivative control (q > 1):

$$\hat{M} = c_2(X_1 - v) + (1 - q)\hat{v}.$$
(3.2)

The intensive form of MM consists of three non-linear ODEs (3.3), (1.12b) and (3.5). The first of them follows from the equations (1.12c) and (3.2):

$$\dot{u} = [c_2(v - X) + q\hat{v}](1 - u) =$$

$$\{c_2(v - X) + q[kmx(1 - u) - d + \hat{x}]\}(1 - u).$$
(3.3)

The equation (1.12b) is borrowed from the EFM. The third ODE (3.5) can be derived through the chain of transformations starting with equation (1.16):

$$\hat{x} = \frac{1}{1-u} \left[\dot{u} + \frac{\varepsilon}{kmx} (d-1+x-\hat{K}) \right] =$$

$$\frac{\dot{u}}{1-u} + \frac{\varepsilon}{(1-u)kmx} (d-1+x-\hat{K}) = \left[c_2 (v-X) + q \hat{v} \right] + \frac{\varepsilon}{(1-u)kmx} (d-1+x-\hat{K}) =$$

$$\frac{\varepsilon}{(1-u)kmx} (d-1+x-\hat{K}) + c_2 (v-X) + q \left[kmx (1-u) - h - n + \hat{x} \right] =$$

$$\frac{c_2 (v-X) + q \left[kmx (1-u) - d \right]}{1-q} + \frac{\varepsilon}{(1-u)(1-q)kmx} (d-1+x-\hat{K}). \quad (3.4)$$

Finally after multiplication of both sides by *x* we get the third ODE as

$$\dot{x} = \frac{c_2(v-X) + q(\hat{K}-d)}{1-q} x + \frac{\varepsilon}{(1-u)(1-q)km} (d - \hat{K} + x - 1), \qquad (3.5)$$

where $\hat{K} = k(1-u)mx$.

A non-trivial stationary state

A positive stationary state for the system of ODEs (3.3), (1.12c) and (3.5) is defined as $E_X = (u_a, X, 1),$ (3.6) where u_a is taken from equation (1.17) of EFM, the stationary (target) employment ratio $X = X_1 - \frac{d}{c_2} > v_G > v_a$ and finally the stationary rate of capacity utilization is identically one as before $x_a = 1.^8$

Correspondingly the two terms of the total wage growth rate \hat{w} – basal \hat{w}^m and stimulating \hat{w}^b – are presented as manifestation of combined proportional control in the respective elementary forms

$$\hat{w}^{m} = c_{2}(v - X)\frac{1 - u}{u} + c_{1}, \qquad (3.7a)$$

$$\hat{w}^{b} = h + q\hat{v}\frac{1 - u}{u} - c_{1} =$$

$$h + qk\left[(1 - u)mx - \frac{d}{k} + \frac{\hat{x}}{k}\right]\frac{1 - u}{u} - c_{1} =$$

$$\left(h + q\frac{1 - u}{u}\hat{x}\right) + qk\left[(1 - u)mx - \frac{d}{k}\right]\frac{1 - u}{u} - c_{1}, \qquad (3.7b)$$

where $c_1 = \text{const} > 0$ can be specified, for example, as $c_1 = h/2$, then $\hat{w}_c^m = \hat{w}_c^b = h/2$, and finally

$$\hat{w} = \hat{w}^m + \hat{w}^b. \tag{3.7}$$

Besides the profit sharing element in the modified form $qk\left[(1-u)mx-\frac{d}{k}\right]\frac{1-u}{u}$, there is a new element reflecting a multiplied effect of growth rate of the rate of capacity utilization $q\frac{1-u}{u}\hat{x}$ on wage growth rate. The sum $\left(h+q\frac{1-u}{u}\hat{x}\right)$ is analogue of an adjusted growth rate of output per worker as if it were dependent on the growth rate of the rate of capacity utilization that is positively associated with a growth rate of employment ratio.

Asymptotical local stability of the stationary state

Table 3.1 plays a supporting role in the current analysis. It contains partial derivatives of \hat{K} with respect to phase variables indispensable for a Jacoby matrix as in the previous similar cases.

⁸ Similar to FM (with exogenous output-capital ratio and exogenous accumulation rate) this model also does not pass the extreme condition test not allowing $u_a > 0$ for $0 \le k \le d/m$.

Table 3.1. Partial derivatives of \hat{K} with respect to phase variables at the stationary state $(u_a, X, 1)$

$\frac{\partial \hat{K}}{\partial u}$	$\frac{\partial \hat{K}}{\partial v}$	$\frac{\partial \hat{K}}{\partial x}$
$-km = -d\frac{1}{1-u_a}$	0	$k(1-u_a)m=d$

For the stationary state $E_X(3.6)$ equation (3.8) defines the Jacoby matrix for k = const:

	$qrac{-d+arepsilon}{1-q}$	$c_2(1-u_a) \frac{1}{1-q}$	$q\frac{1-u_a}{1-q}\left(d+\frac{\varepsilon}{d}-\varepsilon\right)$	
$J_X =$	$\frac{-d+\varepsilon}{(1-q)(1-u_a)}X$	$\frac{1}{1-q}c_2X$	$\frac{1}{1-q} \left(d + \frac{\varepsilon}{d} - \varepsilon \right) X$	(3.8)
	$\frac{-qd+\varepsilon}{(1-q)(1-u_a)}$	$\frac{1}{1-q} c_2$	$\frac{1}{1-q}\left(qd+\frac{\varepsilon}{d}-\varepsilon\right)$	

where for $c_2 > 0$, q > 1 the signs of this matrix elements are defined consequently:

$$J_{11} < 0 \text{ if } \varepsilon > d, \ J_{12} < 0, J_{13} < 0,$$

$$J_{21} < 0 \text{ if } \varepsilon > d, J_{22} < 0, J_{23} < 0,$$

$$J_{31} < 0 \text{ if } \varepsilon > qd, J_{32} < 0, J_{33} < 0.$$

Realistically $\varepsilon > qd$ and moreover $\varepsilon > d$. Thus all elements of this Jacoby matrix including those on the main diagonal are typically negative similar to the previous model in section 2.3 still without exceptions unlike the former. This model is not predator-prey, or Goodwinian, model any more for ($c_2 > 0$, q > 1). Figure 3.1 presents a condensed causal loop structure of MM with polarity typical for a vicinity of the stationary state.



Figure 3.1. A condensed causal loop structure of MM near a stationary state; total number of feedback loops – 8, among them: 1st order – 3 negative, 2nd order – 3 positive, 3rd order – 2 negative

The characteristic polynomial related to the Jacoby matrix J_X given by equation (3.8) is $\lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0 = 0,$ (3.9) where

$$a_{0} = -det(J_{X}) = -(J_{11} J_{22} J_{33} + J_{12} J_{23} J_{31} + J_{21} J_{32} J_{13} - J_{13} J_{22} J_{31} - J_{23} J_{32} J_{11} - J_{12} J_{21} J_{33}),$$
(3.10)

$$a_{1} = -(J_{23} J_{32} + J_{12} J_{21} + J_{13} J_{31} - J_{11} (J_{22} + J_{33}) - J_{22} J_{33}), \qquad (3.11)$$

$$a_2 = -Trace(J_X) = -(J_{11} + J_{22} + J_{33}).$$
(3.12)

The Routh–Hourwitz necessary and sufficient conditions for the local stability are: $a_0 > 0$, $a_2 > 0$ μ $a_1a_2 > a_0$. They are unequivocally satisfied under realistic conditions for $\varepsilon > 0$, q > 1:

$$a_0 = -|J_X| = c_2 v_a \frac{1}{q-1} \varepsilon > 0, \qquad (3.13)$$

$$a_1 = \frac{1}{q-1} (c_2 v_a + q) \varepsilon > 0, \qquad (3.14)$$

$$a_{2} = \frac{c_{2}v_{a}}{q-1} + \varepsilon + \frac{\varepsilon}{d(q-1)} = -\frac{|J_{X}|}{\varepsilon} + \varepsilon + \frac{\varepsilon}{d(q-1)} > 0$$
(3.15)

(hereby all three components are positive); finally

$$a_{1}a_{2} - a_{0} = \frac{1}{q-1} (c_{2}v_{a} + q)\varepsilon \left[\frac{c_{2}v_{a}}{q-1} + \varepsilon + \frac{\varepsilon}{d(q-1)}\right] - c_{2}v_{a} \frac{1}{q-1}\varepsilon = \frac{1}{q-1} \frac{1}{q-1}\varepsilon \left\{c_{2}v_{a} \left[c_{2}v_{a} + \varepsilon(q-1) + \frac{\varepsilon}{d}\right] + c_{2}v_{a} + q\left[\varepsilon(q-1) + \frac{\varepsilon}{d}\right]\right\} > 0$$
(3.16)

for $\varepsilon > 0$, $c_2 > 0$ and q > 1. Thus the stationary state E_X determined by equation (3.6) is asymptotically stable. Typically this stationary state is focus-node as a stable stationary state in extended WFM (section 2.3). As there is inequality $a_1a_2 - a_0 > 0$ sufficient conditions for a birth of a closed orbit through Andronov – Hopf bifurcation are excluded.

The normative Scenario II uses the sub-optimal magnitudes of the control parameters: $c_2 = 0.8823$, q = 7 and $\varepsilon = 2$. Simulations demonstrate that the employment ratio *v* moves to the target X = 0.95 with a very moderate over-shoot whereby $v_{\text{max}} = 0.953$. Figures 3.2a – 3.2c and 3.3 display results that are generally superior to those in extended WFM with its sub-optimal $\delta = 0.3$, $\mu = -2$, $\varepsilon = 1$ for the same initial conditions in Scenario I.



Figure 3.2a. Dynamics for the 'neoclassical' and reinforced stabilisation policies: employment ratio v



Figure 3.2b. Dynamics for the 'neoclassical' and reinforced stabilisation policies: wage – gross w and net $w(1-\delta_1)$



Figure 3.2c. Dynamics for the 'neoclassical' and reinforced stabilisation policies: profit – gross M and net $M(1-\delta_1)$



Figure 3.3. Dynamics for the 'neoclassical' and reinforced stabilisation policies: relative wage – gross u, net $u(1-\delta_1)$, stationary in GM u_G

The questions asked at the end of section 1.3 are resolved. First, due to upgraded profitsharing the dominant positive feedback loops are transformed into negative. More technically, second, two partial derivatives $\frac{\partial \dot{v}}{\partial v}$ and $\frac{\partial \dot{x}}{\partial x}$ estimated at a new stationary state are negative indeed prohibiting self-reinforcing explosive processes contrary to EFM. Third, a 'price' (or better 'prize') of this conversion is a strong gain in a long-term employment ratio *v* as well as elimination of absolute over-accumulation of produced capital *K* that is typical for GM, FM, WFM and extended WFM.

The reinforced stabilization policy may be rightly called a win-win policy in confines of evolving capitalism. Still neoliberal win-lose policies have a strong hold on the economists minds.

3.2 Maintaining capital accumulation and employment through excess income levy

Let us turn to a special two-dimensional case of MM consisting of the equations (3.3a) for \dot{u} and (1.12) for \dot{v} :

$$\dot{u} = \{c_2(v - X) + q[km(1 - u) - d]\}(1 - u).$$
(3.3a)

It has the non-trivial stationary state (u_a , X). For this stationary state equation (3.8a) defines the Jacoby matrix:

$$\widetilde{J}_{X} = \frac{-dq < 0 \quad c_{2}(1-u_{a}) > 0}{-kmX < 0 \quad 0}.$$
(3.8a)

The stationary state (u_a, X) is locally asymptotically stable since $Trace(\tilde{J}_X) = -dq < 0$, $|\tilde{J}_X| = c_2 dX > 0$.

A policy optimization in Vensim is carried out based on a restricted dynamic optimization problem:

Maximize
$$\left[-\int_{1958}^{2021} |v - X| dt - 10^5 IF THEN ELSE(v > X, 1, -1)\right]$$

$$\dot{x} = f(x, c_2, q),$$

 $x_0 = (u_0, v_0) = (0.88, 0.52), X = 0.95 >> v_0,$
 $0.01 \le c_2 = 0.2 \le 1.5, 0.5 \le q = 2 \le 5.$

A sub-optimal solution for a stable focus implies $c_2 = 0.2558$, q = 4. These magnitudes together with the previous magnitudes of the other parameters are used in simulations.

A novel outline of excess income levy

Consider a novel outline of excess income levy within attainable bounds that may enhance long term stability of capital accumulation consciously controlled by main social classes. It suggests the appropriate levy base and appropriate levy rates for primary distribution of income (labour compensation and profit) in an advanced capitalist economy encouraging efficient investment into produced capital, whereby jobs generation serves as engine of more equal and inclusive economic growth than in the above models proposed in the preceding literature.

The notion of excess income levy introduced in (Ryzhenkov 2007) is used in this paper as a general notion for the reduction in pre-levy primary income. The term excess labour compensation levy, in particular, is for the reduction in pre-levy primary labour compensation. The counter-part of

excess labour compensation levy is subsidy (of the same quantity) on pre-levy primary profit. In the opposite case, excess profit levy equals subsidy on labour compensation receivable. It is the state that can levy surcharges on excessive income of labourers (or capitalists) and pay equivalent subsidy. The state plays here the Maxwell Demon's role.

Let w_{pt} is the pre-levy labour compensation taken as the levy base:

$$w_{pt} = w \frac{1 + \hat{w}_{pt}}{1 + \hat{w}\left[\frac{1}{\text{year}}\right] \cdot 1[\text{year}]}.$$
(3.17)

Its rate of change \hat{w}_{pt} is determined according to the combined equation (1.8) in FM. The after-levy labour compensation is denoted as before by w; its rate of change \hat{w} is determined in its turn by the equation the combined equation (3.7) from MM based on the above deterministic form of the modified control law of capital accumulation.

The dynamics of capital accumulation in FM is interpreted as the inertia scenario. An improvement upon this in MM is consequently the normative scenario. Notice that the long-term rate of capital accumulation is the same in both models therefore superior results of the proposed original stabilization policy are not explained by a difference in this rate.

The rate of excess labour compensation levy (as a fraction of unit) is

$$x_w = (\hat{w}_{pt} - \hat{w}) \cdot 1$$
[year]. (3.18)

Applying equations (3.17) and (3.18) an equivalent expression for pre-levy labour compensation can be derived:

$$w_{pt} = w + \frac{w}{1 + \hat{w} \left[\frac{1}{\text{year}}\right] \cdot 1[\text{year}]} x_w.$$
(3.17a)

The overall excess labour compensation levy equals overall subsidy on pre-levy primary profit

$$T_{w} = \frac{x_{w}wL}{1 + \hat{w}\left[\frac{1}{\text{year}}\right] \cdot 1[\text{year}]} = S_{P}.$$
(3.19)

The total profit is now

 $P - (w_{pt}L - T_w) = P - w_{pt}L + S_P = P - wL.$ (3.20)

Using the new stationary employment ratio from the equation (3.6) we get a very elegant formula for the stationary relative excess labour compensation levy (as a fraction of unit)

$$\bar{x}_{w} = [(\hat{w}_{pt})_{b} - \hat{w}_{b}] \cdot 1[\text{year}] = [-g + rX + em(1 - u_{b}) - (d - n)] \cdot 1[\text{year}]$$

$$= r(X - v_{a}) \cdot 1[\text{year}]. \quad (3.21)$$

The share of excess labour compensation levy in net output (i.e., unit excess income levy) is

$$x_P = \frac{T_w}{P} = \frac{S_P}{P} = \frac{wL}{P} \frac{x_w}{1 + \hat{w} \left[\frac{1}{\text{year}}\right] \cdot 1[\text{year}]} =$$

$$\frac{ux_w}{1+\hat{w}\left[\frac{1}{\text{year}}\right]\cdot 1[\text{year}]}.$$
(3.22)

The stationary share of excess labour compensation levy in net output (i.e., unit excess income levy) is

$$\bar{x}_{P} = \frac{\bar{x}_{w}u_{b}}{1 + h\left[\frac{1}{\text{year}}\right] \cdot 1[\text{year}]}.$$
(3.23)

The stationary relative subsidy on pre-levy primary profit is

$$\overline{x}_M = \frac{\overline{x}_P}{1 - u_b}.$$
(3.24)

The labour share in net output is higher than 50 per cent in the model under consideration, so the relative subsidy on pre-levy primary profit exceeds relative excess labour compensation levy in this theoretical model in absolute terms.

Relative wage (brutto) is the sum of relative wage (net) and share of excess labour compensation levy in net output

$$u_{pt} = u + x_{p} = u \left[1 + \frac{x_{w}}{1 + \hat{w} \left[\frac{1}{\text{year}} \right] \cdot 1[\text{year}]} \right] = u \left[\frac{1 + \hat{w}_{pt} \left[\frac{1}{\text{year}} \right] \cdot 1[\text{year}]}{1 + \hat{w} \left[\frac{1}{\text{year}} \right] \cdot 1[\text{year}]} \right].$$
(3.25)

Depending on relation between the target employment ratio X and the stationary employment ratio v_a there are, *ceteris paribus*, three cases:

1) if $X = v_a$ the all three stationary levy (subsidy) ratios \overline{x}_w , \overline{x}_M and \overline{x}_P are zero;

2) if $X > v_a$ these ratios are positive;

3) if $X < v_a$ these ratios are negative.

These three cases are a particular manifestation of the employment ratio – relative labour compensation trade-off. In the second, mostly relevant, case, labourers, having a higher stationary employment ratio than in the inertia scenario, pay levy to the state that provides subsidies to capitalists. In the opposite (third) case when the target employment ratio is lower than the stationary employment ratio in the inertia scenario, capitalists pay levy to the state that provides subsidies to labourers. In the first case, when these both employment ratios are equal, the stationary relative levy (subsidy) is zero. Still excess income levy is pertinent even in this case since employment ratio varies on the transient to the stationary state.

For the relevant previous parameters values, the relative excess labour compensation levy is $\bar{x}_w = r(X - v_a) \cdot 1$ [year] = 2(0.950-0.508) = 0.884. The share of excess labour compensation levy in

net output is $\bar{x}_P = 0.762$ according to (3.23). The relative subsidy on primary profit is $\bar{x}_M = 6.283$ according to (3.24). These quantities are clearly excessive – mostly due to the very low magnitude of the stationary employment ratio $v_a = 0.508$ in the inertia scenario intended for my extreme condition tests.

Still reasonable refinement could be elaborated. We do not treat the problem as a one-shot game between workers and capitalists mediated by the state. This game is repeated again and again. Then rational co-operative strategies may follow.

In my thought experiment, the very process of redistribution of excess income levy by the state will moderate the coefficients of bargained wage term g by capitalists and r by workers (even relatively stronger) in the equation (1.8) of FM. Then x_w and x_p are not so high as in a model without adjustment in g and r. In the process of adaptive adjustment the parameters of the linear Phillips equation (1.8a) are substituted: r_{adj} takes place of r, similarly, g_{adj} takes place of g, thus

$$\dot{r}_{adj} = \eta(r_{stat} - r_{adj}), \qquad (3.26)$$

where initially (t = 1958) $r_{adj} = r$, $\eta > 0$,

$$r_{stat} = zc_2(1 - u_a)/u_a. \tag{3.27}$$

In the same way

$$\dot{g}_{adj} = \eta(g_{stat} - g_{adj}), \tag{3.28}$$

where initially (t = 1958) $g_{adj} = g$,

$$g_{stat} = c_2 X(1 - u_a)/u_a. \tag{3.29}$$

Then

$$z = 1 + \frac{u_a}{c_2 X (1 - u_a)} \left(h - \frac{e}{k} d \right).$$
(3.30)

For $t \to \infty$

$$\frac{g_{adj}}{r_{adj}} \to g_{stat}/r_{stat} = [c_2 X(1 - u_a)/u_a]/[zc_2(1 - u_a)/u_a] = X/z$$
(3.31)

and for $x_p \rightarrow 0, x_w \rightarrow 0$

$$\hat{w}_{pt} \to \hat{w}_a = -g_{stat} + r_{stat}X + em(1 - u_a) = h.$$
 (3.32)

As the relative reduction in r is stronger than in g, there is inequality $g_{stat}/r_{stat} > g/r$.

Now for adjustment in parameters g and r, x_w is determined by equation (3.18) and equation (1.12) governs \dot{v} . Besides that

$$\hat{w}_{pt} = -g_{adj} + r_{adj}v + em(1-u), \tag{3.33}$$

$$\dot{u} = (\hat{w}_{pt} - h - x_w)u. \tag{3.34}$$

The stationary state (u_G , X, g_{stat} , r_{stat}) of this decomposable four-dimensional model is locally asymptotically stable as shown in the simulation experiments. A trivial analytical proof is omitted.

The magnitudes of the parameters in MM and FM are the same as before, additionally $\eta = 0.5$; in particular, sub-optimal $c_2 = 0.2558$ and sub-optimal q = 4, e = 0.1, z = 1.477, $u_a = u_G = 0.8787$, $v_a = 0.508 < v_G = 0.51 << X = 0.95$, $g_{stat} = 0.0335$, $r_{stat} = 0.0521$.

My simulations demonstrate a rather fast convergence of the growth rate of pre-levy labour compensation to the growth rate of post-levy labour compensation as well as narrowing difference



between the relative excess labour compensation levies x_p and x_w that converge to zero in a smooth fashion (Figure 3.4, Panels 2 and 4).

Figure 3.4. Dynamics for the standard profit-sharing and reinforced stabilisation policies: Panel 1 – employment ratio v, Panel 2 – the wage growth rates \hat{w} and \hat{w}_{pt} , Panel 3 – relative wage u, Panel 4 – the relative excess labour compensation levies x_p and x_w

The large gain in the employment ratio due to the reinforced stabilization policy is seen on Panel 1, whereas the standard profit-sharing provides a higher relative wage during the transitional period reported on Panel 3. The analytical and simulation results demonstrate again that the reinforced stabilization policy is a win-win policy. The proposed design for the two-dimensional case can be easily generalized for a three-dimensional case such as the system of ODEs (3.3), (1.12c) and (3.5). Costs of the pre-market co-operation and co-ordination neglected above are to be taken into consideration in a subsequent research.

Conclusion

This paper proves that government expenditures balanced by taxes are tantamount to cutback of long term rate of capital accumulation that is usually implicit in the considered Goodwinian or semi-Goodwinian models. This understanding explains the mystery stated in *Introduction*, namely

why government expenditures balanced by taxes in these Goodwinian or semi-Goodwinian models reduce stationary gross and net labour shares in national income and can even lessen a long term employment ratio under a standard profit-sharing.

By staging extreme conditions tests with a help of the Phillips – Wolfstetter – Flaschel 'capricious' investment function this paper finds out that the standard profit-sharing becomes a failed fix in EFM. Stabilization of the 'capricious' investment function is achieved in extended WFM due to cyclically non-neutral (balanced) government taxes and expenditure. Still this stabilization reduces stationary relative wage compared with that in GM. The standard profit sharing (gross) reinforces stabilization thereby yet the stationary employment ratio becomes also lower than that in GM.

This investigation has a say about the logic of economic calamities in a capitalist economy subject to dominant positive feedback(s). In particular, contrary to the stabilization failure in EFM, the reinforced stabilization policy achieves 'taming of the shrew' by eliminating a destructive dominance of the revealed positive feedback loops in the non-Goodwinian model (MM).

The key element of reinforced stabilization policy via upgraded profit sharing or excess income levy is the targeting of deliberately high employment ratio. This policy does not reduce stationary rate of capital accumulation unlike 'Keynesian' or "neoclassical' stabilization policies. Thus the reason for lowering stationary employment ratio and stationary labour share in relation to GM is eliminated.

Moreover, this reinforced stabilization policy provides superior results related to main economic indicators, including employment, profit, total wage, consumption per head, compared with results based on the standard profit-sharing in the preceding literature. Unlike previous win-lose or lose-lose policies this policy is clearly win-win.

The measures such as proposed employment targeting, upgraded profit-sharing and excess income levy are politically difficult. Still the author spells these options out theoretically in the belief that opponents will not dismiss them outright as 'inconceivable'.

The proposed two alternative forms of reinforced stabilization policy are not comprehensively designed yet since changes in the wage-setting and other relevant institutions implied by supposed overt closed-loop control over capital accumulation as a whole are not discussed. In particular, it is not elaborated, first, whether such a closed-loop control is to be achieved through coercive and/or voluntary cooperation; second, what arrangement of coincidence, coercion and co-adjustment is mostly suited for providing superior social outcomes.

Therefore a future research will be concentrated on elaborating more advanced concrete models that will be tested empirically for particular capitalist economies. Confronting utopian economics with reality-based political economy requires plenty of efforts in system dynamics. Practical implications of the proposed stabilization policies can be discussed with interested parties as well.

For transforming the capitalist society progressively, it is minimally necessary, in my view, to place the profits of banks and major monopolies under conscious public control in the interests of working class. Without the overt controlling of the social reproduction in a rational manner as recommended, attempts to alleviate dynamic inefficiency of capitalism will be far less successful or even doomed to failure. The rational choice under current circumstances is clear: not between "growth and austerity" but between win-lose, lose-lose or win-win strategies as this paper suggests. A more radical solution (transforming capitalist mode of production and transiting to socialism) will be the increasingly stronger (quite conceivable) alternative if the described inferior win-lose and lose-lose strategies remain dominant.

References

Cassidy J. 2009. *How Markets Fail: The Logic of Economic Calamities*. New-York: Farrar, Straus and Giroux.

Fanti L., Manfredi P. 1998. A Goodwin-type growth cycle model with profit-sharing // *Economic Notes* 27: 371–402.

Flaschel P. 2009. The Macrodynamics of Capitalism. Elements for a Synthesis of Marx, Keynes and Schumpeter. Heidelberg: Springer Verlag.

Lordon F. 1995. Cycles et chaos dans un modèle hétérodoxe de croissance endogène // *Revue économique* 46 (6): 1405–1432.

Phillips A. W. 1961. A simple model of employment, money and prices in a growing economy // *Economica* 73: 360–370.

Ryzhenkov A. 2000. Unfolding the Eco-wave. Why Renewal is Pivotal. Chichester a. o.: John Wiley & Sons Ltd.

Idem. 2007. Controlling employment, profitability and proved non-renewable reserves in a theoretical model of the U.S. economy // *Proc. of the 25th International Conference of the System Dynamics Society* (Boston, USA, July 29 – August 2, 2007). The System Dynamics Society, 2007.

URL: http://www.systemdynamics.org/conferences/2007/proceed/papers/RYZHE406.pdf

http://www.systemdynamics.org/conferences/2007/proceed/supp/406.pdf

Idem. 2008. Supposing a control law of capital accumulation for the modern Italian economy // *Proc. of the 26th International Conference of the System Dynamics Society* (Athens, Greece, July 20 - 24, 2008). The System Dynamics Society, 2008. URL:

<u>http://www.systemdynamics.org/conferences/2008/proceed/papers/RYZHE475.pdf</u> http://www.systemdynamics.org/conferences/2008/proceed/supp/475.pdf

Idem. 2010. The structural crisis of capital accumulation in the USA and its *causa prima* // *Proc. of the* 28^{th} *International System Dynamics Conference* (Seoul, Korea, July 25 – 29, 2010). The System Dynamics Society, 2010. URL:

http://www.systemdynamics.org/conferences/2010/proceed/papers/P1353.pdf

http://www.systemdynamics.org/conferences/2010/proceed/supp/S1353.pdf

Idem. 2012. Insights into income policy for enhancing employment and stability of capital accumulation // *Proc. of the 30th International Conference of the System Dynamics Society*, St. Gallen, Switzerland – July 22–26, 2012. URL:

http://www.systemdynamics.org/conferences/2012/proceed/papers/P1460.pdf

http://www.systemdynamics.org/conferences/2012/proceed/supp/S1460.pdf

Idem. 2013. Employment-centred stabilisation policy propelling the economy to "escape velocity" // Proc. of the 31st International Conference of the System Dynamics Society. July 21 – July 25, 2013. System Dynamics Society, Cambridge, Massachusetts USA. URL:

http://www.systemdynamics.org/web.portal?P1170+0

http://www.systemdynamics.org/web.portal?S1170+0

Spencer R. W., Yohe W. P. 1970. The 'crowding out' of private expenditures by fiscal policy actions // Federal Reserve Bank of St. Louis Review 52: 12–24.

Sterman J. D. 2000. Business Dynamics: Systems Thinking and Modeling for a Complex World. Boston a. o.: Irwin / McGraw-Hill.

Wolfstetter E. 1982. Fiscal policy and the classical growth cycle // Zeitschrift für Nationalökonomie (The Journal of Economics) 42: 375–393.

Yoshida H., Asada T. 2007. Dynamic analysis of policy lag in a Keynes–Goodwin model: stability, instability, cycles and chaos // *Journal of Economic Behavior and Organization* 62: 441–469.