Effect of Lead Time on Anchor-and-Adjust Ordering Policy in Continuous Time Stock Control Systems[1]

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Abstract

Anchor-and-adjust is an ordering policy often used in stock control systems. Weight of Supply Line, the relative importance given to the supply line compared to the importance given to the stock, and Stock Adjustment Time, the intended time to close the discrepancy between the desired and current levels of the stock, are the two critical decision parameters of the anchor-and-adjust ordering policy. In this study, we conduct an extensive simulation study using a generic stock management structure and offer suggestions for the selection of these two decision parameters. The decision parameter values are significantly affected from the order and duration of the lead time.

Keywords: anchor-and-adjust; lead time; relative aggressiveness; stock adjustment time; stock management; weight of supply line.

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Introduction

Stock management is a widely encountered task in complex dynamic systems, in which the aim is to alter the level of a stock towards a desired point and maintain it at that point (Diehl and Sterman, 1995; Sterman, 1987a, 1989a, and Chapter 17 in 2000; Sweeney and Sterman, 2000; Yasarcan and Barlas, 2005; Yasarcan, 2010 and 2011). The generic stock management structure captured in Figure 1 can be used to represent a broad range of different stock control systems; see Sterman (1989a) for examples.

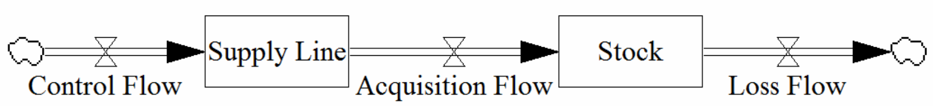

Figure 1. The simplified stock-flow diagram of the generic stock management task

In the generic stock management task, there are two main state variables: *Stock* and *Supply Line* (Figure 1). For example, in an inventory distribution system, *Supply Line* corresponds to in-transit inventory, while in an inventory production system, it corresponds to work-in-process inventory. *Stock* is the net inventory in both systems that *Stock* increases via *Acquisition Flow* and decreases via *Loss Flow*. *Supply Line* increases via *Control Flow* and decreases via *Acquisition Flow*. In an inventory distribution system, *Control Flow* is the orders given to the supplier, while *Acquisition Flow* is the incoming orders. In an inventory production system, *Control Flow* is the production orders, and *Acquisition Flow* is the production completion rate. *Loss Flow* is the sales to the customers in both systems.

Lead time (i.e., supply line delay) is the cause for the existence of a supply line and, thus, the main reason of the challenge in managing a stock (Diehl and Sterman, 1995; Kleinmuntz, 1993; Sterman, 1989a and Chapter 17 in 2000; Yasarcan and Barlas, 2005; Yasarcan, 2010 and 2011). Lead time is defined by its average delay duration and order (Barlas, 2002; Chapter 9 in Forrester, 1961; Mikati, 2010; Chapter 11 in Sterman, 2000; Yasarcan, 2011; Yasarcan and Barlas, 2005a and 2005b; Venkateswaran and Son, 2007; Wikner, 2003). In this paper, the average duration of the lead time is referred to as

Acquisition Delay Time (λ in Sterman, 1989a), and it represents the average lag between the control decisions and their effects on the stock.

Anchor-and-adjust is an ordering policy often used in stock control systems. *Weight of Supply Line* (β in Sterman, 1989a), the relative importance given to the supply line compared to the importance given to the stock, and *Stock Adjustment Time* ($1/\alpha_S$ in Sterman, 1989a), the intended time to close the discrepancy between the desired and current levels of the stock, are the two critical decision parameters of the anchor-and-adjust ordering policy (Sterman, 1989, Chapter 17 in 2000; Sweeney and Sterman, 2000; Yasarcan and Barlas, 2005; Yasarcan, 2011). In stock management, it is a critical issue to obtain a fast and stable response from the stock, which can only be ensured by a proper selection of values for these two parameters.

According to the literature, the supply line can be fully considered by setting *Weight of Supply Line* equal to unity, which reduces the stock management task to a first-order system that cannot oscillate. Hence, when *Weight of Supply Line* is equal to unity, it ensures non-oscillatory stock behavior regardless of the duration and order of the lead time, and therefore, it is often used in stock management (Barlas and Ozevin, 2004; Sterman, 1989a and Chapter 17 in 2000; Yasarcan, 2011; Yasarcan and Barlas, 2005a). Moreover, this value of the weight is optimal for stock management tasks with a discrete supply line delay (Sterman, 1989a).

In most models of inventory systems, lead time is represented as an infinite order delay (discrete delay, fixed pipeline delay) because of the apparent simplicity of its mathematical expression. However, formulating lead time using a continuous delay structure leads to a more accurate representation for most real systems (Chapter 11 in Sterman, 2000; Venkateswaran and Son, 2007; Wikner, 2003; Yasarcan, 2011; Yasarcan and Barlas, 2005a). According to Mikati (2010), assuming a first-order lead time is reasonable when there is enough production capacity in a production-inventory system. Another suggestion is to use a delay structure with an order higher than one rather than using a fixed pipeline delay structure (Venkateswaran and Son, 2007). Wikner (2003, p.

2792) has noted that "... *the third-order delay has proved to be an appropriate compromise between model complexity and model accuracy*".

There are no studies in the literature concerning the optimal value of *Weight of Supply Line* for a stock management task with a continuous lead time. We conduct an extensive simulation study with the intention to fill this gap. "Effect of Delay Order on the Output of the Delay Structure" section aims to assist the non-system dynamicist readers by presenting the dynamic behaviors obtained from delay structures having the same average delay duration, but different orders. In "Stock Management Structure" section, we present the stock-flow diagram and the corresponding equations of the generic stock management task used in the experiments. The terms and decision parameters of the anchor-and-adjust ordering policy are elucidated in section "Anchor-and-adjust Ordering Policy". In this section, we also introduce a new parameter named *Relative Aggressiveness* that reduces the search space and, thus, accelerates the search for the optimal value of *Weight of Supply Line* and enables the analysis of the results. In "Design for Simulation Experiments" section, we describe the simulation settings in detail and give the selected ranges of the values assigned to the experimental parameters during the optimization runs. In section named "Stock Dynamics", we present examples for typical stock dynamics and a non-intuitive result. In "Results" section, contour plots of *Total Penalty* are obtained with respect to *Weight of Supply Line* and *Relative Aggressiveness*. In this section, optimum *Weight of Supply Line* values for a selected range of *Relative Aggressiveness* values and delay orders are also given.

Effect of delay order on the output of the delay structure

This section aims to assist the non-system dynamicist readers by presenting the dynamic behaviors obtained from delay structures having the same average delay duration, but different orders. A delay is the existence of a lag between an input and its resultant output, and it is characterized by its average duration and order. The value of the average delay duration is a positive real number. On the other hand, the order of delay is an integer and ranges from one to infinity. An infinite order delay is also known as discrete delay or fixed delay. To show the differences between the different delay orders, we obtained the dynamics presented in Figure 2 under the following settings:

- The average delay duration is assumed to be five for all cases.
- Initially, input and all outputs of different delay orders are assumed to be zero.
- At time five, there is a step increase in the input from zero to one. Therefore, the input and all of the outputs remain at zero until that time.

As evidenced from the dynamics presented in Figure 2, the output of the discrete delay (the sixth line in the figure) does not show any response at the point of change in the input (i.e., at time 5), but it abruptly catches up with the input (first line) at time 10. Unlike the other orders, the output of the first-order delay (second line) responds immediately to the change in the input. However, after a point, its response lags behind all other responses, thus approaching the input more slowly than the others. The outputs of the remaining delay orders follow patterns between these two extremes (see lines 3, 4, and 5). For more information on delays, see Barlas (2002), Chapter 11 in Sterman (2000), Yasarcan (2011), and Wikner (2003).

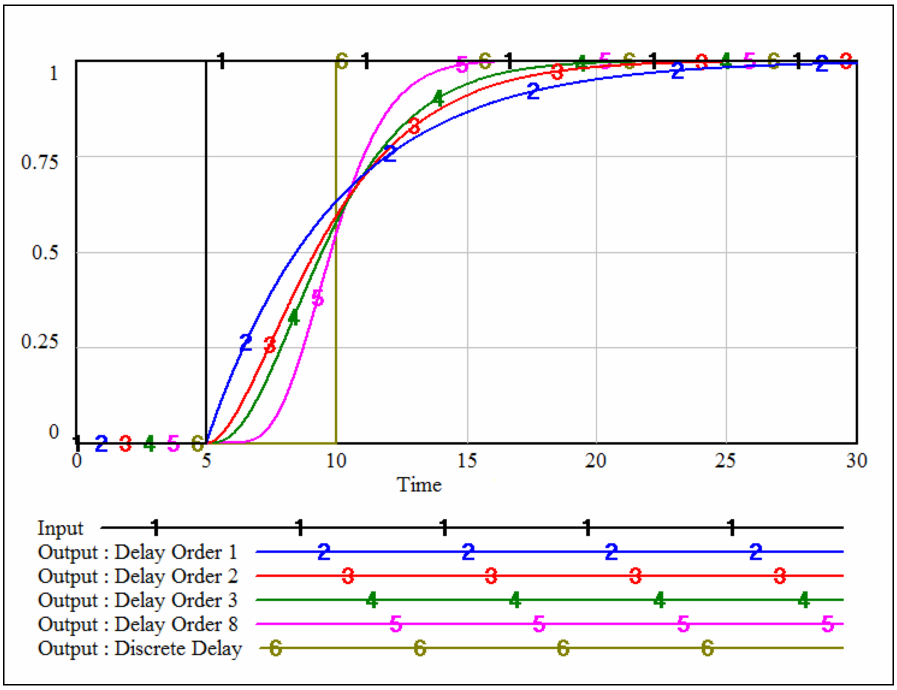

Figure 2. Output behavior for different delay orders

In this study, we used different orders of lead time. As an example, a second-order lead time in a stock management structure is provided in Figure 3.

Stock management structure

In this study, we used a stock management structure with first, second, third, fourth, eighth, and infinite orders of lead time. A stock management structure with a second-order lead time is provided as an example in Figure 3.

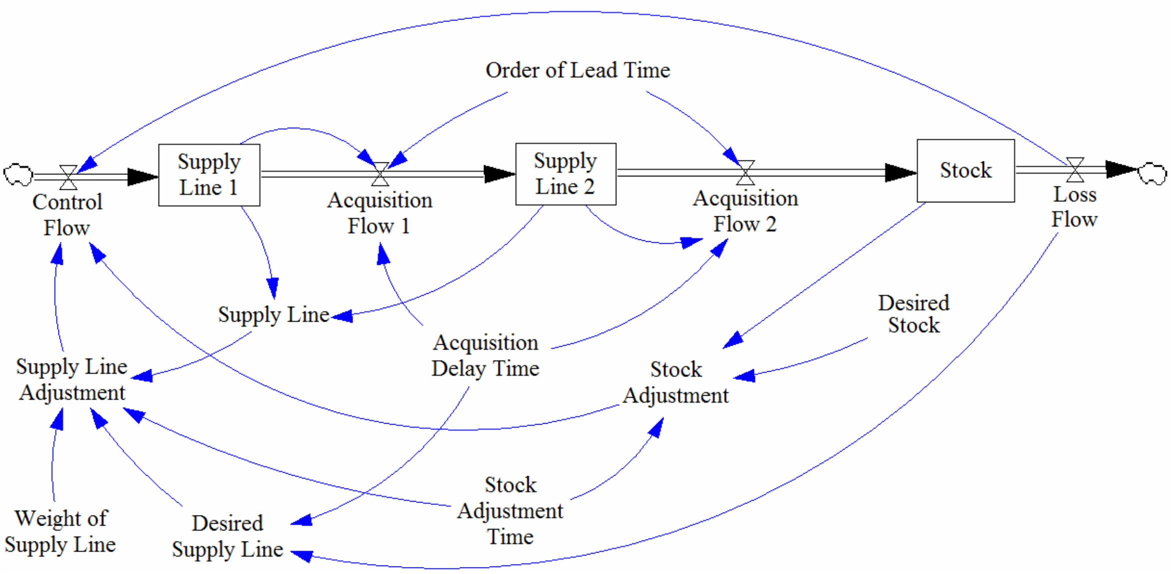

Figure 3. Stock-flow diagram of the stock management task with a 2nd order lead time

As *Stock* is the main state variable in our model, we attempt to maintain *Stock* at a desired level.

$$Stock_0 = Desired\ Stock\ Level \quad [item] \tag{1}$$

$$Stock_{t+DT} = Stock_t + (Acquisition\ Flow\ 2 - Loss\ Flow) \times DT \quad [item] \tag{2}$$

In this study, we use the Euler numerical integration method to simulate the model. *DT* in equations 2, 4, 6, and 18 is the simulation time step. Note that the number of supply line stocks is determined by the order of the lead time. Hence, we have two supply line stocks - *Supply Line 1* and *Supply Line 2*.

$$Supply\,Line\,1_0 = \frac{Desired\ Supply\ Line}{Order\ of\ Lead\ Time} \quad [item] \tag{3}$$

$$Supply\,Line\,1_{t+DT} = Supply\,Line\,1_t + \begin{pmatrix} Control\,Flow \\ -\,Acquisition\,Flow\,1 \end{pmatrix} \times DT \quad [item] \tag{4}$$

$$Supply\,Line\,2_0 = \frac{Desired\ Supply\ Line}{Order\ of\ Lead\ Time} \quad [item] \tag{5}$$

$$Supply\,Line\,2_{t+DT} = Supply\,Line\,2_t + \begin{pmatrix} Acquisition\,Flow\,1 \\ -\,Acquisition\,Flow\,2 \end{pmatrix} \times DT \quad [item] \tag{6}$$

The model is initiated at its equilibrium point, and the state variables (*Stock*, *Supply Line 1*, and *Supply Line 2*) are initiated at their equilibrium levels (Equations 1, 3, and 5). The model is disturbed from its equilibrium by increasing *Desired Stock* by 1 unit at time 1 (Equation 12).

Flow equations are as follows:

$$Loss\ Flow = 2 \quad [item\ /\ time] \tag{7}$$

$$Control\ Flow = \begin{pmatrix} Loss\ Flow + Stock\ Adjustment \\ +\ Supply\ Line\ Adjustment \end{pmatrix} \quad [item/time] \tag{8}$$

$$Acquisition\,Flow\,1 = \frac{Supply\ Line\ 1}{\begin{pmatrix} Acquisition \\ Delay\ Time \end{pmatrix} \Big/ \begin{pmatrix} Order\ of \\ Lead\ Time \end{pmatrix}} \quad [item/time] \tag{9}$$

$$Acquisition\,Flow\,2 = \frac{Supply\ Line\ 2}{\begin{pmatrix} Acquisition \\ Delay\ Time \end{pmatrix} \Big/ \begin{pmatrix} Order\ of \\ Lead\ Time \end{pmatrix}} \quad [item/time] \tag{10}$$

Expectation formation is out of the scope of this work because our aim is to focus on the isolated effect of the control parameters, *Weight of Supply Line* and *Stock Adjustment Time,* on the dynamics of the stock. To eliminate the potential effect of the forecasting method and forecasting parameters, *Loss Flow* is taken as a constant (Equation 7) and assumed to be known by the decision maker. *Control Flow* is a flow variable reflecting the instantaneous control decisions (Equation 8), and it is the input to the supply line (Equation 4). Usually, the expected value of *Loss Flow* is used in the *Control Flow* equation. However, because the exact value of the *Loss Flow* is known by the decision maker and no expectation formation is carried out, the exact value of *Loss Flow* is directly used in the *Control Flow* equation without a loss of generality of the results. The number of acquisition flows is determined by the order of lead time similar to the number of supply line stocks (equations 9 and 10). The last acquisition flow is the output of the supply line, which is also the input to the stock (equations 2 and 6).

Model constants and equations for other variables are as follows:

$$Acquisition\,Delay\,Time = 8 \quad [time] \tag{11}$$

$$Desired\ Stock = \begin{cases} 9, & Time < 1 \\ 10, & Time \geq 1 \end{cases} \quad [item] \tag{12}$$

$$Desired\ Supply\,Line = Acquisition\ Delay\ Time \times Loss\ Flow \quad [item] \tag{13}$$

$$Supply\,Line = Supply\ Line\ 1 + Supply\ Line\ 2 \quad [item] \tag{14}$$

$$\begin{pmatrix} Supply \\ Line \\ Adjustment \end{pmatrix} = \frac{\begin{pmatrix} Weight\ of \\ Supply\ Line \end{pmatrix} \times \begin{pmatrix} Desired\ Supply\,Line \\ -\ Supply\ Line \end{pmatrix}}{Stock\ Adjustment\ Time} \quad [item/time] \tag{15}$$

$$Stock\ Adjustment = \frac{Desired\ Stock\ Level - Stock}{Stock\ Adjustment\ Time} \quad [item/time] \tag{16}$$

Equations 1-7, 9-11, and 14 describe the physical aspects, and equations 8, 12, 13, 15, and 16 describe the decision-making aspects of the stock management structure. The values of the two decision-making parameters (i.e., *Weight of Supply Line* and *Stock Adjustment Time*) are not presented because they are the experimental parameters in this study.

Total Penalty is assumed to be the accumulated absolute difference between the desired and the actual levels of the stock. Equations 17 and 18 reflect this assumption and are used to calculate the total penalty resulting from the different sets of values of *Weight of Supply Line* and *Stock Adjustment Time*. Thus, *Total Penalty* enables comparison of the policies.

$$Total\ Penalty_0 = 0 \quad [item \cdot time] \tag{17}$$

$$Total\ Penalty_{t+DT} = Total\ Penalty_t + \left| \begin{matrix} Desired\ Stock\ Level \\ - Stock \end{matrix} \right| \times DT \quad [item \cdot time] \tag{18}$$

The parameters and variables which are associated with the anchor-and-adjust ordering policy (i.e., *Weight of Supply Line*, *Stock Adjustment Time*, *Control Flow*, *Stock Adjustment*, and *Supply Line Adjustment*) are given as a part of the stock management structure in this section. However, their detailed explanations are reserved for the next section.

Anchor-and-adjust ordering policy

The anchor-and-adjust ordering policy used for the generic stock management task has three terms: expected loss from the stock, stock adjustment (the discrepancy between the desired and actual stock divided by a time parameter), and supply line adjustment (the discrepancy between the desired and actual supply line divided by another time parameter) (Barlas and Ozevin, 2004; Diehl and Sterman, 1995; Sterman, 1987a, 1989a, 1989b, and Chapter 17 in 2000; Yasarcan, 2011; Yasarcan and Barlas, 2005a and 2005b). There are three time parameters in the anchor-and-adjust ordering policy, one for each term. The time parameter used in the expected loss term is ignored in our study because expectation formation is out of the scope of this paper. For expectation formation, see Sterman (1987b). The two other time parameters are *Stock Adjustment Time* ($1/\alpha_S$ in Sterman, 1989a) used in the stock adjustment term and *Supply Line Adjustment Time* ($1/\alpha_{SL}$ in Sterman, 1989a) used in the supply line adjustment term. The existence and stability of oscillations in stock dynamics is determined by the values assigned to these two time parameters for a given lead time. Therefore, assigning adequate values to *Stock Adjustment Time* and *Supply Line Adjustment Time* is critical in obtaining a fast response in stock behavior while simultaneously eliminating the unwanted oscillations.

Alternatively, a weight coefficient can be used in the supply line adjustment term so that a single adjustment time can be used rather than using two separate adjustment times. This coefficient reflects the relative importance given to the supply line compared to the stock. Therefore, this weight is called *Weight of Supply Line* (β in Sterman, 1989a), and it is equal to *Stock Adjustment Time* divided by *Supply Line Adjustment Time*. The supply line can be fully considered by setting *Weight of Supply Line* equal to unity, which corresponds to using the same adjustment time for stock adjustment and supply line adjustment terms. Fully considering supply line means that the decision maker gives the same importance to the discrepancies between the desired and the actual levels of both the stock and its supply line. In the presence of constant or stationary *Loss Flow* (e.g., sales to the customers), giving the same importance to the stock and its supply line effectively reduces the stock management task to a first-order system, which cannot oscillate. Hence, when *Weight of Supply Line* is equal to unity, it ensures non-oscillatory stock behavior regardless of the delay duration and order, and therefore, it is often used in stock management (Barlas and Ozevin, 2004; Sterman, 1989a and Chapter 17 in 2000; Yasarcan and Barlas, 2005a and 2005b). However, we want to mention that non-oscillatory stock behavior does not necessarily imply optimality in all cases.

In discrete-time models[2] and in the presence of discrete lead time, *Stock Adjustment Time* is usually taken as one unit of time to obtain a fast response in the stock dynamics. There is no such usual value in continuous-time models and/or in the presence of continuous lead time. It is also worth noting that independent of the time continuity or discreteness of a model, a low *Stock Adjustment Time* value (i.e., aggressive adjustments) requires a more frequent information update (Venkateswaran and Son, 2007; Yasarcan and Barlas, 2005b). The other critical decision parameter, *Weight of Supply Line*, is often taken as unity in stock management, which is optimum for discrete lead time once the value of *Stock Adjustment Time* is selected satisfactorily low. However, a *Weight of Supply Line* that is equal to unity is not optimum for continuous lead time cases because it leads to an over-damping behavior (i.e., a slow approach of *Stock* to its desired level).

The existence and stability of oscillations in stock dynamics is a function of the order of the lead time, *Acquisition Delay Time* (duration of the lead time), *Weight of Supply Line*, and *Stock Adjustment Time*. In Chapter 5 of his PhD thesis, Yasarcan (2003) reported the critical values of the ratio between the two parameters, *Stock Adjustment Time* and *Acquisition Delay Time*. Those critical values determine the changes in the dynamics of the stock from no-oscillations to stable oscillations and stable oscillations to unstable oscillations. Although the nominal values of *Acquisition Delay Time* and *Stock Adjustment Time* affect the stock behavior, it is their ratio (together with the order of the delay structure and *Weight of Supply Line*) that determines the existence and stability of oscillations. Furthermore, using their ratio reduces the search space of parameters by one dimension. Therefore, we introduce a new parameter, *Relative Aggressiveness*, and define it to be equal to *Acquisition Delay Time* divided by *Stock Adjustment Time* (Equation 19).

$$Relative\ Aggressiveness = \frac{Acquisition\ Delay\ Time}{Stock\ Adjustment\ Time} \quad [dimensionless] \qquad (19)$$

A low *Stock Adjustment Time* value implies aggressive adjustments, while a high value implies smooth adjustments. Therefore, $1/Stock\ Adjustment\ Time$ is a measure of

[2] A discrete-time model is expressed using difference equations and a continuous-time model is expressed using differential or integral equations.

aggressiveness in making adjustments. *Relative Aggressiveness* is directly proportional to $1/\textit{Stock Adjustment Time}$ (Equation 19). Hence, *Relative Aggressiveness* is a measure of aggressiveness in making adjustments relative to *Acquisition Delay Time*. Based on the findings reported in Yasarcan (2003) and the extensive simulation runs conducted as a part of this study, we infer that the nature of the stock behavior is determined by the two dimensionless ratios for a given order of lead time - *Weight of Supply Line* and *Relative Aggressiveness*. Once a reasonable value for *Relative Aggressiveness* is obtained, a sound *Stock Adjustment Time* value can be calculated for any given value of *Acquisition Delay Time* (Equation 20). Thus, introducing *Relative Aggressiveness* puts the selection of *Stock Adjustment Time* into an analytical framework.

$$\textit{Stock Adjustment Time} = \frac{\textit{Acquisition Delay Time}}{\textit{Relative Aggressiveness}} \quad [\textit{time}] \tag{20}$$

In the next section, we describe the simulation settings used in the optimization runs and give the selected ranges of the values assigned to the newly introduced parameter, *Relative Aggressiveness*, and *Weight of Supply Line*.

Design for simulation experiments

The range of *Weight of Supply Line* is selected as [0.0, 1.6], and the range of *Relative Aggressiveness* is selected as [0.1, 6.0]. The region defined by the ranges of these two parameters covers the whole range of stock dynamics. The range of *Weight of Supply Line* is divided into 65 equal distance points whereby the gap between two successive points is 0.025. The range of *Relative Aggressiveness* is divided into 60 equal distance points whereby the gap between two successive points is 0.1. Therefore, the total number of continuous time simulations is 3,900 for each of the delay orders 1, 2, 3, 4, 8, and infinite. In these simulations, the Euler numerical integration method is used. For numerical precision, the simulation time step (*DT* in equations 2, 4, 6, and 18) is set equal to $1/256$. As a result, the numerical error in each generated total penalty value is less than 1% for the given search space.

For a fair comparison of the penalties obtained from different simulation runs, we selected the simulation time length as 250 based on the following considerations:

- The simulation time length should not be unnecessarily long because it directly affects the simulation run time.
- The simulation time length should be long enough to allow the dynamics, which are created by perturbing the model from its equilibrium, to significantly fade away. Therefore, no significant penalty should be incurred after the selected simulation time length. As an aside, the discrepancy between the desired and actual levels of stock diminishes in time for only the stable cases. There exists no such simulation time length for the unstable dynamics because such a dynamic behavior never fades, thus creating an infinite penalty in infinite time. As we focus on the desirable dynamics, which are the stable ones, comparing the unstable dynamics is not a concern in this study.

In this study, we used the same set of values for *Acquisition Delay Time*, *Desired Stock*, and *Loss Flow*, and the same size perturbation in *Desired Stock* for all simulation runs because (1) using a different value for *Desired Stock* or using a different value for *Loss Flow* has no effect on the penalty values when the model is initiated at its equilibrium, (2) a change in *Acquisition Delay Time* or a change in the size of perturbation in *Desired Stock* has a directly proportional effect on the penalty values. Hence, the results obtained in this study are valid for any different initial setting.

It is known that the presence of a supply line delay is one of the main reasons for the difficulty faced in managing a stock. Therefore, eliminating the delay or decreasing its duration (*Acquisition Delay Time*) should be considered to obtain a less complex stock management task (Diehl and Sterman, 1995; Paich and Sterman, 1993; Yasarcan, 2010; Yasarcan and Barlas, 2005b). One should also keep in mind that eliminating or decreasing the duration may not be practically possible, or the associated costs may not be justifiable. In this study, we assume that the duration and order of the lead time remain constant throughout a simulation run.

The findings obtained from the experiment described in this section are presented in "Results" section. We present examples for typical stock dynamics and the associated costs in the next section aiming to increase the understanding of the findings.

Stock dynamics

In Figure 4, we present examples for three typical stock dynamics - non-oscillatory behavior, damping oscillations, and unstable oscillations. The example dynamics and associated penalties are obtained by using a stock management structure with a third-order lead time that has duration of eight units of time.

In this study, *Total Penalty* corresponds to the total area between a dynamic behavior of the stock and the desired level of that stock. In Figure 4, line 1 is the desired stock level, and lines 2, 3, and 4 correspond to non-oscillatory stock behavior, damping oscillations, and unstable oscillations, respectively. Using equations 17 and 18, we obtained penalty values of 34.00 (the area between lines 2 and 1) for the non-oscillatory behavior, 19.00 (the area between lines 3 and 1) for the damping oscillations, and 381.83 (the area between lines 4 and 1) for the unstable oscillations. These penalties are obtained by simulating until time 250, which is the simulation time length selected for this study (see Section 5). However, in Figure 4, we only show the dynamics until time 100 because, one, the amplitude of the unstable behavior becomes too large and dominates the graph making the others indistinguishable and because, two, based on the y-axis range selected for the figure, the distinguishable part of the non-oscillatory behavior and the damping oscillations are completed at approximately 100, after which they are relatively constant. The penalty values for the non-oscillatory behavior and damping oscillations do not change after time 250. However, the unstable oscillations continue to endlessly generate penalty and in a growing fashion. Normally, one would expect damping oscillations to have a higher associated penalty than the penalty of the non-oscillatory behavior. However, we obtained results contradicting this intuitive expectation. The example dynamics we presented in Figure 4 is selected to reflect this unexpected result.

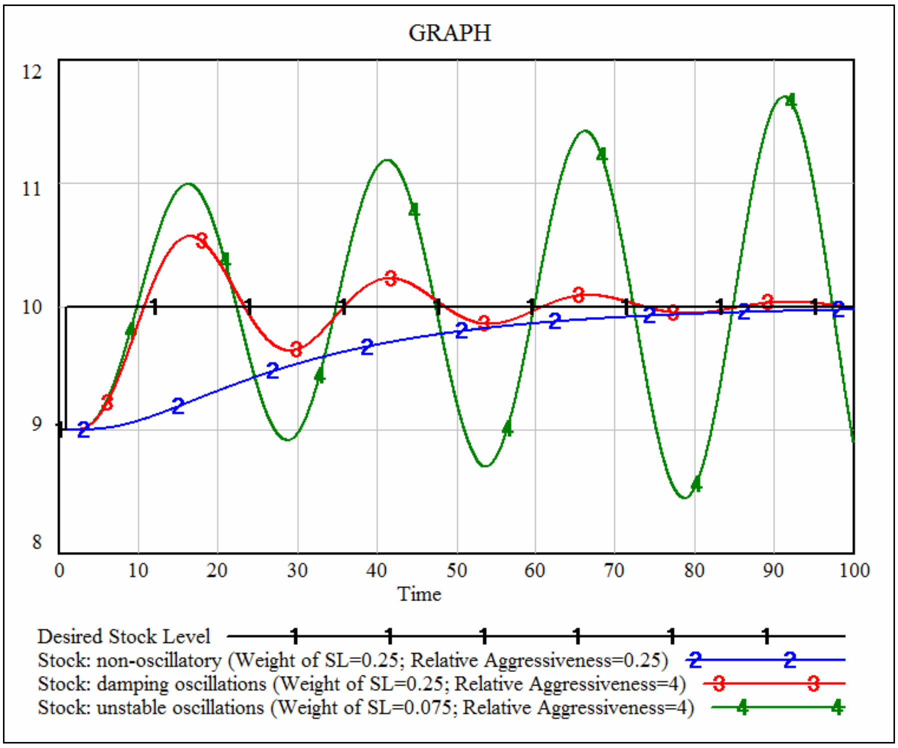

Figure 4. Stock dynamics for different sets of decision parameters

Results

The contour plots in Figure 5 show the relationship between *Weight of Supply Line*, *Relative Aggressiveness*, and *Total Penalty* for the delay orders 1, 2, 3, 4, 8, and infinite. *Total Penalty* is represented by the contour levels in the plots. The darker areas in the contour plots represent lower *Total Penalty* values, and the brighter areas represent higher values. From all contour plots, it can be observed that the effects of *Weight of Supply Line* and *Relative Aggressiveness* on *Total Penalty* are not independent from each other.

In general, both low and high values of *Weight of Supply Line* generate high penalties as evidenced by the white and light gray areas on the left and right sides of the contour plots. Low *Relative Aggressiveness* values also produce high penalties, as represented by the white area on the bottom side of the contour plots. A setting with a high *Relative Aggressiveness* and a low *Weight of Supply Line* generates a high penalty (see the upper left of the contour plots especially for delay orders higher than one), but a setting with a high *Relative Aggressiveness* and a well-selected *Weight of Supply Line* generates a low penalty (see the dark areas in the upper side). Basically, the white area (i.e., high *Total Penalty*) on the far left sides of the contour plots is the result of unstable oscillations. The white area on the undermost side and the light gray area on the far right are caused by over-damped non-oscillatory behavior. The darkest area on these plots is a result of well-selected *Relative Aggressiveness* and *Weight of Supply Line* values. As a side note, a stock management structure with a first-order lead time can never produce unstable oscillations. Therefore, there is no white area in the upper left part of the contour plot belonging to first-order lead time (Figure 5a).

Figure 5 shows that increasing *Relative Aggressiveness* decreases the generated *Total Penalty* values given that *Weight of Supply Line* is adjusted accordingly. There is no theoretical lower limit for *Stock Adjustment Time* and, thus, no theoretical upper limit for *Relative Aggressiveness*. In a real stock management system, other types of delays in addition to lead time, such as the information update delay or decision making delay are also present (Venkateswaran and Son, 2007; Yasarcan, 2011). Such delays can be ignored in a stock management model for the sake of simplicity if they are significantly short compared to lead time. However, *Stock Adjustment Time* should still be carefully selected. Otherwise, an extremely low *Stock Adjustment Time* value, which results in extremely aggressive adjustments, will cause unstable oscillations in a real system. Additionally, increasing *Relative Aggressiveness*, thus, decreasing *Stock Adjustment Time*, has diminishing returns. Therefore, one should never select unreasonably high *Relative Aggressiveness*.

In Figure 6, *Total Penalty* is plotted against *Weight of Supply Line* for delay orders 1, 2, 3, 4, 8, and infinite. For this figure, *Relative Aggressiveness* is fixed as 4, which is a sound example for a reasonable *Relative Aggressiveness* value. According to the figure, when *Weight of Supply Line* is less than 1, a higher order lead time results in a higher *Total Penalty* value compared to the penalty obtained from a lower order lead time. It is also observed that when *Weight of Supply Line* is above 1, all orders of lead time produce the same exact penalty value. For the first-order lead time, the optimum *Weight of Supply Line* is 0.425 for *Relative Aggressiveness* equals to 4 (Figure 6 and Table 1). Interestingly, fully considering the supply line (i.e., *Weight of Supply Line* equal to unity) produces penalty values nearly as bad as completely ignoring the supply line (i.e., *Weight of Supply Line* equal to zero) for the first-order lead time. If the order of lead time is discrete or high, a low *Weight of Supply Line* creates a high *Total Penalty* (for example, observe the penalties when *Weight of Supply Line* is approximately 0.425). Increasing *Weight of Supply Line* beyond 1 is not rational as it creates non-optimum costs for all orders of lead time.

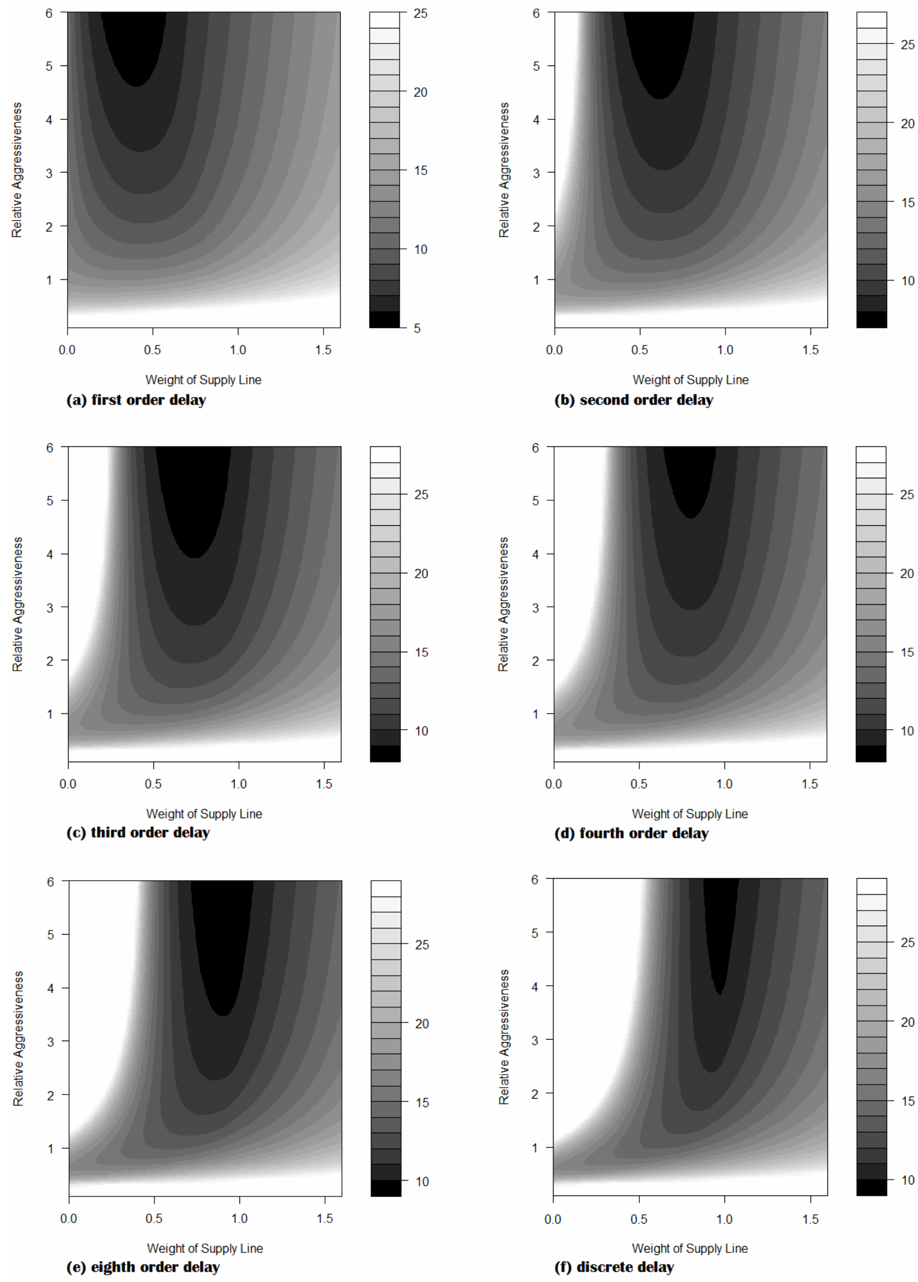

Figure 5. Contour plots of *Total Penalty* values

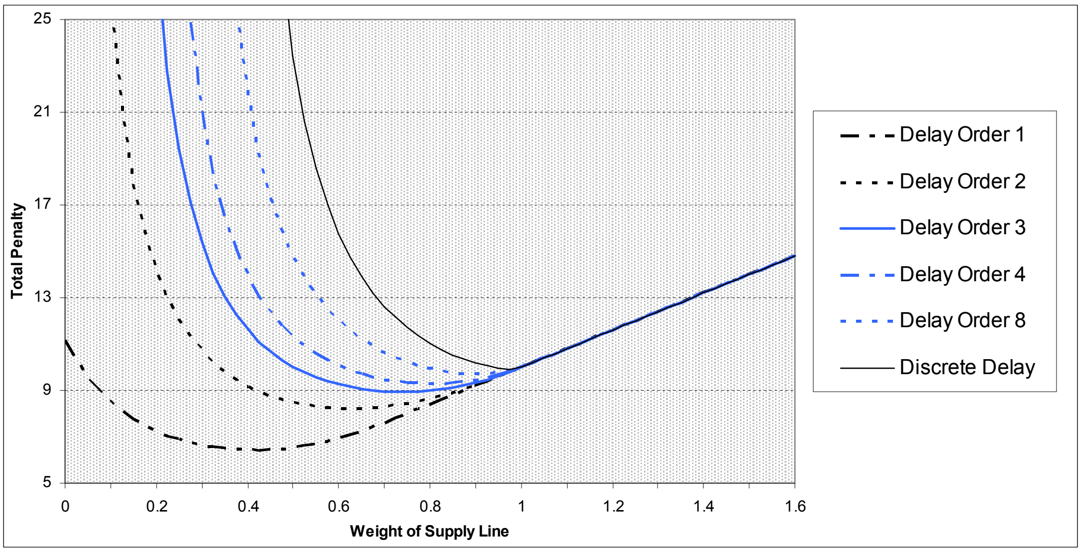

Figure 6. *Total Penalty* plotted against *Weight of Supply Line* when *Relative Aggressiveness* = 4

Table 1 presents the optimum *Weight of Supply Line* values for delay orders 1, 2, 3, 4, 8, and infinite, and for *Relative Aggressiveness* 1, 2, 3, 4, 5, 6, 7, 8, 16, 32, 64, 256, and 1024. For any selected *Relative Aggressiveness* value, the optimum shifts towards unity as the order of lead time increases (see Figure 5, Figure 6, and Table 1). The highest optimum values for *Weight of Supply Line* are obtained from the stock management task with a discrete lead time.

Table 1. Optimum *Weight of Supply Line* values

		Delay Order 1	**Delay Order 2**	**Delay Order 3**	**Delay Order 4**	**Delay Order 8**	**Discrete Delay**
Relative Aggressiveness	1	0.325	0.475	0.525	0.550	0.600	0.625
	2	0.425	0.625	0.725	0.775	0.825	0.900
	3	0.425	0.625	0.750	0.800	0.900	0.950
	4	0.425	0.625	0.725	0.800	0.900	0.975
	5	0.400	0.600	0.725	0.800	0.900	0.975
	6	0.375	0.575	0.700	0.800	0.925	1.000
	7	0.350	0.575	0.700	0.775	0.900	1.000
	8	0.350	0.550	0.700	0.775	0.900	1.000
	16	0.275	0.475	0.625	0.725	0.900	1.000
	32	0.200	0.375	0.575	0.700	0.875	1.000
	64	0.150	0.300	0.525	0.650	0.875	1.000
	256	0.075	0.200	0.475	0.625	0.850	1.000
	1024	0.050	0.125	0.450	0.625	0.850	1.000

Conclusions

In stock management, it is a critical issue to obtain a fast and stable response from the stock, which can only be ensured by a proper selection of values for *Stock Adjustment Time* and *Weight of Supply Line*. There is no theoretical lower limit for *Stock Adjustment Time* and decreasing it decreases the *Total Penalty* given that *Weight of Supply Line* is adjusted accordingly. In a real stock management system, other types of delays in addition to lead time, such as the information update delay or decision making delay are also present. Such delays can be ignored in a stock management model for the sake of simplicity if they are significantly short compared to lead time. However, *Stock Adjustment Time* should still be carefully selected. Otherwise, an extremely low *Stock Adjustment Time* value, which results in extremely aggressive adjustments, will cause unstable oscillations in a real system. Additionally, decreasing *Stock Adjustment Time*, has diminishing returns. Therefore, one should never select unreasonably low *Stock Adjustment Time*.

Although the nominal values of *Acquisition Delay Time* and *Stock Adjustment Time* affect the stock behavior, it is their ratio (together with the order of the delay structure and *Weight of Supply Line*) that determines the existence and stability of oscillations. Furthermore, using their ratio reduces the search space of parameters by one dimension. Therefore, we introduce a new parameter, *Relative Aggressiveness*, and define it to be equal to *Acquisition Delay Time* divided by *Stock Adjustment Time*. After selecting a practically low value of *Stock Adjustment Time* without ignoring the aforementioned concerns about its selection, that value can be used in the anchor-and-adjust ordering policy. Based on this value of *Stock Adjustment Time*, the value for *Relative Aggressiveness* can be calculated using Equation 19, which then is used to obtain the value of the other critical decision parameter of the anchor-and-adjust ordering policy, *Weight of Supply Line* (Figure 5 and Table 1).

Weight of Supply Line equal to unity ensures non-oscillatory stock behavior regardless of the delay duration and order, and therefore, it is often used in stock management. However, according to the results presented in this paper, a non-oscillatory stock behavior does not necessarily imply optimality in continuous time stock control systems. For example, fully considering the supply line (i.e., *Weight of Supply Line* equal to unity) produces penalty values nearly as bad as completely ignoring the supply line (i.e., *Weight of Supply Line* equal to zero) for the first-order lead time. Furthermore, for an extremely high *Relative Aggressiveness* value and, thus, an extremely low *Stock Adjustment Time* value, which results in extremely aggressive adjustments, the optimum *Weight of Supply Line* value becomes closer to zero rather than unity for the first and second orders (see Table 1).

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References

Barlas Y. 2002. System Dynamics: Systemic Feedback Modeling for Policy Analysis. *Knowledge for Sustainable Development: An Insight into the Encyclopedia of Life Support Systems*. UNESCO/EOLSS, Paris, France, Oxford, UK, 1131-1175.

Barlas Y, Ozevin MG. 2004. Analysis of Stock Management Gaming Experiments and Alternative Ordering Formulations. *Systems Research and Behavioral Science* **21**(4): 439-470.

Diehl E, Sterman JD. 1995. Effects of Feedback Complexity on Dynamic Decision Making. *Organizational Behavior and Human Decision Processes* **62**(2): 198-215.

Forrester JW. 1961. *Industrial Dynamics*. Cambridge: MIT Press.

Kleinmuntz DN. 1993. Information Processing and Misperceptions of the Implications of Feedback in Dynamic Decision Making. *System Dynamics Review* **9**(3): 223–237.

Mikati N. 2010. Dependence of Lead Time on Batch Size studied by a System Dynamics Model. *International Journal of Production Research* **48**(18): 5523-5532.

Mutallip A. 2013. *Deciding on Parameter Values of Anchor and Adjust Heuristic in Stock Management*, M.Sc. Thesis, Bogazici University, Istanbul.

Paich M, Sterman JD. 1993. Boom, Bust, and Failures to Learn in Experimental Markets. *Management Science* **39**(12): 1439-1458.

Sterman JD. 1987a. Testing Behavioral Simulation Models by Direct Experiment. *Management Science* **33**(12): 1572-1592.

Sterman JD. 1987b. Systems Simulation: Expectation Formation in Behavioral Simulation Models. *Behavioral Science* **32**(3): 190-211.

Sterman JD. 1989a. Modeling Managerial Behavior: Misperceptions of Feedback in a Dynamic Decision Making Experiment. *Management Science* **35**(3): 321-339.

Sterman JD. 1989b. Misperceptions of Feedback in Dynamic Decision Making. *Organizational Behavior and Human Decision Processes* **43**(3): 301-335.

Sterman JD. 2000. *Business Dynamics: Systems Thinking and Modeling for a Complex World*. Irwin/McGraw-Hill, Boston, MA.

Sweeney LB, Sterman JD. 2000. Bathtub Dynamics: Initial Results of a Systems Thinking Inventory. *System Dynamics Review* **16**(4): 249-286.

Venkateswaran J, Son YJ. 2007. Effect of Information Update Frequency on the Stability of Production-Inventory Control Systems. *International Journal of Production Economics* **106**(1): 171–190.

Wikner J. 2003. Continuous-time Dynamic Modelling of Variable Lead Times. *International Journal of Production Research* **41**(12): 2787–2798.

Yasarcan H. 2003. *Feedback, Delays and Non-linearities in Decision Structures*, PhD Thesis, Bogazici University.

Yasarcan H. 2010. Improving Understanding, Learning, and Performances of Novices in Dynamic Managerial Simulation Games. *Complexity* **15**(4): 31-42.

Yasarcan H. 2011. Stock Management in the Presence of Significant Measurement Delays. *System Dynamics Review* **27**(1): 91–109.

Yasarcan H, Barlas Y. 2005a. A Generalized Stock Control Formulation for Stock Management Problems Involving Composite Delays and Secondary Stocks. *System Dynamics Review* **21**(1): 33–68.

Yasarcan H, Barlas Y. 2005b. Stable Stock Management Heuristics when the Outflow is Proportional to the Control Stock. *Proceedings of The 23rd International System Dynamics Conference*, Boston, MA.