Pitfalls of Multi Method Modelling: Concepts and Views

Abstract

In the formal modelling of problems, a variety of techniques and approaches have been developed. Although at times this diversity has caused misunderstandings between groups there are now many examples of dialogue between approaches and fruitful ventures as the result. These experiences are gradually developing the principles of a multi method form of modelling, which, seeks to analyse, accommodate and benefit from the features of more than one modelling approach in a single study. However, different modelling systems have different underlying principles and it is important to recognise these in order to avoid the unsafe translation of ideas and prejudices from one system to another. This paper looks at two seductive pitfalls: Comparing system dynamic models with analytical differential equation models and treating the concept of 'randomness' as if it were the same across modelling systems The potential for misunderstandings is underlined with a few examples from established literature.

1. Introduction

Modelling based on sound mathematical principles is well established in what Wigner (1960) calls its "unreasonable effectiveness". Modelling when applied appropriately support the solution of problems and the understanding systems where such qualities are lacking. Within the sphere of model construction and realisation many approaches and techniques exist, each suggesting different ways of looking at a given problem. As such models are now, typically, solved numerically using software they provide a very direct realisation of the metaphor claiming computers as "bicycles for the mind"; extending the limits of human reasoning to the questions within our conception, yet previously beyond our reach.

There are a growing number of examples where different forms of modelling enter a dialog in search of some fruitful co-operation. Early examples of these conversations were often sterile or bruising affairs. Recently however, a succession of studies has explored the relationship between different modelling methods, mapping both their differences and areas of common interest. While this work has prompted some interesting discussions, reshaping and transcending methodological boundaries, modellers would do well to tread carefully. Beyond the variously distinct methods, literature which tackles abstract modelling in general terms warns that there are dangers in our assumptions that ideas will translate neatly between those different worlds. Our basis for discussion will be 'touchstone texts', established literature for the field, rather than the modellers opinions or prejudices.

This paper aims to explore a number of dangers, or pitfalls, which come into play when comparing and combining modelling approaches. The first pitfall explored is the differences in models based on their conceptualisation. To illustrate this we compare a System Dynamics model of the type described by Sterman (2000) and an Ordinary Differential Equation model of the type described by Boyce & DiPrima (2005) and present how the different approaches impact what should technically be equivalent studies. Next we focus on the concept of randomness and how it is presented in a variety of modelling approaches. Using an inherently stochastic problem, the tossing of coins, key underlying concepts in each method are exposed. Additional some examples of incautious comparisons of those falling into the traps, are examined.

2. Conceptualisation in SD and ODE models

Consider the proposition: "How can System Dynamics (SD) models and Ordinary Differential Equation (ODE) models be compared?" To many this question is plainly a tautology. Surely SD models are simply a form of ODE model and those who are not aware of this are sadly under educated. Indeed, in mean company such a question may even attract contempt or derision. Reactions also lean the other way too; ODE and SD models are not made to be compared. One is a simulation with a graphical form the other is simply an equation to be solved analytically or using a computer based tool. However on closer examination the issue is not so straight forward.

ODEs as described in texts such as Boyce & DiPrima (2005) or Mesterton-Gibbons (1995) are a fundamental approach for applied mathematicians they are used to predict the change in relationships between quantities and are also used across a wide range of areas including applied sciences. As well as their equation form they may also be presented in their 'box diagram' form.

$$\frac{dR}{dt} = \alpha R - \gamma RF \qquad \frac{dF}{dt} = \delta RF - \beta F$$

The system of equations for this Volterra population model above would be represented in box form as seen in figure 1:

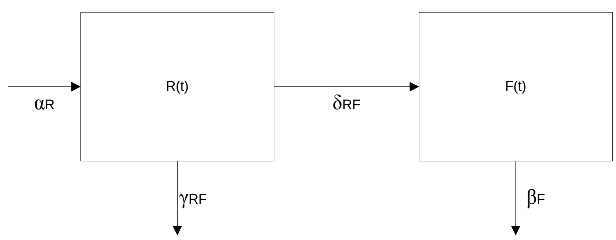


Figure 1 Box diagram for Volterra population model

Their long heritage means that they are often still solved analytically and in contrast to SD models guidelines on their formulation are covered only briefly in the literature. They have a plethora of classifications many related more complex variants. None the less, Forrester (1961) states that, SD models are essentially systems of ODEs typically with non linear properties. One might assume therefore that valid ODE models and valid SD models would be equivalent. Such an assumption would, I suggest, be a pitfall as it ignores the important role of conceptualisation inherent in the two different styles.

Consider, for example, the differential equation model for the spread of diabetes in Morocco of Boutayeb, Twizell et al. (2004) presented as a box diagram in Figure 2.

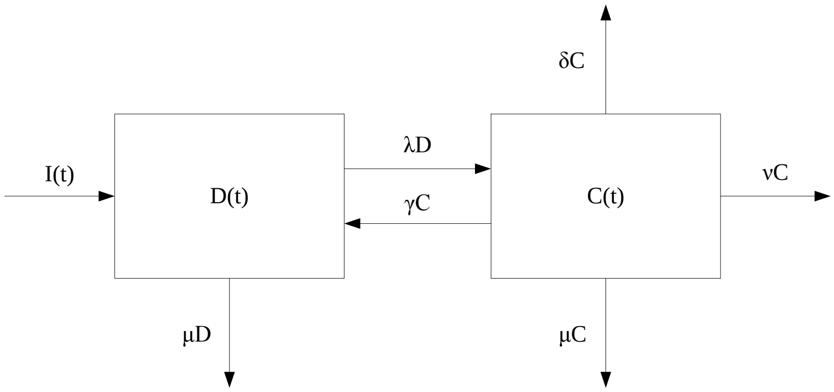


Figure 2 Differential equation model from Boutayeb (2004)

I is the diabetes incidence rate, D is the population with diabetes, C is the population with treatable complications. Constant coefficients are used to describe the fractional rates of exchange between populations; μ is the natural mortality rate, λ is the rate at which complications are developed, δ is the rate at which complications become severe and untreatable, ν is the rate of mortality due to complications, γ is that rate at which people with complications recover through treatment.

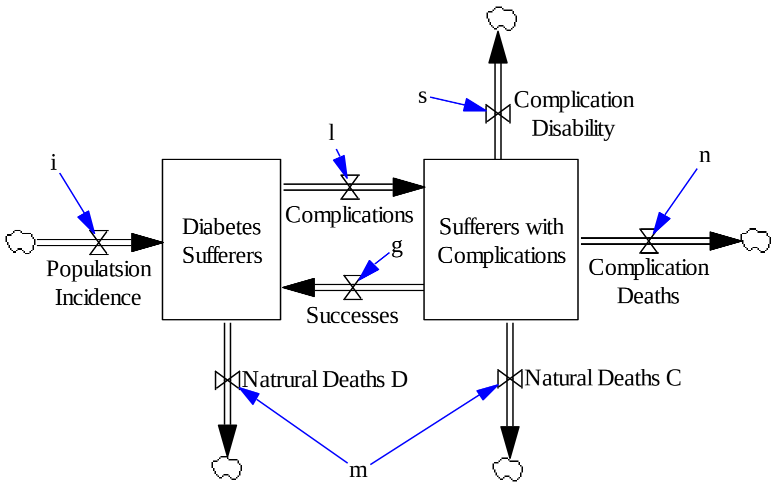


Figure 3 Diagram from Boutayeb et al (2004) adapted to system dynamics form

In an experiment undertaken at the 2006 UK System Dynamics Society annual meeting, participants were asked to look at a System Dynamics model translated directly from Boutayeb, Twizell et al. (2004) presented in Figure 3. The original model uses first order differential equations, the same as those underlying an SD model and therefore the two models can be considered exactly equivalent. The participants, comments and complaints focussed on the following issuesl:

- Lack of sophistication in segmenting populations.
- Lack of causal relationships accounting for the rates of movement.
- Lack of points of intervention; areas in the model where change could be affected.

These observations could be explained in terms of the differences such as tractability; The simplicity of segmentation may be related to tractability issues. Since no endogenous feedback hypothesis is required the causal structure, that would typically be included supporting the dynamic hypothesis of a System Dynamics model, is unnecessary. The missing points of intervention are unnecessary in a differential equation model as the iterative experimentation cycle, typical in a based method simulation method is not applied. However perhaps there is something more at the root of these observations; an implicit assumption about the purpose of modelling.

The model of Boutayeb, Twizell et al. (2004)is adequate for its stated purpose; *"to show that investment in primary health care is a necessary and cost-effective strategy [in order] to control the incidences of diabetes and its complications"*. Its model is very concise, particularly the equation form and therefore easily communicable to an audience of mathematicians. The intention of the model is to describe the level of incidence rather than to investigate the dynamics of the problem or the effects of policy changes.

The example may demonstrate a relationship between the purpose, what the model is intended to do, the conceptualisation, which part of the problem are important, and the process, how the purpose is realised. All three are influenced by the choice of method. In Boutayeb, Twizell (2004) the segmentation of the Differential Equation model was addressed in a separate section by redefining the model using partial differential equations with age as a second independent variable. This approach has no direct equivalent in System Dynamics and therefore could not be suggested by practitioners of that method.

On the whole then, although the models appear to be equivalent the guidelines for using the two different methods, in terms of how you reason about the problem, collect and manage data differ enough the a good enough model in one world falls short in the other. The pitfall here is to underestimate the subtle complexity the established modes of conceptualising the problem have on the finished model. The next section looks at how five different modelling methods manage to incorporate the concept of randomness and into their models.

3. Considering the Role of Randomness

A typical division when considering the properties of modelling systems, is made between the stochastic and the deterministic methods. Conventional wisdom would suggest that such categories make it easier to choose between methods, or judge approaches for their compatibility in advance of their use. However the role of randomness in modelling varies considerably. A closer examination of the issue in each case reveals some underlying principles different systems of modelling. In this section we shall consider five different approaches Differential Equation Modelling, System Dynamics Modelling, Stochastic Process Modelling, Econometric modelling and Discrete Event Simulation modelling.

Differential Equation (DE) Modelling

A superficial assessment of DE modelling, typically classified as a continuous, deterministic approach, finds that the approach does not use randomness in models. This perception may be because classical DE models are often applied to problems that are intrinsically continuous and deterministic, however differential equations, as a field of mathematical study, are very diverse. A special class of DE, Stochastic Differential Equation (SDE) models, include at least one stochastic process term in their definition. The role of the stochastic term is typically to model error or variation from the deterministic value of the equation (Gard 1988, Øksendal 2003). Additionally difference equations, which model similar patterns of behaviour using a discrete time step, are well established and (Edwards 2001) among others demonstrates their use although their appearance in modelling texts is less frequently. The majority of DE models used in teaching, and perhaps in practice, are classical DE models rather than difference equations or SDEs.

Classical DE modelling is often applied to problems, such as motion, mechanics and other applications in the physical sciences and was originally, as (Boyce, DiPrima 2005) notes, the main reason for their development. In these applications continuous, deterministic behaviour is satisfactorily described by continuous deterministic models. However for models where uncertainty, or human agency, is present such as population growth, epidemics and similar problems, continuous deterministic descriptions appear inadequate. For example the problem of whether an infectious illness is caught on exposure appears to be a discrete, probabilistic problem.

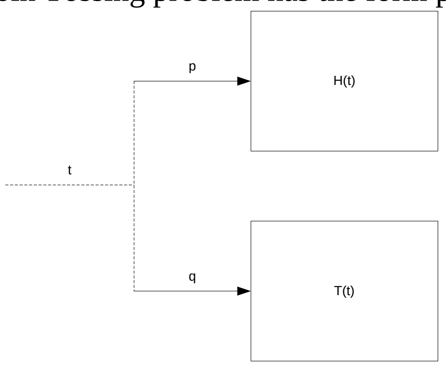
The use of classical DE models in such problems rests on what (Dym 1980) call the ‘continuum hypothesis’ whereby the model is formulated so that the main subject is treated as a field or continuum regardless of its observable composition. Using the example of a traffic flow analysis model (Dym 1980) details some of the requirements and restrictions the continuum hypothesis places on the formulation of the model. For example measures of traffic flow, and traffic density are in the model are based on aggregate values with appropriate scaling to maintain the continuum hypothesis. The DE model in this case has a necessarily macroscopic outlook. With the continuum hypotheses established, by virtue of the Strong Law of Large Numbers, a probability distribution is replaced by a flow equivalent to the mean value of the distribution.

The Coin Tossing Problem

To illustrate the role of randomness in different systems of modelling consider an inherently stochastic problem, such as tossing coins. Imagine a scenario where coins arrive at a rate *t*. The probability of a coin showing heads is *p* and the probability of showing tails is *q*. Those that show heads are placed in basket *H* and those showing tails placed in basket *T*. Models can be made in each system to investigate the contents of the baskets *H* and *I* after a fixed period of time *t* has elapsed, assuming that both assuming *p* and *q* have the value 0.5.

The DE model for the Coin Tossing problem has the form presented in Figure 4:

$$\frac{dH}{dt} = p \quad \frac{dT}{dt} = q$$



$$H(t) = pt \quad T(t) = qt$$

Figure 4 Differential equation model of the Coin Tossing Problem

In accordance with the continuum hypothesis, assuming *p* and *q* to be 0.5 does not, in this case, assert that all the coins are fair, rather that on average they are fair.

System Dynamics (SD) Modelling

SD models have a strong relationship with classical DE models and in practice representing randomness in both approaches is similar. The effect of variations considered random in the micro scale may be averaged into flows as with DE models. For example modelling the coin tossing problem using SD constructs (Figure 5) appears very similar to the DE approach. Both rely on the continuum hypothesis although in the case of SD implicitly so.

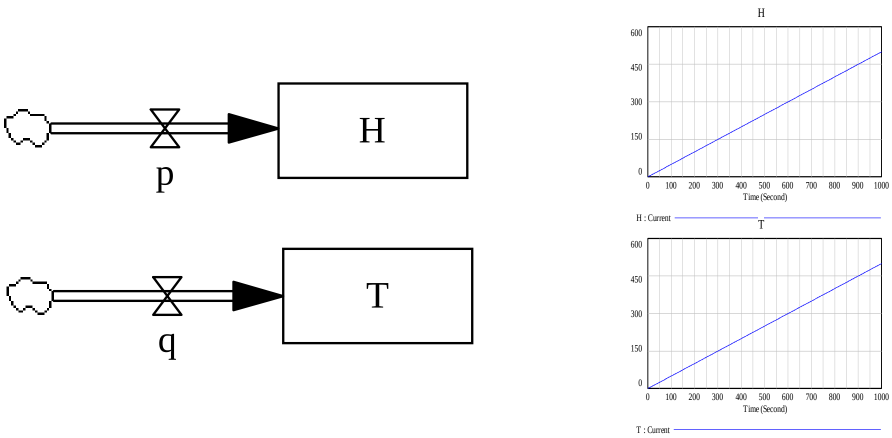


Figure 5 System dynamics model of the coin tossing problem

Randomness is rarely included explicitly in models and the reasons why capture some important properties of the approach. Because of the way SD models are structured where, random inputs to the system do occur they are often smoothed out by the effect of delays and aggregation (Forrester 1961). SD modelling also assumes an endogenous feedback-based hypothesis about the behaviour of the problem. The premise is that the behaviour of the problem can be explained by the interaction of internal variables. Complex system behaviour may be explained through the non-linear dynamics of a model's feedback structure whereas such behaviour may be described as randomness in the absence of any other available explanation. The endogenous feedback hypothesis favours the inclusion of a closed information feedback loop in the model, including as many of the causal variables as necessary to understand the behaviour of the problem. As a result (Sterman 2000) argues that genuinely random behaviour is uncommon: *"Many variables appear to vary randomly. In most situations, randomness is a measure of our ignorance, not intrinsic to the system…. When we say there are random variations in , say, the demand for a firms product, what we actually mean is that we don't know the reasons for these variations…people tend to call the residual random as if the customers were somehow rolling dice to decide whether to buy the product*"

However, the effect of exogenous randomness is treated differently. After the feedback properties in the model have been established (Forrester 1961) recommends the use of randomness or "noise signals" in the system inputs to test the its robustness. (Sterman 2000) describes how noise can be used to excite the latent dynamics in a model or unfreeze a system stuck in local optima.

Stochastic Process (SP) Modelling

It may fairly be claimed that randomness is a pre-requisite in order for SP models to be used effectively. The form and structure of SP models is diverse, discrete and continuous forms are equally common and SP models can also consider the stochastic properties of individual occurrences as well as macro level trends. However in all SP models randomness is the key generating principle by which problems are described, represented and solved. An integral part of the approach is to characterise and bound randomness in the behaviour or state of the problem over time. The use of SP modelling in uncovering deterministic properties of stochastic problems is a common application. Mesterton-Gibbons (1995) and Bunday (1986) demonstrate the use of calculus and SP modelling find the underlying stationary distributions of some SP models.

Uncertainty, or randomness, in the problem is modelled by random variables in the SP model. As Edwards (2001) describes, an appropriate pattern or distribution must be assigned to each variable in the model building process. Once constructed the SP model may provide incite into three key issues; What are the possible behaviours or states for the problem? What is the probable behaviour or state for the problem? How probable is a given state or behaviour?

In discrete models, without the continuum hypothesis restriction, it is relatively easy to model micro level uncertainty. For example for in the Coin Tossing problem the behaviour of an individual coin can be described at micro level using a simple Markov chain model (Figure 6);

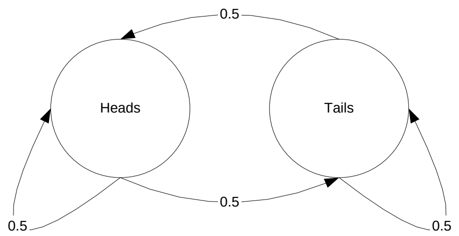


Figure 6 Markov chain model for tossing an individual coin

However the Coin Tossing problem as originally presented could be modelled using the binomial distribution. This is a discrete probability distribution that models events with two possible outcomes.

$$H(t) \sim B(t, p) \quad T(t) \sim B(t, q)$$

$$P(H(t) = x) = \frac{t!}{x!(t-x)!} p^t q^{t-x}$$

$$\overline{H}(1000) = tp = 500$$

Based on the original values of *p* and *q* the model expects 500 occurrences of heads from 1000 instances of tossing the coin. It is possible to test the validity of the model using statistical methods using samples from the population if necessary. Notably in this model the assumption is that all the coins are fair. In the case of the Coin Tossing problem, with certain limits, the continuous normal distribution it is considered an acceptable approximation to the discrete binomial distribution creating a continuum hypothesis for the model.

Econometric Modelling

Most principles of economic theory are deterministic, describing a concrete relationship between variables that behave, *ceteris paribus*, as described by theory.
Begg, Fischer et al. (2005) provides the example of the Keynesian macro-economic consumption function relating spending, with household income.

$$C = A + cY$$

Where C is total Consumption, A Autonomous consumption, c the marginal propensity to consume and Y household income.
However econometric modelling and econometric methods, integrating theory with data and statistical techniques is very diverse with many variations available to the modeller. All econometric models are based on observed data and the role of randomness in data analysis is central, if occasionally not fully acknowledged; many courses in econometric modelling require advanced study of statistical distributions and probability theory as a prerequisite. Structural models must account for uncertainty in the relationship between the sample and population data. An analysis of sampling theory is beyond the scope of this study however Cuthbertson, Taylor et al. (1992) and Greene (2003) address how properties of sample data affect the estimates of the β coefficients in regression models.

Structural models also must account for uncertainty, usually called error or disturbance, in the accuracy of the observations. Cuthbertson, Taylor et al. (1992) describes the structure of a Classic Linear Regression Model (CLRM) of the Keynesian consumption function.

$$y_t = \beta_1 + \beta_2 x + \varepsilon_t$$

Where the ε_t term represents the quantity of error.

The most common approach in structural models, such as CLRMs, is to consider this error as stochastic, belonging to a normal distribution with a constant finite variance and a mean of value of zero. Like the other structural variables error terms are considered to be independent of each other and non-auto correlated. Greene (2003) examines the statistical principles that support these assumptions and examines the issues of auto correlated error in the context of time series.

Cuthbertson, Taylor et al. (1992) describes time series data, used to create econometric models, as the realisation of a stochastic process. As such, time series models such as AR, MA and ARMA models, described in Chapter 3, propose a structure for a stochastic process that produced the time series data. AR models consider the effect of autoregressive stochastic elements over a specific number of time periods. MA models consider variation on a mean value plus stochastic elements over a specific number of time periods. ARMA models include both properties. Terms in the proposed models still require residual coefficients to be calculated form observed data.

Although stationary time series models are the most common use of stochastic models in econometrics texts (Greene 2003) also includes non stationary processes such as random walks. Few texts advise under what circumstances such models are necessary however. The use of time series modelling in econometrics is a method for focussing solely on behaviour rather than cause or correlation they are therefore used analyse variables with poorly understood or innumerable influences, such as stock prices.

The practical use of randomness in econometric models then is either as a tool to account for error or as a method to analyse data without proposing a causal structure for the underlying behaviour. Modelling the Coin Tossing problem using econometrics requires data observed by tossing a real coin and the proposal of a model that fits the observed behaviour. (Cuthbertson, Taylor et al. 1992) describes an equivalent problem where observed data is used to produce the Maximum Likelihood (LM) of the overall behaviour of the coins.

It is hypothesised that the data belongs to a binomial distribution. The total number of heads observed is H and the unknown probability of a single observation of heads is Π. From the definition of the binomial distribution the probability, P, of H observations in n tosses is given by.

$$P = \frac{n!}{H!(n-H)!}\Pi^{H-1}(1-\Pi)^{n-H}$$

Based on experimental observations where 518 occurrences of heads in 1000 trials. (Cuthbertson, Taylor et al. 1992) suggests that the modeller may experiment with values of Π as 0.1, 0.2, 0.3 etc in order to find the value that given the highest value of P based on H=518 n=1000 however demonstrates that the LM, can be found by taking the first derivative, equating it to zero and simplifying to give.

$$\overline{\Pi} = \frac{H}{n}$$

$$\frac{H}{n} = \frac{518}{1000} = 0.518$$

Discrete Event Simulation (DES) Modelling

For DES models, by contrast, stochastic behaviour is a key generating mechanism. DES modelling is supported by the assumption, that accurate imitation of the problem is sufficient to understand its behaviour and randomness, real or metaphorical, has a direct role in both the internal description of the model and the overall behaviour of the problem. Although software is a flexible medium and DES models are able to include any behaviour that can be encoded, deterministic or stochastic, from the literature implementing random behaviour is clearly central to DES modelling. The description and implementation of random behaviour within the DES model is the subject more than 70 pages in (Banks 2001) and almost 200 pages in(Law, Kelton 1991).

The implementation of randomness in DES models is distinct to that in SP models. Whereas SP models conceptualise randomness, DES models actually recreate the random behaviour. DES models reproduce the behaviour rather than just describe it. The behaviour of the elements within the problem, described by random processes, is imitated in the model. The generated behaviour is then analysed to understand how the elements interact to create the overall behaviour of the problem.. In many ways randomness in DES models could be considered the inversion of the econometric time series approach; Rather than starting with data, proposing structure and analysing the relationship in DES stochastic structure is proposed and integrated, data sets are the then generated and these are analysed, using statistical methods, to understand the overall behaviour.

A DES model of the coin tossing problem, created using Arena, simulated the tossing of individual coins, randomly assigning values of heads or tails. Each run of the model, using a different set of random numbers, produces a different outcome and over successive runs collecting and statistically summarising the data the original distribution is confirmed.

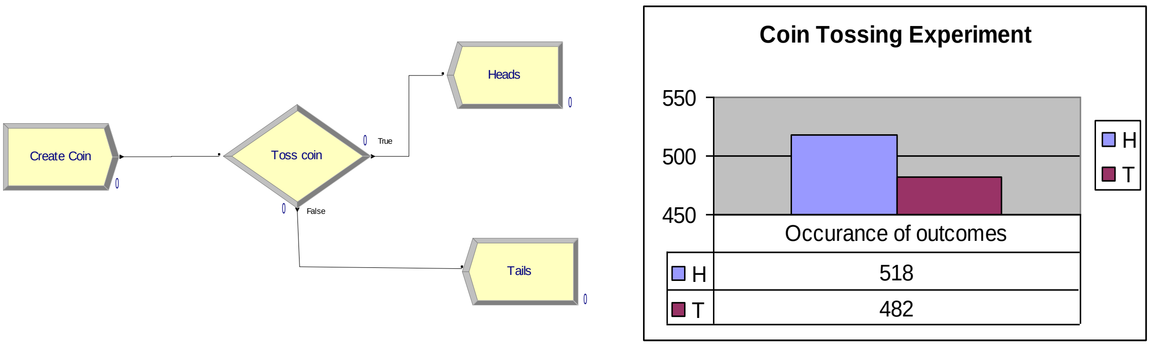


Figure 7 Discrete event simulation model of the coin tossing problem

Establishing distributions in the problem and choosing appropriate distributions to generate behaviour in the DES model is therefore very important. (Law, Kelton 1991) provided examples of 18 different probability distributions that may be used in models.

An insufficient knowledge of the workings and literature of other modelling methods can lead to confusion and increased risk of modelling failure. Unfortunately poorly formed models produce results just as the well formed ones do, the difference is their usefulness. Our final excursion looks at the kind of mistakes it is possible to make if your view of other modelling methods is developed without sufficient reference to that systems core principles and techniques. They may cause the practitioner to misfomulate the model misread the results and miss important truths in the comparison.

4. Conceptual Traps for the Unwary

Modellers of different backgrounds are often prone to misunderstanding other approaches because of misconceptions of how those methods really work. Most modellers will have at least a superficial knowledge of other approaches and this of itself may be the cause of some confusion. (Meadows 1985) suggests that common misunderstandings, and conflict, in comparing models and modelling approaches stems from an unconscious tendency to judge the models and methods of one system according to the principles and assumptions of another. For example, for practitioners using methods where a significant amount of data is collected prior to modelling, models formed from conceptualised relationships may seem to lack an evidence base. For practitioners of methods that emphasise causal mechanisms, models based on randomness may seem less than convincing.

It is also possible that the full range and ability of an approach is not recognised even by its practitioners. Structural econometric models, for example, are sometimes portrayed as proposing linear relationships between the independent variables of the problem however, as (Greene 2003) makes clear, the underlying theory requires that only a linear function of the variable is compared, provided that the number of observations tested is sufficiently large. Other statistical techniques may be used to relax the assumptions of the simplest forms of linear regression. Such misconceptions are perhaps understandable in observers from other methods however it s not uncommon for practitioners to not fully understand the capabilities and assumptions of their own method. (Meadows 1985) refers to as the “selective blindness” of working within a particular system.

Flawed Analysis, Misleading Conclusions

In some cases a superficial appreciation of a particular method, coupled with analysis based on the values of a different one, are the cause of the misleading conclusions. (Atherton, Borne 1992), a general reference work on modelling and simulation discusses system dynamics during the entry for ‘Ordinary Differential Equations’. *“Biologists and Sociologists [...] are often not well trained in numerical mathematics. For such individuals Forrester developed his method of rates and levels”.*
This is apparently a misunderstanding; arguably rates and levels have a more significant role in system dynamics modelling than simply to make up for a lack of mathematical training on the part of the modeller. Many relationships have non-linear properties which require considerable skill to conceptualise and solve mathematically. Rates and levels provide a form of conceptualisation that allows the modeller to focus on the qualities of the problem rather than the mathematical detail of the model. They also enable actors in the problem, who may not be trained in mathematics, to contribute to the construction and verification of the model.

On reproducing the method from (Forrester 1961) for calculating level equations (Atherton, Borne 1992) states *“This is obviously nothing but a reformulation of Eulers’ integration. However, persons with weak mathematical background seem to* *be more at ease with the terms rate and level than with the term differential equation"*. The failure of (Atherton, Borne 1992) to recognise any benefit of the approach, aside of an avoidance of complicated mathematics, is apparent. The use of Euler's integration, as the simplest and most well known numerical approximation, appears to play a role in this reasoning. Students of differential equations are well aware of its weaknesses and the relative strengths of more accurate alternatives such as Runge-Kutta (h5). (Boyce, DiPrima 2005) devote a whole chapter to comparing Euler and other numerical solutions to differential equations. Although it is not certain (Atherton, Borne 1992) may be reflecting on the choice of an elementary numerical approximation as evidence of mathematical naivety. However, the position of (Atherton, Borne 1992) overlooks that many system dynamics modellers are aware of other approximations and indeed the short comings of the Euler method. They offer the following reasoning for its use *"In models of social and human systems the errors in initial conditions, parameters and especially model specification are large and the data [the model may be compared with] are often corrupted by significant measurement error. In such cases Euler's errors are inconsequential"* (Sterman 2000). The Euler method is not integral to the system dynamics approach and most modern SD software offers a choice of numerical algorithms including both Euler and Runge-Kutta (h5). In the choice between methods it is left to the modeller to prioritise model execution time or accuracy of calculation.

On the limitations of the Euler method (Sterman 2000) states *"Euler integration is simple and adequate for many applications. However there are some systems and some model purposes, particularly in engineering and physics where Euler is not appropriate"*. The comments of (Atherton, Borne 1992) may be based on the assumption that physical problems are the primary application area for both systems of modelling however this is not the case.

Conflicting Assumptions

In discussing the problem of using of expected or mean values in a model rather than considering the stochastic behaviour (Bartholomew 1973) states *"Such calculations tell us, in an average sense what would happen if the model were allowed to operate and in this sense may be said to simulate the process…extensive use has been made of [such simulation techniques] by Forrester and his colleagues at the Massachusetts Institute of Technology"*. The author appears to overlook two critical assumptions of in system dynamics modelling; Firstly, the Strong Law of Large numbers is deemed to apply and on that premise a distribution may be substituted by its mean. Secondly, in system dynamics modelling the basis of the dynamic behaviour is the complex causal relationships rather than stochastics. It is assumed, implicitly, that the phenomena of interest are behaviours produced by the interaction of causal trends and therefore simulation "in an average sense" is an appropriate way to examine them. The methods of (Bartholomew 1973) by contrast implicitly assume that phenomena of interest are produced by the interaction of stochastic behaviours in the problem.

The cases cited above demonstrate that, whatever the facts unguarded comments may live on to haunt their owners, if nothing else live on. The essence of the multi method approach is that understanding concepts from other methods improves one's own practice. Rash comments are as much a pitfall as misunderstanding the problem in this case.

5. Conclusions

The opening section discussed the promise of increased fruitful dialogue between methods developing to establish a diverse skill set for modellers based on multiple perspectives The key role of conceptualisation was identified as a discriminating factor in transferring knowledge, skills and good practice. Expanding the range of techniques we saw how random features of a problem were incorporated into different models, suggesting different uses according to the properties of the problem. Finally we looked at how even the most experience and senior practitioners can be caught out if now fully aware of the quality of the comparisons they are making. Although multi method approaches are still gaining credibility there are already some guidelines we can identify to avoid the most obvious mistakes.

Future work in this area would include the exploration other areas such as how different systems, use data and the role of iteration in the development of models. Techniques currently gaining popularity such as agent based models would also be interesting for comparison.

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