Visual integration with stock-flow models: How far can intuition carry us?

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Abstract

Doing integral calculus is not easy for most students, and the way it is commonly taught in schools has attracted considerable criticism. In this paper we argue that stock–flow models have the potential to improve this state of affairs. We summarize and interpret previous research and the results of some of our own studies to explore how an intuitive understanding of (and teaching) integral calculus might be possible, based on such stock–flow models: They might be used for doing "visual integration" without calculations. Unfortunately, stock-flow tasks themselves seem to be quite difficult to solve for many people, and most attempts to make them more intuitive and easily solvable have not met with much success. There might, however, be some potential in using animated representations. In any case, a good starting point for students to eventually be able to perform visual integration in an intuitive way and to arrive at a deeper understanding of integral calculus seems to be to present flows as a succession of changes in stocks.

Doing integral calculus is not easy for most students; and the way it is commonly taught in schools has attracted considerable criticism. From a rather general perspective, it has been lamented that teaching often focuses on the calculation itself while neglecting whether students really comprehend what they do (e.g. Borneleit, Danckwerts, Henn, & Weigand, 2001; Hußmann & Prediger, 2010). As a consequence, students might not develop adequate mental models and process tasks solely automatically without a deeper understanding (Danckwerts & Vogel,, 1992; Davis & Vinner, 1986; Bürger & Malle, 2000; Blum, 2000; Hahn & Prediger, 2004; Tall, 1992; Tall & Vinner, 1981). Based on these considerations there have been some suggestions to improve the traditional math education by emphasizing the importance of basic mental representations (Biehler, 1985; Davis & Vinner, 1986; Hußmann & Prediger, 2010; Hahn & Prediger, 2008). It might, for instance, be much easier and more intuitive to do integral calculus *not* with numbers and formulas but rather by imagining the changes over time for the quantities in question. And this *visual integration* might work better the more similar the representations used to convey an integration task are to processes and objects students are already familiar with.

In this chapter, we deal with a specific approach to visual integration, using so-called *stock-flow models* (Sterman, 2000). Stock-flow models have been examined in cognitive science and decision research; in our view, they also have the potential to improving the teaching of integral calculus. The main aim in this chapter is to summarize and interpret previous research and our own studies in regard to how an intuitive understanding (and teaching) of integral calculus might be possible, based on such stock-flow models. We begin by discussing the potential uses of intuition in math education. Then we briefly explain what stock-flow models are and proceed to summarize previous attempts to make such stock-flow models intuitively understandable to students. After that we report results from our own studies and finish with some potential implications of this research for teaching integral calculus.

Math education and intuition

The well known *heuristics and biases* research program initiated by Tversky and Kahneman (1974) claimed that the intuitions we have about solving mathematical tasks are not always helpful and demonstrated numerous “fallacies” based on wrong intuitions. Meanwhile, however, a consensus seems to have been reached that there are both valid and invalid intuitions and that an important precondition for the development of valid intuitions is the opportunity to learn the regularities that exist in the environment (Kahneman & Klein, 2009). In recent years, the idea that intuitions can be a powerful basis for good judgments and decisions has been widely publicized by several best-selling books (e.g., Gigerenzer, 2008; Gladwell, 2007). This idea, that valid intuitions can be very helpful in solving problems has also repeatedly been propagated in the context of math education (e.g., Brown & Campione, 1994; Senge, Cambron-McCabe, Lucas, Smith, Dutton, & Kleiner, 2000). For instance, Ebersbach and Wilkening (2007) found that young children already possess intuitive knowledge about the characteristics of nonlinear growth, long before these functions are taught in school. As another example, Fischbein (1994) demonstrated that multiplication can be performed in an intuitive and in a non-intuitive way. He constructed text problems in which the solution was to multiply two numbers, 15 and 0.75. If the formulation of the problem asked for multiplying 0.75 by 15, about 75% of his fifth-, seventh- and ninth grade students solved it correctly, whereas the solution rate for multiplying 15 by 0.75 was only about 25%. Fischbeins's explanation for the discrepancy was that, in the first case (in contrast to the second), students could use an intuition about multiplication: *If a number is multiplied it becomes larger*. Fischbein (1995) argued that there exist intuitions that are especially useful in learning about probabilities. And indeed, there is ample evidence that such valid statistical intuitions do exist (Sedlmeier, 1999; 2007). Fischbein (1975) also claimed that schooling, with its overwhelming emphasis on deterministic explanations about the world might considerably weaken childrens' valid intuitions about probabilities. Engel and Sedlmeier (2005) took this claim to the test and found evidence consistent with it: Irrespective of type of school (Hauptschule, Realschule, and Gymnasium), solution rates decreased with amount of schooling (see Green 1983, 1991 for similar results). However, there is also good news: valid statistical intuitions can be re-activated even in adults by using suitable external represention s (e.g., Sedlmeier, 2000; Sedlmeier & Gigerenzer, 2001; Sedlmeier & Hilton, 2012).

What kinds of external representations are these? Norman (1993) argues that such representations must follow the *naturalness principle*: „Experiential cognition is aided when the properties of the representation match the properties of the thing being represented"(p. 72). In many cases, the essential manipulation to evoke valid intuitions is to use a suitable visual or graphical format that represents the task in hand well. But changing the order in which numbers are presented (such as in Fischbein's "multiplication study" mentioned above) may already have strong effects.

There are basically two explanations for why representation influences cognition. The first is an evolutionary account: over millennia, our perception has become sensitive to the structure of the environment and therefore, external representations that represent well the part of the environment dealt with in a given task automatically evoke intuitive ways to deal with the respective problems (e.g., Cosmides, 1989; Cosmides & Tooby, 1996). The second explanation for the facilitating effects of external representations rests on learning processes (Frensch & Rünger, 2003). Such an explanation has, for instance, been presented in the form of a computational model (a neural network) for intuitions about probabilities (Sedlmeier, 1999; 2002; 2007). Assuming that intuitions arise at least partly as a result of learning processes might be regarded as the more convincing explanation because this assumption allows us to explain intuitions as a result of the experiences one has within one's life-time (Sedlmeier, 2005). For instance, Bayes' formula, presented in its usual format, is totally non-intuitive for most students but is intuitive for most statisticians because they were exposed to it and thought about it in varied ways and circumstances for many years: expertise can make originally non-intutive representations intuitive (Hogarth, 2001). Such an explanation is also consistent with "street-math" results that show that South American street vendors and carpenters can perform relatively complex calculations with extensive exposure to suitable external representations but with little or no formal schooling (Nunes, Schliemann, & Carraher, 1993).

Sometimes, like in the case of valid statistical intuitions, this expertise is acquired in our daily routines, by monitoring the relative frequencies with which things and events occur (Sedlmeier, 2007). This might also hold for processes that can be represented by stock-flow models.

Stock-flow models

In everyday life, we are often faced with judgments about changing quantities where the underlying processes can be represented by simple *stock-flow models*. Let us illustrate this kind of model with some examples: An obvious one is a bank account. Here, the *stock* consists of a certain sum of money, which over time can be increased by some *inflow* (e.g, salary, interest rates) and decreased by some *outflow* (e.g., expenses, bank fees). Another example for a stock is the concentration of carbon dioxide in the atmosphere, which is responsible for global warming. In this case, the inflow consists of naturally occurring emissions of carbon dioxide and of man-made emissions (e.g. from burning oil, gas or coal), and the outflow consists of the absorption of carbon dioxide by biomass (e.g. plants and trees), and the oceans. But we don't have to move outside ourselves to encounter stock-flow systems: Our weight can be considered a stock that is increased by the intake of calories (inflow) and decreased by the energy consumption of our body (outflow).

To make judgments about the patterns in the quantities over time, we need to integrate the quantity at the beginning of the time interval in question and quantities that are added and subtracted to the initial one over a given time span. Stock-flow tasks usually differ somewhat from the way integral-calculus tasks are presented in school: textbooks usually only present the netflow, that is, the difference between in- and outflow, and only ask for the stock at the end of the process (e.g., the bank account, the carbon dioxide concentration, or the body weight *after* some period of time). However, distinguishing between the two kinds of flow and examining intermediary states might be quite informative for students, too, because of the ubiquity and everyday-relevance of stock-flow tasks. Because we are confronted with such tasks so frequently, one might expect that we have developed some ability that lets us make these judgments at least partly in an intuitive way. If there are such valid intuitions, they might prove to be very helpful in teaching integral calculus in schools.

Intuition and stock-flow models: previous results

If students have valid intuitions about change and accumulation, then presenting integral-calculus tasks in a format that corresponds to these intuitions should facilitate their understanding. One might argue that stock-flow tasks as used in the literature do exactly this: they “translate” abstract integration tasks into a format that should be more easily understood by problem solvers. Before we briefly discuss the results obtained in previous research, let us look at a typical stock-flow task, a variant of the *bathtub-task* and its usual representation (e.g., Sterman & Booth Sweeney, 2005).

The task is usually introduced by presenting some form of bathtub such as in Figure 1. The Figure depicts the stock, that is, the amount of water in the tub and it shows the two potential flows: the inflow indicated by the tap at the upper left and the outflow indicated by the drain at the lower right of Figure 1.

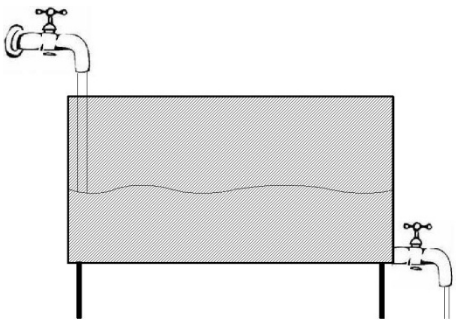

Figure 1: Representation of the general setup of a typical stock-flow task, the *bathtub task*.

After (or with) this figure, usually a *flow diagram* is presented, such as the one in Figure 2. The present flow diagram indicates a constant outflow of 50 liters per minute but a variable inflow of 75 liters per minute for the first four minutes that diminishes to 25 liters per minute for the next four minutes and repeats this pattern once.

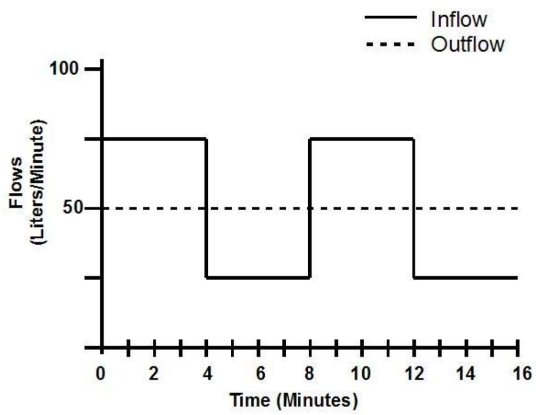

Figure 2: Flow diagram for the bathtub task.

From that information, participants are to draw the alteration of the quantity of water in the tub over time, starting, say, from 100 liters. The correct solution is shown in Figure 3.

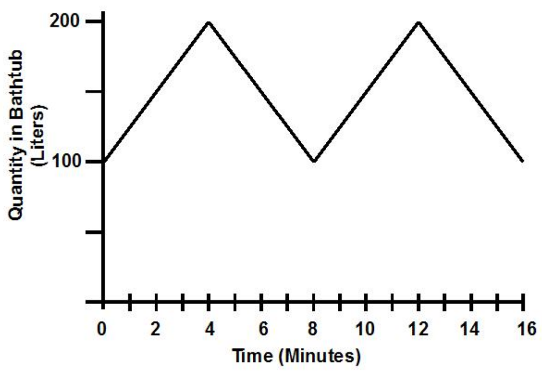

Figure 3: Correct solution for the version of the bathtub task specified by the flow diagram in Figure 2.

Everybody is familiar with a bathtub, so one might expect the existence of valid intuitions for solving tasks of this kind. However, initial results were quite discouraging: the usual solution rates for tasks of this kind, even for mathematically trained participants were often markedly below 50% correct answers (e.g., Booth Sweeney & Sterman, 2007; Sterman & Booth Sweeney, 2005). Following these disappointing results, several studies tried modifications of the task representations. One modification was to offer other, possibly more easily understandable representations such as bar graphs or tables depicting the pattern of both in- and outflows instead of the commonly used line graph. Another modification concerned the task contents: for instance, instead of water, tasks dealt with persons or cars, that is, with countable entities (which might be easier to deal with). In addition to the representational changes, there were also other attempts at improving solution rates: monetary incentives were offered to increase participants' motivation, and in some studies, participants received feedback about whether their solutions were correct and were offered the possibility to change them. None of these manipulations had a noticeable effect (for overview see Cronin & Gonzalez, 2007; Cronin, Gonzalez, & Sterman, 2009). Moreover, it seems that even domain-specific experience does not make a difference in these kinds of tasks (Brunstein, Gonzalez, & Kanter, 2010).

Our attempts at increasing the intuitive understanding of stock-flow tasks

The rather surprising outcomes of studies that attempted to improve the understanding of stock-flow tasks lead us to try further modifications concerning the representation of such tasks that might make them more intuitive. We report the results from three selected kinds of studies that examined (i) the use of animated representations, (ii) possible inconsistencies between the parts that are commonly used to present stock-flow tasks, and (iii) a way of reconciling potential inconsistences in the usual task representation.

Does animation help?

Stock-flow tasks usually involve some kind of change or movement over time. However, the usual way to represent such tasks (e.g., Figures 1 to 3) only involves static graphical representations. Previous attempts to optimize the representation of stock-flow tasks (e.g. bar graphs, tables etc.) might have failed because these task representations might not have triggered adequate mental representations in participants. In three studies (Schwarz & Sedlmeier, 2013) we showed participants animated simulations of stock-flow systems before we had them work on commonly used stock flow tasks. The studies were conducted with different samples, partly different manipulations, and with a variety of tasks. The results, however, were quite comparable: Figure 4 shows that, over all, seeing an animated ("Simulation") instead of a static ("Control") stock-flow task did not consistently increase solution rates in subsequently presented stock-flow tasks.

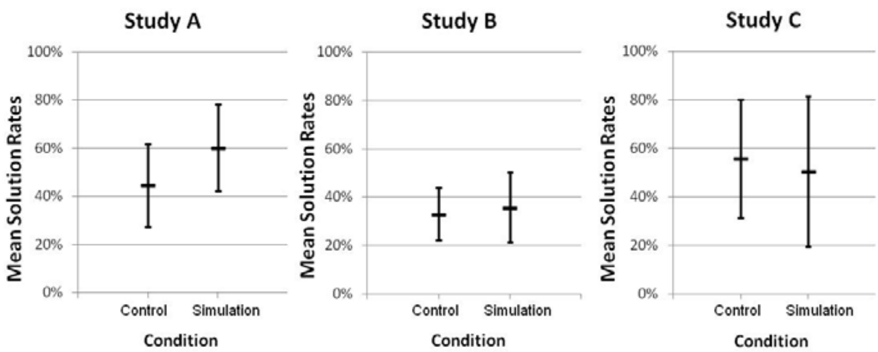

Figure 4: Mean solution rates of animated simulations and a control condition. Error bars represent 95% confidence intervals.

These results should, however, not be taken to be the last word on the potential facilitating effect of animated representations. After all, there might still be possibilities to improve the animations. Moreover, there seem to be some gender differences concerning the impact of animated representations (more beneficial for males); and substituting animated for static representations also increases the amount of information to be processed by participants. So the non-impact of animated representations on solution rates shown in Figure 4 might at least partly be due to gender effects and information overload. We are currently exploring both potential explanations.

Might less actually be more?

In most previous studies, flow diagrams, showing both in- and outflow, such as the one in Figure 2 have been used. Participants had to infer changes in stock over time, using the information shown in the flow diagram, and to draw these changes into a (then empty) solution diagram such as the one shown in Figure 3. At a first glance, the *frames* for Figures 2 and 3 look quite alike and partipants might implicitly conclude that these two diagrams are to contain similar kinds of information. Of course, this is not the case: the flow diagram contains rates (e.g. liters per minute) whereas the stock diagram contains quantities (e.g., liters). Previous results indeed indicate that quite a number of participants might not make a distinction between the two different kinds of information and seem to use an invalid intuition by drawing changes in stock in analogy to changes in flows – a result often termed “correlation heuristic” (e.g., Cronin et al., 2009). The simplest way to prevent participants’ use of such a correlation heuristic would be to just omit the flow diagram and instead describe the flows only verbally. However, omitting the flow diagram also reduces the amount of information available to participants and violates Meyer’s (2001) well-known *multi-media principle*, according to which people learn better from a combination of words and pictures than from words alone. Therefore, without the diagram, solution rates should decrease if participants use the information contained therein properly. In contrast, if participants tend to use the correlation heuristic, solution rates should increase if the flow diagram is omitted. Figure 5 shows the results from a study (Brockhaus & Sedlmeier, 2013), in which one group obtained only a verbal description of the flows (“No graph”, n = 32) and the other obtained the usual flow diagram (“Graph”, n = 32). In this study, seven criteria were used to judge the degree to which the solution was correct (solutions were, for instance judged as partly correct if participants recognized the points at which the stock decreased or increased but drew wrong quantities). Omitting the flow diagram (“No graph”) increased the solution rates, indicating that less information (no flow diagram) seems to have had beneficial effects in that this manipulation prohibited the use of the correlation heuristic. In the condition without a flow diagram, 45% of participants solved the task completely correctly as compared to 28% in the (commonly used) condition with a flow diagram.

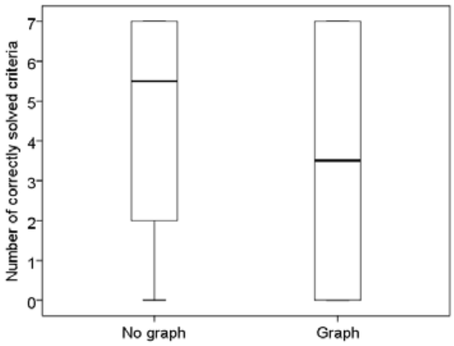

Figure 5: Boxplots that compare the performances of the groups with (“Graph”) and without (“No graph”) a flow diagram.

This result indicates that the presentation of both rates and quantities in a similar graphical way might be a serious impediment to understanding stock-flow tasks intuitively. But even if the flow diagram is omitted, the solution rates (45% completely correct solutions) do not indicate an intuitive task understanding.

Can flow be understood as a succession of stocks?

If participants have difficulties understanding the difference between rates and quantities, as indicated in the previous paragraph, one might try to find some kind of connection or transition between the two. One might, for instance, conceive of flow as a succession of stocks. This idea is illustrated in Figure 6. Here, the “flow,” both in and out, is successively accumulated over some (discrete) time steps (one minute each); and the result of this stepwise accumulation is shown as a *change* in the stock (here contained in a quite narrow “tub”). For instance, after the 1st minute, the stock has increased by 9 units (upper row of stocks – “Water flowing into the tub”), and at the same time it also has decreased by 6 units (lower row of stocks – “Water flowing out of the tub”) yielding an accumulated net-flow of 3 units. In the second time step, in- and outflow remain constant, so after 2 minutes, the netflow has accumulated to 6 units, etc. If participants can imagine (and understand) these changes well, they should be able to arrive at the correct stock after the 12 minutes used in this task.

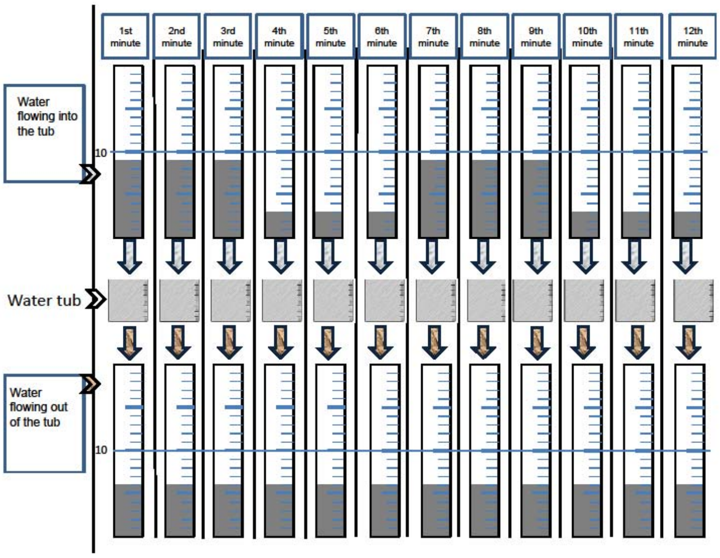

Figure 6: The modified flow diagram (as a succession of changes in stocks). The amount of the inflow and outflow after every minute is shown in the tub itself.

Participants' understanding of the task was assessed using again a succession of stocks that was depicted in the same format (Figure 7). Here, an initial amount of water in the tub had to be modified according to the in- and outflows depicted in Figure 6.

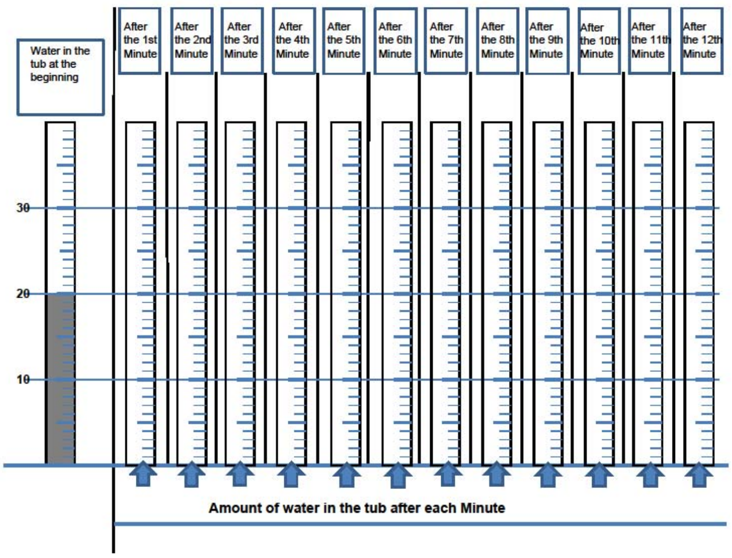

Figure 7: Modified solution diagram to draw the changes in stock over time as a succession of stocks.

The modified version of the bath-tub task (both diagrams presented as a succession of stocks) was compared to the usual baseline condition (see Figure 2) in a further study (Brockhaus & Sedlmeier, 2013). In this study, a scoring system was used that yielded 2 points for a completely correct solution. As the boxplots in Figure 8 show, the median number of points in the modified condition is 2 (indicating a completely correct solution) as compared to a median number of 0 points in the baseline condition. In the modified condition, 83.3% of participants (25 out of 30) solved the tasks completely correctly as compared to only 30% (9 out of 30) in the baseline condition.

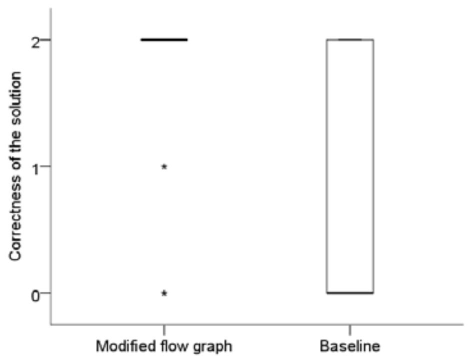

Figure 8: Boxplots that compare the performance in the condition with the modified representation ("Modified flow graph") and the classical flow diagram with a line graph ("Baseline"). In the condition with the modified flow graph, the solutions of 2 and 3 participants were scored as having 1 and 0 points, respectively (depicted as extreme outliers in the left boxplot).

Presenting the flow as a succession of stocks was clearly helpful and seems to have prevented the use of an invalid intuition, that is, to use the pattern in the flow diagram as the basis for one's response in the solution diagram (the correlation heuristic). The present manipulation seems to have enabled participants to determine the netflow step by step and to update the stock accordingly by imagining the respective changes and then adding up the changes in the stock. In other words: participants performed visual integration successfully.

How Far Can Intuition Carry Us?

It would have been nice if we could have shown that stock-flow tasks *per se* provide an easy and intuitive way to understanding integral calculus. Such stock-flow tasks can be found throughout daily life and therefore carry the pontential to considerably increase students' motivation to deal with this rather difficult mathematical topic. Unfortunately, previous results indicate that the intuitive approach to understanding integral calculus by using visual integration might be rather limited. In contrast to other mathematical domains such as probability theory and others mentioned above, where valid intuitions have been identified, information about changes over time and especially the integration of (possibly changing) rates into changing quantities might not so easily trigger valid intuitions. On the contrary, we found that it is the usual way to present stock-flow tasks, showing a flow-diagram and having participants draw their solution into a similarly looking solution diagram that might actually make the task more difficult by evoking *invalid* intuitions, one of which has been termed correlation heuristic in the literature. Having to deal with information about both rates and quantities at the same time may constitute the main obstacle in doing visual integration intuitively. Whereas the potential benefits of animated over static representations are not clear yet, our results indicate that students might need hints that clarify the transition between rates and quantities. They might also need more structure as they apparently profit from a step-by-step representation of the results of in- and outflows if one of theses flows changes over time. Presenting flow as a succession of changes in stocks might be a good starting point for students to eventually be able to perform visual integration in an intuitive way and to arrive at a deeper understanding of integral calculus.

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